Search space reduction criterion based on derivatives in Global Optimization algorithms [9]

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Interval Global Optimization algorithms are based on a Branch and Bound scheme using the following five rules: bounding, termination, selection, subdivision and elimination. The research in interval Global Optimization algorithms try to determine the appropiated B&B rules. An example is the selection of a specific subdivision rule [1]. The elimination rule is one of the most investigated. The simplest elimination rules are the midpoint and monotonicity tests [4]. Most of the proposals have been devised to improve the efficiency based on the derivative information, such as monotonicity, concavity and Newton Method tests [2, 3]. Here we develop a new elimination and subdivision technique which also uses derivative information in one dimensional functions.

This paper investigates interval Global Optimization algorithms for solving the box constrained Global Optimization problem:

$$\min_{x \in X} f(x), \tag{6}$$

where the interval $X = [\underline{x}, \overline{x}] \subseteq \mathbb{R}$ is the search region, and $f(x) : X \subset \mathbb{R} \to \mathbb{R}$ is the objective function. The global minimum value of f is denoted by f^*, and the set of global minimizer points of f on X by X^*. That is,

$$f^* = \min_{x \in X} f(x) \qquad \text{and} \qquad X^* = \{x^* \mid f(x^*) = f^*\}.$$

Herein real numbers are denoted by $x, y, \ldots$, and a real bounded and closed interval by $X = [\underline{x}, \overline{x}]$, where $\underline{x} = \min X$ and $\overline{x} = \max X$. The set of compact intervals is denoted by $\mathbb{I} := \{[a, b] \mid a \leq b,\ a, b \in \mathbb{R}\}$. A function $F : \mathbb{I} \to \mathbb{I}$ is called *inclusion function* of f in $X \subseteq \mathbb{R} \to \mathbb{R}$, if $x \in X$ implies $f(x) \in F(X)$. In other words, $f(X) \subseteq F(X)$, where $f(X)$ is the range of the function f on X. It is assumed in the present study that the inclusion function of the objective function is available (possibly given by interval arithmetic).

For a given interval X, we denote $f_- = \overline{F}(\underline{x})$ and $f_+ = \overline{F}(\overline{x})$. When we express a real number as an interval, we shall usually retain the simpler noninterval notation. For example x in place of $[x, x]$ [2].

Let's denote the derivative of the inclusion function in the interval X by $G(X) = F'(X) = [\underline{G}, \overline{G}]$, the straight line with slope $\underline{G}$ at point $\underline{x}$ by L, the straight line width slope $\overline{G}$ at point $\overline{x}$ by U, and the intersection between L and U by (x_m, y_m)

The new ideas are described in Algorithm 1.

The algorithm is based on a more efficient (in comparison with traditional approaches) usage of the search information about the lower and upper bounds of the first derivative. A graphical example of algorithm 1 is shown in Figure 7.

Extensive numerical examples will be presented and compared with traditional interval Global Optimization algorithm using Newton method.

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Algorithm 1 Description of Algorithm

proc $ISBD(F, X, \epsilon, T, Q) \equiv$

F *Inclusion Function of F*

$Q = \{\}, T := (X)$ *Final and Work Lists*

$\tilde{f} = min\{f_-, f_+\}$

$G(X) := [\underline{G}, \overline{G}]$

Calculate L and U

(x_m, y_m) =L∩U

while ($T \neq \{\}$)

  $T := T - \{X\}$

  if $y_m < \tilde{f}$

    $f_m = f(x_m)$

    if $(f_m < \tilde{f})$

      $\tilde{f} = f(x_m)$

      $X_{new} = [L \cap \tilde{f}, U \cap \tilde{f}]$

    if $(U \cap \tilde{f}) \in [\underline{x}, x_m]$

      $X_{left} = [\underline{x}, U \cap \tilde{f}]$

      $G(X_{left}) := [\underline{G}, \overline{G}]$

      Calculate L and U

      (x_m, y_m) =L∩U

      if ($w(X_{left}) < \epsilon$) *Termination Criterion*

        then $Q := Q + \{X_{left}\}$

        else $T := T + \{X_{left}\}$

    if $(L \cap \tilde{f}) \in [\underline{x}, x_m]$

      $X_{rigth} = [L \cap \tilde{f}, \overline{x}]$

      $G(X_{rigth}) := [\underline{G}, \overline{G}]$

      Calculate L and U

      (x_m, y_m) =L∩U

      if ($w(X_{rigth}) < \epsilon$) *Termination Criterion*

        then $Q := Q + \{X_{rigth}\}$

        else $T := T + \{X_{rigth}\}$

end

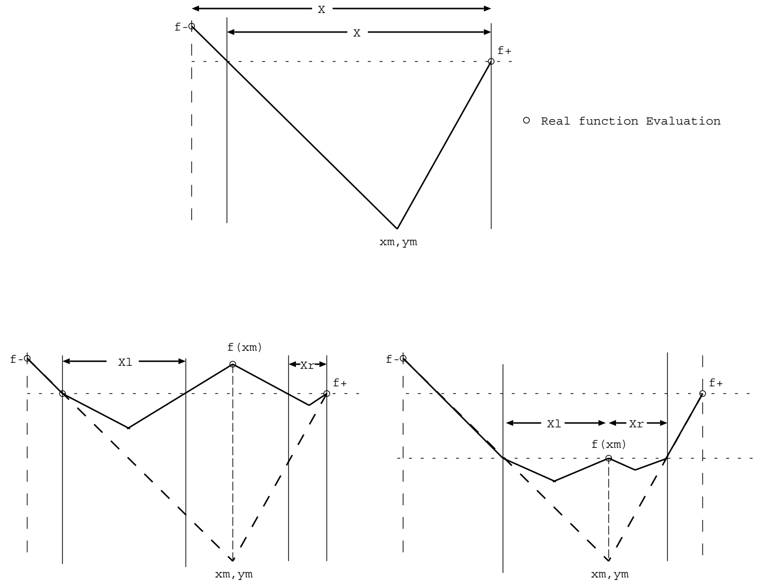

Figure 7: Example of algorithm execution

References

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