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FULLY DIFFERENTIAL DECAY RATE OF A STANDARD MODEL HIGGS BOSON INTO TWO b-JETS AT NNLO*

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We present a general method for computing QCD jet cross sections at next-to-next-to-leading order accuracy, called CoLoRFulNNLO. We also discuss how to combine the predictions for the production of a Standard Model Higgs boson and its decay into a b-quark pair, both computed at the next-to-next-to-leading order accuracy, to predict precisely the kinematic distributions of b-jets emerging in the process $pp \to H + X \to b\bar{b} + X$.

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1. Introduction

The discovery of a new boson [1, 2] seems to complete the experimental verification of the validity of the Standard Model (SM). All measured properties of this particle are consistent with the SM predictions: it is a $J^P = 0^+$ particle which couples to other bosons and fermions in accordance with the masses of those within the uncertainty of the measurements. Its measured width is also in agreement with the prediction $\Gamma_H = (4.07 \pm 0.16_{\rm theo})$ MeV.

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Nevertheless, the precision of the current measurements still allows that this particle is the first observed member of an extended Higgs sector that is more natural from the theoretical point of view. For instance, supersymmetric extensions offer solutions to open questions in the fundamental physical description of Nature, such as the hierarchy problem, or the elementary constituents of cold dark matter.

Distinction of different Higgs sectors requires high precision measurements and predictions. In this paper, we focus on the latter. As the width of the Higgs boson is very narrow, the narrow width approximation (NWA) is excellent, which means that in order to make a high-precision prediction for the process $pp \to H + X \to b\bar{b} + X$, we need high precision prediction for both production $pp \to H + X$ and decay $H \to b\bar{b}$, which can be fused according to the formula

$$\frac{d\sigma}{dO_{b\bar{b}}} = \left[\sum_{n=0}^{\infty} \frac{dd^2\sigma^{(n)}_{pp\to H+X}}{dp_{\perp,H}d\eta_H}\right] \times \left[\frac{\sum_{n=0}^{\infty} d\Gamma^{(n)}_{H\to b\bar{b}} \Big/ dO_{b\bar{b}}}{\sum_{n=0}^{\infty} \Gamma^{(n)}_{H\to b\bar{b}}}\right] \times \mathrm{Br}\left(H \to b\bar{b}\right) , \tag{1}$$

where $O_{b\bar{b}}$ denotes some kinematic variable of the emerging b-jets, such as rapidity or transverse momentum. The branching ratio $\mathrm{Br}(H \to b\bar{b})$ is known with better than 1% accuracy [4]. The production cross section is known at the next-to-next-to-leading order (NNLO) in QCD perturbation theory for the inclusive cross section [5, 6], as well as for X being a vector boson [7, 8], or a jet [9]. In this paper, we focus on the $H \to b\bar{b}$ process which is, by now, also known to NNLO accuracy [10, 11].

2. Method

The CoLoRFulNNLO method (Completely Local subtRactions for Fully differential predictions at NNLO) is a general subtraction scheme for computing QCD jet cross sections at the NNLO accuracy. At present, it can be applied for processes of massless partons and without coloured particles in the initial state. In developing this scheme [12, 13], we set the following requirements useful for efficient numerical implementations and automation:

— the subtractions should be defined and recorded explicitly for all degrees of freedom (momentum, spin, colour and flavour) and for arbitrary processes — we use the colour state formalism of Ref. [3];

— the method should be general, valid in any order of perturbation theory;

— the subtractions should be fully local in the multidimensional phase space, therefore, the correctness of the subtraction terms can be checked in arbitrarily chosen phase-space points, which is needed for mathematical rigour and useful for numerical efficiency;

— the method should yield fully differential predictions in four space-time dimensions (so arbitrary detector cuts can be employed and matching to shower Monte Carlo programs is possible);

— the subtractions can be constrained over the phase space (as physical predictions cannot depend on such constraints, this provides an important check of the predictions and a tool for optimization).

The cross section for m-jet production in the perturbation theory can be written as $\sigma = \sigma_{\rm LO} + \sigma_{\rm NLO} + \sigma_{\rm NNLO} + \ldots$ The cross section at LO is the integral of the fully differential Born cross section of m final-state particles times the jet function $J_m(\{p_i\})$ that depends on the final-state momenta and defines the m-jet physical observable. The fully differential NLO correction is a sum of two terms:

$$d\sigma_{\rm NLO} = d\sigma^{\rm R}_{m+1} J_{m+1} + d\sigma^{\rm V}_{m} J_m \,, \tag{2}$$

where the real correction $d\sigma^{\rm R}_{m+1} J_{m+1}(\{p_i\})$ is the differential Born cross section of $m+1$ particles, allowing one particle becoming unresolved (collinear to another one or soft, meaning a particle with vanishing energy) making the m-particle and $(m{+}1)$-particle final states kinematically degenerate, while the virtual correction $d\sigma^{\rm V}_{m} J_m(\{p_i\})$ is the interference of the one-loop amplitude of m partons with the corresponding Born one. Both of these terms are separately singular even after ultraviolet renormalization due to integrations over the unresolved momenta (either real, or virtual in the loop), which lead to infrared singularities. According to the Kinoshita–Lee–Nauenberg theorem, their sum is finite if J defines an infrared-safe physical observable.

The third term $\sigma_{\rm NNLO}$ in the perturbative expansion of the cross section is a sum of three contributions: *(i)* the double real correction $d\sigma^{\rm RR}_{m+2} J_{m+2}(\{p_i\})$, which is essentially the fully differential Born cross section of $m+2$ particles, allowing one or two particles becoming unresolved, making either the m-particle or the $(m+1)$-particle final states kinematically degenerate with the $(m+2)$-particle final state; *(ii)* the real-virtual correction $d\sigma^{\rm RV}_{m+1} J_{m+1}(\{p_i\})$, which is the interference of the Born amplitude and one-loop amplitudes for $(m+1)$-particle final states, containing explicit poles emerging in the loop integral and allowing one particle becoming unresolved; *(iii)* the double virtual correction $d\sigma^{\rm VV}_{m} J_m(\{p_i\})$ that contains the two-loop corrections to the amplitude for m-particle final states. These three contributions are singular even after ultraviolet renormalization due to integrations over the unresolved real and loop momenta, but their sum is finite if J defines an infrared-safe physical observable. Our goal is to construct a reorganization of the three contributions into three finite integrals without changing the sum. For this purpose, we use the same key concepts that were the basis of general methods for computing cross sections at NLO: *(i)* the known infrared and collinear factorization of QCD matrix elements (in tree amplitudes [14, 15] and in their one-loop versions [16, 17]) and *(ii)* mappings of the phase space which allow for integrating over the unresolved real particles independently of the jet function [12, 13]. In addition, we also use *(iii)* a process independent way of disentangling overlapping singularities of the amplitudes both among various doubly-unresolved phase-space regions as well as singly- and doubly-unresolved ones. A solution valid at NNLO was presented in Ref. [18], and a general one, valid at any order in perturbation theory in [19].

After these steps, the sum of the three divergent contributions can be written as a sum of three finite ones, $\sigma_{\rm NNLO} = \sigma_{m+2} + \sigma_{m+1} + \sigma_m$, where each can be computed in four dimensions using Monte Carlo integrations. Symbolically, the fully differential contributions can be written as

$$d\sigma_{m+2} = \left\{ d\sigma^{\rm RR}_{m+2} J_{m+2} - d\sigma^{\rm RR,A_2}_{m+2} J_m - \left[d\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - d\sigma^{\rm RR,A_{12}}_{m+2} J_m \right] \right\}_{\epsilon=0} , \tag{3}$$

$$d\sigma_{m+1} = \left\{ \left[d\sigma^{\rm RV}_{m+1} + \int_1 d\sigma^{\rm RR,A_1}_{m+2} \right] J_{m+1} - \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}_{\epsilon=0} , \tag{4}$$

$$d\sigma_m = \left\{ d\sigma^{\rm VV}_m + \int_2 \left[d\sigma^{\rm RR,A_2}_{m+2} - d\sigma^{\rm RR,A_{12}}_{m+2} \right] + \int_1 \left[d\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 d\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\}_{\epsilon=0} J_m , \tag{5}$$

where the $\rm A_1$ terms regularize the one-particle (single) unresolved singularities, the $\rm A_2$ terms regularize the double unresolved singularities, while the $\rm A_{12}$ terms have two purposes: to regularize the double unresolved singularities in the RR,$\rm A_1$ subtraction and also the single unresolved ones in the RR,$\rm A_2$ subtraction.

The simultaneous factorization of the phase space and the matrix elements allows for integrating out the momenta and sum over the spin, colour and flavour degrees of freedom of the unresolved particles. We denote all these steps symbolically by $\int_1$ and $\int_2$ where the index shows the number of unresolved particles. This procedure leads to integrated subtraction terms that have to be added to cross sections with less particles in the final state (Eqs. (3)–(5)), which leads to cancellation of the ϵ poles that emerge in the loop integrals when the divergent integrals in $d = 4$ dimensions are regulated by dimensional regularization in $d = 4–2\epsilon$ dimensions. The Laurent-expansion of these integrals contains poles starting at $1/\epsilon^4$. We have checked the cancellation of the leading $(1/\epsilon^4)$ and subleading $(1/\epsilon^3)$ poles for an arbitrary number m of jets, and the cancellation of all poles for $m = 2$ and 3 analytically.

3. Predictions

We have implemented the CoLoRFulNNLO method for computing the fully differential decay rate of a Higgs boson into a $b\bar{b}$-pair. In our implementation, we can constrain the phase space of the subtractions by choosing the value of a dimensionless parameter $\alpha_0 < 1$, while $\alpha_0 = 1$ means no constraint. The three contributions σ_n $(n = m,\ m+1,\ m+2)$ depend on α_0. However, physical predictions must not depend on α_0, therefore, checking that the full prediction is independent of α_0 gives a strong check of correctness. In Fig. 1, we compare the distribution of the pseudorapidity of the

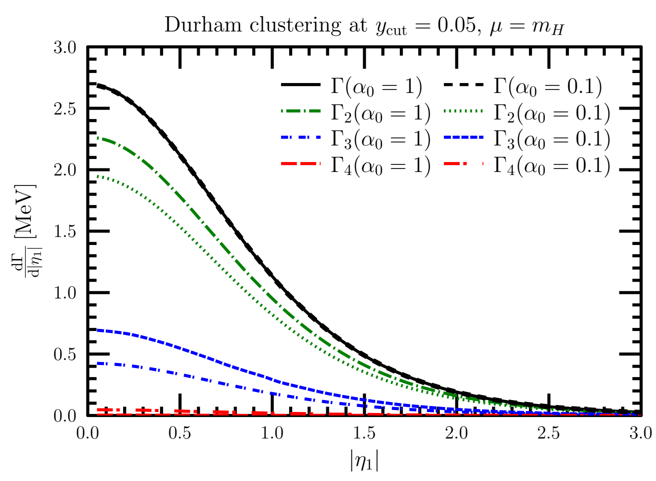

Fig. 1. Rapidity distribution of the hardest jet: dependence of the various contributions on α_0.

leading jet (measured from an arbitrarily fixed axis in the rest frame of the Higgs boson and clustered with the Durham algorithm [20] at $y_{\rm cut} = 0.05$) with ($\alpha_0 = 0.1$) and without ($\alpha_0 = 1$) phase-space restriction. The predictions are shown at a renormalization scale $\mu_{\rm R} = \mu_0 = m_H$. Clearly, the independent contributions depend on α_0, but their sum does not.

The independence of the physical prediction of α_0 can be used for reducing the time needed for the full computation because with constrained subtractions, one does not have to compute all of them for each phase-space point. For the $H \to b\bar{b}$ process, the average number of subtraction calls ($\langle N_{\rm sub}\rangle$) is reduced significantly if $\alpha_0 = 0.1$ ($\langle N_{\rm sub}\rangle = 14.5$) compared to the computation with $\alpha_0 = 1$ ($\langle N_{\rm sub}\rangle = 52$), which is reflected in a reduction by a factor of 2.5 of the CPU time needed to complete the computation leading to similar numerical precision.

Turning to physical predictions for the b-jets, one finds that for massless b-quarks standard jet algorithms, such as the $k_\perp$ or anti-$k_\perp$ algorithms are not infrared safe beyond NLO accuracy [21]. The origin of IR unsafety is the splitting of a gluon into a b-quark pair. In the double real emission such a splitting leads to three b-jets in the final state (two from the Higgs boson and one from the gluon) if the b-quarks are nearly collinear, when the matrix element is singular and requires regularization by subtraction. However, the subtraction term contains only two b-jets and a gluon jet, therefore its contribution falls into a different bin from that of the double real emission in the singular limit, leading to uncancelled singularities. To make infrared safe predictions with tagged jets, we use the flavour-$k_\perp$ algorithm of Ref. [21] to model b-tagging of the jets. This algorithm amounts to assigning an additive flavour number $+1$ to a b-quark and -1 to an anti-b-quark and 0 to all other partons. Combining a $b\bar{b}$-pair results in a flavourless pseudo-particle. In this way, the collinear splitting of a gluon into a $b\bar{b}$-pair in the double real emission contribution gives a flavourless jet, just like the corresponding subtraction term does, so the two contributions (double real and the subtraction) fall into the same bin. We demonstrate that this algorithm gives stable predictions at NNLO in Fig. 2 where the rapidity and the transverse momentum distribution of the hardest b-jet in the rest frame of the Higgs boson is shown together with the dependence of the predictions on the renormalization scale varied around the default one (m_H) in the range of $[\mu_0/2, 2\mu_0]$. The bands at LO and NLO accuracy overlap marginally, but the NNLO band lies inside the NLO one over the whole kinematic range.

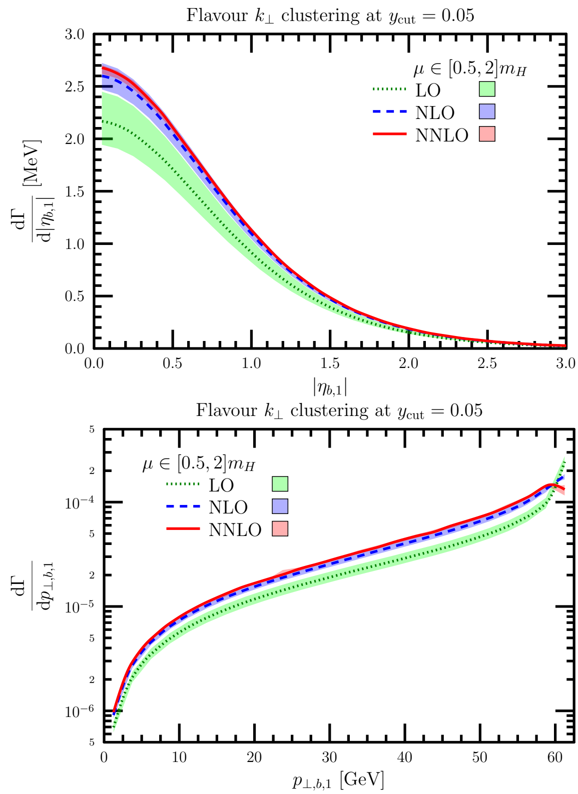

Fig. 2. Kinematic distributions of the hardest b-jet in the rest frame of the Higgs boson: rapidity η and transverse momentum $p_\perp$. The bands represent the variation of the scale at LO, NLO and NNLO around the default one in the range of $[\mu_0/2, 2\mu_0]$.

4. Conclusions

In this paper, we presented a subtraction scheme for computing fully differential cross sections at the NNLO accuracy (presently only for processes without coloured particles in the initial state). The subtractions are fully local, exact and explicit in colour. We presented the first application of the method for the process $H \to b\bar{b}$, for which we have shown the cancellation of the ϵ poles analitically and also the independence of the physical predictions of the unphysical constraint on the subtractions. We used the flavour-$k_\perp$ algorithm to make predictions for distributions of tagged b-jets, for which the original $k_\perp$-algorithm is not infrared safe.

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