Determining Initial Bound by "Ray-method" in Branch and Bound Procedure

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It is very hard to solve an optimization problem if the problem contains integrality constraints. Such problems are integer programming problems, which can be solved by various algorithms. The most widespread algorithm is the Branch and Bound algorithm, which similarly to other methods, builds a search tree. Following from the structure of the problem, sometimes these methods obtain a result only at the expense of building a large tree. It may also happen that it take a lot of time to recognise that the problem does not have an integer solution. Therefor, the demand to reduce the size of the tree built by the algorithms is understandable. The paper aims to develop a reliable method, that can reduce the size of the search tree. The idea is based on the results of the theoretical research of the Computing Centre of the Russian Academy of Sciences.[1] It is difficult to adapt the main theoretical results into a computer environment. As a result these methods in original form are not applicable yet. Based on the theorems, that have been worked out recently, a new method, which can be relatively easily implemented has been developed, which has proved to be efficient on the basis of tests. By using the solution of the relaxation problem, the algorithm gives a bound for the value of the objective function of the problem with integer conditions, thereby creating an initial bound for the solvers, wich build a search tree. The key to the efficiency of the algoritms based on the ray-method is to create a feasible integer solution relatively quickly. One characteristic of the tree is that the optima get worse as we move down on its branches. If we find a feasible integer solution on one of the nodes to the original problem, we use the objective value of this solution as a bound. If a node has to be branched, but its objective value is already worse then our current bound we do not have to perform this operation, and we say this node is fathomed. However, we do not have such a bound at the starting node. The algorithms based on the ray-method aim to create a initial bound before we start to build the search tree. As a result, we get the opportunity to cut the unnecessary branches as soon as possible. This means that we do not have to build a big part of the tree in practice. It is essential to be able to implement these methods, since they serve to accelerate an applied method, namely Branch and Bound. It is obvious that, if calculating the initial bound takes more time then building the part of the tree, which can be cut with this bound, then we have not improved the efficiency. The method presented in this paper is relatively simple, easy to implement, which has proved to be efficient during the tests so far.

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References

[1] Khachaturov, V.R., Combinatorial methods and algorithms for solving problems of discrete optimization with large dimensionality, Nauka, 2000.