A New Approach to Neural Network Design

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The radial basis function (RBF) neural networks were proposed in the mid-80s for classification and approximation. These networks work with a single hidden layer in most of the cases. The basic idea behind them is that the function or the class is approximated by a combination of basis functions, what differs significantly from zero on a small bounded region. This permits the local tuning of the RBF networks.

The parameters of the basis functions are fixed in the basic version of the RBF networks, and the optimization is done on the level of the weights by that these functions are combined to obtain the approximation. In more advanced versions, it is permitted the modification of these parameters too.

Starting from the mid 80s, it was established that the RBF neural networks have the universal approximation property with respect to the continuous functions, and error bounds were calculated for these networks.

A newer approach is proposed in this paper with respect to the approximation properties of these networks. By this approach it is possible to estimate the number of the necessary hidden nodes and their internal parameters, and to evaluate how appropriate is the application of the RBF neural networks for the approximation of classes of functions.

The orthogonalization of the RBF neural networks in fact means that we look for an orthogonalization method of the Gaussian functions. The idea is that if we can find a proper orthogonalization then we can apply the theory of approximations with combinations of orthogonal units.

By orthogonalization of the set F of the Gaussian functions of form $f : R^n \to R, f(x) = e^{-\frac{||x-c||^2}{2r^2}}$, we mean that we to find an infinite subset $F_0 \subset F$, and an internal product, such that we have for all $f_1, f_2 \in F_0$ that

$$< f_1, f_2 >= \begin{cases} c_{f_1} \; if \; f_1 = f_2 \\ 0 \;\; if \; f_1 \neq f_2 \end{cases} .$$

We note the Hilbert space generated by this internal product by H. Then we can apply the theory of the orthogonal approximations within H. The paper presents the basic results in this respect.

We define the Gaussian spectrum of the goal function and we present a network design method based on the analysis of this spectrum. Using this theoretical network design framework we present applications to function approximation and time series prediction. We found in concordance with our expectations that the spectrum-based neural networks perform much better than those with randomly selected parameters.

Further work should be done in order to generalize the proposed approach to other types of neural networks.