The structure of pairing strategies for k-in-a-row type games

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The positional game k-in-a-row is played by two players on the infinite (chess)board. In the classical version of the game the players alternately put their own marks to previously unmarked squares, and whoever reaches a *winning set* (k-consecutive squares horizontally, vertically or diagonally) first of his own marks, wins. In the Maker-Breaker game Maker is the one who tries to occupy a winning set while Breaker only tries to prevent Maker's win. For different values of k, there are several winning strategies either for Maker or for Breaker. One group of those are *pairing (paving) strategies*.

A pairing strategy generally means that the possible moves of a game are paired up; if one player plays one, the other player plays its pair. A winning pairing strategy of Breaker in the k-in-a-row type games is such a pairing of the squares of the board that each winning set contains at least one pair. Using the above mentioned pairing strategy, Breaker gets at least one square from each winning set, therefore wins the game. We showed that Breaker can have a winning pairing strategy for the k-in-a-row game, if $k \geq 9$.

In the case of $k = 9$ there are only highly symmetric (8- or 16-toric) pairings of the board, we list all (194543) 8-toric pairings. Among those 194543 different pairings we can define alternating paths and cycles which give us a natural link between two pairings: Two pairings are neighboring if we can get them from each other by alternating only one path or cycle.

We analyze the network of pairings which is a huge and sparse graph (194543 nodes and 532107 edges). It is triangle free and has 14 components. One of those components is a giant component containing 194333 nodes and there are some components which forms the net of a 4D cube.

We investigate another similar game by considering the k-in-a-row game on the hexagonal board. For that game Breaker has winning pairing strategies if $k \geq 7$. For $k = 7$ we list 26 different 6-toric pairings – and these are the only ones – which gives us a similar but smaller graph as before.

At the end we higlight some possible extension to higher dimensions and the issues arising from those.

References

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