Healy, M. J. R. 1951. Latin rectangle designs for 2n factorial experiments on 32 plots. *The Journal of Agricultural Science.* 41 (4), pp. 315-316.

LATIN RECTANGLE DESIGNS FOR 2^n FACTORIAL EXPERIMENTS ON 32 PLOTS

By M. J. R. HEALY

Rothamsted Experimental Station, Harpenden, Herts

Many agricultural experiments are carried out at a number of different centres in order to investigate conditions over the whole of some region. Normally a very high degree of precision is unnecessary at the individual centres, and in any case, as many of the centres will often be commercial farms, the experiments cannot contain large numbers of plots. In practice with 2^n factorial experiments, it is often convenient to use thirty-two plots in a 4×8 pattern on the ground. Provided interactions of three or more factors can be assumed negligible, Latin rectangle designs can be used which enable fertility trends in two directions to be eliminated simultaneously. The designs differ from the somewhat similar semi-Latin square designs in that they provide unbiased estimates of error.

(1) *Three factors.* The simplest design is in four blocks of eight plots with no confounding, each block being in a 4×2 pattern. Alternatively, the Latin rectangle design given below can be used; each row contains one complete replicate, while the three-factor interaction is confounded with columns:

(1)	*a*	*b*	*ab*	*c*	*ac*	*bc*	*abc*
ab	*b*	*a*	(1)	*abc*	*bc*	*ac*	*c*
ac	*c*	*abc*	*bc*	*a*	(1)	*ab*	*b*
bc	*abc*	*c*	*ac*	*b*	*ab*	(1)	*a*

Rows and columns should be completely randomized before use.

(2) *Four factors.* In this case there will be two complete replicates. The four-factor interaction can be confounded with rows, while partial confounding with columns is necessary if information on all two-factor interactions is to be retained. The best system of confounding is

Rows $ABCD$
Columns 1–4 AB, ACD, BCD
5–8 CD, ABC, ABD

This gives the following design:

(1)	*bd*	*ac*	*abcd*	*ab*	*ad*	*bc*	*cd*
abc	*a*	*bcd*	*d*	*acd*	*c*	*abd*	*b*
cd	*bc*	*ad*	*ab*	(1)	*bd*	*ac*	*abcd*
abd	*acd*	*b*	*c*	*bcd*	*abc*	*d*	*a*

in which rows and columns are to be randomized; the two replicates are not kept separate in the randomization. In the analysis, AB and CD will be estimated from the four columns in which they are not confounded.

(3) *Five factors.* Thirty-two plots will now accommodate a single replicate, interactions of three or more factors providing the estimate of error. If two of the two-factor interactions can be sacrificed, a design can be used with confounding as follows:

Rows ABC, ADE, $BCDE$
Columns AC, DE, ABD, ABE, BCD, BCE, $ACDE$

This gives the following design:

(1)	*bc*	*de*	*bcde*	*ace*	*abe*	*acd*	*abd*
abc	*a*	*abcde*	*ade*	*be*	*ce*	*bd*	*cd*
bde	*cde*	*b*	*c*	*abcd*	*ad*	*abce*	*ae*
acde	*abde*	*ac*	*ab*	*d*	*bcd*	*e*	*bce*

Rows and columns are randomized, and the analysis is straightforward.

If information must be retained on all two-factor interactions a form of partial confounding can still be used. In effect, the design is considered as two complementary half-replicates with a different confounding in each, as follows:

Rows ABC, CDE, $ABDE$
Columns two half-replicates, $BCE \equiv I$
1–4 $AB \equiv ACE$, $CD \equiv BDE$, $ADE \equiv ABCD$
5–8 $AC \equiv ABE$, $DE \equiv BCD$, $ABD \equiv ACDE$

and the design is:

(1)	*ace*	*bcd*	*abde*	*bce*	*acd*	*ab*	*de*
abe	*bc*	*acde*	*d*	*abd*	*e*	*bcde*	*ac*
cde	*ad*	*be*	*abc*	*ae*	*bd*	*c*	*abcde*
abcd	*bde*	*a*	*ce*	*cd*	*abce*	*ade*	*b*

As before, the four partially confounded interactions are estimated from the four columns in which they are not confounded.

(4) *Six or more factors.* Similar designs for six or seven factors can be devised, but these provide no estimate of error apart from two-factor interactions.

NUMERICAL EXAMPLE

The partially confounded design for five factors is here superimposed on a sugar-beet uniformity trial conducted by Immer (1932). The randomized plan, with yields in lb. per plot of $\frac{1}{60}$ acre, is shown below:

bc 542	*abd* 587	*bcde* 583	*acde* 576	*abe* 549	*d* 562	*ac* 576	*e* 569
ad 629	*ae* 615	*c* 634	*be* 594	*cde* 637	*abc* 623	*abcde* 643	*bd* 629
ace 562	*bce* 596	*ab* 624	*bcd* 627	(1) 639	*abde* 628	*de* 645	*acd* 651
bde 604	*cd* 638	*ade* 609	*a* 634	*abcd* 615	*ce* 586	*b* 605	*abce* 618

The partially confounded interaction totals, each based on sixteen plots, are:

AB +94; *CD* +34; *AC* −57; *DE* +159.

and the analysis of variance is

	D.F.	S.S.	M.S.
Rows	3	16938·3	
Columns	7	3255·5	
Main effects	5	4383·2	
2-factor interactions:			
Not confounded	6	891·2	388·4
Partially confounded	4	2407·6	
Remainder	6	474·3	
Total	31	28350·1	

As no treatments were in fact applied, the treatments and remainder sums of squares can be pooled to give a mean square of 388·4. It is perhaps unfortunate that the randomization chosen should have been among the 1% of all randomizations giving ostensibly significant 'treatment' effects. The uniformity trial used above has been analysed in several different ways by Yates (1937), and the above may be compared with his Table 31.

SUMMARY

Designs are given for factorial experiments with three, four or five factors at two levels each, using thirty-two plots in a 4×8 lay-out on the ground. The effects of both rows and columns can be eliminated from the estimate of error. Provided that three-factor interactions can be ignored, information can be retained on all two-factor interactions.

REFERENCES

Immer, F. R. (1932). *J. Agric. Res.* **44**, 649–668.
Yates, F. (1937). *Tech. Commun. Bur. Soil Sci., Harpenden*, no. 35.

(*Received* 27 *April* 1950)