

Elementary Decomposition of Soliton Automata

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A soliton automaton is the mathematical model of so called soliton valves in certain carbohydrate molecules, and has the potential to serve as a future molecular switching device. The underlying object of a soliton automaton is a soliton graph, which is an undirected graph G having a perfect internal matching, i.e., a matching that covers all the vertices of G with degree at least two. Such vertices are called internal, whereas vertices with degree one are called external in G . The states of the automaton are all the perfect internal matchings of G , and transitions are defined by making alternating walks connecting two external vertices in G .

The main contribution of this paper is a decomposition of soliton automata according to the global internal structure of their underlying soliton graphs. For practical reasons, all edges of soliton graphs are assumed to be accessible by an external alternating path, i.e., alternating path starting from an external vertex. The elementary components – maximal connected subgraphs spanned by allowed edges – of G can be grouped into disjoint families in a way that:

1. Each family has a unique gate member through which all other members can be accessed by external alternating paths.
2. For each family, all external alternating paths reaching that family arrive in vertices belonging to the same canonical class of the gate.
3. Each family has an ear decomposition by which all non-gate members are located on an appropriate thread around the gate.

The families above can be arranged in a fixed partial order reflecting the order in which they are accessible by external alternating paths. The corresponding soliton automaton, too, can be decomposed into component automata determined by these families. For each internal family F , the component automaton defined by F will have a transition between any two of its different states. A transition from any particular state to itself, however, will exist if and only if the family represents a non-bipartite subgraph.