Lifting and Separation Procedures for the Cut Polytope

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In this joint work with Gerhard Reinelt we describe a new separation and lifting approach for the cut polytope on arbitrary graphs exploiting known methods for dense and complete graphs respectively. Furthermore, we introduce a preliminary implementation using the branch&cut framework ABACUS.

Separation and lifting

The *max-cut problem* is one of the most studied combinatorial optimization problems. It is known to be $\mathcal{NP}$-hard and has a number of interesting applications, such as optimal design of VLSI circuits or the identification of configurations of minimal energy of so-called *spin glasses.*

Like for most of the hard combinatorial optimization problems, the technique that is typically used to solve max-cut problems to optimality is *branch&cut.* But its effectiveness heavily depends on the quality and speed of the separation routines used for the approximation of the *cut polytope*. The better part of the currently known methods are related to the max-cut problem on *dense* and *complete* graphs respectively and can only be successfully applied to relatively small problem instances.

A trivial way to exploit this knowledge for arbitrary graphs would be the artifical completion by adding zero-weighted edges. In this context the number of nodes is crucial for both basic practicability and duration of the separation.

Our approach uses a graph shrinking procedure to reduce the number of nodes prior to the artificial completion. Violated inequalities found on the resulting complete graph are then translated to the original graph by means of projection and lifting.

Implementation

A preliminary version has been implemented using the programming language C++ and the branch&cut framework *ABACUS*. It contains various separation routines, amongst others a variant of so-called *target cuts.*

References

[1] M. Jünger, G. Reinelt, G. Rinaldi. Lifting and separation procedures for the cut polytope, *Technical Report*