

Sum of Gaussian Functions Based 3d Point Cloud Registration

Bálint Fodor

Acquiring real-world 3d point cloud data is getting more and more accessible as RGB-D cameras (like Microsoft Kinect), Time-of-Flight cameras and other range imaging devices get cheaper. These products can also provide the depth information for the pixels of a captured frame. The 3d point set of a video frame is positioned in the coordinate system of the camera, which means that the range images acquired while moving the device will be represented in separate coordinate systems.

To get a comprehensive 3d model in the same coordinate system the frames should be transformed based on the pose information of the camera. In most of the cases however we do not have direct knowledge about the relative, nor the absolute translation and orientation of the camera.

We assume that subsequent frames contain overlapping parts of the same 3d object. Finding these parts the transformation between the range images can be estimated. The main problem is finding good matchings between points of the overlapping parts, while obtaining such a matching the rigid transformation between the parts can be easily calculated with well studied, closed form solutions[1].

Point cloud registration is one of the earliest problems risen in the 3d computer vision communities. Several approaches and formulations have been investigated, but due to the under-determined nature of the problem the topic has still relevance. A detailed categorization of the point registration problem can be found in the inspiring work of Tam et. al.[2].

The main idea of our proposed method is turning the point sets into continuous 3d vector-scalar functions. The score of a transformation between the functions is calculated by the space integral of the multiplication of the two functions. Of course the higher the score the better the transformation is. So the matching and transform estimation problems are together treated as a function fitting task.

It is important to note that we are not seeking for a global maximum during the fitting task, but aiming to get stuck in a local maximum where two overlapping regions are merging. Thus a fair initial transformation guess is essential.

For the two point sets we form corresponding continuous functions. It is done by replacing all points in a set with a 3d Gaussian function. The center of the Gaussian will be the original point. The resulting function for a point set will be a sum of Gaussian functions. Thus the points of a set $\{a_i\}$ are replaced by the function:

$$f_a(x) = \sum_{i=1}^n e^{-\frac{(x-a_i)^2}{w}} \quad (1)$$

where x is a 3d variable and w is a parameter of the Gaussian function that controls the 'slimness' of the function.

We define the rigid transformation as a rotation, translation and scale of the 3d point set x and denote with $t(x, p)$, where p is the parameter vector of the transformation. The score function for the parameters p is defined by the integral of the function multiplication:

$$S(p) = \int_{\mathbb{R}^3} f_a(x) \cdot t(f_b(x), p) dx \quad (2)$$

where f_a and f_b are the corresponding functions to the point sets $\{a_i\}$ and $\{b_j\}$.

Now the problem is to find a local maximum of $S(p)$ given an initial guess parameter p_0 . Fortunately, the equation can be simplified due to special properties of the Gaussian functions.

The proposed method was tested on synthetic data. Figure 1a. shows the same object in two different poses. The blue model is the original one and the green one is to be transformed. Both the models were treated as point clouds by taking the vertices of the objects. Each object consists of 505 vertices.

Figure 1b. shows the resulting transformation after 100 iterations of the steepest gradient optimization. The initial guess for the transformation was set to identity, so no translation, rotation or scale was considered. The test took 33s to run on a 2.4 GHz Intel Core i5 CPU without utilizing parallel computation.

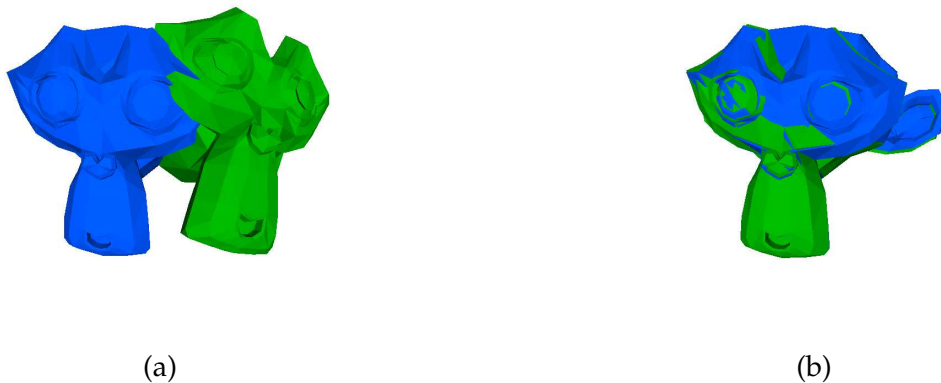


Figure 1: (a) initial poses of the models (b) final transformation

References

- [1] BKP Horn, HM Hilden, and S Negahdaripour. Closed-form solution of absolute orientation using orthonormal matrices. *JOSA A*, 5(July), 1988.
- [2] Gary K L Tam, Zhi-Quan Cheng, Yu-Kun Lai, Frank C Langbein, Yonghuai Liu, David Marshall, Ralph R Martin, Xian-Fang Sun, and Paul L Rosin. Registration of 3D point clouds and meshes: a survey from rigid to nonrigid. *IEEE transactions on visualization and computer graphics*, 19(7):1199–217, July 2013.