About the axiomatization of first- and second-order spatio-temporal logics [14]

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First first-order temporal logics, introduced in the sixties, root in the theory of first-order modal logics and nowadays are widely used in the theory of specification and verification of arbitrary computational systems. The temporal logic presents a tool for formulating and proving changing-through-time properties of a computational device (either software or hardware). Another possibility is writing specifications directly in a temporal logic language and allow an automated process to plan or construct an appropriate computing device. The temporal logics build on first-order logic have dramatically greater expressive power than logics based on propositional logic but the price of this power is non-axiomatizability, at least of most of these logics.
There is a diversified legion of first-order temporal logics:

1., we can decide what type of time notion is to be used.

2., as in the first-order modal logics, we have to choose what part of language is changing in time (for example, the interpretation of predicates but not the domain is changing).

3., we ca As an introduction to the methods of this paper we fill this lack for the case of $\mathbb{R}$, (we use in this proof only monadic predicate symbols which seems to be a sharpening of existing proofs in the literature) further we show that the first-order temporal logics over $\mathbb{Q}$ are axiomatizable. Reynolds [2] axiomatized some first-order temporal logics over $\mathbb{Q}$ with temporal operators $Until$ and $Since$ and proved completeness in a quite novel way. We give a short proof for the axiomatizability of first-order temporal logics with arbitrary temporal connectives over $\mathbb{Q}$. (Over $\mathbb{Q}$, $Until$ and $Since$ cannot express arbitrary temporal connectives.) A general reason of this is the ω-categoricity of the first-order theory $(\mathbb{Q}, <)$. In this way we shall show – by a rather simple and short translation – the recursive enumerability of the set of theorems of that theory. The price of simplicity is that we do not give any nice inner axiom system written using axioms and deduction rules. n choose some different temporal-logical connectives.

The first-order temporal logic using classical structures (like $\mathbb{N}, \mathbb{Z}, \mathbb{R}$) for time are usually non-axiomatizable. In [1] Garson provided some proofs of the non-axiomatizability over $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$, but his basic first-oder theory was other than classical the so-called free logic. Using classical logic, Garson's proof works only for $\mathbb{N}$ and $\mathbb{Z}$.

After that, our interest turns over temporal logics motivated by the relativistic space-time. At the present we do not know about computational devices that have relativistic features but nobody can exclude them from the future development.

Combining a result of van Benthem [3], that the first-order theory $(\mathbb{Q} \times \mathbb{Q}, L)$ is ω-categorical, with the last method, we are in the position of stating that with such a time notion, the first-order temporal logics are recursively enumerable and hence these logics are axiomatizable. The relation L is a kind of causal connectivity. This time notion (rather spa Further, we investigate the monadic second-order theory of this time notion. We prove it to be not recursively enumerable. It is surprising at first glance because the monadic second-order theory of $(\mathbb{Q}, <)$ is known to be decidable. In fact, one existential quantification over subsets is enough. This means the non-axiomatizability of the $\forall\exists$-fragment of this theory.

We select here only some references:

References

[1] James W. Garson: Quantification in modal logic. Handbook of philosophical logic Vol. II. (ed. Gabbay, Guenthner) 1984.

[2] Mark Reynolds: Axiomatising First-Order Temporal Logic: Until and Since over linear time. Studia Logica 57/1996.

[3] Johan van Benthem: The logic of time. Reidel, Synthese Library 156. 1983.

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