On Length of Directing Words of Automata

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A finite automaton $\mathbf{A} = (A, X, \delta)$ is directable if there exist an input word q and a state b such that $aq = b$ for every state a of automaton. The word q is called directing word. In 1964 Cerny [1] conjectured that a directable automaton with n-state must have a directing word of length less or equal to $(n-1)^2$. Recently, this conjecture is proved in cases $n \leq 5$, for cyclic automata with prime number of states and for monotonic automata. There are some special cases of directable automata where the upper bound is less then $(n-1)^2$ (for example the bound is $n-1$ for the commutative, definite and nilpotent directable automata). In general case the conjecture is not proved, and the best known upper bound is $\mathbf{O}(n^3)$.

Let $A'x = \{ax | a \in A'\}$ and $A'x^{-1} = \{a | ax \in A'\} (A' \subseteq A)$. The symbol x is called idempotent if $Axx = x$, and x is a simple idempotent if there exists only one $a \in A$ such that $|\{a\}x^{-1}| > 1$. It has been proved by Rystsov [4] that a directable automata with simple idempotent and n states must have an input word with length less or equal to $2(n-1)^2$.

In our paper the directable automata having a simple idempotent is studied.

Let $\mathbf{A} = (A, X, \delta)$ be an automaton, x an input symbol and $A' = \{a_0, a_1, \ldots, a_{m-1}\} \subseteq A$. A' is called an x-cycle if $a_i = a_{(i+1) \mod m}$ for every state $a_i \in A'$, the length of A' is m.

First we will assume that the directable automata has an input symbol x (without the simple idempotent) for which A is an x-cycle. We will prove that the minimal length of their directing words is not greater than $(n-1)^2$.

In the second case we assume that there exists a state a for which $\{a\}$ is an x-cycle and $A - \{a\}$ is an x-cycle too. In this case we can also prove that the minimal length of the directing words is not greater than $(n-1)^2$.

References

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