Analysis of a hyperbolic optimization method

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In our works concerning the analytic representation of ECG signals using complex rational functions we had to face an optimization problem which is defined on the complex unit disc, i.e. on the set $\{ z \in \mathbb{C} : |z| < 1 \}$. It turned out that the function to be optimized has several local minima, sophisticated methods are required to reach an optimal solution. We have developed and analysed many algorithms suitable for this problem, see e.g. [1] and [2].

This work to be presented here is aimed at the study of another algorithm, which is based on the hyperbolic version and variation of the well-known gradient method. Note that the complex unit disc also provides the so-called Poincaré disc model of the Bolyai–Lobachevsky hyperbolic geometry. We will also make advantage of the fact that the congruent transforms in this geometry can be expressed by the means of some rational complex functions, namely the Blaschke functions, defined as

$$B_{\delta,a}(z) := \delta \frac{z-a}{1-\overline{a}z}. \qquad (\delta, a, z \in \mathbb{C}, |\delta| = 1, |a| < 1, z \neq 1/\overline{a})$$

Our aim is to create a MATLAB implementation of this proposed algorithm, to present some numerical experiments and to analyse the method and its convergence properties in the case of some simple test functions.

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References

[1] S. Fridli, L. Lócsi, F. Schipp, Rational Function Systems in ECG Processing, *Proc. 13th Int. Conf. Computer Aided Systems Theory (EUROCAST 2011), Part I (eds. R. Moreno–Díaz et. al.)*, Springer LNCS 6927 (2011), 88–95.

[2] S. Fridli, P. Kovács, L. Lócsi, F. Schipp, Rational Modeling of Multi-lead QRS Complexes in ECG Signals, *Annales Univ. Sci. Budapest., Sect. Comp.*, **37** (2012), 145–155.