Potential Clouds in Robust Design

Martin Fuchs and Arnold Neumaier

The goal of robust design is to safeguard reliably against worst-case scenarios while seeking an optimal design. An engineer typically faces the task to develop a product which satisfies given requirements formulated as design constraints. Output of the engineer's work should be an optimal design with respect to a certain design objective. In many cases this is the cost or the mass of the designed product. An algorithmic method for design optimization functions as decision making support for engineers. The attempt of autonomous design has been made trying to capture the reasoning of the system experts. For more complex kinds of structures, e.g., a spacecraft component or a whole spacecraft, the design process addresses several different engineering fields, so the design optimization becomes multidisciplinary. An interaction between the involved disciplines is necessary. The resulting overall optimization process is known as multidisciplinary design optimization (MDO). Particularly MDO benefits from autonomous optimization methods for decision support that bridge the gap between different technical backgrounds. The difficulties arising during design optimization can be of most complex nature: multiobjective, multilevel, mixed integer nonlinear programming (MINLP) optimization problems with discontinuities or strong nonlinearities are involved. Standard optimization techniques cannot be used to solve such problems.

In many cases, in particular in early design phases, it is common engineering practice to handle uncertainties by assigning intervals, or safety margins, to the uncertain variables, usually combined with an iterative process of refining the intervals while converging to a robust optimal design. The refinement of the intervals is done by experts who assess whether the worst-case scenario, that has been determined for the design at the current stage of the iteration process, is too pessimistic or too optimistic. How to assign the intervals and how to choose the endpoint of the assigned intervals to get the worst-case scenario is usually not computed but assessed by an expert. The goal of the whole iteration includes both optimization of the design and safeguarding against uncertainties. The achieved design can thus be qualified as robust. Apart from interval assignments there are further methods to handle uncertainties in design processes: e.g., fuzzy clustering, simulation techniques like Monte Carlo.

Real life applications of uncertainty methods disclose various problems. The dimension of many uncertain real life scenarios is very high which causes severe computational problems, famous as the curse of dimensionality. Even given the knowledge of the multivariate probability distributions the numerical computation of the error probabilities becomes very expensive, if not impossible. Often standard simulation techniques are used to tackle the dimensionality issue, as the computational effort they require seems to be independent of the dimension. Advancements have been made based on sensitivity analysis, or on α-level optimization.

Frequently, especially in early design phases, data are scarce, though a large amount of data would be required to use traditional methods to estimate high dimensional probability distributions. Simulation techniques like Monte Carlo also require a large amount of information to be reliable, or unjustified assumptions on the uncertainties have to be made. However, mostly there are only interval bounds on the uncertain variables, sometimes probability distributions for single variables without correlation information. The lack of information typically causes standard simulation based methods to underestimate the effects of the uncertain tails of the probability distribution. Similarly, a reduction of the problem to an interval analysis after assigning intervals to the uncertain variables as described before (e.g., 3 σ boxes) entails a loss of valuable uncertainty information which would actually be available, maybe unformalized, but is not at all involved in the uncertainty model.

To overcome the problem of high dimensions in real life applications and the problem of incomplete information we make use of the concept of potential clouds. Potential clouds combine ideas from p-boxes, fuzzy sets, interval and optimization methods. The clouds can be weaved into an optimization problem formulation as confidence regions constraints. The computational effort is still tractable in higher dimensions. Remarkably potential clouds even enable an a posteriori information update for experts, even if an expert is unable to give a formal description of his knowledge. Unformalized knowledge is available, e.g., if an expert does not know correlations exactly, but can formulate a statement like 'if variable a has a large value then variable b cannot have a low value'. Thus he is able to exclude irrelevant scenarios, although he is unable to give a formal description. This can be performed in a graphical user interface as an interaction between the uncertainty modeling and the optimization phase. The new information can be processed by means of potential clouds.

The basic concept of our methodology can be summarized in three essential steps within an iterative framework. First, the expert provides the underlying system model, given as a black-box model, and all a priori available uncertainty information on the input variables of the model. Second, the information is processed to generate a potential cloud. Parameterized by given confidence levels, the clouds provide a nested collection of regions of relevant scenarios affecting the worst-case for a given design and thus produce safety constraints for the optimization. Third, optimization methods minimize a certain objective function (e.g., cost, mass) subject to the functional constraints which are represented by the system model, and subject to the safety constraints from the cloud. To this end we have developed heuristic optimization techniques. The results of the optimization are returned to the expert, who is given an interactive possibility to provide additional uncertainty information a posteriori and to rerun the procedure, adaptively improving the uncertainty model.

References

[1] D. Dubois and H. Prade. *Possibility Theory: An Approach to Computerized Processing of Uncertainty*. New York: Plenum Press, 1986.

[2] S. Ferson. *Ramas Risk Calc 4.0 Software: Risk Assessment with Uncertain Numbers*. Lewis Publishers, U.S., 2002.

[3] M. Fuchs and A. Neumaier. Potential based clouds in robust design optimization. *Journal of statistical theory and practice, special issue on imprecision*, 2008. accepted, preprint available on-line at: `http://www.martin-fuchs.net/publications.php`.

[4] B. Moeller and M. Beer. Fuzzy randomness: uncertainty in civil engineering and computational mechanics. *Springer-Verlag Berlin Heidelberg*, 2004.

[5] A. Neumaier. Clouds, fuzzy sets and probability intervals. *Reliable Computing 10*, pages 249–272, 2004. `http://www.mat.univie.ac.at/ neum/ms/cloud.pdf`.