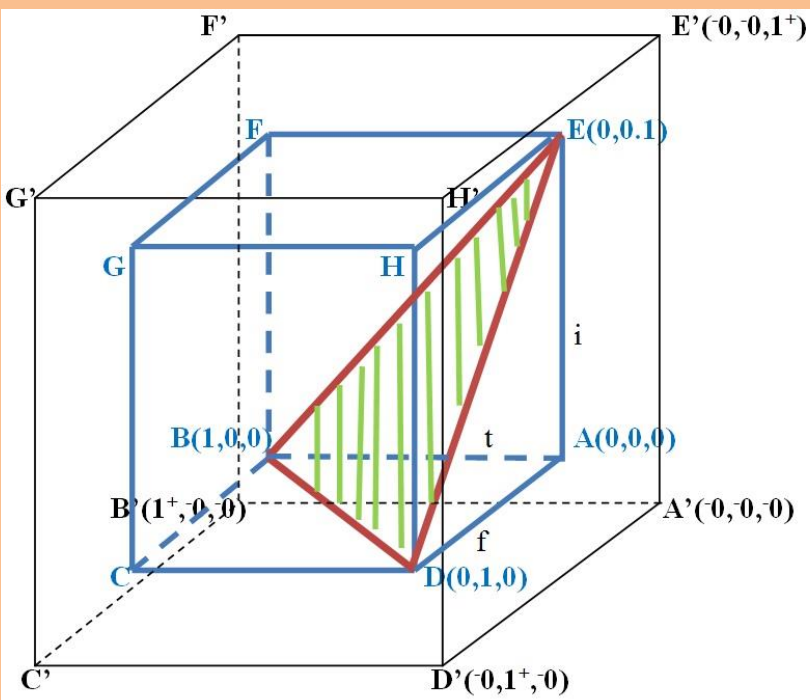
Full Issue

Neutrosophic Sets and Systems

Vol. 1, 2013

Neutrosophic Sets and Systems

An International Journal in Information Science and Engineering



ISSN 2331-6055 (print)
ISSN 2331-608X (online)

ISSN 2331-6055 (print) *ISSN 2331-608X (online)*

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An International Journal in Information Science and Engineering

Volume 1 **2013**

Contents

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An International Journal in Information Science and Engineering

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Neutrosophic Sets and Systems has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Neutrosophic Measure and Neutrosophic Integral

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Abstract. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. We now introduce for the first time the notions of neutrosophic measure and neutrosophic integral. Neutrosophic Science means development and applications of neutrosophic logic/set/measure/integral/ probability etc. and their applications in any field. It is possible to define the neutrosophic measure and consequently the neutrosophic integral and neutrosophic probability in many ways, because there are various types of indeterminacies, depending on the problem we need to solve. Indeterminacy is different from randomness. Indeterminacy can be caused by physical space materials and type of construction, by items involved in the space, or by other factors. Neutrosophic measure is a generalization of the classical measure for the case when the space contains some indeterminacy. Neutrosophic Integral is defined on neutrosophic measure. Simple examples of neutrosophic integrals are given.

Keywords: neutrosophy, neutrosophic measure, neutrosophic integral, indeterminacy, randomness, probability.

1 Introduction to Neutrosophic Measure

1.1 Introduction

Let <A> be an item. <A> can be a notion, an attribute, an idea, a proposition, a theorem, a theory, etc.

And let <antiA> be the opposite of <A>; while <neutA> be neither <A> nor <antiA> but the neutral (or indeterminacy, unknown) related to <A>.

For example, if <A> = victory, then <antiA> = defeat, while <neutA> = tie game.

If <A> is the degree of truth value of a proposition, then <antiA> is the degree of falsehood of the proposition, while <neutA> is the degree of indeterminacy (i.e. neither true nor false) of the proposition.

Also, if <A> = voting for a candidate, <antiA> = voting against that candidate, while <neutA> = not voting at all, or casting a blank vote, or casting a black vote. In the case when <antiA> does not exist, we consider its measure be null {m(antiA)=0}. And similarly when <neutA> does not exist, its measure is null { m(neutA) = 0}.

1.2 Definition of Neutrosophic Measure

We introduce for the first time the scientific notion of neutrosophic measure.

Let X be a neutrosophic space, and Σ a σ-neutrosophic algebra over X. A *neutrosophic measure* ν is defined by for neutrosophic set $A \in \Sigma$ by

$\nu : X \to R^3$,

$$\nu(A) = (m(A), m(neutA), m(antiA)), \tag{1}$$

with *antiA* = the opposite of A, and *neutA* = the neutral (indeterminacy) neither A nor anti A (as defined above);

for any $A \subseteq X$ and $A \in \Sigma$,

m(A) means *measure of the determinate part of A;*

m(neutA) means *measure of indeterminate part of A;*

and *m(antiA)* means *measure of the determinate part of antiA*;

where ν is a function that satisfies the following two properties:

a) Null empty set: $\nu(\Phi) = (0,0,0)$.

b) Countable additivity (or σ-additivity): For all countable collections $\{A_n\}_{n \in L}$ of disjoint neutrosophic sets in Σ, one has:

$$\nu\left(\bigcup_{n\in L} A_n\right) = \left(\sum_{n\in L} m(A_n), \sum_{n\in L} m(neutA_n), \sum_{n\in L} m(antiA_n) - (n-1)m(X)\right)$$

where X is the whole neutrosophic space,

and (2)

$$\sum_{n\in L} m(antiA_n) - (n-1)m(X) = m(X) - \sum_{n\in L} m(A_n) = m(\bigcap_{n\in L} antiA_n).$$

1.3 Neutrosophic Measure Space

A neutrosophic measure space is a triplet (X, Σ, ν).

1.4 Normalized Neutrosophic Measure

A neutrosophic measure is called normalized if

4 $\nu(X) = (m(X), m(neutX), m(antiX)) = (x_1, x_2, x_3)$,

with $x_1 + x_2 + x_3 = 1$,

4

$$\text{and } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \tag{3}$$

Where, of course, X is the whole neutrosophic measure space.

1.5 Finite Neutrosophic Measure Space

Let $A \subset X$. We say that $\nu(A) = (a_1, a_2, a_3)$ is finite if all a_1, a_2, and a_3 are finite real numbers.

A *neutrosophic measure space* (X, Σ, ν) is called *finite* if $\nu(X) = (a, b, c)$ such that all *a, b,* and *c* are finite (rather than infinite).

1.6 σ-Finite Neutrosophic Measure

A neutrosophic measure is called σ-finite if X can be decomposed into a countable union of neutrosophically measurable sets of fine neutrosophic measure.

Analogously, a set A in X is said to have a σ-finite neutrosophic measure if it is a countable union of sets with finite neutrosophic measure.

1.7 Neutrosophic Axiom of Non-Negativity

We say that the neutrosophic measure ν *satisfies the axiom of non-negativity*, if:

$$\forall A \in \Sigma,$$

$$\nu(A) = (a_1, a_2, a_3) \geq 0 \text{ if } a_1 \geq 0, a_2 \geq 0, \text{ and } a_3 \geq 0. \quad (4)$$

While a neutrosophic measure ν, that satisfies only the null empty set and countable additivity axioms (hence not the non-negativity axiom), takes on at most one of the $\pm\infty$ values.

1.8 Measurable Neutrosophic Set and Measurable Neutrosophic Space

The members of Σ are called measurable neutrosophic sets, while (X, Σ) is called a measurable neutrosophic space.

1.9 Neutrosophic Measurable Function

A function $f : (X, \Sigma_X) \to (Y, \Sigma_Y)$, mapping two measurable neutrosophic spaces, is called *neutrosophic measurable function* if $\forall B \in \Sigma_Y, f^{-1}(B) \in \Sigma_X$ (the inverse image of a neutrosophic Y-measurable set is a neutrosophic X-measurable set).

1.10 Neutrosophic Probability Measure

As a particular case of neutrosophic measure ν is th neutrosophic probability measure, i.e. a neutrosophic measure that measures probable/possible propositions

$$^{-}0 \leq \nu(X) \leq 3^{+}, \quad (5)$$

where X is the whole neutrosophic probability sample space.

We use nonstandard numbers, such 1^{+} for example, to denominate the absolute measure (measure in all possible worlds), and standard numbers such as *1* to denominate the relative measure (measure in at least one world). Etc.

We denote the neutrosophic probability measure by $\mathcal{NP}$ for a closer connection with the classical probability $\mathcal{P}$.

1.11 Neutrosophic Category Theory

The neutrosophic measurable functions and their neutrosophic measurable spaces form a neutrosophic category, where the functions are arrows and the spaces objects.

We introduce the neutrosophic category theory, which means the study of the neutrosophic structures and of the neutrosophic mappings that preserve these structures.

The classical category theory was introduced about 1940 by Eilenberg and Mac Lane.

A neutrosophic category is formed by a class of neutrosophic objects $X, Y, Z, \ldots$ and a class of neutrosophic morphisms (arrows) $\nu, \xi, \omega, \ldots$ such that:

a) If $Hom(X, Y)$ represent the neutrosophic morphisms from X to Y, then $Hom(X, Y)$ and $Hom(X', Y')$ are disjoint, except when $X = X'$ and $Y = Y'$;

b) The composition of the neutrosophic morphisms verify the axioms of

i) Associativity: $(\nu \circ \xi) \circ \omega = \nu \circ (\xi \circ \omega)$

ii) Identity unit: for each neutrosophic object X there exists a neutrosophic morphism denoted id_X, called *neutrosophic identity* of X such that $id_X \circ \nu = \nu$ and $\xi \circ id_X = \xi$

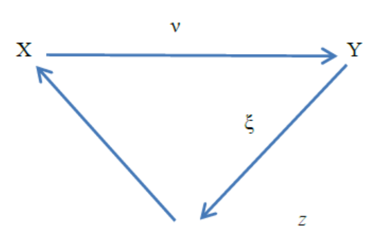

Fig. 2

1.12 Properties of Neutrosophic Measure

a) Monotonicity.

If A_1 and A_2 are neutrosophically measurable, with $A_1 \subseteq A_2$, where

$\nu(A_1) = (m(A_1), m(neutA_1), m(antiA_1))$,

and $\nu(A_2) = (m(A_2), m(neutA_2), m(antiA_2))$,

then

$$m(A_1) \leq m(A_2), m(neutA_1) \leq m(neutA_2), m(antiA_1) \geq m(antiA_2) \quad (6)$$

Let $\nu(X) = (x_1, x_2, x_3)$ and $\nu(Y) = (y_1, y_2, y_3)$. We say that $\nu(X) \leq \nu(Y)$, if $x_1 \leq y_1$, $x_2 \leq y_2$, and $x_3 \geq y_3$.

b) Additivity.

If $A_1 \cap A_2 = \Phi$, then $\nu(A_1 \cup A_2) = \nu(A_1) + \nu(A_2)$, (7)

where we define

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, a_3 + b_3 - m(X)) \quad (8)$$

where X is the whole neutrosophic space, and

$$a_3 + b_3 - m(X) = m(X) - m(A) - m(B) = m(X) - a_1 - a_2$$
$$= m(antiA \cap antiB). \quad (9)$$

1.13 Neutrosophic Measure Continuous from Below or Above

A *neutrosophic measure* ν *is continuous from below* if, for $A_1, A_2, \ldots$ neutrosophically measurable sets with $A_n \subseteq A_{n+1}$ for all n, the union of the sets A_n is neutrosophically measurable, and

$$\nu\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} \nu(A_n) \quad (10)$$

And a *neutrosophic measure* ν *is continuous from above* if for $A_1, A_2, \ldots$ neutrosophically measurable sets, with $A_n \supseteq A_{n+1}$ for all n, and at least one A_n has finite neutrosophic measure, the intersection of the sets A_n and neutrosophically measurable, and

$$\nu\left(\bigcap_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} \nu(A_n). \quad (11)$$

1.14 Generalizations

Neutrosophic measure is a generalization of the *fuzzy measure*, because when $m(neutA) = 0$ and $m(antiA)$ is ignored, we get

$$\nu(A) = (m(A), 0, 0) \equiv m(A) \quad (12)$$

and the two fuzzy measure axioms are verified:

a) If $A = \Phi$, then $\nu(A) = (0,0,0) \equiv 0$

b) If $A \subseteq B$, then $\nu(A) \le \nu(B)$.

The neutrosophic measure is practically a triple classical measure: a classical measure of the determinate part of a neutrosophic object, a classical part of the indeterminate part of the neutrosophic object, and another classical measure of the determinate part of the opposite neutrosophic object. Of course, if the indeterminate part does not exist (its measure is zero) and the measure of the opposite object is ignored, the neutrosophic measure is reduced to the classical measure.

1.15 Examples

Let's see some examples of neutrosophic objects and neutrosophic measures.

a) If a book of *100* sheets (covers included) has *3* missing sheets, then

$$\nu(book) = (97, 3, 0) \quad (13)$$

where ν is the neutrosophic measure of the book number of pages.

b) If a surface of *5 × 5* square meters has cracks of *0.1 × 0.2* square meters, then $\nu(surface) = (24.98, 0.02, 0)$, (14), where ν is the neutrosophic measure of the surface.

c) If a die has two erased faces then

$$\nu(die) = (4, 2, 0), \quad (14)$$

where ν is the neutrosophic measure of the die's number of correct faces.

d) An approximate number N can be interpreted as a neutrosophic measure $N = \underline{d} + \underline{i}$, where $\underline{d}$ is its determinate part, and $\underline{i}$ its indeterminate part. Its anti part is considered *0*.

For example if we don't know exactly a quantity q, but only that it is between let's say $q \in [0.8, 0.9]$, then $q = 0.8 + i$, where *0.8* is the determinate part of q, and its indeterminate part $i \in [0, 0.1]$.

We get a negative neutrosophic measure if we approximate a quantity measured in an inverse direction on the *x*-axis to an equivalent positive quantity.

For example, if $r \in [-6, -4]$, then $r = -6 + i$, where -6 is the determinate part of r, and $i \in [0, 2]$ is its indeterminate part. Its anti part is also 0.

e) Let's measure the truth-value of the proposition

G = "through a point exterior to a line one can draw only one parallel to the given line".

The proposition is incomplete, since it does not specify the type of geometrical space it belongs to. In an *Euclidean geometric space* the proposition *G* is true; in a *Riemannian geometric space* the proposition *G* is false (since there is no parallel passing through an exterior point to a given line); in a *Smarandache geometric space* (constructed from mixed spaces, for example from a part of Euclidean subspace together with another part of Riemannian space) the proposition *G* is indeterminate (true and false in the same time).

$$\nu(G) = (1,1,1). \quad (15)$$

f) In general, not well determined objects, notions, ideas, etc. can become subject to the neutrosophic theory.

2 Introduction to Neutrosophic Integral

2.1 Definition of Neutrosophic Integral

Using the neutrosophic measure, we can define a neutrosophic integral.

The neutrosophic integral of a function *f* is written as:

$$\int_X f d\nu \quad (16)$$

where *X* is the a neutrosophic measure space, and the integral is taken with respect to the neutrosophic measure ν.

Indeterminacy related to integration can occur in multiple ways: with respect to value of the function to be integrated, or with respect to the lower or upper limit of integration, or with respect to the space and its measure.

2.2 First Example of Neutrosophic Integral: Indeterminacy Related to Function's Values

Let *fN: [a, b]* → *R* (17)

where the neutrosophic function is defined as:

fN (x) = g(x)+i(x) (18)

with *g(x)* the determinate part of *fN(x)*, and *i(x)* the indeterminate part of *fN(x)*,where for all *x* in *[a, b]* one has: $i(x)\in[0,h(x)], h(x)\geq 0$. (19)

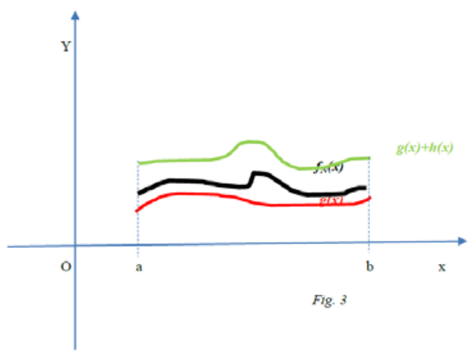

Fig. 3

Therefore the values of the function fN(x) are approximate, i.e. $f_N(x)\in[g(x), g(x)+h(x)]$. (20)

Similarly, the neutrosophic integral is an approximation:

$$\int_a^b f_N(x)d\nu = \int_a^b g(x)dx + \int_a^b i(x)dx \tag{21}$$

1.10 Second Example of Neutrosophic Integral: Indeterminacy Related to the Lower Limit

Suppose we need to integrate the function

f: X → *R* 22)

on the interval *[a, b]* from *X*, but we are unsure about the lower limit *a*. Let's suppose that the lower limit "*a*" has a

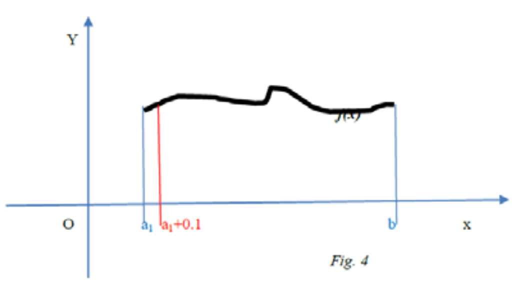

Fig. 4

determinant part "a_1" and an indeterminate part ε, i.e.

$$a = a_1+\varepsilon \tag{23}$$

where

$$\varepsilon\in[0, 0.1]. \tag{24}$$

Therefore

$$\int_a^b {}_X f d\nu = \int_{a_1}^b f(x)dx - i_1 \tag{25}$$

where the indeterminacy i_1 belongs to the interval:

$$i_1 \in [0, \int_{a_1}^{a_1+0.1} f(x)dx]. \tag{26}$$

Or, in a different way:

$$\int_a^b {}_X f d\nu = \int_{a_1+0.1}^b f(x)dx + i_2 \tag{27}$$

where similarly the indeterminacy i_2 belongs to the interval:

$$i_2 \in [0, \int_{a_1}^{a_1+0.1} f(x)dx]. \tag{28}$$

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Received: November 12, 2013. Accepted: December 5, 2013

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Another Form of Correlation Coefficient between Single Valued Neutrosophic Sets and Its Multiple Attribute Decision-Making Method

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Abstract. A single valued neutrosophic set (SVNS), which is the subclass of a neutrosophic set, can be considered as a powerful tool to express the indeterminate and inconsistent information in the process of decision making. Then, correlation is one of the most broadly applied indices in many fields and also an important measure in data analysis and classification, pattern recognition, decision making and so on. Therefore, we propose another form of correlation coefficient between SVNSs and establish a multiple attribute decision making method using the correlation coefficient of SVNSs under single valued neutrosophic environment. Through the weighted correlation coefficient between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, two illustrative examples are employed to illustrate the actual applications of the proposed decision-making approach.

Keywords: Correlation coefficient; Single valued neutrosophic set; Decision making.

1 Introduction

To handle the indeterminate information and inconsistent information which exist commonly in real situations, Smarandache [1] firstly presented a neutrosophic set from philosophical point of view, which is a powerful general formal framework and generalized the concept of the classic set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set [1, 2]. In the neutrosophic set, a truth-membership, an indeterminacy-membership, and a falsity-membership are represented independently. Its functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \rightarrow\ ]^-0, 1^+[$, $I_A(x): X \rightarrow\ ]^-0, 1^+[$, and $F_A(x): X \rightarrow\ ]^-0, 1^+[$. Obviously, it will be difficult to apply in real scientific and engineering areas. Therefore, Wang et al. [3] proposed the concept of a single valued neutrosophic set (SVNS), which is the subclass of a neutrosophic set, and provided the set-theoretic operators and various properties of SVNSs. Thus, SVNSs can be applied in real scientific and engineering fields and give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. However, the correlation coefficient is one of the most frequently used tools in engineering applications. Therefore, Hanafy et al. [4] introduced the correlation of neutrosophic data. Then, Ye [5] presented the correlation coefficient of SVNSs based on the extension of the correlation coefficient of intuitionistic fuzzy sets and proved that the cosine similarity measure of SVNSs is a special case of the correlation coefficient of SVNSs, and then applied it to single valued neutrosophic multicriteria decision-making problems. Hanafy et al. [6] presented the centroid-based correlation coefficient of neutrosophic sets and investigated its properties. Recently , S. Broumi and F. Smarandache [8] Correlation coefficient of interval neutrosophic set and investigated its properties.

In this paper, we propose another form of correlation coefficient between SVNSs and investigate its properties. Then, a multiple attribute decision-making method using the correlation coefficient of SVNSs is established under single valued neutrosophic environment. To do so, the rest of the paper is organized as follows. Section 2 briefly describes some concepts of SVNSs. In Section 3, we develop another form of correlation coefficient between SVNSs and investigate its properties. Section 4 establishes a multiple attribute decision-making method using the correlation coefficient of SVNSs under single valued neutrosophic environment. In Section 5, two illustrative examples are presented to demonstrate the applications of the developed approach. Section 6 contains a conclusion and future research.

2 Some concepts of SVNSs

Smarandache [1] firstly presented the concept of a neutrosophic set from philosophical point of view and gave the following definition of a neutrosophic set.

Definition 1 [1]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Obviously, it is difficult to apply in practical problems. Therefore, Wang et al. [3] introduced the concept of a SVNS, which is an instance of a neutrosophic set, to apply in real scientific and engineering applications. In the following, we introduce the definition of a SVNS [3].

Definition 2 [3]. Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. Thus, A SVNS A can be expressed as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}.$$

Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 3 [3]. The complement of a SVNS A is denoted by A^c and is defined as

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in X\}.$$

Definition 4 [3]. A SVNS A is contained in the other SVNS B, $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every x in X.

Definition 5 [3]. Two SVNSs A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

3 Correlation coefficient of SVNSs

Motivated by another correlation coefficient between intuitionistic fuzzy sets [7], this section proposes another form of correlation coefficient between SVNSs as a generalization of the correlation coefficient of intuitionistic fuzzy sets [7].

Definition 6. For any two SVNSs A and B in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, another form of correlation coefficient between two SVNSs A and B is defined by

$$N(A,B) = \frac{C(A,B)}{\max\{C(A,A), C(B,B)\}} \tag{1}$$

$$= \frac{\sum_{i=1}^{n}\left[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)\right]}{\max\left\{\sum_{i=1}^{n}\left[T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)\right], \sum_{i=1}^{n}\left[T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)\right]\right\}}$$

Theorem 1. The correlation coefficient $N(A, B)$ satisfies the following properties:

(1) $N(A, B) = N(B, A)$;

(2) $0 \le N(A, B) \le 1$;

(3) $N(A, B) = 1$, if $A = B$.

Proof. (1) It is straightforward.

(2) The inequality $N(A, B) \ge 0$ is obvious. Thus, we only prove the inequality $N(A, B) \le 1$.

$$N(A,B) =$$
$$\sum_{i=1}^{n}\left[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)\right]$$
$$= T_A(x_1)\cdot T_B(x_1) + T_A(x_2)\cdot T_B(x_2) + \ldots + T_A(x_n)\cdot T_B(x_n)$$
$$+ I_A(x_1)\cdot I_B(x_1) + I_A(x_2)\cdot I_B(x_2) + \ldots + I_A(x_n)\cdot I_B(x_n)$$
$$+ F_A(x_1)\cdot F_B(x_1) + F_A(x_2)\cdot F_B(x_2) + \ldots + F_A(x_n)\cdot F_B(x_n)$$

According to the Cauchy–Schwarz inequality:

$$(x_1\cdot y_1 + x_2\cdot y_2 + \ldots + x_n\cdot y_n)^2$$
$$\le (x_1^2 + x_2^2 + \ldots + x_n^2)\cdot(y_1^2 + y_2^2 + \ldots + y_n^2),$$

where $(x_1, x_2, \ldots, x_n) \in R^n$ and $(y_1, y_2, \ldots, y_n) \in R^n$, we can obtain

$$[(N(A,B)]^2 \le \sum_{i=1}^{n}\left[T_A^2(x_i) + I_A^2(x_n) + F_A^2(x_n)\right]$$
$$\cdot\sum_{i=1}^{n}\left[T_B^2(x_i) + +I_B^2(x_i) + F_B^2(x_i)\right]$$
$$= N(A,A)\cdot N(B,B).$$

Thus, $N(A,B) \le [N(A,A)]^{1/2}\cdot[N(B,B)]^{1/2}$.

Then, $N(A,B) \le \max\{N(A,A), N(B,B)\}$.

Therefore, $N(A, B) \le 1$.

(3) If $A = B$, there are $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ for any $x_i \in X$ and $i = 1, 2, \ldots, n$. Thus, there are $N(A, B) = 1$.

In practical applications, the differences of importance are considered in the elements in the universe. Therefore, we need to take the weights of the elements x_i ($i = 1, 2,\ldots, n$) into account. Let w_i be the weight for each element x_i ($i = 1, 2,\ldots, n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then we have the following weighted correlation coefficient between the SVNSs A and B:

$$W(A,B) = \frac{\sum_{i=1}^{n} w_i\left[T_A(x_i)\cdot T_B(x_i) + I_A(x_i)\cdot I_B(x_i) + F_A(x_i)\cdot F_B(x_i)\right]}{\max\left\{\sum_{i=1}^{n} w_i\left[T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)\right], \sum_{i=1}^{n} w_i\left[T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)\right]\right\}} \tag{2}$$

If $w = (1/n, 1/n,\ldots, 1/n)^T$, then Eq. (2) reduce to Eq. (1). Note that $W(A, B)$ also satisfy the three properties of Theorem 1.

Theorem 2. Let w_i be the weight for each element x_i (i = 1, 2,…, n), $w_i \in$ [0, 1], and $\sum_{i=1}^{n} w_i = 1$, then the weighted correlation coefficient $W(A, B)$ defined in Eq. (2) also satisfies the following properties:

(1) $W(A, B) = W(B, A)$;
(2) $0 \le W(A, B) \le 1$;
(3) $W(A, B) = 1$, if $A = B$.

Since the process to prove these properties is similar to that in Theorem 1, we do not repeat it here.

4 Decision-making method using the correlation coefficient of SVNSs

This section proposes a single valued neutrosophic multiple attribute decision-making method using the proposed correlation coefficient of SVNSs.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2,\ldots, C_n\}$ be a set of attributes. Assume that the weight of an attribute C_j (j = 1, 2,…, n), entered by the decision-maker, is w_j, $w_j \in$ [0, 1] and $\sum_{j=1}^{n} x_j = 1$. In this case, the characteristic of an alternative A_i (i = 1, 2,…, m) with respect to an attribute C_j (j = 1, 2,…, n) is represented by a SVNS form:

$$A_i = \{\langle C_j, T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C, j = 1,2,\ldots,n\},$$

where $T_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j) \in$ [0, 1] and $0 \le T_{A_i}(C_j) + I_{A_i}(C_j) + F_{A_i}(C_j) \le 3$ for $C_j \in C$, j = 1, 2, …, n, and i = 1, 2, …, m.

For convenience, the values of the three functions $T_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j)$ are denoted by a single valued neutrosophic value (SVNV) $a_{ij} = \langle t_{ij}, i_{ij}, f_{ij}\rangle$ (i = 1, 2, …, m; j = 1, 2,…, n), which is usually derived from the evaluation of an alternative A_i with respect to an attribute C_j by the expert or decision maker. Thus, we can establish a single valued neutrosophic decision matrix $D = (a_{ij})_{m\times n}$:

$$D = (a_{ij})_{m\times n} = \begin{bmatrix} \langle t_{11}, i_{11}, f_{11}\rangle & \langle t_{12}, i_{12}, f_{12}\rangle & \cdots & \langle t_{1n}, i_{1n}, f_{1n}\rangle \\ \langle t_{21}, i_{21}, f_{21}\rangle & \langle t_{22}, i_{22}, f_{22}\rangle & \cdots & \langle t_{2n}, i_{2n}, f_{2n}\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle t_{m1}, i_{m1}, f_{m1}\rangle & \langle t_{m2}, i_{m2}, f_{m2}\rangle & \cdots & \langle t_{mn}, i_{mn}, f_{mn}\rangle \end{bmatrix}.$$

In the decision-making method, the concept of ideal point has been used to help identify the best alternative in the decision set. The ideal alternative provides a useful theoretical construct against which to evaluate alternatives. Generally, the evaluation attributes can be categorized into two kinds, benefit attributes and cost attributes. Let H be a collection of benefit attributes and L be a collection of cost attributes. An ideal SVNV can be defined by an ideal element for a benefit attribute in the ideal alternative A^* as

$$a_j^* = \langle t_j^*, i_j^*, f_j^*\rangle = \langle \max_i(t_{ij}), \min_i(i_{ij}), \min_i(f_{ij})\rangle \text{ for } j \in H,$$

while an ideal SVNV can be defined by an ideal element for a cost attribute in the ideal alternative A^* as

$$a_j^* = \langle t_j^*, i_j^*, f_j^*\rangle = \langle \min_i(t_{ij}), \max_i(i_{ij}), \max_i(f_{ij})\rangle \text{ for } j \in L.$$

Then, by applying Eq. (2) the weighted correlation coefficient between an alternative A_i (i = 1, 2, …, m) and the ideal alternative A^* is given by

$$W(A_i, A^*) = \frac{\sum_{j=1}^{n} w_j \left[t_{ij}\cdot t_j^* + i_{ij}\cdot i_j^* + f_{ij}\cdot f_j^*\right]}{\max\left\{\sum_{j=1}^{n} w_j \left[t_{ij}^2 + i_{ij}^2 + f_{ij}^2\right], \sum_{j=1}^{n} w_j \left[(t_j^*)^2 + (i_j^*)^2 + (f_j^*)^2\right]\right\}} \quad (3)$$

Then, the bigger the measure value $W(A_i, A^*)$ (i = 1, 2, . . . , m) is, the better the alternative A_i is, because the alternative A_i is close to the ideal alternative A^*. Through the weighted correlation coefficient between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

5 Illustrative examples

In this section, two illustrative examples for the multiple attribute decision-making problems are provided to demonstrate the application of the proposed decision-making method.

5.1 Example 1

Now, we discuss the decision-making problem adapted from the literature [5]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the three attributes: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is the environmental impact, where C_1 and C_2 are benefit attributes and C_3 is a cost attribute. The weight vector of the three attributes is given by $w = (0.35, 0.25, 0.4)^T$. The four possible alternatives are to be evaluated under the above three attributes by the form of SVNVs.

For the evaluation of an alternative A_i with respect to an attribute C_j (i =1, 2, 3, 4; j =1, 2, 3), it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to an attribute C_1, he or she may say that the possibility in which the statement is good is 0.4 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.2. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.4, 0.2, 0.3\rangle$. Thus, when the four possible alternatives with respect to the above three attributes are evaluated by the expert, we can obtain the following single valued neutrosophic decision matrix D:

$$D=\begin{bmatrix}\langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.8,0.2,0.5\rangle\\ \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.2,0.8\rangle\\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.3,0.8\rangle\\ \langle 0.7,0.0,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.3,0.8\rangle\end{bmatrix}.$$

Then, we utilize the developed approach to obtain the most desirable alternative(s).

From the single valued neutrosophic decision matrix, we can obtain the following ideal alternative:

$$A^*=\{\langle C_1,0.7,0.1,0.1\rangle,\langle C_2,0.6,0.1,0.2\rangle,\langle C_3,0.5,0.3,0.8\rangle\}.$$

By using Eq. (3), we can obtain the values of the weighted correlation coefficient $W(A_i, A^*)$ (i =1, 2, 3, 4):

$W(A_1, A^*) = 0.8016$, $W(A_2, A^*) = 0.9510$, $W(A_3, A^*) = 0.8588$, and $W(A_4, A^*) = 0.9664$.

Thus, the ranking order of the four alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1$. Therefore, the alternative A_4 is the best choice among the four alternatives.

5.2 Example 2

A multi-criteria decision making problem is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A = \{A_1, A_2, A_3, A_4\}$ whose core competencies are evaluated by means of the four attributes: (1) C_1 is the level of technology innovation; (2) C_2 is the control ability of flow; (3) C_3 is the ability of management; (4) C_4 is the level of service, where C_1, C_2 and C_2 are all benefit attributes. Assume that the weight vector for the four attributes is $w = (0.3, 0.25, 0.25, 0.2)^T$.

The proposed decision making method is applied to solve this problem for selecting suppliers.

For the evaluation of an alternative A_i (i =1, 2, 3, 4) with respect to a criterion C_j (j =1, 2, 3, 4), it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to a criterion C_1, he or she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.1. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.5, 0.1, 0.3\rangle$. Thus, when the four possible alternatives with respect to the above four attributes are evaluated by the similar method from the expert, we can obtain the following single valued neutrosophic decision matrix D:

$$D=\begin{pmatrix}\langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.3,0.2,0.1\rangle\\ \langle 0.4,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.9,0.0,0.1\rangle & \langle 0.5,0.3,0.2\rangle\\ \langle 0.4,0.3,0.1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.6,0.2,0.2\rangle\\ \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.7,0.2,0.1\rangle\end{pmatrix}$$

Then, we employ the developed approach to obtain the most desirable alternative(s).

From the single valued neutrosophic decision matrix, we can obtain the following ideal alternative:

$$A^*=\{\langle C_1,0.6,0.1,0.1\rangle,\langle C_2,0.5,0.1,0.3\rangle,\\ \langle C_3,0.9,0.0,0.1\rangle,\langle C_4,0.7,0.2,0.1\rangle\}.$$

By applying Eq. (3), we can obtain the values of the weighted correlation coefficient $W(A_i, A^*)$ (i =1, 2, 3, 4):

$W(A_1, A^*) = 0.7998$, $W(A_2, A^*) = 0.8756$, $W(A_3, A^*) = 0.7580$, and $W(A_4, A^*) = 0.7532$.

Thus, the ranking order of the four alternatives is $A_2 \succ A_1 \succ A_3 \succ A_4$. Therefore, the alternative A_2 is the best choice among the four alternatives.

From the two examples, we can see that the proposed single valued neutrosophic multiple attribute decision-making method is more suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations.

6 Conclusion

In this paper, we proposed another form of the correlation coefficient between SVNSs. Then a multiple attribute decision-making method has been established in single valued neutrosophic setting by means of the weighted correlation coefficient between each alternative and the ideal alternative. Through the correlation coefficient, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, two illustrative examples illustrated the applications of the developed approach. Then the technique proposed in this paper is suitable for handling decision-making problems with single value neutrosophic information and can provide a useful way for decision-makers. In the future, we shall continue working in the applications of the correlation coefficient between SVNSs to other domains, such as data analysis and classification, pattern recognition, and medical diagnosis.

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Received: Nov. 1st, 2013. Accepted: Nov. 20, 2013

Soft Neutrosophic Group

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Abstract.In this paper we extend the neutrosophic group and subgroup to soft neutrosophic group and soft neutrosophic subgroup respectively. Properties and theorems related to them are proved and many examples are given.

Keywords: Neutrosophic group,neutrosophic subgroup,soft set,soft subset,soft group,soft subgroup,soft neutrosophic group, soft ,neutrosophic subgroup.

1 Introduction

The concept of neutrosophic set was first introduced by Smarandache [13,16] which is a generalization of the classical sets, fuzzy set [18], intuitionistic fuzzy set [4] and interval valued fuzzy set [7]. Soft Set theory was initiated by Molodstov as a new mathematical tool which is free from the problems of parameterization inadequacy. In his paper [11], he presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability. Later on many researchers followed him and worked on soft set theory as well as applications of soft sets in decision making problems and artificial intelligence. Now, this idea has a wide range of research in many fields, such as databases [5, 6], medical diagnosis problem [7], decision making problem [8], topology [9], algebra and so on.Maji gave the concept of neutrosophic soft set in [8] and later on Broumi and Smarandache defined intuitionistic neutrosophic soft set. We have worked with neutrosophic soft set and its applications in group theory.

2 Preliminaries

2.1 Nuetrosophic Groups

Definition 1 $[14]$ *Let* $(G,*)$ *be any group and let* $\langle G \cup I \rangle = \{a + bI : a,b \in G\}$ *. Then neutrosophic group is generated by* I *and* G *under* $*$ *denoted by* $N(G) = \{\langle G \cup I \rangle, *\}$. I *is called the neutrosophic element with the property* $I^2 = I$ *. For an integer* n *,* $n + I$ *and* nI *are neutrosophic elements and* $0.I = 0$.

I^{-1} , the inverse of I is not defined and hence does not exist.

Theorem 1 $[14]$ *Let* $N(G)$ *be a neutrosophic group. Then*

1) $N(G)$ in general is not a group;
2) $N(G)$ always contains a group.

Definition 2 *A pseudo neutrosophic group is defined as a neutrosophic group, which does not contain a proper subset which is a group.*

Definition 3 *Let* $N(G)$ *be a neutrosophic group. Then,*

1) A proper subset $N(H)$ of $N(G)$ is said to be a neutrosophic subgroup of $N(G)$ if $N(H)$ is a neutrosophic group, that is, $N(H)$ contains a proper subset which is a group.
2) $N(H)$ is said to be a pseudo neutrosophic subgroup if it does not contain a proper subset which is a group.

Example 1 $(N(Z),+)$, $(N(Q),+)$ $(N(R),+)$ *and* $(N(C),+)$ *are neutrosophic groups of integer, rational, real and complex numbers, respectively.*

Example 2 *Let* $Z_7 = \{o,1,2,...,6\}$ *be a group under addition modulo* 7 .
$N(G) = \{\langle Z_7 \cup I \rangle, '+' \bmod ulo 7\}$ *is a neutrosophic group which is in fact a group. For* $N(G) = \{a + bI : a,b \in Z_7\}$ *is a group under* `$+$' *modulo* 7 .

Definition 4 *Let* $N(G)$ *be a finite neutrosophic group. Let* P *be a proper subset of* $N(G)$ *which under the* *operations of* $N(G)$ *is a neutrosophic group. If* $o(P) / o(N(G))$ *then we call* P *to be a Lagrange neutrosophic subgroup.*

Definition 5 $N(G)$ *is called weakly Lagrange neutrosophic group if* $N(G)$ *has at least one Lagrange neutrosophic subgroup.*

Definition 6 $N(G)$ *is called Lagrange free neutrosophic group if* $N(G)$ *has no Lagrange neutrosophic subgroup.*

Definition7 *Let* $N(G)$ *be a finite neutrosophic group. Suppose* L *is a pseudo neutrosophic subgroup of* $N(G)$ *and if* $o(L) / o(N(G))$ *then we call* L *to be a pseudo Lagrange neutrosophic subgroup.*

Definition 8 *If* $N(G)$ *has at least one pseudo Lagrange neutrosophic subgroup then we call* $N(G)$ *to be a weakly pseudo Lagrange neutrosophic group.*

Definition 9 *If* $N(G)$ *has no pseudo Lagrange neutrosophic subgroup then we call* $N(G)$ *to be pseudo Lagrange free neutrosophic group.*

Definition 10 *Let* $N(G)$ *be a neutrosophic group. We say a neutrosophic subgroup* H *of* $N(G)$ *is normal if we can find* x *and* y *in* $N(G)$ *such that* $H = xHy$ *for all* $x, y \in N(G)$ *(Note* $x = y$ *or* $y = x^{-1}$ *can also occur).*

Definition 11 *A neutrosophic group* $N(G)$ *which has no nontrivial neutrosophic normal subgroup is called a simple neutrosophic group.*

Definition 12 *Let* $N(G)$ *be a neutrosophic group. A proper pseudo neutrosophic subgroup* P *of* $N(G)$ *is said to be normal if we have* $P = xPy$ *for all* $x, y \in N(G)$ *. A neutrosophic group is said to be pseudo simple neutrosophic group if* $N(G)$ *has no nontrivial pseudo normal subgroups.*

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition13 $[11]$ *A pair* (F, A) *is called a soft set over* U *where* F *is a mapping given by* F : $A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e -elements of the soft set (F, A), or as the set of e-approximate elements of the soft set.

Example 3 *Suppose that* U *is the set of shops.* E *is the set of parameters and each parameter is a word or sentence. Let*

$$E = \left\{ \begin{array}{l} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{array} \right\}.$$

Let us consider a soft set (F, A) *which describes the attractiveness of shops that Mr.* Z *is taking on rent. Suppose that there are five houses in the universe* $U = \{h_1, h_2, h_3, h_4, h_5\}$ *under consideration, and that* $A = \{e_1, e_2, e_3\}$ *be the set of parameters where*

e_1 stands for the parameter 'high rent,

e_2 stands for the parameter 'normal rent,

e_3 stands for the parameter 'in good condition.

Suppose that

$F(e_1) = \{h_1, h_4\}$,

$F(e_2) = \{h_2, h_5\}$,

$F(e_3) = \{h_3, h_4, h_5\}$.

The soft set (F, A) is an approximated family $\{F(e_i), i = 1,2,3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Thus, we have the soft set (F, A) as a collection of approximations as below:

$$(F, A) = \{ \text{ high rent } = \{h_1, h_4\}, \text{ normal rent } = \{h_2, h_5\}, \text{ in good condition } = \{h_3, h_4, h_5\}\}.$$

Definition 14 $[3]$ *. For two soft sets* (F, A) *and*

(H,B) *over* U , (F,A) *is called a soft subset of* (H,B) *if*

1) $A \subseteq B$ and
2) $F(e) \subseteq H(e)$, for all $e \in A$.

This relationship is denoted by $(F,A) \subset (H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A) \supset (H,B)$.

Definition 15 $[3]$. *Two soft sets* (F,A) *and* (H,B) *over* U *are called soft equal if* (F,A) *is a soft subset of* (H,B) *and* (H,B) *is a soft subset of* (F,A) .

Definition 16 *Let* $[3]$ (F,A) *and* (G,B) *be two soft sets over a common universe* U *such that* $A \cap B \neq \phi$. *Then their restricted intersection is denoted by* $(F,A) \cap_R (G,B) = (H,C)$ *where* (H,C) *is defined as* $H(c) = F(c) \cap G(c)$ *for all* $c \in C = A \cap B$.

Definition 17 $[3]$ *The extended intersection of two soft sets* (F,A) *and* (G,B) *over a common universe* U *is the soft set* (H,C) *, where* $C = A \cup B$ *, and for all* $e \in C$, $H(e)$ *is defined as*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F,A) \cap_{\varepsilon} (G,B) = (H,C)$.

Definition 18 $[3]$ *The restricted union of two soft sets* (F,A) *and* (G,B) *over a common universe* U *is the soft set* (H,C) *, where* $C = A \cup B$ *, and for all* $e \in C$, $H(e)$ *is defined as the soft set* $(H,C) = (F,A) \cup_R (G,B)$ *where* $C = A \cap B$ *and* $H(c) = F(c) \cup G(c)$ *for all* $c \in C$.

Definition 19 $[3]$ *The extended union of two soft sets* (F,A) *and* (G,B) *over a common universe* U *is the soft set* (H,C) *, where* $C = A \cup B$ *, and for all* $e \in C$, $H(e)$ *is defined as*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F,A) \cup_{\varepsilon} (G,B) = (H,C)$.

2.3 Soft Groups

Definition 20 $[2]$ *Let* (F,A) *be a soft set over* G . *Then* (F,A) *is said to be a soft group over* G *if and only if* $F(x) \prec G$ *forall* $x \in A$.

Example 4 *Suppose that* $G = A = S_3 = \{e,(12),(13),(23),(123),(132)\}$. *Then* (F,A) *is a soft group over* S_3 *where*

$$\begin{aligned} &F(e) = \{e\}, \\ &F(12) = \{e,(12)\}, \\ &F(13) = \{e,(13)\}, \\ &F(23) = \{e,(23)\}, \\ &F(123) = F(132) = \{e,(123),(132)\}. \end{aligned}$$

Definition 21 $[2]$ *Let* (F,A) *be a soft group over* G . *Then*

1) (F,A) is said to be an identity soft group over G if $F(x) = \{e\}$ for all $x \in A$, where e is the identity element of G and
2) (F,A) is said to be an absolute soft group if $F(x) = G$ for all $x \in A$.

Definition 22 *The restricted product* (H,C) *of two soft groups* (F,A) *and* (K,B) *over* G *is denoted by the soft set* $(H,C) = (F,A)\overset{\wedge}{\circ}(K,B)$ *where* $C = A \cap B$ *and* H *is a set valued function from* C

to $P\left(G\right)$ *and is defined as* $H(c) = F(c)K(c)$ *for all* $c \in C$ *. The soft set* $\left(H,C\right)$ *is called the restricted soft product of* $\left(F,A\right)$ *and* $\left(K,B\right)$ *over* G.

3 Soft Neutrosophic Group

Definition 23 *Let* $N\left(G\right)$ *be a neutrosophic group and* $\left(F,A\right)$ *be soft set over* $N\left(G\right)$ *.Then* $\left(F,A\right)$ *is called soft neutrosophic group over* $N\left(G\right)$ *if and only if* $F\left(x\right) \prec N\left(G\right)$ *, for all* $x \in A$.

Example 5 *Let*

$$N\left(Z_4\right) = \left\{\begin{matrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I, \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{matrix}\right\}$$

be a neutrosophic group under addition modulo 4 *. Let* $A = \left\{e_1, e_2, e_3, e_4\right\}$ *be the set of parameters, then* $\left(F,A\right)$ *is soft neutrosophic group over* $N\left(Z_4\right)$ *where*

$$F\left(e_1\right) = \left\{0,1,2,3\right\}, F\left(e_2\right) = \left\{0,I,2I,3I\right\},$$
$$F\left(e_3\right) = \left\{0,2,2I,2+2I\right\},$$
$$F\left(e_4\right) = \left\{0,I,2I,3I,2,2+2I,2+I,2+3I\right\}.$$

Theorem 2 *Let* $\left(F,A\right)$ *and* (H,A) *be two soft neutrosophic groups over* $N\left(G\right)$ *. Then their intersection* $\left(F,A\right) \cap \left(H,A\right)$ *is again a soft neutrosophic group over* $N\left(G\right)$ *.*

Proof The proof is straightforward.

Theorem 3 *Let* $\left(F,A\right)$ *and* $\left(H,B\right)$ *be two soft neutrosophic groups over N(G). If* $A \cap B = \phi$ *, then* $\left(F,A\right) \cup \left(H,B\right)$ *is a soft neutrosophic group over* $N\left(G\right)$ *.*

Theorem 4 *Let* $\left(F,A\right)$ *and* $\left(H,A\right)$ *be two soft neutrosophic groups over* $N\left(G\right)$ *. If* $F\left(e\right) \subseteq H\left(e\right)$ *for all* $e \in A$ *, then* $\left(F,A\right)$ *is a soft neutrosophic subgroup of* $\left(H,A\right)$ *.*

Theorem 5 *The extended union of two soft neutrosophic groups* $\left(F,A\right)$ *and* $\left(K,B\right)$ *over* $N\left(G\right)$ *is not a soft neutrosophic group over* $N\left(G\right)$ *.*

Proof Let $\left(F,A\right)$ and $\left(K,B\right)$ be two soft neutrosophic groups over $N\left(G\right)$. Let $C = A \cup B$, then for all $e \in C$, $(F,A) \cup_{\varepsilon} (K,B) = (H,C)$ where

$$H\left(e\right) \begin{cases} = F\left(e\right) & \text{If } e \in A - B, \\ = K\left(e\right) & \text{If } e \in B - A, \\ = F\left(e\right) \cup K\left(e\right) & \text{If } e \in A \cap B. \end{cases}$$

As union of two subgroups may not be again a subgroup. Clearly if $e \in C = A \cap B$, then $H\left(e\right)$ may not be a subgroup of $N\left(G\right)$. Hence the extended union $\left(H,C\right)$ is not a soft neutrosophic group over $N\left(G\right)$.

Example 6 *Let* $\left(F,A\right)$ *and* $\left(K,B\right)$ *be two soft neutrosophic groups over* $N\left(Z_2\right)$ *under addition modulo* 2 *, where*

$$F\left(e_1\right) = \left\{0,1\right\}, F\left(e_2\right) = \left\{0,I\right\}$$

And

$$K\left(e_2\right) = \left\{0,1\right\}, K\left(e_3\right) = \left\{0,1+I\right\}.$$

Then clearly their extended union is not a soft neutrosophic group as $H\left(e_2\right) = F\left(e_2\right) \cup K\left(e_2\right) = \left\{0,1,I\right\}$ *is not a subgroup of* $N\left(Z_2\right)$ *.*

Theorem 6 *The extended intersection of two soft neutrosophic groups over* $N\left(G\right)$ *is soft neutrosophic group over* $N\left(G\right)$ *.*

Theorem 7 *The restricted union of two soft neutrosophic groups* $\left(F,A\right)$ *and* $\left(K,B\right)$ *over* $N\left(G\right)$ *is not a soft neutrosophic group over* $N\left(G\right)$.

Theorem 8 *The restricted intersection of two soft neutrosophic groups over* $N\left(G\right)$ *is soft neutrosophic group over* $N\left(G\right)$ *.*

Theorem 9 *The restricted product of two soft neutrosoph-*

ict groups (F,A) and (K,B) over $N(G)$ is a soft neutrosophic group over $N(G)$.

Theorem 10 *The AND operation of two soft neutrosophic groups over $N(G)$ is soft neutrosophic group over $N(G)$.*

Theorem 11 *The OR operation of two soft neutrosophic groups over $N(G)$ may not be a soft neutrosophic group.*

Definition 24 *A soft neutrosophic group which does not contain a proper soft group is called soft pseudo neutrosophic group.*

Example 7 *Let* $N(Z_2)=\langle Z_2\cup I\rangle=\{0,1,I,1+I\}$ *be a neutrosophic group under addition modulo* 2. *Let* $A=\{e_1,e_2,e_3\}$ *be the set of parameters, then* (F,A) *is a soft pseudo neutrosophic group over* $N(G)$ *where*

$$F(e_1)=\{0,1\},$$
$$F(e_2)=\{0,I\},$$
$$F(e_3)=\{0,1+I\}.$$

Theorem 12 *The extended union of two soft pseudo neutrosophic groups (F,A) and (K,B) over $N(G)$ is not a soft pseudo neutrosophic group over $N(G)$.*

Example 8 *Let* $N(Z_2)=\langle Z_2\cup I\rangle=\{0,1,I,1+I\}$ *be a neutrosophic group under addition modulo* 2. *Let* (F,A) *and* (K,B) *be two soft pseudo neutrosophic groups over* $N(G)$*, where*

$$F(e_1)=\{0,1\},F(e_2)=\{0,I\},$$
$$F(e_3)=\{0,1+I\}.$$

And

$$K(e_1)=\{0,1+I\},K(e_2)=\{0,1\}.$$

Clearly their restricted union is not a soft pseudo neutrosophic group as union of two subgroups is not a subgroup.

Theorem 13 *The extended intersection of two soft pseudo neutrosophic groups (F,A) and (K,B) over $N(G)$ is again a soft pseudo neutrosophic group over $N(G)$.*

Theorem 14 *The restricted union of two soft pseudo neutrosophic groups (F,A) and (K,B) over $N(G)$ is not a soft pseudo neutrosophic group over $N(G)$.*

Theorem 15 *The restricted intersection of two soft pseudo neutrosophic groups (F,A) and (K,B) over $N(G)$ is again a soft pseudo neutrosophic group over $N(G)$.*

Theorem 16 *The restricted product of two soft pseudo neutrosophic groups (F,A) and (K,B) over $N(G)$ is a soft pseudo neutrosophic group over $N(G)$.*

Theorem 17 *The AND operation of two soft pseudo neutrosophic groups over $N(G)$ soft pseudo neutrosophic soft group over $N(G)$.*

Theorem 18 *The OR operation of two soft pseudo neutrosophic groups over $N(G)$ may not be a soft pseudo neutrosophic group.*

Theorem19 *Every soft pseudo neutrosophic group is a soft neutrosophic group.*

Proof The proof is straight forward.

Remark 1 *The converse of above theorem does not hold.*

Example 9 *Let* $N(Z_4)$ *be a neutrosophic group and* (F,A) *be a soft neutrosophic group over* $N(Z_4)$. *Then*

$$F(e_1)=\{0,1,2,3\},F(e_2)=\{0,I,2I,3I\},$$
$$F(e_3)=\{0,2,2I,2+2I\}.$$

But (F,A) *is not a soft pseudo neutrosophic group as* (H,B) *is clearly a proper soft subgroup of* (F,A). *where*

$$H(e_1)=\{0,2\},H(e_2)=\{0,2\}.$$

Theorem 20 (F,A) *over* $N(G)$ *is a soft pseudo* *neutrosophic group if* $N(G)$ *is a pseudo neutrosophic group.*

Proof Suppose that $N(G)$ be a pseudo neutrosophic group, then it does not contain a proper group and for all $e \in A$, the soft neutrosophic group (F,A) over $N(G)$ is such that $F(e) \prec N(G)$. Since each $F(e)$ is a pseudo neutrosophic subgroup which does not contain a proper group which make (F,A) is soft pseudo neutrosophic group.

Example 10 *Let* $N(Z_2) = \langle Z_2 \cup I \rangle = \{0,1,I,1+I\}$ *be a pseudo neutrosophic group under addition modulo* 2. *Then clearly* (F,A) *a soft pseudo neutrosophic soft group over* $N(Z_2)$ *, where*

$$F(e_1) = \{0,1\}, F(e_2) = \{0,I\},$$
$$F(e_3) = \{0,1+I\}.$$

Definition 25 *Let* (F,A) *and* (H,B) *be two soft neutrosophic groups over* $N(G)$ *. Then* (H,B) *is a soft neutrosophic subgroup of* (F,A) *, denoted as* $(H,B) \prec (F,A)$ *, if*

1) $B \subset A$ and
2) $H(e) \prec F(e)$, for all $e \in A$.

Example 11 *Let* $N(Z_4) = \langle Z_4 \cup I \rangle$ *be a soft neutrosophic group under addition modulo* 4 *, that is*

$$N(Z_4) = \left\{ \begin{array}{l} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I, \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{array} \right\}.$$

Let (F,A) *be a soft neutrosophic group over* $N(Z_4)$ *, then*

$$F(e_1) = \{0,1,2,3\}, F(e_2) = \{0,I,2I,3I\},$$
$$F(e_3) = \{0,2,2I,2+2I\},$$
$$F(e_4) = \{0,I,2I,3I,2,2+2I,2+I,2+3I\}.$$

(H,B) is a soft neutrosophic subgroup of (F,A), where

$$H(e_1) = \{0,2\}, H(e_2) = \{0,2I\},$$
$$H(e_4) = \{0,I,2I,3I\}.$$

Theorem 21 *A soft group over* G *is always a soft neutrosophic subgroup of a soft neutrosophic group over* $N(G)$ *if* $A \subset B$.

Proof Let (F,A) be a soft neutrosophic group over $N(G)$ and (H,B) be a soft group over G. As $G \subset N(G)$ and for all $b \in B, H(b) \prec G \subset N(G)$. This implies $H(e) \prec F(e)$, for all $e \in A$ as $B \subset A$. Hence $(H,B) \prec (F,A)$.

Example 12 *Let* (F,A) *be a soft neutrosophic group over* $N(Z_4)$ *, then*

$$F(e_1) = \{0,1,2,3\}, F(e_2) = \{0,I,2I,3I\},$$
$$F(e_3) = \{0,2,2I,2+2I\}.$$

Let $B = \{e_1, e_3\}$ *such that* $(H,B) \prec (F,A)$ *, where*

$$H(e_1) = \{0,2\}, H(_3) = \{0,2\}.$$

Clearly $B \subset A$ *and* $H(e) \prec F(e)$ *for all* $e \in B$.

Theorem 22 *A soft neutrosophic group over* $N(G)$ *always contains a soft group over G.*

Proof The proof is followed from above Theorem.

Definition 26 *Let* (F,A) *and* (H,B) *be two soft pseudo neutrosophic groups over* $N(G)$ *. Then* (H,B) *is called soft pseudo neutrosophic subgroup of* (F,A) *, denoted as* $(H,B) \prec (F,A)$ *, if*

1) $B \subset A$
2) $H(e) \prec F(e)$, for all $e \in A$.

Example 13 *Let* (F,A) *be a soft pseudo neutrosophic group over* $N(Z_4)$ *, where*

$$F(e_1) = \{0, I, 2I, 3I\}, F(e_2) = \{0, 2I\}.$$

Hence $(H,B) \prec (F,A)$ where

$$H(e_1) = \{0, 2I\}.$$

Theorem 23 *Every soft neutrosophic group* (F,A) *over* $N(G)$ *has soft neutrosophic subgroup as well as soft pseudo neutrosophic subgroup.*

Proof Straightforward.

Definition 27 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *, then* (F,A) *is called the identity soft neutrosophic group over* $N(G)$ *if* $F(x) = \{e\}$, *for all* $x \in A$ *, where* e *is the identity element of* G *.*

Definition 28 *Let* (H,B) *be a soft neutrosophic group over* $N(G)$ *, then* (H,B) *is called Full-soft neutrosophic group over* $N(G)$ *if* $F(x) = N(G)$ *, for all* $x \in A$ *.*

Example 14 *Let*

$$N(R) = \left\{ \begin{matrix} a + bI \ : \ a, b \in R \text{ and} \\ I \text{ is indeterminacy} \end{matrix} \right\}$$

is a neutrosophic real group where R *is set of real numbers and* $I^2 = I$ *, therefore* $I^n = I$ *, for* n *a positive integer. Then* (F,A) *is a Full-soft neutrosophic real group where*

$$F(e) = N(R), \text{ for all } e \in A$$

Theorem 24 *Every Full-soft neutrosophic group contain absolute soft group.*

Theorem 25 *Every absolute soft group over* G *is a soft neutrosophic subgroup of Full-soft neutrosophic group over* $N(G)$ *.*

Theorem 26 *Let* $N(G)$ *be a neutrosophic group. If order of* $N(G)$ *is prime number, then the soft neutrosophic group* (F,A) *over* $N(G)$ *is either identity soft neutrosophic group or Full-soft neutrosophic group.*

Proof Straightforward.

Definition 29 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. If for all* $e \in A$ *, each* $F(e)$ *is Lagrange neutrosophic subgroup of* $N(G)$ *, then* (F,A) *is called soft Lagrange neutrosophic group over* $N(G)$ *.*

Example 15 *Let* $N(Z_3 / \{0\}) = \{1, 2, I, 2I\}$ *is a neutrosophic group under multiplication modulo* 3 *. Now* $\{1,2\}$ *,* $\{1, I\}$ *are subgroups of* $N(Z_3 / \{0\})$ *which divides order of* $N(Z_3 / \{0\})$ *. Then the soft neutrosophic group*

$$(F,A) = \{F(e_1) = \{1,2\}, F(e_2) = \{1, I\}\}$$

is an example of soft Lagrange neutrosophic group.

Theorem 27 *If* $N(G)$ *is Lagrange neutrosophic group, then* (F,A) *over* $N(G)$ *is soft Lagrange neutrosophic group but the converse is not true in general.*

Theorem 28 *Every soft Lagrange neutrosophic group is a soft neutrosophic group.*

Proof Straightforward.

Remark 2 *The converse of the above theorem does not hold.*

Example 16 *Let* $N(G) = \{1, 2, 3, 4, I, 2I, 3I, 4I\}$ *be a neutrosophic group under multiplication modulo* 5 *and* (F,A) *be a soft neutrosophic group over* $N(G)$ *, where*

$$F(e_1) = \{1, 4, I, 2I, 3I, 4I\}, F(e_2) = \{1, 2, 3, 4\},$$
$$F(e_3) = \{1, I, 2I, 3I, 4I\}.$$

But clearly it is not soft Lagrange neutrosophic group as $F(e_1)$ *which is a subgroup of* $N(G)$ *does not divide order of* $N(G)$ *.*

Theorem 29 *If* $N(G)$ *is a neutrosophic group, then the soft Lagrange neutrosophic group is a soft neutrosophic group.*

Proof Suppose that $N(G)$ be a neutrosophic group and (F,A) be a soft Lagrange neutrosophic group over $N(G)$. Then by above theorem (F,A) is also soft neutrosophic group.

Example 17 *Let* $N(Z_4)$ *be a neutrosophic group and* (F,A) *is a soft Lagrange neutrosophic group over* $N(Z_4)$ *under addition modulo* 4 *, where*

$$F(e_1)=\{0,1,2,3\}, F(e_2)=\{0,I,2I,3I\},$$
$$F(e_3)=\{0,2,2I,2+2I\}.$$

But (F,A) *has a proper soft group* (H,B) *, where* $H(e_1)=\{0,2\}, H(e_3)=\{0,2\}$.

Hence (F,A) is soft neutrosophic group.

Theorem 30 *Let* (F,A) *and* (K,B) *be two soft Lagrange neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.

Theorem 31 *Let* (F,A) *and* (H,B) *be two soft Lagrange neutrosophic groups over* $N(G)$ *.Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is not soft Lagrange neutrosophic group over $N(G)$.
2) Their OR operation $(F,A)\vee(K,B)$ is not a soft Lagrange neutrosophic group over $N(G)$.

Definition 30 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F,A) *is called soft weakly Lagrange neutrosophic group if atleast one* $F(e)$ *is a Lagrange neutrosophic subgroup of* $N(G)$ *, for some* $e\in A$.

Example 18 *Let* $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ *be a neutrosophic group under multiplication modulo* 5 *, then* (F,A) *is a soft weakly Lagrange neutrosophic group over* $N(G)$ *, where*

$$F(e_1)=\{1,4,I,2I,3I,4I\}, F(e_2)=\{1,2,3,4\},$$
$$F(e_3)=\{1,I,2I,3I,4I\}.$$

As $F(e_1)$ *and* $F(e_3)$ *which are subgroups of* $N(G)$ *do not divide order of* $N(G)$.

Theorem 32 *Every soft weakly Lagrange neutrosophic group* (F,A) *is soft neutrosophic group.*

Remark 3 *The converse of the above theorem does not hold in general.*

Example 19 *Let* $N(Z_4)$ *be a neutrosophic group under addition modulo* 4 *and* $A=\{e_1,e_2\}$ *be the set of parameters, then* (F,A) *is a soft neutrosophic group over* $N(Z_4)$ *, where*

$$F(e_1)=\{0,I,2I,3I\}, F(e_2)=\{0,2I\}.$$

But not soft weakly Lagrange neutrosophic group over $N(Z_4)$.

Definition 31 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F,A) *is called soft Lagrange free neutrosophic group if* $F(e)$ *is not Lagrangeneutrosophic subgroup of* $N(G)$ *, for all* $e\in A$.

Example 20 *Let* $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ *be a neutrosophic group under multiplication modulo* 5

and then $\left(F,A\right)$ *be a soft Lagrange free neutrosophic group over* $N\left(G\right)$ *, where*

$$F\left(e_1\right)=\left\{1,4,I,2I,3I,4I\right\},F\left(e_2\right)=\left\{1,I,2I,3I,4I\right\}.$$

As $F\left(e_1\right)$ *and* $F\left(e_2\right)$ *which are subgroups of* $N\left(G\right)$ *do not divide order of* $N\left(G\right)$ *.*

Theorem 33 *Every soft Lagrange free neutrosophic group* $\left(F,A\right)$ *over* $N\left(G\right)$ *is a soft neutrosophic group but the converse is not true.*

Definition 32 *Let* $\left(F,A\right)$ *be a soft neutrosophic group over* $N\left(G\right)$ *. If for all* $e\in A$ *, each* $F\left(e\right)$ *is a pseudo Lagrange neutrosophic subgroup of* $N\left(G\right)$ *, then* $\left(F,A\right)$ *is called soft pseudo Lagrange neutrosophic group over* $N\left(G\right)$ *.*

Example 21 *Let* $N\left(Z_4\right)$ *be a neutrosophic group under addition modulo* 4 *and* $A=\left\{e_1,e_2\right\}$ *be the set of parameters, then* $\left(F,A\right)$ *is a soft pseudo Lagrange neutrosophic group over* $N\left(Z_4\right)$ *where*

$$F\left(e_1\right)=\left\{0,I,2I,3I\right\},F\left(e_2\right)=\left\{0,2I\right\}.$$

Theorem 34 *Every soft pseudo Lagrange neutrosophic group is a soft neutrosophic group but the converse may not be true.*

Proof Straightforward.

Theorem 35 *Let* $\left(F,A\right)$ *and* $\left(K,B\right)$ *be two soft pseudo Lagrange neutrosophic groups over* $N\left(G\right)$ *. Then*

1) Their extended union $\left(F,A\right)\cup_{\varepsilon}\left(K,B\right)$ over $N\left(G\right)$ is not a soft pseudo Lagrange neutrosophic group over $N\left(G\right)$.
2) Their extended intersection $\left(F,A\right)\cap_{\varepsilon}\left(K,B\right)$ over $N\left(G\right)$ is not pseudo Lagrange neutrosophic soft group over $N\left(G\right)$.
3) Their restricted union $\left(F,A\right)\cup_R\left(K,B\right)$ over $N\left(G\right)$ is not pseudo Lagrange neutrosophic soft group over $N\left(G\right)$.
4) Their restricted intersection $\left(F,A\right)\cap_R\left(K,B\right)$ over $N\left(G\right)$ is also not soft pseudo Lagrange neutrosophic group over $N\left(G\right)$.
5) Their restricted product $\left(F,A\right)\overset{\wedge}{\underset{\circ}{}}\left(K,B\right)$ over $N\left(G\right)$ is not soft pseudo Lagrange neutrosophic group over $N\left(G\right)$.

Theorem 36 *Let* $\left(F,A\right)$ *and* $\left(H,B\right)$ *be two soft pseudo Lagrange neutrosophic groups over* $N\left(G\right)$ *. Then*

1) Their AND operation $\left(F,A\right)\wedge\left(K,B\right)$ is not soft pseudo Lagrange neutrosophic group over $N\left(G\right)$.
2) Their OR operation $\left(F,A\right)\vee\left(K,B\right)$ is not a soft pseudo Lagrange neutrosophic soft group over $N\left(G\right)$.

Definition 33 *Let* $\left(F,A\right)$ *be a soft neutrosophic group over* $N\left(G\right)$ *. Then* $\left(F,A\right)$ *is called soft weakly pseudo Lagrange neutrosophic group if atleast one* $F\left(e\right)$ *is a pseudo Lagrange neutrosophic subgroup of* $N\left(G\right)$ *, for some* $e\in A$ *.*

Example 22 *Let* $N\left(G\right)=\{1,2,3,4,I,2I,3I,4I\}$ *be a neutrosophic group under multiplication modulo* 5 *Then* $\left(F,A\right)$ *is a soft weakly pseudo Lagrange neutrosophic group over* $N\left(G\right)$ *, where*

$$F\left(e_1\right)=\left\{1,I,2I,3I,4I\right\},F\left(e_2\right)=\left\{1,I\right\}.$$

As $F\left(e_1\right)$ *which is a subgroup of* $N\left(G\right)$ *does not divide order of* $N\left(G\right)$ *.*

Theorem 37 *Every soft weakly pseudo Lagrange neutrosophic group* $\left(F,A\right)$ *is soft neutrosophic group.*

Remark 4 *The converse of the above theorem is not true in general.*

Example 23 *Let $N(Z_4)$ be a neutrosophic group under addition modulo 4 and $A=\{e_1,e_2\}$ be the set of parameters, then (F,A) is a soft neutrosophic group over $N(Z_4)$, where*

$$F(e_1)=\{),I,2I,3I\},F(e_2)=\{0,2I\}.$$

But it is not soft weakly pseudo Lagrange neutrosophic group.

Theorem 38 *Let (F,A) and (K,B) be two soft weakly pseudo Lagrange neutrosophic groups over $N(G)$. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.

Definition 34 *Let (F,A) be a soft neutrosophic group over $N(G)$. Then (F,A) is called soft pseudo Lagrange free neutrosophic group if $F(e)$ is not pseudo Lagrange neutrosophic subgroup of $N(G)$, for all $e\in A$.*

Example 24 *Let $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ be a neutrosophic group under multiplication modulo 5 Then (F,A) is a soft pseudo Lagrange free neutrosophic group over $N(G)$, where*

$$F(e_1)=\{1,I,2I,3I,4I\},F(e_2)=\{1,I,2I,3I,4I\}.$$

As $F(e_1)$ and $F(e_2)$ which are subgroups of $N(G)$ do not divide order of $N(G)$.

Theorem 39 *Every soft pseudo Lagrange free neutrosophic group (F,A) over $N(G)$ is a soft neutrosophic group but the converse is not true.*

Theorem 40 *Let (F,A) and (K,B) be two soft pseudo Lagrange free neutrosophic groups over $N(G)$. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $N(G)$ is not pseudo Lagrange free neutrosophic soft group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.

Definition 35 *A soft neutrosophic group (F,A) over $N(G)$ is called soft normal neutrosophic group over*

$N(G)$ *if* $F(e)$ *is a normal neutrosophic subgroup of* $N(G)$, *for all* $e \in A$.

Example 25 *Let* $N(G) = \{e, a, b, c, I, aI, bI, cI\}$ *be a neutrosophic group under multiplicationwhere* $a^2 = b^2 = c^2 = e, bc = cb = a, ac = ca = b, ab = ba = c$. *Then* (F, A) *is a soft normal neutrosophic group over* $N(G)$ *where*

$$F(e_1) = \{e, a, I, aI\},$$
$$F(e_2) = \{e, b, I, bI\},$$
$$F(e_3) = \{e, c, I, cI\}.$$

Theorem 42 *Every soft normal neutrosophic group* (F, A) *over* $N(G)$ *is a soft neutrosophic group but the converse is not true.*

Theorem 42 *Let* (F, A) *and* (H, B) *be two soft normal neutrosophic groups over* $N(G)$. *Then*

1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $N(G)$ is not soft normal neutrosophic group over $N(G)$.
2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $N(G)$ is soft normal neutrosophic group over $N(G)$.
3) Their restricted union $(F, A) \cup_R (K, B)$ over $N(G)$ is not soft normal neutrosophic group over $N(G)$.
4) Their restricted intersection $(F, A) \cap_R (K, B)$ over $N(G)$ is soft normal neutrosophic group over $N(G)$.
5) Their restricted product $(F, A)_{\circ}^{\wedge} (K, B)$ over $N(G)$ is not soft normal neutrosophic soft group over $N(G)$.

Theorem 43 *Let* (F, A) *and* (H, B) *be two soft normal neutrosophic groups over* $N(G)$. *Then*

1) Their AND operation $(F, A) \wedge (K, B)$ is soft normal neutrosophic group over $N(G)$.
2) Their OR operation $(F, A) \vee (K, B)$ is not soft normal neutrosophic group over $N(G)$.

Definition 36 *Let* (F, A) *be a soft neutrosophic group over* $N(G)$. *Then* (F, A) *is called soft pseudo normal neutrosophic group if* $F(e)$ *is a pseudo normal neutrosophic subgroup of* $N(G)$, *for all* $e \in A$.

Example 26 *Let* $N(Z_2) = \langle Z_2 \cup I \rangle = \{0, 1, I, 1 + I\}$ *be a neutrosophic group under addition modulo* 2 *and let* $A = \{e_1, e_2\}$ *be the set of parameters, then* (F, A) *is soft pseudo normal neutrosophic group over* $N(G)$, *where*

$$F(e_1) = \{0, I\}, F(e_2) = \{0, 1 + I\}.$$

As $F(e_1)$ *and* $F(e_2)$ *are pseudo normal subgroup of* $N(G)$.

Theorem 44 *Every soft pseudo normal neutrosophic group* (F, A) *over* $N(G)$ *is a soft neutrosophic group but the converse is not true.*

Theorem 45 *Let* (F, A) *and* (K, B) *be two soft pseudo normal neutrosophic groups over* $N(G)$. *Then*

1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $N(G)$ is not soft pseudo normal neutrosophic group over $N(G)$.
2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $N(G)$ is soft pseudo normal neutrosophic group over $N(G)$.
3) Their restricted union $(F, A) \cup_R (K, B)$ over $N(G)$ is not soft pseudo normal neutrosophic

group over $N(G)$.

4) Their restricted intersection $(F,A)\cap_R (K,B)$ over $N(G)$ is soft pseudo normal neutrosophic group over $N(G)$.

5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft pseudo normal neutrosophic group over $N(G)$.

Theorem 46 Let (F,A) *and* (K,B) *be two soft pseudo normal neutrosophic groups over* $N(G)$. *Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is soft pseudo normal neutrosophic group over $N(G)$.

2) Their OR operation $(F,A)\vee(K,B)$ is not soft pseudo normal neutrosophic group over $N(G)$.

Definition 37 *Let* $N(G)$ *be a neutrosophic group. Then* (F,A) *is called soft conjugate neutrosophic group over* $N(G)$ *if and only if* $F(e)$ *is conjugate neutrosophic subgroup of* $N(G)$, *for all* $e\in A$.

Example 27 *Let*

$$N(G)=\left\{\begin{matrix}0,1,2,3,4,5,I,2I,3I,4I,5I,\\1+I,2+I,3+I,...,5+5I\end{matrix}\right\}$$

be a neutrosophic group under addition modulo 6 *and let* $P=\{0,3,3I,3+3I\}$ *and* $K=\{0,2,4,2+2I,4+4I,2I,4I\}$ *are conjugate neutrosophic subgroups of* $N(G)$. *Then* (F,A) *is soft conjugate neutrosophic group over* $N(G)$, *where*

$$F(e_1)=\{0,3,3I,3+3I\},$$
$$F(e_2)=\{0,2,4,2+2I,4+4I,2I,4I\}.$$

Theorem 47 *Let* (F,A) *and* (K,B) *be two soft conjugate neutrosophic groups over* $N(G)$. *Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft conjugate neutrosophic group over $N(G)$.

2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is again soft conjugate neutrosophic group over $N(G)$.

3) Their restricted union $(F,A)\cup_R(K,B)$ over $N(G)$ is not soft conjugate neutrosophic group over $N(G)$.

4) Their restricted intersection $(F,A)\cap_R(K,B)$ over $N(G)$ is soft conjugate neutrosophic group over $N(G)$.

5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft conjugate neutrosophic group over $N(G)$.

Theorem 48 *Let* (F,A) *and* (K,B) *be two soft conjugate neutrosophic groups over* $N(G)$. *Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is again soft conjugate neutrosophic group over $N(G)$.

2) Their OR operation $(F,A)\vee(K,B)$ is not soft conjugate neutrosophic group over $N(G)$.

Conclusion

In this paper we extend the neutrosophic group and subgroup,pseudo neutrosophic group and subgroup to soft neutrosophic group and soft neutrosophic subgroup and respectively soft pseudo neutrosophic group and soft pseudo neutrosophic subgroup. The normal neutrosophic subgroup is extended to soft normal neutrosophic subgroup. We showed all these by giving various examples in order to illustrate the soft part of the neutrosophic notions used.

Acknowledgements

We would like to thank Editors and referees for their kind suggestions that helped to improve this paper.

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Received:November 5, 2013. Accepted:November 28, 2013.

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Neutrosophic Examples in Physics

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Abstract: Neutrosophy can be widely applied in physics and the like. For example, one of the reasons for 2011 Nobel Prize for physics is "for the discovery of the accelerating expansion of the universe through observations of distant supernovae", but according to neutrosophy, there exist seven or nine states of accelerating expansion and contraction and the neutrosophic state in the universe. Another two examples are "a revision to Gödel's incompleteness theorem by neutrosophy" and "six neutral (neutrosophic) fundamental interactions". In addition, the "partial and temporary unified theory so far" is discussed (including "partial and temporary unified electromagnetic theory so far", "partial and temporary unified gravitational theory so far", "partial and temporary unified theory of four fundamental interactions so far", and "partial and temporary unified theory of natural science so far").

Keywords: Neutrosophy, application, neutrosophic example, physics, partial and temporary unified theory so far.

1 Introduction

Neutrosophy is proposed by Prof. Florentin Smarandache in 1995.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[without necessarily connection between them.

More information about Neutrosophy may be found in references [1-4].

Now we discuss the neutrosophic examples in physics and the like.

2 Discussion on "the accelerating expansion of the universe"

One of the reasons for 2011 Nobel Prize for physics is "for the discovery of the accelerating expansion of the universe through observations of distant supernovae". But according to neutrosophy, "the accelerating expansion of the universe" is debatable.

Supposing that "the expansion of the universe" is an idea <A>, its opposite or negation <Anti-A> should be "the contraction of the universe", and the spectrum of "neutralities" <Neut-A> should be "the stable state of the universe" (i.e. the state located between the two extremes, supporting neither expansion nor contraction).

In fact, the area nearby a black hole is in the state of contraction, because the mass of black hole (or similar black hole) is immense, and it produces a very strong gravitational field, so that all matters and radiations (including the electromagnetic wave or light) will be unable to escape if they enter to a critical range around the black hole.

The viewpoint of "the accelerating expansion of the universe" unexpectedly turns a blind eye to the fact that partial universe (such as the area nearby a black hole) is in the state of contraction.

As for "the stable state of the universe", it should be located at the transition area between expansion area and contraction area.

Again, running the same program to the state of "the expansion of the universe", supposing that "the accelerating expansion of the universe" is an idea <A>, its opposite or negation <Anti-A> should be "the decelerating expansion of the universe", and the spectrum of "neutralities" <Neut-A> should be "the uniform expansion of the universe" (i.e. the state located between the two extremes, supporting neither accelerating expansion nor decelerating expansion).

Similarly, running the same program to the state of "the contraction of the universe", it can be divided into three cases: "the accelerating contraction of the universe", "the decelerating contraction of the universe", and "the uniform contraction of the universe".

To sum up, there exist seven states in the universe: accelerating expansion, decelerating expansion, uniform expansion, accelerating contraction, decelerating contraction, uniform contraction, and stable state.

In addition, according to neutrosophy, another kind of seven states is as follows: long-term expansion, short-term expansion, medium-term expansion, long-term contraction, short-term contraction, medium-term contraction, and stable state.

It should be noted that, the stable state can be also divided into three cases, such as: "long-term stable state", "short-term stable state", and "medium-term stable state"; thus there exist nine states in the universe.

Considering all possible situations, besides these seven or nine states, due to the limitations of human knowledge, there may be other unknown states.

From this example we can see that, all of the absolute, solitary and one-sided viewpoints are completely wrong. But with the help of neutrosophy, many of these mistakes may be avoided.

3 A revision to Gödel's incompleteness theorem by neutrosophy

According to reference [4], the main contents of the revision are as follows.

As well-known, neutrosophy paves the way to consider all possible situations. But we can see that in the proof of Gödel's incompleteness theorem, all possible situations are not considered.

First, in the proof, the following situation is not considered: wrong results can be deduced from some axioms. For example, from the axiom of choice a paradox, the doubling ball theorem, can be deduced, which says that a ball of volume 1 can be decomposed into pieces and reassembled into two balls both of volume 1. It follows that in certain cases, the proof of Gödel's incompleteness theorem may be faulty.

Second, in the proof of Gödel's incompleteness theorem, only four situations are considered, that is, one proposition can be proved to be true, cannot be proved to be true, can be proved to be false, cannot be proved to be false and their combinations such as one proposition can neither be proved to be true nor be proved to be false. But those are not all possible situations. In fact, there may be many kinds of indeterminate situations, including it can be proved to be true in some cases and cannot be proved to be true in other cases; it can be proved to be false in some cases and cannot be proved to be false in other cases; it can be proved to be true in some cases and can be proved to be false in other cases; it cannot be proved to be true in some cases and cannot be proved to be false in other cases; it can be proved to be true in some cases and can neither be proved to be true, nor be proved to be false in other cases; and so on.

Because so many situations are not considered, we may say that the proof of Gödel's incompleteness theorem is faulty, at least, is not one with all sided considerations.

In order to better understand the case, we consider an extreme situation, where one proposition as shown in Gödel's incompleteness theorem can neither be proved, nor disproved. It may be assumed that this proposition can be proved in 9999 cases, only in 1 case it can neither be proved, nor disproved. We will see whether or not this situation has been considered in the proof of Gödel's incompleteness theorem.

Some people may argue that, this situation is equivalent to the one where the proposition can neither be proved, nor disproved. But the difference lies in the distinction between the part and the whole. If one case may represent the whole situation, many important theories cannot be applied. For example the general theory of relativity involves singular points; the law of universal gravitation does not allow the case where the distance r is equal to zero. Accordingly, whether or not one may say that the general theory of relativity and the law of universal gravitation cannot be applied as a whole? Similarly, the situation also cannot be considered as the one that can be proved. But, this problem may be easily solved with the neutrosophic method.

Moreover, if we apply the Gödel's incompleteness theorem to itself, we may obtain the following possibility: in one of all formal mathematical axiom systems, the Gödel's incompleteness theorem can neither be proved, nor disproved.

If all possible situations can be considered, the Gödel's incompleteness theorem can be improved in principle. But, with our boundless universe being ever changing and being extremely complex, it is impossible "considering all possible situations". As far as "considering all possible situations" is concerned, the Smarandache's neutrosophy is quite good, possibly, the best. Therefore this paper proposes to revise the Gödel's incompleteness theorem into the incomplete axiom with Smarandache's neutrosophy.

Considering all possible situations with Smarandache's neutrosophy, one may revise the Gödel's Incompleteness theorem into the incompleteness axiom: Any proposition in any formal mathematical axiom system will represent, respectively, the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[.

4 Six neutral (neutrosophic) fundamental interactions

As well-known, according to the present understanding, there are four fundamental interactions or forces: gravitational, electromagnetic, weak and strong interaction.

While, in accordance with the neutrosophy theory that between an entity and its opposite there exist intermediate entities, thus besides the existing four fundamental interactions there must exist six neutral (neutrosophic) fundamental interactions (as six new forms of fundamental interaction). For example, between strong interaction and weak interaction there exists intermediate interaction, namely neutral (neutrosophic) strong-weak fundamental interaction (NSW fundamental interaction), it neither strong interaction nor weak interaction, but something in between. Similarly, considering other five pairs of opposite interactions: strong and electromagnetic fundamental interaction, strong and gravitational fundamental interaction, weak and electromagnetic fundamental interaction, weak and gravitational fundamental interaction, and electromagnetic and gravitational fundamental interaction respectively, other five neutral (neutrosophic) fundamental interactions are as follows: neutral (neutrosophic) strong-electromagnetic fundamental interaction (NSE fundamental interaction), neutral (neutrosophic) strong-gravitational fundamental interaction (NSG fundamental interaction), neutral (neutrosophic) weak-electromagnetic fundamental interaction (NWE fundamental interaction), neutral (neutrosophic) weak-gravitational fundamental interaction (NWG fundamental interaction) and neutral (neutrosophic) electromagnetic-gravitational fundamental interaction (NEG fundamental interaction).

Thus, there may be ten fundamental interactions all together.

5 Several unified theories

Whether or not the unified theory can be existed? According to neutrosophy, there are three cases as follows: the unified theory can be existed, the unified theory cannot be existed, and the neutrosophic case (such as the "partial and temporary unified theory so far").

Now we discuss the "partial and temporary unified theory so far" .

What is the "unified theory"? In 1980, Stephen Hawking once claimed, physicists have seen the outline of "final theory", this theory of everything can express all laws of nature with a single and beautiful mathematical model, perhaps that it is so simple and can be written on a T-shirt.

In other words, for any field, the strict "unified theory" refers to that all the laws of this field can be expressed in a single mathematical model.

If following this concept to understand the strict "unified theory", we have to say, such a "unified theory" is simply cannot be existed. In other words, there is only "partial and temporary unified theory so far".

Now we discuss that the strict "unified electromagnetic theory" cannot be existed.

5.1 Why the strict "unified electromagnetic theory" cannot be existed and applying least square method to establish "partial and temporary unified electromagnetic theory so far"

It might be argued that Maxwell's equations are "unified electromagnetic theory". Facing with this argument, we ask three questions. First, whether or not all the electromagnetic laws can be included or derived by Maxwell's equations? Second, whether or not the later appeared high temperature superconductivity problem and the like can be solved by Maxwell's equations? Third, whether or not the faster-than-light（FTL）problems can be solved by Maxwell's equations? If negative answers were given to these three questions, then it should be acknowledged that Maxwell's equations are not strict "unified electromagnetic theory", but only "partial and temporary unified electromagnetic theory".

Based on the same reason, the "theory of the unified weak and electromagnetic interaction" cannot be existed, and there is only "partial and temporary theory of the unified weak and electromagnetic interaction so far".

Now we establish the "partial and temporary unified electromagnetic theory so far".

First of all, for any field, applying least square method to establish this field’s "partial and temporary unified theory so far" (the corresponding expression is "partial and temporary unified variational principle so far").

Supposing that for a certain domain Ω, we already establish the following general equations

$$F_i = 0 \quad (i = 1,2 \to n) \qquad (1)$$

On boundary V, the boundary conditions are as follows

$$B_j = 0 \quad (j = 1,2 \to m) \qquad (2)$$

Applying least square method, for this field and the domains and boundary conditions the "partial and temporary unified theory so far" can be expressed in the following form of "partial and temporary unified variational principle so far"

$$\Pi = \sum_{1}^{n} W_i \int_{\Omega} F_i^2 d\Omega + \sum_{1}^{m} W_j' \int_V B_j^2 dV = \min\nolimits_0 \quad (3)$$

where: $\min_0$ was introduced in reference [5], indicating the minimum and its value should be equal to zero. W_i and W_j' are suitable positive weighted constants; for the simplest cases, all of these weighted constants can be taken as 1. If only a certain equation is considered, we can only make its corresponding weighted constant is equal to 1 and the other weighted constants are all equal to 0.

By using this method, we already established the "partial and temporary unified water gravity wave theory so far" and the corresponding "partial and temporary unified water gravity wave variational principle so far" in reference [6]; and established the "partial and temporary unified theory of fluid mechanics so far" and the corresponding "partial and temporary unified variational principle of fluid mechanics so far" in reference [7].

Some scholars may said, this is simply the application of least square method, our answer is: the simplest way may be the most effective way.

It should be noted that, due to that time we cannot realize that the strict "unified theory" cannot be existed, therefore in references [6] and [7], the wrong ideas that "unified water gravity wave theory", "unified water gravity wave variational principle", "unified theory of fluid mechanics" and "unified variational principle of fluid mechanics" were appeared. Now we correct these mistakes in this paper.

It should also be noted that, Eq.(2) can be included in Eq.(1), therefore we will only discuss Eq.(1), rather than discuss Eq.(2).

Now we write Maxwell's equations as follows

$F_1 = 0$, in domain Ω_1

where : $F_1 = \nabla \bullet D - \rho$

$F_2 = 0$, in domain Ω_2

where : $F_2 = \nabla \times E + \partial B / \partial t$

$F_3 = 0$, in domain Ω_3

where : $F_3 = \nabla \bullet B$

$F_4 = 0$, in domain Ω_4

where : $F_4 = \nabla \times H - j - \partial D / \partial t$

In addition, for isotropic medium, the following equations should be added

$F_5 = 0$, in domain Ω_5

where : $F_5 = D - \varepsilon_0 \varepsilon_r E$

$F_6 = 0$, in domain Ω_6

where : $F_6 = B - \mu_0 \mu_r H$

$F_7 = 0$, in domain Ω_7

where : $F_7 = j - \gamma E$

Besides these equations, the Coulomb's law reads

$F_8 = 0$, in domain Ω_8

where : $F_8 = f - \dfrac{k q_1 q_2}{r^2}$, according to the experimental data, k =9.0×10^{9}N·m²/C².

Due to the limited space, other equations of electromagnetism are no longer listed. Also, a number of conservation equations (such as the equation of conservation of energy), and a number of laws (such as the law of composition of velocities), are also no longer listed. All of them will be discussed below.

In addition, some solitary equations established only for the solitary points or special cases can be written as follows

$$S_j = 0 \qquad (j = 1,2 \to m) \qquad (4)$$

For example, the scale factor in the Coulomb's law can be written as the following solitary equation

$S_1 = 0$

where : $S_1 = k - 9.0\times10^9$N·m²/C²。

Another example is that, in plasma problem, the shielding distance (Debye distance) can be written as the following solitary equation

$S_2 = 0$

where : $S_2 = D - \sqrt{\varepsilon_0 kT / ne^2}$.

Also due to limited space, other electromagnetic solitary equations are no longer listed.

For the reason that some solitary equations cannot be run the integral process, they will be run the square sum process.

Applying least square method, "partial and temporary unified electromagnetic theory so far" can be expressed in the following form of "partial and temporary unified electromagnetic variational principle so far"

$$\Pi_{\text{EM}} = \sum_{1}^{n} W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_{1}^{m} W_j' S_j^2 = \min\nolimits_0 \quad (5)$$

where: the subscript EM denotes that the suitable scope is the electromagnetism, all of the equations $F_i = 0$ denote so far discovered (derived) all of the equations related to electromagnetism, all of the equations $S_i = 0$ denote so far discovered (derived) all of the solitary equations related to electromagnetism, and W_i and W_j' are suitable positive weighted constants.

Clearly, here n and m are all very large integers.

5.2 Applying least square method to establish "partial and temporary unified gravitational theory so far"

Firstly, it should be noted that, for different gravitational problems, the different formulas or different gravitational theories should be applied. The "universal gravitational formulas or equations" actually cannot be existed. For this conclusion, many scholars do not realize it. In addition, all of the different gravitational formulas can be written as the form of Eq.(1) (namely the form that the right side of the expression is equal to zero).

The first formula should be mentioned is Newton's universal gravitational formula

$$F=-\frac{GMm}{r^2} \qquad (6)$$

It can be written as the following form

$$F_1=0 \qquad (6')$$

where : $F_1=F+\frac{GMm}{r^2}$

Prof. Hu Ning derived an equation according to general relativity, with the help of Hu's equation and Binet's formula, in reference [8] we derived the following improved Newton's formula of universal gravitation

$$F=-\frac{GMm}{r^2}-\frac{3G^2M^2mp}{c^2r^4} \qquad (7)$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object m moving around the object M along with a curve, and the value of p is given by: $p = a(1-e^2)$ (for ellipse), $p = a(e^2-1)$ (for hyperbola), $p = y^2/2x$ (for parabola).

This formula can give the same results as given by general relativity for the problem of planetary advance of perihelion and the problem of gravitational defection of a photon orbit around the Sun.

It can be written as the following form

$$F_2=0 \qquad (7')$$

where : $F_2=F+\frac{GMm}{r^2}+\frac{3G^2M^2mp}{c^2r^4}$

It should be noted that, according to Eq.(6) and Eq.(7) the FTL can be existed.

In some cases, we should also consider the following gravitational formula including three terms

$$F=-\frac{GMm}{r^2}\left(1+\frac{3GMp}{c^2r^2}+\frac{wG^2M^2p^2}{c^4r^4}\right) \qquad (8)$$

where: w is a constant to be determined.

It can be written as the following form

$$F_3=0 \qquad (8')$$

where: $F_3=F+\frac{GMm}{r^2}\left(1+\frac{3GMp}{c^2r^2}+\frac{wG^2M^2p^2}{c^4r^4}\right)$

But for the example that a small ball rolls along the inclined plane in the gravitational field of the Earth, all of the above mentioned formulas cannot be applied. In reference [5], we present the following gravitational formula with the variable dimension fractal form (the fractal dimension is variable, instead of constant).

$$F=-GMm/r^{2-\delta} \qquad (9)$$

where : $\delta=1.206\times10^{-12}u$, u is the horizon distance that the small ball rolls.

It can be written as the following form

$$F_4=0 \qquad (9')$$

where : $F_4=F+GMm/r^{2-\delta}$

In addition, the gravitational field equations of Einstein's theory of general relativity, and the gravitational formula and gravitational equations derived by other scholars, can also be written as the form of Eq.(1) (namely the form that the right side of the expression is equal to zero).

In some cases, when dealing with gravitational problem, we should also consider some principle of conservation, such as the principle of conservation of energy. Here we write the principle of conservation of energy as the form of Eq.(1) (namely the form that the right side of the expression is equal to zero). So do the other principles of conservation.

In references [9], we discussed two cases to apply the principle of conservation of energy directly and indirectly.

To apply the principle of conservation of energy directly is as follows.

Supposing that the initial total energy of a closed system is equal to $W(0)$, and for time t the total energy is equal to $W(t)$, then according to the principle of conservation of energy, it gives

$$W(0)=W(t) \qquad (10)$$

It can be written as the following form

$$F_5=\frac{W(t)}{W(0)}-1=0 \qquad (11)$$

To apply the principle of conservation of energy indirectly is as follows.

Supposing that we are interested in a special physical quantity Q , not only it can be calculated by using the principle of conservation of energy, but also can be calculated by using other gravitational formula. For distinguishing the values, let's denote the value given by other laws as Q , while denote the value given by the principle of conservation of energy as Q' , then the equation to apply the principle of conservation of energy indirectly is as follows

$$F_6 = \frac{Q}{Q'} - 1 = 0 \qquad (12)$$

Now we discuss some solitary equations established only for the solitary points or special cases.

The first one is the solitary equation about the gravitational constant.

$$S_1 = G - 6.67\times 10^{-11}\ \mathrm{N{\bullet}m^2/kg^2} = 0 \qquad (13)$$

The second one is considering the deflection angle for the problem of gravitational defection of a photon orbit around the Sun.

By using general relativity or improved Newton's formula of universal gravitation (namely Eq.(7)), the deflection angle ϕ_0 reads

ϕ_0 =1.75”

However, according to the experiment, we should have ϕ =1.77±0.20, taking the average, it gives

ϕ =1.77”

According to this expression, the corresponding solitary equation is as follows

$$S_2 = \phi - 1.77'' = 0 \qquad (14)$$

Other solitary equations include: the solitary equations established by the values of planetary advance of perihelion, the solitary equations established by the unusual values of gravity at different times during total solar eclipse, and the like. Due to the limited space, they are no longer listed.

Applying least square method, "partial and temporary unified gravitational theory so far" can be expressed in the following form of "partial and temporary unified gravitational variational principle so far"

$$\Pi_{\mathrm{G}} = \sum_1^n W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_1^m W_j' S_j^2 = \min{}_0 \qquad (15)$$

where: the subscript G denotes that the suitable scope is the gravity, all of the equations $F_i = 0$ denote so far discovered (derived) all of the equations related to gravity, all of the equations $S_i = 0$ denote so far discovered (derived) all of the solitary equations related to gravity, and W_i and W_j' are suitable positive weighted constants.

It should be noted that, as we establish "partial and temporary unified theory so far" and the corresponding "partial and temporary unified variational principle so far", the including phenomenon is allowed. For example, the three terms gravitational formula Eq.(8) includes Eq.(7), while Eq.(7) includes Eq.(6). But we still consider these three equations simultaneously. This is because that, in some cases Eq.(7) is more convenient; as for Eq.(6), it is enough in most cases, moreover, putting Eq.(6) at the most prominent position, express our respect to Newton who is the greatest scientist in the history. In addition, the coexisting phenomenon is also allowed. For example, the gravitational formulas of classical mechanics, the gravitational field equations of Einstein's theory of general relativity, and the equations of other gravitational theories are coexisting. For the solution that is satisfying two or more than two theories simultaneously, or solving the problems in different fields simultaneously, and the like, we will discuss them in other papers (such solutions may only be reached with the method of variational principle).

Now we discuss the applications of variational principle Eq.(15).

Example 1. Setting $W_2 = 1$ and $W_1' = 1$ in variational principle Eq.(15), and other weighted constants are all equal to 0, namely applying Eq.(7) and Eq.(13) to derive the changing rule for the gravitational coefficient G' (instead of the gravitational constant G) and make the gravitational formula in accordance with the inverse square law.

In references [10], changing Eq.(7) into the following form in accordance with the inverse square law

$$F = -\frac{G'Mm}{r^2}$$

It gives

$$-\frac{G'Mm}{r^2} = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4}$$

Then we have the changing rule for the gravitational coefficient G' as follows

$$G' = G\left(1 + \frac{3GMp}{c^2r^2}\right) \qquad (16)$$

For problem of Mercury’s advance of perihelion, we have

$$(1 + 5.0381\times 10^{-8})G \le G' \le (1 + 1.1623\times 10^{-7})G$$

For problem of gravitational defection of a photon orbit around the Sun, we have

$$G \le G' \le 2.5G$$

Example 2. Setting $W_4 = 1$ and $W_6 = 1$ in variational principle Eq.(15), and other weighted constants are all equal to 0, namely applying Eq.(9) and Eq.(12) to determine the unknown δ in Eq.(9).

According to Eq.(12), variational principle Eq.(15) can be simplified into the following form applied the law of conservation of energy indirectly

$$\Pi = \int_{x_1}^{x_2} \left(\frac{Q}{Q'} - 1\right)^2 dx = \min{}_0 \qquad (17)$$

The solution procedure can be found in reference [9]. For the final optimum approximate solution, the value of Π calculated by the improved universal gravitational formula and improved Newton's second law is equal to 0.1906446, it is only 0.033% of the value of Π_0 calculated by the original universal gravitational formula and original Newton's second law.

Example 3. Setting $W_3 = 1$ and $W_2' = 1$ in variational principle Eq.(15), and other weighted constants are all equal to 0, namely applying Eq.(8) and Eq.(14) to determine the unknown w in Eq.(8).

The solution procedure can be found in reference [10], the final result is as follows.

The range of value of w is as follows

$0.08571 \leq w \leq 0.42857$

Taking the average, it gives

$w = 0.25714$

For the problem of gravitational defection of a photon orbit around the Sun, the general relativity cannot give the solution that is exactly equal to the experimental value, while the method presented in this paper can do so.

It should be noted that, for variation principle Eq.(15), if there is an exact solution, then its right side can be equal to 0, here the variational principle Eq.(15) is exactly equivalent to $F_i = 0$ and $S_i = 0$ (see example 1 and example 3). If there is only an approximate solution, the right side of variational principles Eq.(15) can only be approximately equal to 0, at this moment we can apply the appropriate optimization method to seek the best approximate solution, and the effect of the solution can be judged according to the extent that the value of Π is close to 0 (see example 2).

5.3 Other "partial and temporary unified theory so far", especially "partial and temporary unified theory of natural science so far"

To extend the above mentioned method, we can get various "partial and temporary unified theory so far".

For unified dealing with the problems of four fundamental interactions, applying least square method, "partial and temporary unified theory of four fundamental interactions so far" can be expressed in the following form of "partial and temporary unified variational principle of four fundamental interactions so far"

$$\Pi_{\text{G.E.S.W}} = \sum_{1}^{n} W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_{1}^{m} W_j' S_j^2 = \min{}_0 \quad (18)$$

where: the subscript G.E.S.W denotes that the suitable scope is the four fundamental interactions, all of the equations $F_i = 0$ denote so far discovered (derived) all of the equations related to four fundamental interactions, all of the equations $S_i = 0$ denote so far discovered (derived) all of the solitary equations related to four fundamental interactions, and W_i and W_j' are suitable positive weighted constants.

For unified dealing with the problems of natural science, applying least square method, "partial and temporary unified theory of natural science so far" can be expressed in the following form of "partial and temporary unified variational principle of natural science so far"

$$\Pi_{\text{NATURE}} = \sum_{1}^{n} W_i \int_{\Omega_i} F_i^2 d\Omega_i + \sum_{1}^{m} W_j' S_j^2 = \min{}_0 \quad (19)$$

where: the subscript NATURE denotes that the suitable scope is all of the problems of natural science, all of the equations $F_i = 0$ denote so far discovered (derived) all of the equations related to natural science, all of the equations $S_i = 0$ denote so far discovered (derived) all of the solitary equations related to natural science, and W_i and W_j' are suitable positive weighted constants.

In this way, the theory of everything to express all of natural laws, described by Hawking that a single equation could be written on a T-shirt, is partially and temporarily realized in the form of "partial and temporary unified variational principle of natural science so far".

As already noted, for "partial and temporary unified theory so far" and the corresponding "partial and temporary unified variational principle so far", the including phenomenon and coexisting phenomenon are allowed. Here we would like to point out that, besides the including process and coexisting process, the neutrosophic one, namely the simplifying process is also allowed. For example, the first simplifying result of "partial and temporary unified theory of natural science so far" is "theory of conservation of energy", it can be expressed in the following form of "first simplifying variational principle for partial and temporary unified theory of natural science so far" (it is shorted as "variational principle of conservation of energy").

$$\Pi_{\text{NATURE}}^{\text{SIMPLE}-1} = \int_{t_1}^{t_2} (W(t)/W(0)-1)^2 dt = \min{}_0 \quad (20)$$

This "variational principle of conservation of energy" can be applied for unified dealing with many problems in physics, mechanics, astronomy, biology, engineering, and even many issues in social science. For example, in reference [11], based on "theory of conservation of energy", for some cases we derived Newton's second law, the law of universal gravitation, and the like.

Further topics are finding more simplifying processes (simplifying variational principles) and their combinations. These will make "partial and temporary unified theory of natural science so far" simpler, clearer, more perfect, and more practical.

For this purpose, the neutrosophy will give very important contribution.

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Received: October 25th, 2013. Accepted: November 3rd, 2013.

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Filters via Neutrosophic Crisp Sets

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Abstract. In this paper we introduce the notion of filter on the neutrosophic crisp set, then we consider a generalization of the filter's studies. Afterwards, we present the important neutrosophic crisp filters. We also study several relations between different neutrosophic crisp filters and neutrosophic topologies. Possible applications to database systems are touched upon.

Keywords: Filters; Neutrosophic Sets; Neutrosophic crisp filters; Neutrosophic Topology; Neutrosophic Crisp Ultra Filters; Neutrosophic Crisp Sets.

1 Introduction

The fundamental concept of neutrosophic set, introduced by Smarandache in [6, 7, 8] and studied by Salama in [1, 2, 3, 4, 5, 9, 10], provides a groundwork to mathematically act towards the neutrosophic phenomena which exists pervasively in our real world and expand to building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts, such as the neutrosophic crisp set theory.

2 Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [6, 7, 8] and Salama et al. [1, 2, 3, 4, 5, 9, 10]. Smarandache introduced the neutrosophic components T, I, and F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]^-0, 1^+\right[$ is the non- standard unit interval.

3 Neutrosophic Crisp Filters

3.1 Definition 1

First we recall that a neutrosophic crisp set A is an object of the form A = <A$_1$, A$_2$, A$_3$>, where A$_1$, A$_2$, A$_3$ are subsets of X, and

$$A_1 \cap A_2 = \phi, A_1 \cap A_3 = \phi, A_2 \cap A_3 = \phi.$$

Let Ψ be a neutrosophic crisp set in the set X. We call Ψ a neutrosophic crisp filter on X if it satisfies the following conditions:

(N_1) Every neutrosophic crisp set in X, containing a member of Ψ, belongs to Ψ.

(N_2) Every finite intersection of members of Ψ belongs to Ψ.

(N_3) ϕ_N is not in Ψ.

In this case, the pair (X, Ψ) is neutrosophically filtered by Ψ.

It follows from (N_2) and (N_3) that every finite intersection of members of Ψ is not ϕ_N (not empty). We obtain the following results.

3.2 Proposition 1

The conditions (N_2) and *(N$_1$)* are equivalent to the following two conditions:

(N_{2a}) The intersection of two members of Ψ belongs to Ψ.

(N_{1a}) X_N belongs to Ψ.

3.3 Proposition 1.2

Let Ψ be a non-empty neutrosophic subsets in X satisfying (N_1).

Then,

(1) $X_N \in \Psi$ iff $\Psi \neq \phi_N$;

(2) $\phi_N \notin \Psi$ iff $\Psi \neq$ all neutrosophic crisp subsets of X.

From the above Propositions (1) and (2), we can characterize the concept of neutrosophic crisp filter.

3.4 Theorem 1.1

Let Ψ be a neutrosophic crisp subsets in a set X. Then Ψ is neutrosophic crisp filter on X, if and only if it satisfies the following conditions:

(i) Every neutrosophic crisp set in X, containing a member of Ψ, belongs to Ψ.

(ii) If $A, B \in \Psi$, then $A \cap B \in \Psi$.

(iii) $\Psi^X \neq \Psi \neq \phi_N$.

Proof: It's clear.

3.5 Theorem 1.2

Let $X \neq \phi$. Then the set $\{X_N\}$ is a neutrosophic crisp filter on X. Moreover if A is a non-empty neutrosophic crisp set in X, then $\{B \in \Psi^X : A \subseteq B\}$ is a neutrosophic crisp filter on X.

Proof: Let $N = \{B \in \Psi^X : A \subseteq B\}$. Since $X_N \in \Psi$ and $\phi_N \notin \Psi$, $\phi_N \neq \Psi \neq \Psi^X$. Suppose $U, V \in \Psi$, then $A \subseteq U, A \subseteq V$. Thus $A_1 \subseteq U_1 \cap V_1$, $A_2 \subseteq U_2 \cap V_2$ or $A_2 \subseteq U_2 \cup V_2$, and $A_3 \subseteq U_3 \cup V_3$ for all $x \in X$. So $A \subseteq U \cap V$ and hence $U \cap V \in N$.

4 Comparison of Neutrosophic Crisp Filters

4.1 Definition 2

Let Ψ_1 and Ψ_2 be two neutrosophic crisp filters on a set X. We say that Ψ_2 is finer than Ψ_1, or Ψ_1 is coarser than Ψ_2, if $\Psi_1 \subset \Psi_2$.

If also $\Psi_1 \neq \Psi_2$, then we say that Ψ_2 is strictly finer than Ψ_1, or Ψ_1 is strictly coarser than Ψ_2.

We say that two neutrosophic crisp filters are comparable if one is finer than the other. The set of all neutrosophic crisp filters on X is ordered by the relation: Ψ_1 coarser than Ψ_2, this relation inducing the inclusion relation in Ψ^X.

4.2 Proposition 2

Let $(\Psi_j)_{j \in J}$ be any non-empty family of neutrosophic crisp filters on X. Then $\Psi = \cap_{j \in J} \Psi_j$ is a neutrosophic crisp filter on X. In fact, Ψ is the greatest lower bound of the neutrosophic crisp set $(\Psi_j)_{j \in J}$ in the ordered set of all neutrosophic crisp filters on X.

4.3 Remark 2

The neutrosophic crisp filter induced by the single neutrosophic set X_N is the smallest element of the ordered set of all neutrosophic crisp filters on X.

4.4 Theorem 2

Let A be a neutrosophic set in X. Then there exists a neutrosophic filter $\Psi(A)$ on X containing A if for any given finite subset $\{S_1, S_2, ..., S_n\}$ of A, the intersection $\cap_{i=1} S_i \neq \phi_N$. In fact $\Psi(A)$ is the coarsest neutrosophic crisp filter containing A.

Proof $(\Rightarrow)$ Suppose there exists a neutrosophic filter $\Psi(A)$ on X containing A. Let B be the set of all finite intersections of members of A. Then by axiom (N_2), $B \subset \Psi(A)$. By axiom (N_3), $\phi_N \notin \Psi(A)$. Thus for each member B of B, we get that the necessary condition holds

$(\Leftarrow)$ Suppose the necessary condition holds.

Let $\Psi(A) = \{A \in \Psi^X : A \text{ contains a member of B}\}$, where B is the family of all finite intersections of members of A. Then we can easily check that $\Psi(A)$ satisfies the conditions in Definition 1. We say that the neutrosophic crisp filter $\Psi(A)$ defined above is generated by A, and A is called a *sub-base* of $\Psi(A)$.

4.5 Corollary 2.1

Let Ψ be a neutrosophic crisp filter in a set X, and A a neutrosophic set. Then there is a neutrosophic crisp filter $\Psi^{/}$ which is finer than Ψ and such that $A \in \Psi^{/}$ if and A is a neutrosophic set. Then there is a neutrosophic crisp filter $\Psi^{/}$ which is finer than Ψ and such that $A \in \Psi^{/}$ iff $A \cap U \neq \phi_N$ for each $U \in \Psi$.

4.6 Corollary 2.2

A set φ_N of a neutrosophic crisp filter on a non-empty set X, has a least upper bound in the set of all neutrosophic crisp filters on X if for all finite sequence $(\Psi_j)_{j \in J}, 0 \leq j \leq n$ of elements of φ_N and all $A_j \in \Psi_j (1 \leq j \leq n)$, $\cap_{j=1} A_j \neq \phi_N$.

4.7 Corollary 2.3

The ordered set of all neutrosophic crisp filters on a non-empty set X is inductive.

If Λ is a sub-base of a neutrosophic filter N on X, then Ψ is not in general the set of neutrosophic sets in X containing an element of Λ; for Λ to have this property it is necessary and sufficient that every finite intersection of members of Λ should contain an element of Λ. Hence, we have the following results.

4.8 Theorem 3

Let β be a set of neutrosophic crisp sets on a set X. Then the set of neutrosophic crisp sets in X containing an element of β is a neutrosophic crisp filter on X if β possesses the following two conditions:

(β_1) The intersection of two members of β contain a member of β.

(β_2) $\beta \neq \phi_N$ and $\phi_N \notin \beta$.

4.9 Definition 3

Let Λ and β be two neutrosophic sets on X satisfying conditions (β_1) and (β_2). We call them bases of neutrosophic crisp filters they generate. We consider two neutrosophic bases equivalent, if they generate the same neutrosophic crisp filter.

4.10 Remark 3

Let Λ be a sub-base of neutrosophic filter Ψ. Then the set β of finite intersections of members of Λ is a base of a neutrosophic filter Ψ.

4.11 Proposition 3.1

A subset β of a neutrosophic crisp filter Ψ on X is a base of Ψ if every member of Ψ contains a member of β.

Proof $(\Rightarrow)$ Suppose β is a base of N. Then clearly, every member of Ψ contains an element of β. $(\Leftarrow)$ Suppose the necessary condition holds. Then the set of neutrosophic sets in X containing a member of β coincides with Ψ by reason of $(\Psi_j)_{j \in J}$.

4.12 Proposition 3.2

On a set X, a neutrosophic crisp filter $\Psi^{/}$ with base $\beta^{/}$ is finer than a neutrosophic crisp filter Ψ with base β if every member of β contains a member of $\beta^{/}$.

Proof: This is an immediate consequence of Definitions 2 and 3.

4.13 Proposition 3.3

Two neutrosophic crisp filters bases β and $\beta^{/}$ on a set X are equivalent if every member of β contains a member of $\beta^{/}$ and every member of $\beta^{/}$ and every member of $\beta^{/}$ contains a member of β.

5 Neutrosophic Crisp Ultrafilters

5.1 Definition 4

A neutrosophic ultrafilter on a set X is a neutrosophic crisp filter Ψ such that there is no neutrosophic crisp filter on X which is strictly finer than Ψ (in other words, a maximal element in the ordered set of all neutrosophic crisp filters on X).

Since the ordered set of all neutrosophic crisp filters on X is inductive, Zorn's lemma shows that:

5.2 Theorem 4

Let Ψ be any neutrosophic ultrafilter on a set X; then there is a neutrosophic ultrafilter other than Ψ.

5.3 Proposition 4

Let Ψ be a neutrosophic ultrafilter on a set X. If A and B are two neutrosophic subsets such that $A \cup B \in \Psi$, then $A \in \Psi$ or $B \in \Psi$.

Proof: Suppose not. Then there are neutrosophic sets A and B in X such that $A \notin \Psi, B \notin \Psi$ and $A \cup B \in \Psi$ Let $\Lambda = \{M \in \Psi^X : A \cup M \in \Psi\}$. It is straightforward to check that Λ is a neutrosophic crisp filter on X, and Λ is strictly finer than Ψ, since $B \in \Lambda$.This contradiction proves the hypothesis that Ψ is a neutrosophic crisp ultrafilter.

5.4 Corollary 4

Let Ψ be a neutrosophic crisp ultrafilter on a set X and let $(\Psi_j)_{1 \leq j \leq n}$ be a finite sequence of neutrosophic crisp sets in X. If $\underset{j=1}{\cup} \Psi_j \in \Psi$, then at least one of the Ψ_j belongs to Ψ.

5.5 Definition 5

Let A be a neutrosophic crisp set in a set X. If U is any neutrosophic crisp set in X, then the neutrosophic crisp set $A \cap U$ is called trace of U on A, and it is denoted by U_A. For all neutrosophic crisp sets U and V in X, we have $(U \cap V)_A = U_A \cap V_A$.

5.6 Definition 6

Let A be a neutrosophic crisp set in a set X. Then the set Λ_A of traces $A \in \Psi^X$ of members of Λ is called the trace of Λ on A.

5.7 Proposition 5

Let Ψ be a neutrosophic crisp filter on a set X and $A \in \Psi^X$. Then the trace Ψ_A of Ψ on A is a neutrosophic crisp filter if each member of Ψ intersects with A.

Proof: The result in Definition 6 shows that Ψ_A satisfies (N_2). If $M \cap A \subset P \subset A$, then $P = (M \cup P) \cap A$. Thus Ψ_A satisfies (N_1). Hence Ψ_A is a neutrosophic crisp filter if it satisfies (N_3), i.e. if each member of Ψ intersects with A.

5.8 Definition 7

Let Ψ be a neutrosophic crisp filter on a set X and $A \in \Psi^X$. If the trace is Ψ_A of Ψ on A, then Ψ_A is said to be induced by Ψ and A.

5.9 Proposition 6

Let Ψ be a neutrosophic crisp filter on a set X inducing a neutrosophic filter N_A on $A \in \Psi^X$. Then the trace β_A on A of a base β of Ψ is a base of Ψ_A.

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Received: November 12, 2013. Accepted: December 5, 2013

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Communication vs. Information, an Axiomatic Neutrosophic Solution

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Abstract. Study represents an application of the neutrosophic method, for solving the contradiction between communication and information. In addition, it recourse to an appropriate method of approaching the contradictions: Extensics, as the method and the science of solving the contradictions.

The research core is the reality that the scientific research of communication-information relationship has reached a dead end. The bivalent relationship communication-information, information-communication has come to be contradictory, and the two concepts to block each other.

After the critical examination of conflicting positions expressed by many experts in the field, the extensic and inclusive hypothesis is issued that information is a form of communication. The object of communication is the sending of a message. The message may consist of thoughts, ideas, opinions, feelings, beliefs, facts, information, intelligence or other significational elements. When the message content is primarily informational, communication will become information or intelligence.

The arguments of supporting the hypothesis are:

a) linguistic (the most important being that there is "communication of information" but not "information of communication"; also, it is clarified and reinforced the over situated referent, that of the communication as a process),

b) systemic-procedural (in the communication system is developing an information system; the informing actant is a type of communicator, the information process is a communication process),

c) practical (the delimitation eliminates the efforts of disparate and inconsistent understanding of the two concepts),

d) epistemological arguments (the possibility of inter-subjective thinking of reality is created), linguistic arguments,

e) logical and realistic arguments (it is noted the situation that allows to think coherently in a system of concepts - derivative series or integrative groups)

f) and arguments from historical experience (the concept of communication has temporal priority, it appears 13 times in Julius Caesar's writings).

In an axiomatic conclusion, the main arguments are summarized in four axioms: three are based on the pertinent observations of specialists, and the fourth is a relevant application of Florentin Smarandache's neutrosophic theory.

Keywords: neutrosophy, communication, information, message, extensics

1. Clarification on the used methodological tool

With the Extensics as a science of solving the conflicting issues, "extensical procedures" will be used to solve the contradiction. In this respect, considering that the matter-elements are defined, their properties will be explored ("The key to solve contradictory problems, Wen Cai argues, the founder of Extensics (Cai, 1999, p. 1540), is the study of properties about matter-elements"). According to „The basic method of Extensics is called extension methodology" (...), and "the application of the extension methodology in every field is the extension engineering methods" (Weihai Li & Chunyan Yang, 2008, p. 34).

With neutrosophic, linguistic, systemic, and hermeneutical methods, grafted on "extension methodology" a) are "open up the things", b) is marked "divergent nature of matter-element", c) "extensibility of matter-element" takes place and c) "extension communication" allows a new inclusion perspective to open, a sequential ranging of things to emphasize at a higher level and the contradictory elements to be solved. "Extension" is, as postulated by Wen Cai (Cai, 1999, p. 1538) "opening up carried out".

2. The subject of communication: the message. The subject of informing: the information. The information thesis as species of message

In order to finish our basic thesis that of the information as a form of communication, new arguments may be revealed which corroborate with those previously mentioned. As phenomena, processes, the communication and information occur in a unique communication system. In communication, information has acquired a specialized profile. In the information field, the intelligence, in his turn, strengthened a specific, detectable, identifiable and discriminative profile. It is therefore acceptable under the pressure of practical argument that one may speak of a general communication system which in relation to the message sent and configured in the communication process could be imagined as information system or intelligence system. Under the influence of the systemic assumption that a (unitary) communicator transmits or customize transactionally with another (receiving) communicator a message, one may understand the communicational system as the interactional unit of the factors that exerts and fulfill the function of communicating a message.

In his books "Messages: building interpersonal communication skills" (attained in 1993 its fourth edition and in 2010 its twelfth) and "Human Communication" (2000), Joseph De Vito (the renowned specialist who has proposed the name "Communicology" for the sciences of communication - 1978), develops a concept of a simple and productive message. The message is, as content, what is communicated. As a systemic factor, it is emerging as what is communicated. To remember in this context is that the German Otto Kade insisted that what it is communicated to receive the title of "release". According to Joseph De Vito, through communication meanings are transmitted. "The communicated message" is only a part of the meanings (De Vito, 1993, p. 116). Among the shared meanings feelings and perceptions are found (De Vito J., 1993, p. 298). Likewise, information can be communicated (De Vito, 1990, p. 42), (De Vito, 2000, p. 347) (also, Fârte, 2004; Ciupercă, 2009; Cojocaru, Bragaru & Ciuchi, 2012; Cobley & Schulz, 2013).

In a "message theory" called "Angelitics", Rafael Capurro argues that the message and information are concepts that designate similar but not identical phenomena. In Greek "Angelia" meant message; from here, "Angelitics" or theory of the message (Angelitics is different from Angeologia dealing, in the field of religion and theology, with the study of angels). R. Capurro set four criteria for assessing the relationship between message and information. The similarity of the two extends over three of them. The message, as well as the information, is characterized as follows: „is supposed to bring something new and/or relevant to the receiver; can be coded and transmitted through different media or messengers; is an utterance that gives rise to the receiver's selection through a release mechanism of interpretation". "The difference between these two is the next: „a message is sender-dependent, i.e. it is based on a heteronomic or assymetric structure. This is not the case of information: we receive a message but we ask for information" (http://www.capurro.de/angeletics_zkm.html) (see also, Capurro, 2011; Holgate, 2011). To request information is to send a message of requesting information. Therefore, the message is similar to the information in this respect too. In our opinion, the difference between them is from genus to species: information is a species of message. The message depends on the transmitter and the information, as well. Information is still a specification of the message, is an informative message. C. Shannon asserts that the message is the defining subject of the communication. He is the stake of the communication because „the fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point" (Shannon, 1948, p. 31).

The communication process is in fact the "communication" of a complex and multilayered message. 'Thoughts, interests, talents, experiences"(Duck & McMahan, 2011, p. 222), "information, ideas, beliefs, feelings "(Wood, 2009, p. 19 and p. 260) can be found in a message. G. A. Miller, T. M. Newcomb and Brent R. Ruben consider that the subject of communication is information: "Communication - Miller shows – means that information is passed from one place to another" (Miller, 1951, p. 6). In his turn, T. M. Newcomb asserts: „very communication act is viewed as a transmission of information" (Newcomb, 1966, p. 66) and Brent R. Ruben argues: „Human communication is the process through which individuals in relationships, groups, organizations and societies create, transmit and use information to relate to the environment and one another" (Ruben, 1992, p. 18).

Professor Nicolae Drăgulănescu, member of the American Society of Information Science and Technology, is the most important of Romanian specialists in the Science of information. According to him, "communicating information" is the third of the four processes that form the "informational cycle", along with generating the information, processing/storing the information and the use of information. The process of communication, Nicolae Drăgulanescu argues, is one of the processes whose object is the information (http://ndragulanescu.ro/publicatii/CP54.pdf, p. 8) (also, Drăgulănescu, 2002; Drăgulănescu, 2005). The same line is followed by Gabriel Zamfir too; he sees the information as "what is communicated in one or other of the available languages" (Zamfir, 1998, p. 7), as well as teacher Sultana Craia: communication is a "process of transmitting a piece of information, a message" (Craia, 2008, p. 53). In general, it is accepted that information means transmitting or receiving information. However, when speaking of transmitting information, the process is considered not to be information but communication. Therefore, it is created the appearance that the information is the product and communication would only be the transmitting process. Teodoru Ştefan, Ion Ivan şi Cristian Popa assert: "Communication is the process of transmitting information, so the ratio of the two categories is from the basic product to its transmission" (Popa, Teodoru & Ivan I., 2008, p. 22). The professors Vasile Tran and Irina Stănciugelu see communication as an "exchange of information with symbolic content" (Tran & Stănciugelu, 2003, p. 109). The communication is an over-ranged concept and an ontological category more extended than informing or information. On the other hand, information is generated even in the global communication process. From this point of view, information (whose subject-message is information) is a regional, sectorial communication. Information is that communication whose message consists of new, relevant, pertinent and useful significances, i.e. of information. This position is shared by Doru Enache too (Enache, 2010, p. 26).

The position set by Norbert Wiener, consolidated by L. Brillouin and endorsed by many others makes from the information the only content of the message. N. Wiener argues that the message "contains information" (Wiener N., 1965, p. 16), L. Brillouin talks about "information contained in the message" (Brillouin, 2004, p. 94 and p. 28).

Through communication "information, concepts, emotions, beliefs are conveyed" and communication "means (and subsumes) information" (Rotaru, 2007, p.10). Well-known teachers Marius Petrescu and Neculae Năbârjoiu consider that the distinction between communication and information must be achieved depending on the message. A communication with an informational message becomes information. As a form of communication, information is characterized by an informative message and a "message is informative as long as it contains something unknown yet" (Petrescu & Năbârjoiu, 2006, p. 25). One of the possible significant elements that could form the message content is thus the information as well. Other components could be thoughts, ideas, beliefs, knowledge, feelings, emotions, experiences, news facts. Communication is "communicating" a message regardless of its significant content.

3. The information thesis as a form of communication

The question of the relationship between communication and information as fields of existence is the fingerprint axis of communication and information ontology. The ontological format allows two formulas: the existence in the act and the virtual existence. The ontological component of the concepts integrates a presence or a potency and an existential fact or at a potential of existence (Zins, 2007; Allo, 2007; Stan, 2009; Burgin, 2010; Case, 2013).

In addition to the categorial-ontological element, in the nuclear ratio of communication-information concepts it shows comparative specificities and regarding attributes and characteristics, on three components, epistemological, methodological and hermeneutical.

In a science which would have firmly taken a strong subject, a methodology and a specific set of concepts, this ontological founding decision would be taken in an axiom. It is known that, in principle, axioms solve within the limits of that type of argument called evidence (clear and distinct situation), the relations between the systemic, structural, basic concepts. Specifically, in Extensics, scientists with an advanced vision, substantiated by professor Wen Cai, axioms govern the relationship between two matter-elements with divergent profiles. For the communication and information issues that have occurred relatively recently (about three quarters of a century) in subjects of study or areas of scientific concern not a scientific authority to settle the issue was found. The weaknesses of these sciences of soft type are visible even today when after non accredited proposals of science ("comunicology" - communicology Joseph De Vito, "communicatics," - "comunicatique" of Metayer G., informatology - Klaus Otten and Anthony Debons, 1970) it was resorted to the remaining in the ambiguity of validating the subject "The sciences of communication and information" or "The sciences of information and communication", enjoying the support of some courses, books, studies and dictionaries (Toma, 1999; Tudor, 2001; Strechie, 2009; Ţenescu, 2009).

This generic vision of unity and cohesion wrongs both the communication and information (Vlăduţescu, 2004; Vlăduţescu, 2006). In practice, the apparent unjust overall, integrative, altogether treatment has not an entirely and covering confirmation. In almost all humanist universities of the world the faculties and the communication courses are prevailing, including those of Romania and China. Professor Nicolae Drăgulănescu ascertained in what Romania is concerned, that in 20 colleagues communication (with various denominations) is taught and in only two the informing-information is taught.

The main perspectives from which the contradictory relationship of communication-information was approached are the ontological, the epistemological and the systemic. In most cases, opinions were incidental. When it was about the dedicated studies, the most common comparative approach was not programmatically made on one or more criteria and neither directly and applied.

In his study "Communication and Information" (19 March 9, pp. 3-31), J. R. Schement starts from the observation that "in the rhetoric of the Information Age, the communication and information converge in synonymous meanings." On the other hand, he retains that there are specialists who declare in favor of stating a firming distinction of their meanings. To clarify exactly the relationship between the two phenomena, i.e. concepts, he examines the definitions of information and communication that have marked the evolution of the "information studies" and the "communication studies". For informing (information) three fundamental themes result: information-as-thing (M. K. Buckland), information-as-process (N. J. Belkin - 1978, R. M. Hayes, Machlup & Mansfield, Elstner - 2010 etc.), Information-as-product-of - manipulation (C. J. Fox, R. M. Hayes). It is also noted that these three subjects involve the assessing of their issuers, a "connection to the phenomenon of communication". In parallel, from examining the definitions of communication it is revealed that the specialists "implicitly or explicitly introduce the notion of information in defining communication". There are also three the central themes of defining communication: communication-as-transmission (C. Shannon, W. Weaver, E. Emery, C. Cherry, B. Berelson, G. Steiner), communication-as-sharing-process (R. S. Gover, W. Schramm), communication-as-interaction (G. Gerbner, L. Thayer). Comparing the six thematic nodes, Schement emphasizes that the link between information and communication is "highly complex" and dynamic "information and communication is ever present and connected" (Schement, 1993, p. 17). In addition, in order that “information exist, the potential for communication must be present”. The result at the ontological level of these findings is that the existence of information is (strictly) conditioned by the presence of communication. That is for the information to occur communication must be present. Communication will precede and always condition the existence of information. And more detailed: communication is part of the information ontology. Ontologically, information occurs in communication also as potency of communication (Vlăduțescu, 2002). J. R. Schement is focused on finding a way to census a coherent image leading to a theory of communication and information ("Toward a Theory of Communication and Information" - Schement, 1993, p. 6). He avoids to conclusively asserting the temporal and linguistic priority, the ontological precedence and the amplitude of communication in relation to information. The study concludes that

1. "Information and communication are social structures" ("two words are used as interchangeable, even as synonyms" – it is argued) (Schement, 1993, p. 17),

2. "The study of information and communication share concepts in common" (in both of them communication, information, "symbol, cognition, content, structure, process, interaction, technology and system are to be found" - Schement, 1993, p. 18),

3. "Information and communication form dual aspects of a broader phenomenon" (Schement J.R., 1993, p. 18).

In other words, we understand that: a) linguistically ("words", "terms", "notions", "concepts", "idea of") communication and information are synonyms; b) as area of study the two resort the same conceptual arsenal. Situation produced by these two elements of the conclusion allows, in our opinion, a hierarchy between communication and information. If it is true that ontologically and temporally the communication precedes information, if this latter phenomenon is an extension smaller than the first, if eventual sciences having communication as object, respectively information, benefit from the one and the same conceptual vocabulary, then the information can be a form of communication. Despite this line followed coherently by the linguistic, categorical-ontological, conceptual and definitional epistemological arguments brought in the reasoning, the third part of the conclusion postulates the existence of a unique phenomenon which would include communication and information (3. "Information and communication form two aspects of the same phenomenon "- Schement JR, 1993, p. 18). This phenomenon is not named. The conclusive line followed by the arguments and the previous conclusive elements enabled us to articulate information as one of the forms of communication. Confirmatively, the fact that J. R. Schement does not name a phenomenon situated over communication and information, gives us the possibility of attracting the argument in order to strengthen our thesis that information is a form of communication. That is because a category of phenomena encompassing communication and information cannot be found. J. R. Schement tends towards a leveling perspective and of convergence in the communication and information ontology. Instead, M. Norton supports an emphasized differentiation between communication and information. He belongs to those who see communication as one of the processes and one of the methods "for making information available". The two phenomena "are intricately connected and have some aspects that seem similar, but they are not the same" (Norton, 2000, p. 48 and p. 39). Harmut B. Mokros and Brent R. Ruben (1991) lay the foundation of a systemic vision and leveling understanding of the communication-information relationship. Taking into account the context of reporting as a core element of the internal structure of communication and information systems, they mark the information as a criterion for the radiography of relationship. The systemic-theoretical non-linear method of research founded in 1983 by B. R. Ruben is applied to the subject represented by the phenomena of communication and information. Research lays in the "Information Age" and creates an informational reporting image. The main merit of the investigation comes from the relevance given to the non-subordination between communication and information in terms of a unipolar communication that relates to a leveling information. Interesting is the approach of information in three constituent aspects: "informatione" (potential information - that which exists in a particular context, but never received a significance in the system), "information" (active information in the system) and "information" (information created socially and culturally in the system). The leveling information is related to a unified communication (Hofkirchner, 2010; Floridi, 2011; Fuchs, 2013; Hofkirchner, 2013). On each level of information there is communication. Information and communication is co-present: communication is inherent to information. Information has inherent properties of communication. Research brings a systemic-contextual elucidation to the relationship between communication and information and only subsidiarily a firm ontological positioning. In any case: in information communication never misses.

In the most important studies of the professor Stan Petrescu: "Information, the fourth weapon" (1999) and "About intelligence. Espionage-Counterespionage" (2007), information is understood as "a type of communication" (Petrescu, 1999, p. 143) and situated in the broader context of "knowledge on the internal and international information environment " (Petrescu, 2007, p. 32).

4. Axiomatic conclusion: four axioms of communication-information ontology

4.1. The message axiom.

We call the ontological segregation axiom on the subject or the Tom D. Wilson - Solomon Marcus' axiom, the thesis that not any communication is information, but any information is communication. Whenever the message contains information, the communicational process will acquire an informational profile. Moreover, the communicational system becomes informational system. Derivatively, the communicator becomes the "informer" and the communicational relationship turns into informational relationship. The interactional basis of society, even in the Information Age, is the communicational interaction. Most social interactions are non-informational. In this respect, T. D. Wilson has noted: „We frequently receive communications of facts, data, news, or whatever which leave us more confused than ever. Under formal definition, these communications contain no information" (Wilson, 1987, p. 410). Academician Solomon Marcus takes into account the undeniable existence of a communication "without a transfer of information" (Marcus, 2011a, p. 220; Marcus, 2011b). For communications that do not contain information we do not have a separate and specific term. Communications containing information or just information are called informing.

Communication involves a kind of information, but as Jean Baudrillard stated (Apud Dâncu, 1999, p. 39), "it is not necessarily based on information". More specifically, any communication contains cognition that can be knowledge, data or information. Therefore, in communication, information may be missing, may be adjacent, incidental or collateral. Communication can be informational in nature or its destination. That communication which by its nature and organization is communication of information is called informing.

The main process ran in Information System is informing. The function of such a system is to inform. The actants can be informants, producers-consumers of information, transmitters of information, etc. The information action takes identity by the cover enabled onto-categorial by the verb "to inform". In his turn, Petros A. Gelepithis considers the two concepts, communication and information to be crucial for "the study of information system" (Gelepithis, 1999, p. 69).

Confirming the information axiom as post reductionist message, as reduced object of communication, Soren Brier substantiates: „communication system actually does not exchange information" (Brier, 1999, p. 96). Sometimes, within the communication system information is no longer exchanged.

However, communication remains; communication system preserves its validity, which indicates and, subsequently, proves that there can be communication that does not involve information (Bates, 2006; Dejica, 2006; Chapman & Ramage, 2013).

On the other hand, then

a) when in the Information System functional principles such as "need to know"/"need to share" are introduced,

b) when running processes for collecting, analyzing and disseminating information,

c) when the beneficiaries are deciders, "decision maker", "ministry", "government", "policymakers" and

d) when the caginess item occurs, this Information System will become Intelligence System (see Gill, Marrin & Phytian, 2009, p. 16, p. 17, p. 112, p. 217), (Sims & Gerber, 2005, p. 46, p. 234; Gill P.& Phytian, 2006, p. 9, p. 236, p. 88; Johnson, 2010, p. 5, p. 6, p. 61, p. 392, p. 279; Maior, 2009; Maior, 2010). Peter Gill shows that "Secrecy is the Key to Understanding the essence of intelligence" (Gill, 2009, p. 18), and Professor George Cristian Maior emphasizes: "in intelligence, collecting and processing information from secret sources remain essential" (Major, 2010, p. 11).

Sherman Kent, W. Laqueur, M. M. Lowenthal, G.-C. Maior etc. start from a complex and multilayered concept of intelligence, understood as meaning knowledge, activity, organization, product, process and information. Subsequently, the question of ontology, epistemology, hermeneutics and methodology of intelligence occurs. Like Peter Gill, G.-C. Maior does pioneering work to separate the ontological approach of intelligence from the epistemological one and to analyze the "epistemological foundation of intelligence" (Maior, 2010, p. 33 and p. 43).

The intelligence must be also considered in terms of ontological axiom of the object. In this regard, noticeable is that one of its meanings, perhaps the critical one, places it in some way in the information area. In our opinion, the information that has critical significance for accredited operators of the state, economic, financial and political power, and holds or acquires confidential, secret feature is or becomes intelligence. Information from intelligence systems can be by itself intelligence or end up being intelligence after some specialized processing. "Intelligence is not just information that merely exists" (Marinică & Ivan, 2010, p. 108), Mariana Marinică and Ion Ivan assert, it is acquired after a "conscious act of creation, collection, analysis, interpretation and modeling information" (Marinică & Ivan, 2010, p. 105).

4.2. Linguistic axiom.

A second axiom of communication-information ontological segregation can be drawn in relation to the linguistic argument of the acceptable grammatical context. Richard Varey considers that understanding "the difference between communication and information is the central factor" and finds in the linguistic context the criterion to validate the difference: „we speak of giving information **to** while communicate **with** other" (Varey, 1997, p. 220). The transmission of information takes place "to" or to someone, and communication takes place "with". Along with this variant of grammatical context it might also emerge the situation of acceptability of some statements in relation to the object of the communication process, respectively the object of the information process.

The statement "to communicate a message, information" is acceptable. Instead, the statement "to inform communication" is not. The phrase "communication of messages-information" is valid, but the phrase "informing of communication", is not. Therefore, language bears knowledge and "lead us" (Martin Heidegger states) to note that, linguistically, communication is more ontological extensive and that information ontology is subsumed to it (Henno, 2013; Gîfu & Cristea, 2013; Gorun & Gorun, 2011).

The ontical and ontological nature of language allows it to express the existence and to achieve a functional-grammatical specification. Language allows only grammatical existences. As message, the information can be "communicated" or "communicable". There is also the case in which a piece of information cannot be "communicated" or "communicable". Related, communication cannot be "informed". The semantic field of communication is therefore larger, richer and more versatile (Ştefan Buzărnescu, 2006). Communication allows the "incommunicable".

4.3. Teleological axiom.

In addition to the axiom of segregating communication, of informing in relation to the object (message), it may be stated as an axiom a Magoroh Maruyama's contribution to the demythologization of information. In the article "Information and Communication in Poly Epistemological System" in "The Myths of Information", he states: „The transmission of information is not the purpose of communication. In Danish culture, for example, the purpose of communication is frequently to perpetuate the familiar, rather than to introduce new information" (Maruyama, 1980, p. 29).

The ontological axiom of segregation in relation to the purpose determines information as that type of communication with low emergence in which the purpose of the interaction is transmitting information.

4.4. The neutrosophic communication axiom.

Understanding the frame set by the three axioms, we find that some communicational elements are heterogeneous and neutral in relation to the criterion of informativity. In a speech some elements can be suppressed without the message suffering informational alterations. This means that some message-discursive meanings are redundant; others are not essential in relation to the orexis-the practical course or of practical touch in the order of reasoning. Redundancies and non-nuclear significational components can be elided and informational and the message remains informationally unchanged. This proves the existence of cores with neutral, neutrosophic meanings. (In the epistemological foundations of the concept of neutrosophy we refer to Florentin Smarandache's work, *A Unifying Field in Logics, Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*, 1998) (Smarandache, 1998; Smarandache, 1999; Smarandache, 2002; Smarandache, 2005; Smarandache, 2010a; Smarandache, 2010b; Smarandache & Păroiu, 2012).

On the operation of this phenomenon are based the procedures of textual contraction, of grouping, of serial registration, of associating, summarizing, synthesizing, integrating.

We propose to understand by neutrosophic communication that type of communication in which the message consists of and it is based on neutrosophic significational elements: non-informational, redundant, elidable, contradictory, incomplete, vague, imprecise, contemplative, non-practical, of relational cultivation. Informational communication is that type of communication whose purpose is sharing an informational message. The issuer's fundamental approach is, in informational communication, to inform. To inform is to transmit information or, specifically, in the professor's Ilie Rad words: "to inform, that is just send information" (Moldovan, 2011, p. 70) (also, Rad, 2005; Rad, 2008). In general, any communication contains some or certain neutrosophic elements, suppressible, redundant, elidable, non-nuclear elements. But when neutrosophic elements are prevailing communication is no longer informational, but neutrosophic. Therefore, the neutrosophic axiom allows us to distinguish two types of communication: neutrosophic communication and informational communication. In most of the time our communication is neutrosophic. The neutrosophic communication is the rule. The informational communication is the exception. In the ocean of the neutrosophic communication, diamantine islands of informational communication are distinguished.

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Received: November 08, 2013. Accepted: November 29, 2013.

NSS

A Novel Image Segmentation Algorithm Based on Neutrosophic Filtering and Level Set

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Abstract. Image segmentation is an important step in image processing and analysis, pattern recognition, and machine vision. A few of algorithms based on level set have been proposed for image segmentation in the last twenty years. However, these methods are time consuming, and sometime fail to extract the correct regions especially for noisy images. Recently, neutrosophic set (NS) theory has been applied to image processing for noisy images with indeterminant information. In this paper, a novel image segmentation approach is proposed based on the filter in *NS* and level set theory. At first, the image is transformed into NS domain, which is described by three membership sets (*T*, *I* and *F*). Then, a filter is newly defined and employed to reduce the indeterminacy of the image. Finally, a level set algorithm is used in the image after filtering operation for image segmentation. Experiments have been conducted using different images. The results demonstrate that the proposed method can segment the images effectively and accurately. It is especially able to remove the noise effect and extract the correct regions on both the noise-free images and the images with different levels of noise.

Keywords: Image segmentation, Neutrosophic set, Directional alpha-mean filter, Level set.

1 Introduction

Image segmentation is an essential process and is also one of the most difficult tasks in image processing field. It is defined as a process dividing an image into different regions such that each region is homogeneous, but the union of any two adjacent regions is not homogeneous.

Image segmentation approaches are based on either discontinuity and/or homogeneity. The approaches based on discontinuity tend to partition an image by detecting isolated points, lines and edges according to the abrupt changes of the intensities. The approaches based on homogeneity include thresholding, clustering, region growing, and region splitting and merging [1].

Neutrosophy set (NS) provides a powerful tool to deal with the indeterminacy, and the indeterminacy is quantitatively described using a membership [2]. In neutrosophic set, a set A is described by three subsets: <A>, <Neut-A> and <Anti-A>, which is interpreted as truth, indeterminacy, and falsity set. It provides a new tool to describe the image with uncertain information, which had been applied to image processing techniques [3], such as image segmentation, thresholding and denoise.

In this paper, we proposed a novel image segmentation method based on NS theory. The image is mapped into NS domain and a new filter, directional alpha-mean filter is defined in NS domain, and used to remove the indeterminance on the image. Finally, the image on NS domain is segmented using the method based on level set active contour model.

The remainder of this paper is organized as follows. The next section describes the neutrsosophic image, the directional alpha-mean filter, and the segmentation algorithm integrated with level set model. Section three reports the experiments and the relevant discussion. Concluding remarks are drawn in Section four.

2 Proposed method

2.1 Neutrosophic image

An image might have a few indeterminate regions, such as noise, shadow, and boundary. It is hard for the classic sets to interpret the indeterminate regions on images clearly. In a neutrosophic set, a subset *I*, is named as indeterminate set and employed to interpret the indeterminacy in the image.

A neutrosophic image is described by three membership sets *T*, *I* and *F*. The pixel $P(i,j)$ in the image domain is transformed into the neutrosophic set domain, denoted as $P_{NS}(i,j)$ and $P_{NS}(i,j)=\{T(i,j), I(i,j), F(i,j)\}$ (*T*, *I* and *F* are the membership values belonging to bright pixel set, indeterminate set and non-bright pixel set, respectively, which are defined as follows [4]:

$$T(i,j) = \frac{\overline{g}_w(i,j) - \overline{g}_{w\min}}{\overline{g}_{w\max} - \overline{g}_{w\min}} \tag{1}$$

$$I(i,j) = \frac{\delta(i,j) - \delta_{\min}}{\delta_{\max} - \delta_{\min}} \tag{2}$$

$$F(i,j) = 1 - T(i,j) \tag{3}$$

where $\overline{g}_w(i,j)$ is the mean value of intensity in the local neighborhood, whose size is $w \times w$. $\delta(i,j)$ is the absolute value of the difference between intensity $g(i,j)$ and its local mean value $\overline{g}_w(i,j)$. The value of I measures the indeterminacy degree of P_{NS}.

2.2 Directional α-mean operation

In [3], an α-mean operation was defined on a neutrosophic image, and it removed noise efficiently. However, it might blur the image and reduce the contrast, which could reduce the performance of the segmentation. To overcome this drawback, a directional α-mean operation (denoted as DAM) is newly proposed to remove the noise effect and preserve the edges at the same time.

The function of the directional mean filter DMF is defined as [5]:

$$DMF(i,j) = \begin{cases} R_1 & |G_{Th}(i,j) - G_{Tv}(i,j)| \le \sigma \\ R_2 & G_{Th}(i,j) - G_{Tv}(i,j) > \sigma \\ R_3 & G_{Tv}(i,j) - G_{Th}(i,j) > \sigma \end{cases} \tag{4}$$

where $G_{Th}(i,j)$ and $G_{Tv}(i,j)$ are the norm of the gradient at (i,j) of the subset T at the horizontal and vertical direction, respectively. σ is a threshold value and selected as 0.01here.

The directional α-mean filter $DAMF$ is defined using the subset T and I as:

$$DAMF(i,j) = \begin{cases} T(i,j) & I(i,j) < \alpha \\ DMF(i,j) & I(i,j) \ge \alpha \end{cases} \tag{5}$$

2.3 Level set

Level set method was proposed in [6], and applied for image segmentation [7]. The level set method tracks the evolution of the boundaries between different objects, which are embedded as the zero level set.

The level set active contour models can be divided into two classes: edge based and region based. The edge based model tries to find a curve with the maximum edge indicator value which can minimize the energy function $J(C)$ [8]:

$$J(C) = \int |C'(s)| g(Im(C(s))) ds \tag{6}$$

where *g()* is an edge indicator function, C is the boundary, and it can be represented implicitly as the zero level set of a true positive function $\phi : \Omega \to R$, Ω is the domain of image. The evolution equation of boundary C can be derived as:

$$\begin{cases} \frac{\partial \phi}{\partial t} = |\nabla \phi| \left(div\left(g(Im) \frac{\nabla \phi}{|\nabla \phi|} \right) + v(Im) \right) \\ \phi(0,x,y) = \phi_0(x,y) \quad \text{in } \Omega \end{cases} \tag{7}$$

where *v()* is a term for increasing the evolution speed to reach the boundary.

The region based model uses the inside/outside mean values to compose the energy function [9]:

$$\begin{aligned} F(C,c_1,c_2) = {} & \mu_1 L(C) + \mu_2 S(C) \\ & + \lambda_1 \int_{S_1} |Im - c_1|^2 dS_1 \\ & + \lambda_2 \int_{S_2} |Im - c_2|^2 dS_2 \end{aligned} \tag{8}$$

where c_1 and c_2 are the mean intensities of the regions inside and outside the boundary C, respectively. L and S are the length of C and the area inside C. S_1 and S_2 are the region inside and outside of C, respectively. The associated level set flow can be represented as:

$$\begin{cases} \frac{\partial \phi}{\partial t} = \delta(\phi) \left[\begin{array}{l} \mu div\left(\frac{\nabla \phi}{|\nabla \phi|} \right) - v - \\ \lambda_1 (Im - c_1)^2 + \lambda_2 (Im - c_2)^2 \end{array} \right] \\ \phi(0,x,y) = \phi_0(x,y) \\ \frac{\delta(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial n} = 0 \end{cases} \tag{9}$$

where $\delta()$ is the Dirac function, and n denotes the exterior normal to the boundary $\partial\Omega$. *div()* is the divergence function on the image.

Usually, edge based approaches often suffer from noise, especially, when the image has a low signal/noise ratio; while the region based approaches are more adaptive to noise or vanishing boundaries due to considering the entire information of the regions to build an energy function.

2.4 Segmentation algorithm based on neutrosophic set and level set (NSLS)

A segmentation algorithm is proposed based on the directional α-mean filter and level set on neutrosophic image. Firstly, the image is transferred into the NS domain. Then, the DAMF is processed in the NS image. Finally, the boundary of region is segmented using the level set active contour algorithm based on the region model. The energy function is defined using the T subset after DAMF processing.

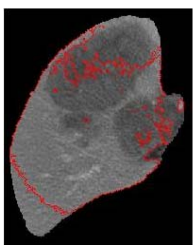
$$F(C,c_1,c_2)=\mu_1 L(C)+\mu_2 S(C) + \lambda_1\int_{S_1}|T'-c_1|^2 dS_1 + \lambda_2\int_{S_2}|T'-c_2|^2 dS_2 \quad (10)$$

3 Experimental results and discussion

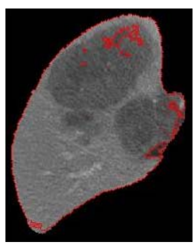
To test the performance of the proposed method, a few of images and images with different noise levels are employed. The NFLS method is compared with that of the segmentation algorithm based on the traditional level set [9], which is noted as TLS.

(a) Original image.

(b) Segmentation result of NSLS at 800 iterations.

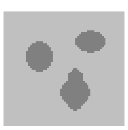


(c) Segmentation result of TLS at 800 iterations.

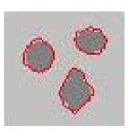


(d) Segmentation result of TLS at 1500 iterations.

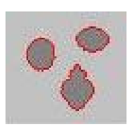
Figure 1: Comparison result on the “liver tumour” image with different iterations.



(a) Original image.

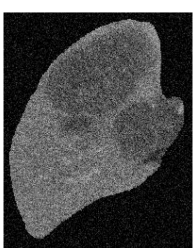


(b) The segmentation result of NSLS at 50 iterations.

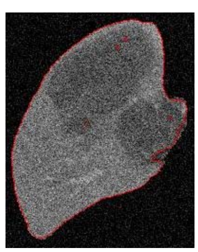


(c) The segmentation result of TLS at 50 iterations.

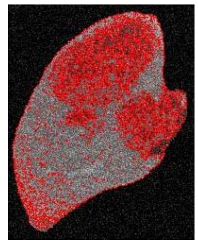
Figure 2: Comparison result on the “cells” image with same iterations.



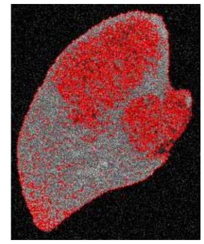
(a) Image with Gaussian noise (variance = 25).



(c) Segmentation result of NSLS at 1000 iterations.



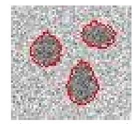
(d) Segmentation result of TLS at 1500 iterations.



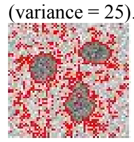
(e) Segmentation result of TLS at 2000 iterations.

Figure 3: Comparison result on the noisy “liver tumour” image with different iterations.

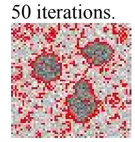
(a) Image with Gaussian noise (variance = 25).



(b) Segmentation result of NSLS at 50 iterations.



(c) Segmentation result of TLS at 50 iterations.



(d) Segmentation result of LS at 200 iterations.

Figure 4: Comparison result on the noisy “three cells” image with different iterations.

An experiment is performed to compare the time consumption of NSLS and LS methods. The NSLS takes less than 33 seconds per image on average for an AMD Phenom(tm) 9500 Quad-core Processor, 2.2 GHz. Table 1 compares the computational time on different images for different algorithms. The NSLS takes less iteration and fewer CPU times than the TLS method.

Image	TLS		NSLS	
	Iteration	CPU time (s)	Iteration	CPU time (s)
Liver tumour	1500	99.57	800	53.03
Cells	50	2.88	50	4.56
Noisy liver tumour	2000	255.34	1000	66.01
Noisy cells	200	13.19	50	4.56

Table 1: Comparison of the CPU times on images with and without noise.

From the comparisons, it can be seen clearly that the NFLS method has better performance on the image segmentation than the traditional level set method with high segmentation accuracy and low iteration time.

On noisy images, the NFLS segments the most objects with entire shape, while the performances of the traditional method are affected by the noise and some objects are divided into several regions. The results by the NSLS are smoother and more connected. Furthermore, the boundary’ position and orientation are more accurate. The outperformance benefits from the fact that the NSLS approach handles the indeterminacy of the images well and DAMF operation remove the effect of noise and other indeterminant information, and preserve the determinant information in NS domain.

4 Conclusion

In this paper, a novel image segmentation approach is proposed based on neutrosophic filtering and level set theory. The image is transformed into neutrosophic set domain, and described using three membership sets (*T, I* and *F*). The directional alpha-mean filter (DAMF) is employed to reduce the image's indeterminacy, and the image is segmented on the *T* subset after DAMF processing using level set algorithm. The experimental results show that the proposed method can perform better on clear images and noisy images, due to the fact that the proposed approach can handle the indeterminacy of the images well. The proposed method can be used widely in many image processing applications.

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Received: October 31, 2013. Accepted: November 17, 2013.

NSS

Neutrosophic Crisp Points & Neutrosophic Crisp Ideals

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Abstract. The purpose of this paper is to define the so called "neutrosophic crisp points" and "neutrosophic crisp ideals", and obtain their fundamental properties. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Crisp Point, Neutrosophic Crisp Ideal.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories, generalizing both their crisp and fuzzy counterparts. The idea of "neutrosophic set" was first given by Smarandache [12, 13]. In 2012 neutrosophic operations have been investigated by Salama at el. [4 - 10]. The fuzzy set was introduced by Zadeh [13]. The intuitionstic fuzzy set was introduced by Atanassov [1, 2, 3]. Salama at el. [9] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [11]. Here we shall present the crisp version of these concepts.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular the work of Smarandache in [12, 13], and Salama at el. [4 -10].

3 Neutrosophic Crisp Points

One can easily define a natural type of neutrosophic crisp set in X, called "neutrosophic crisp point" in X, corresponding to an element $p \in X$:

3.1 Definition

Let X be a nonempty set and $p \in X$. Then the neutrosophic crisp point p_N defined by $p_N = \left\langle \{p\}, \phi, \{p\}^c \right\rangle$ is called a neutrosophic crisp point (NCP for short) in X, where NCP is a triple ({only one element in X}, the empty set,{the complement of the same element in X}).

Neutrosophic crisp points in X can sometimes be inconvenient when expressing a neutrosophic crisp set in X in terms of neutrosophic crisp points. This situation will occur if $A = \left\langle A_1, A_2, A_3 \right\rangle$, and $p \notin A_1$, where A_1, A_2, A_3 are three subsets such that $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, $A_{21} \cap A_3 = \phi$. Therefore we define the vanishing neutrosophic crisp points as follows:

3.2 Definition

Let X be a nonempty set, and $p \in X$ a fixed element in X. Then the neutrosophic crisp set $p_{N_N} = \left\langle \phi, \{p\}, \{p\}^c \right\rangle$ is called "vanishing neutrosophic crisp point" (VNCP for short) in X, where VNCP is a triple (the empty set,{only one element in X},{the complement of the same element in X}).

3.1 Example

Let $X = \{a, b, c, d\}$ and $p = b \in X$. Then $p_N = \left\langle \{b\}, \phi, \{a, c, d\} \right\rangle$ Now we shall present some types of inclusions of a neutrosophic crisp point to a neutrosophic crisp set:

3.3 Definition

Let $p_N = \left\langle \{p\}, \phi, \{p\}^c \right\rangle$ be a NCP in X and $A = \left\langle A_1, A_2, A_3 \right\rangle$ a neutrosophic crisp set in X.

(a) p_N is said to be contained in A ($p_N \in A$ for short) iff $p \in A_1$.

(b) Let p_{N_N} be a VNCP in X, and $A = \left\langle A_1, A_2, A_3 \right\rangle$ a neutrosophic crisp set in X. Then p_{N_N} is said to be contained in A ($p_{N_N} \in A$ for short) iff $p \notin A_3$.

3.1 Proposition

Let $\{D_j : j \in J\}$ is a family of NCSs in X. Then

(a_1) $p_N \in \bigcap_{j \in J} D_j$ iff $p_N \in D_j$ for each $j \in J$.

(a_2) $p_{N_N} \in \bigcap_{j \in J} D_j$ iff $p_{N_N} \in D_j$ for each $j \in J$.

(b_1) $p_N \in \bigcup_{j \in J} D_j$ iff $\exists j \in J$ such that $p_N \in D_j$.

(b_2) $p_{N_N} \in \underset{j\in J}{\cap} D_j$ iff $\exists j \in J$ such that $p_{N_N} \in D_j$.

Proof
Straightforward.

3.2 Proposition

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$.

b) $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof
Obvious.

3.4 Proposition

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then

$A = (\cup \{p_N : p_N \in A\}) \cup (\cup \{p_{NN} : p_{NN} \in A\})$.

Proof

It is sufficient to show the following equalities:
$A_1 = (\cup\{p\ \} : p_N \in A\}) \cup (\cup\{\phi : p_{NN} \in A\})$, $A_3 = \phi$
and $A_3 = (\cap\{\{p\ \}^c : p_N \in A\}) \cap (\cap\{\{p\}^c : p_{NN} \in A\})$,
which are fairly obvious.

3.4 Definition

Let $f : X \to Y$ be a function.

(a) Let p_N be a nutrosophic crisp point in X. Then the image of p_N under f, denoted by $f(p_N)$, is defined by $f(p_N) = \langle \{q\}, \phi, \{q\}^c \rangle$, where $q = f(p\ )$.

(b) Let p_{NN} be a VNCP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \langle \phi, \{q\}, \{q\}^c \rangle$, where $q = f(p\ )$.

It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$, where $q = f(p\ )$, and it is exactly the same meaning of the image of a NCP under the function f.

$f(p_{NN})$ is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p\ )$.

3.4 Proposition

Any NCS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup\{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup\{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \langle A_1, A_2, A_3 \rangle$, then $\underset{N}{A} = \langle x, A_1, \phi, A_1^c \rangle$ and $\underset{NN}{A} = \langle x, \phi, A_2, A_3 \rangle$.

3.5 Proposition

Let $f : X \to Y$ be a function and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then we have $f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

4 Neutrosophic Crisp Ideal Subsets

4.1 Definition

Let X be non-empty set, and L a non–empty family of NCSs. We call L a neutrosophic crisp ideal (NCL for short) on X if

i. $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

ii. $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic crisp ideal L is called a σ-neutrosophic crisp ideal if $\{M_j\}_{j\in \mathrm{N}} \leq L$, implies $\underset{j\in J}{\cup} M_j \in L$ (countable additivity).

The smallest and largest neutrosophic crisp ideals on a non-empty set X are $\{\phi_N\}$ and the NSs on X. Also, NCL_f, NCL_c are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NCS : B \subseteq A\}$ is an NCL on X. This is called the principal NCL of all NCSs, denoted by NCL$\langle A \rangle$.

4.1 Remark

i. If $X_N \notin L$, then L is called neutrosophic proper ideal.

ii. If $X_N \in L$, then L is called neutrosophic improper ideal.

iii. $\varphi_N \in L$.

4.1 Example

Let $X = \{a, b, c\}$, $A = \langle \{a\}, \{a, b, c\}, \{c\} \rangle$, $B = \langle \{a\}, \{a\}, \{c\} \rangle$, $C = \langle \{a\}, \{b\}, \{c\} \rangle$, $D = \langle \{a\}, \{c\}, \{c\} \rangle$, $E = \langle \{a\}, \{a, b\}, \{c\} \rangle$, $F = \langle \{a\}, \{a, c\}, \{c\} \rangle$, $G = \langle \{a\}, \{b, c\}, \{c\} \rangle$. Then the family $L = \{ \phi_N, A, B, D, E, F, G \}$ of NCSs is an NCL on X.

4.2 Definition

Let L_1 and L_2 be two NCLs on X. Then L_2 is said to be finer than L_1, or L_1 is coarser than L_2, if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1, or L_1 is strictly coarser than L_2.

Two NCLs said to be comparable, if one is finer than the other. The set of all NCLs on X is ordered by the relation: L_1 is coarser than L_2; this relation is induced the inclusion in NCSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear. $L_j = \langle A_{j1}, A_{j2}, A_{j3} \rangle$.

4.1 Proposition

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic crisp ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are neutrosophic crisp ideals on X, where

$\bigcap_{j \in J} L_j = \left\langle \bigcap_{j \in J} A_{j1}, \bigcap_{j \in J} A_{j2}, \bigcup_{j \in J} A_{j3} \right\rangle$ or

$\bigcap_{j \in J} L_j = \left\langle \bigcap_{j \in J} A_{j1}, \bigcup_{j \in J} A_{j2}, \bigcup_{j \in J} A_{j3} \right\rangle$ and

$\bigcup_{j \in J} L_j = \left\langle \bigcup_{j \in J} A_{j1}, \bigcup_{j \in J} A_{j2}, \bigcap_{j \in J} A_{j3} \right\rangle$ or

$\bigcup_{j \in J} L_j = \left\langle \bigcup_{j \in J} A_{j_1}, \bigcap_{j \in J} A_{j_2}, \bigcap_{j \in J} A_{j_3} \right\rangle$.

In fact, L is the smallest upper bound of the sets of the L_j in the ordered set of all neutrosophic crisp ideals on X.

4,2 Remark

The neutrosophic crisp ideal defined by the single neutrosophic set ϕ_N is the smallest element of the ordered set of all neutrosophic crisp ideals on X.

4.2 Proposition

A neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ in the neutrosophic crisp ideal L on X is a base of L iff every member of L is contained in A.

Proof

(Necessity) Suppose A is a base of L. Then clearly every member of L is contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic crisp subsets in X contained in A coincides with L by the Definition 4.3.

4.3 Proposition

A neutrosophic crisp ideal L_1, with base $A = \langle A_1, A_2, A_3 \rangle$, is finer than a fuzzy ideal L_2 with base $B = \langle B_1, B_2, B_3 \rangle$, iff every member of B is contained in A.

Proof

Immediate consequence of the definitions.

4.1 Corollary

Two neutrosophic crisp ideals bases A, B, on X, are equivalent iff every member of A is contained in B and vice versa.

4.1 Theorem

Let $\eta = \langle A_{j1}, A_{j2}, A_{j3} \rangle : j \in J$ be a non-empty collection of neutrosophic crisp subsets of X. Then there exists a neutrosophic crisp ideal $L(\eta) = \left\{ A \in NCS : A \subseteq \bigcup_{j \in J} A_j \right\}$ on X for some finite collection $\{A_j : j = 1, 2, ..., n \subseteq \eta\}$.

Proof

It's clear.

4.3 Remark

The neutrosophic crisp ideal L(η) defined above is said to be generated by η and η is called sub-base of L(η).

4.2 Corollary

Let L_1 be a neutrosophic crisp ideal on X and A ∈ NCSs, then there is a neutrosophic crisp ideal L_2 which is finer than L_1 and such that A∈ L_2 iff $A \cup B \in L_2$ for each B∈ L_1.

Proof

It's clear.

4.2 Theorem

If $L = \{\phi_N, \langle A_1, A_2, A_3 \rangle\}$ is a neutrosophic crisp ideals on X, then:

i) $[\ ]L = \{\phi_N, \langle A_1, A_2, A_3^c \rangle\}$ is a neutrosophic crisp ideals on X.

ii) $\langle\ \rangle L = \{\phi_N, \langle A_3, A_2, A_1^c \rangle\}$ is a neutrosophic crisp ideals on X.

Proof

Obvious.

4.3 Theorem

Let $A = \langle A_1, A_2, A_3 \rangle \in L_1$, and $B = \langle B_1, B_2, B_3 \rangle \in L_2$, where L_1 and L_2 are neutrosophic crisp ideals on X, then A*B is a neutrosophic crisp set:

$A * B = \langle A_1 * B_1, A_2 * B_2, A_3 * B_3 \rangle$ where

$A_1 * B_1 = \cup\{\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle\}$,

$A_2 * B_2 = \cap\{\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle\}$ and

$A_3 * B_3 = \cap\{\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle\}$.

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Received: October 01, 2013. Accepted: December 01, 2013.

Several Similarity Measures of Neutrosophic Sets

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Abstract. Smarandache (1995) defined the notion of neutrosophic sets, which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set. In this paper, we first develop some similarity measures of neutrosophic sets. We will present a method to calculate the distance between neutrosophic sets (NS) on the basis of the Hausdorff distance. Then we will use this distance to generate a new similarity measure to calculate the degree of similarity between NS. Finally we will prove some properties of the proposed similarity measures.

Keywords- Neutrosophic Set, Matching Function, Hausdorff Distance, Similarity Measure.

1 Introduction

Smarandache introduced a concept of neutrosophic set which has been a mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data [1, 2].The concept of similarity is fundamentally important in almost every scientific field. Many methods have been proposed for measuring the degree of similarity between fuzzy sets (Chen, [11]; Chen et al., [12]; Hyung, Song, & Lee, [14]; Pappis& Karacapilidis, [10]; Wang, [13]...). But these methods are unsuitable for dealing with the similarity measures of neutrosophic set (NS). Few researchers have dealt with similarity measures for neutrosophic set and single valued neutrosophic set ([3, 4,17,18]), (i.e. the crisp neutrosophic sets, where the components T, I, F are all crisp numbers). Recently, Jun [3] discussed similarity measures on interval neutrosophic set (which an instance of NS) based on Hamming distance and Euclidean distance and showed how these measures may be used in decision making problems. Furthermore, A.A.Salama [4] defined the correlation coefficient, on the domain of neutrosophic sets, which is another kind of similarity measurement. In this paper we first extend the Hausdorff distance to neutrosophic set which plays an important role in practical application, especially in many visual tasks, computer assisted surgery and so on. After that a new series of similarity measures has been proposed for neutrosophic set using different approaches.

Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, psychology [5], handwriting recognition [6], decision making [7], coding theory etc.

This paper is organized as follows: Section2 briefly reviews the definition of Hausdorff distance and the neutrosophic set. Section 3 presents the new extended Hausdorff distance between neutrosophic sets. Section 4 provides the new series of similarity measure between neutrosophic sets, some of its properties are discussed. In section 5 a comparative study was done. Finally the section 6 outlines some conclusions.

2 Preliminaries

In this section we briefly review some definitions and examples which will be used in the rest of the paper.

2.1Definition: Hausdorff Distance

The Hausdorff distance (Nadler, 1978) is the maximum distance of a set to the nearest point in the other set. More formal description is given by the following

Given two finite sets $A = \{a_1, ..., a_p\}$ and $B = \{b_1, ..., b_q\}$, the Hausdorff distance H (A, B) is defined as:

$$H(A, B) = \max \{h(A, B), h(B, A)\} \quad (1)$$

where

$$H(A, B) = \max_{a \in A} \min_{b \in B} d(a, b) \quad (2)$$

a and b are elements of sets A and B respectively; d (a, b) is any metric between these elements.

The two distances h (A, B) and h (B, A) are called directed Hausdorff distances.

The function h (A, B) (the directed Hausdorff distance from A to B) ranks each element of A based on its distance to the nearest element of B, and then the largest ranked such element (the most mismatched element of A) specifies the value of the distance. Intuitively, if h(A, B) = c, then each element of A must be within distance c of some element of B, and there also is some element of A that is exactly distance c from the nearest element of B (the most mismatched element). In general h (A, B) and h (B, A) can attain very different values (the directed distances are not symmetric).

Let us consider the real space R, for any two intervals A= $[a_1,a_2]$ and B= $[b_1,b_2]$, the Hausdorff distance H(A,B) is given by

$$H(A, B) = \max\{|a_1 - b_1|, |a_2 - b_2|\} \quad (3)$$

2.2 Definition (see [2]). Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x: $T_{A(x)}, I_{A(x)}, F_{A(x)}$ >,x ∈ U}, where the functions T, I, F : U→$]^-0,1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ U to the set A with the condition.

$$^-0 \le T_{A(x)} + I_{A(x)} + F_{A(x)} \le 3^+. \quad (4)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

2.3 Definition (see [18]): Let X be a space of points (objects) with generic elements in X denoted by x (Wang et al., 2010). An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈[0, 1].

When X is continuous, an SVNS A can be written as

$$A=\int_X \frac{<T_A(x), I_A(x), F_A(x),>}{x}, x \in X. \quad (5)$$

When X is discrete, an SVNS A can be written as

$$A=\sum_1^n \frac{<T_A(x_i), I_A(x_i), F_A(x_i),>}{x_i}, x_i \in X. \quad (6)$$

2.4 Definition (see [2,18]). A neutrosophic set or single valued neutrosophic set (SVNS) A is contained in another neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$.

2.5 Definition (see [2]). The complement of a neutrosophic set A is denoted by A^c and is defined as $T_{A^c(x)} = F_{A(x)}$, $I_{A^c(x)} = I_{A(x)}$, and $F_{A^c(x)} = T_{A(x)}$ for every x in X.

A complete study of the operations and application of neutrosophic set can be found in [1] [2] [18].

In this paper we are concerned with neutrosophic sets whose T_A, I_A and F_A values are single points in [0, 1] instead of subintervals/subsets in [0, 1].

3 Extended Hausdorff Distance Between Two Neutrosophic Sets

Based on the Hausdorff metric, Eulalia Szmidt and Janusz Kacprzyk defined a new distance between intuitionistic fuzzy sets and/or interval-valued fuzzy sets in[8], taking into account three parameter representation (membership, non-membership values, and the hesitation margins) of A-IFSs which fulfill the properties of the Hausdorff distances. Their definition is defined by:

$$H_3(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\} \quad (7)$$

where A = {< x, $\mu_A(x)$, $\nu_A(x)$, $\pi_A(x)$ >} and B = {< x, $\mu_B(x)$, $\nu_B(x)$, $\pi_B(x)$>}.

The terms and symbols used in [8] are changed so that they are consistent with those in this section.

In this paper we are interested in extending the Hausdorff distance formulation in constructing a new distance for neutrosophic set due to its simplicity in the calculation.

Let X={x_1,x_2, …, x_n} be a discrete finite set. Consider a neutrosophic set A in X, where $T_{A(x_i)}$, $I_{A(x_i)}$, $F_{A(x_i)}$ ∈ [0, 1], for every x_i ∈ X, represent its membership, indeterminacy, and non-membership values respectively denoted by A = {< x, $T_{A(x_i)}$, $I_{A(x_i)}$, $F_{A(x_i)}$ >}.

Then we propose a new distance between A ∈ NS and B ∈ NS defined by

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\} \quad (8)$$

Where $d_H(A,B)$ = H (A, B) denote the extended Hausdorff distance between two neutrosophic sets A and B.

Let A, B and C be three neutrosophic sets. For all x_i ∈ X we have:

$$d_H(A,B) = H(A,B) = \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_{B(x_i)}|, |F_A(x_i) - F_B(x_i)|\} \quad (9)$$

The same between A and C are written as:

For all x_i ∈ X

H (A, C)

=

$\max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$ (10)

and between B and C is written as:

For all $x_i \in X$

H (B , C) = $\max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\}$ (11)

3.1 Proposition:

The above defined distance $d_H(A, B)$ between NS A and B satisfies the following properties (D1-D4):

(D1) $d_H(A, B) \geq 0$. (12)

(D2) $d_H(A, B)$ =0 if and only if A = B; for all A, B $\in$ NS. (13)

(D3) $d_H(A, B) = d_H(B, A)$. (14)

(D4) If A⊆B⊆C, C is an NS in X, then

$d_H(A, C) \geq d_H(A, B)$ (15)

And

$d_H(A, C) \geq d_H(B, C)$ (16)

Remark: Let A, B $\in$ NS, A⊆ B if and only if , for all x_i in X

$T_A(x_i) \leq T_B(x_i), I_A(x_i) \geq I_B(x_i), F_A(x_i) \geq F_B(x_i)$ (17)

It is easy to see that the defined measure $d_H(A, B)$ satisfies the above properties (D1)-(D3). Therefore, we only prove (D4).

Proof of (D4) for the extended Hausdorff distance between two neutrosophic sets. Since

A⊆ B ⊆ C implies , for all x_i in X

$T_A(x_i) \leq T_B(x_i) \leq T_C(x_i), I_A(x_i) \geq I_B(x_i) \geq I_C(x_i), F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$

We prove that $d_H(A, B) \leq d_H(A, C)$ (18)

α - If $|T_A(x_i) - T_C(x_i)| \geq |I_A(x_i) - I_C(x_i)| \geq |F_A(x_i) - F_C(x_i)|$ (19)

Then

H (A, C) = $|T_A(x_i) - T_C(x_i)|$ but we have

(i) For all x_i in X, $|I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)|$

(ii) (20)

$\leq |T_A(x_i) - T_C(x_i)|$

And , $\forall\ x_i \in X$

$|F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|$ (21)

$\leq |T_A(x_i) - T_C(x_i)|$

(iii) $\forall\ x_i \in X$ 55, 55

$|I_B(x_i) - I_C(x_i)| \leq |I_A(x_i) - I_C(x_i)|$ (22)

$\leq |T_A(x_i) - T_C(x_i)|$

And ,for all x_i in X

$|F_B(x_i) - F_C(x_i)| \leq |F_A(x_i) - F_C(x_i)|$

$\leq |T_A(x_i) - T_C(x_i)|$ (23)

On the other hand we have, $\forall\ x_i \in X$

(iv) $|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|$ (24)

and $|T_B(x_i) - T_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$

Combining (i), (ii), and (iii) we obtain

Therefore, for all x_i in X

$\frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}$

$\leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

And

$\frac{1}{n}\sum_1^n \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\}$

$\leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

That is

$d_H(A, B) \leq d_H(A, C)$ and $d_H(B, C) \leq d_H(A, C)$. (25)

β - If $|T_A(x_i) - T_C(x_i)| \leq |F_A(x_i) - F_C(x_i)| \leq |I_A(x_i) - I_C(x_i)|$ (26)

Then

H (A, C) = $|I_A(x_i) - I_C(x_i)|$ but we have $\forall\ x_i \in X$ (27)

(a) $|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|$ (28)

$\leq |I_A(x_i) - I_C(x_i)|$

And $|F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|$

$\leq |I_A(x_i) - I_C(x_i)|$

(b) $|T_B(x_i) - T_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$ (29)

$\leq |I_A(x_i) - I_C(x_i)|$

And $|F_B(x_i) - F_C(x_i)| \leq |F_A(x_i) - F_C(x_i)|$ (30)

$\leq |I_A(x_i) - I_C(x_i)|$

On the other hand we have $\forall\ x_i \in X$:

(c) $|I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)|$ and (31)

$|I_B(x_i) - I_C(x_i)| \leq |I_A(x_i) - I_C(x_i)|$

Combining (a) and (c) we obtain:
Therefore, $\forall\ x_i \in X$

$\frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}$
$\leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$
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And

$\frac{1}{n}\sum_1^n \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\}$

$\leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

That is

$d_H(A,B) \leq d_H(A,C)$ and $d_H(B,C) \leq d_H(A,C)$

(32)

γ - If
$|T_A(x_i) - T_C(x_i)| \leq |I_A(x_i) - I_C(x_i)| \leq |F_A(x_i) - F_C(x_i)|$
(33)

Then

H (A, C) = $|F_A(x_i) - F_C(x_i)|$ but we have for all x_i in X
(34)

(a) $|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|$
(35)

$\leq |F_A(x_i) - F_C(x_i)|$

and $|I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)|$ (36)

$\leq |F_A(x_i) - F_C(x_i)|$

(b) $\forall\ x_i \in X$ $|T_B(x_i) - T_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$
(37)

$\leq |F_A(x_i) - F_C(x_i)|$

and $\forall\ x_i \in X$ $|I_B(x_i) - I_C(x_i)| \leq |I_A(x_i) - I_C(x_i)|$
(38)

$\leq |F_A(x_i) - F_C(x_i)|$

On the other hand we have for all x_i in X

(c) $\forall\ x_i \in X$ $|F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|$
(39)
and
$|F_B(x_i) - F_C(x_i)| \leq |F_A(x_i) - F_C(x_i)|$
(40)

Combining (a), (b), and (c) we obtain

Therefore, for all x_i in X

$\frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}$

$\leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

.

And

$\frac{1}{n}\sum_1^n \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\}$

$\leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

That is

$d_H(A,B) \leq d_H(A,C)$ and $d_H(B,C) \leq d_H(A,C)$.
(41)

From α, β , and γ, we can obtain the property (D4).

3.2 Weighted Extended Hausdorff Distance Between Two Neutrosophic Sets.

In many situations the weight of the element $x_i \in X$ should be taken into account. Usually the elements have different importance. We need to consider the weight of the element so that we have the following weighted distance between NS. Assume that the weight of $x_i \in X$ is w_i where X={x_1, x_2,.., x_n}, $w_i \in [0,1]$, i={1,2,3,.., n} and $\sum_1^n w_i$ =1. Then the weighted extended Hausdorff distance between NS A and B is defined as:

$$d_{HW}(A,B) = \sum_1^n w_i\ d_H(A(x_i), B(x_i)) \quad (42)$$

It is easy to check that $d_{HW}(A,B)$ satisfies the four properties D1-D4 defined above.

4 Some new similarity measures for neutrosophic sets

The distance measure between two NS is used in finding the similarity between neutrosophic sets. We found in the literature different similarity measures, and we extend them to neutrosophic sets (NS), several of them defined below: Liu [9] also gave an axiom definition for the similarity measure of fuzzy sets, which also can be expressed for neutrosophic sets (NS) as follow:

4.1.Definition: Axioms of a Similarity Measure

A mapping S:NS(X)×NS(X)→[0,1], NS(X) denotes the set of all NS in X={x_1,x_2,…,x_n}, S(A, B) is said to be the degree of similarity between A∈ NS and B∈ NS, if S(A,B) satisfies the properties of conditions (P1-P4):

(P1) S (A, B) = S (B, A). (43)

(P2) S(A,B) = (1,0,0) = $\underline{1}$ .If A = B for all A,B ∈ NS. (44)

(P3) $S_T(A,B) \geq 0$, $S_I(A,B) \geq 0$, $S_F(A,B) \geq 0$. (45)

(P4) If A⊆B⊆C for all A, B, C ∈ NS, then S (A, B) ≥S (A, C) and S (B, C) ≥ S (A, C). (46)

Numerical Example:

Let A ≤ B ≤ C. with $T_A \leq T_B \leq T_C$ and $I_A \geq I_B \geq I_C$ and $F_A \geq F_B \geq F_C$ for each x_i∈ NS.

For example:

A= { x_1 (0.2, 0.5, 0.6); x_2 (0.2, 0.4, 0.4) }

B= { x_1 (0.2, 0.4, 0.4); x_2 (0.4, 0.2, 0.3) }

C= { x_1 (0.3, 0.3, 0.4); x_2 (0.5, 0.0, 0.3) }

In the following we define a new similarity measure of neutrosophic set and discuss its properties.

4.2 Similarity Measures Based on the Set – Theoretic Approach.

In this section we extend the similarity measure for intuitionistic and fuzzy set defined by Hung and Yung [16] to neutrosophic set which is based on set-theoretic approach as follow.

4.2.Definition: Let A,B be two neutrosophic sets in X={x_1,x_2,.., x_n}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {< x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of X in A and B respectively, then the similarity measure between the neutrosophic sets A and B can be evaluated by the function

For all x_i in X

$$S_T(A,B) = \left(\sum_1^N \left[\frac{\min(T_A(x_i),T_B(x_i))}{Max(T_A(x_i),T_B(x_i))}\right]\right)/n \quad (47)$$

$$S_I(A,B) = 1 - \left(\sum_1^N \left[\frac{\min(I_A(x_i),I_B(x_i))}{Max(I_A(x_i),I_B(x_i))}\right]\right)/n \quad (48)$$

$$S_F(A,B) = 1 - \left(\sum_1^N \left[\frac{\min(F_A(x_i),F_B(x_i))}{Max(F_A(x_i),F_B(x_i))}\right]\right)/n \quad (49)$$

and $S(A,B) = (S_T(A,B), S_I(A,B), S_F(A,B))$ (50)

where

$S_T(A,B)$ denote the degree of similarity (where we take only the T's).

$S_I(A,B)$ denote the degree of indeterminate similarity (where we take only the I's).

$S_F(A,B)$ denote degree of nonsimilarity (where we take only the F's).

Min denotes the minimum between each element of A and B.

Max denotes the minimum between each element of A and B.

Proof of (P4) for the (1).

Since A⊆B⊆C implies, for all x_i in X

$$T_A(x_i) \leq T_B(x_i) \leq T_C(x_i), I_A(x_i) \geq I_B(x_i) \geq I_C(x_i), F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$$

Then, for all x_i in X

$$\frac{\min(T_A(x_i),T_B(x_i))}{Max(T_A(x_i),T_B(x_i))} = \frac{T_A(x_i)}{T_B(x_i)} \quad (51)$$

$$\frac{\min(T_A(x_i),T_C(x_i))}{Max(T_A(x_i),T_C(x_i))} = \frac{T_A(x_i)}{T_C(x_i)} \quad (52)$$

$$\frac{\min(T_B(x_i),T_C(x_i))}{Max(T_B(x_i),T_C(x_i))} = \frac{T_B(x_i)}{T_C(x_i)} \quad (53)$$

Therefore, for all x_i in X

$$\frac{T_A(x_i)}{T_C(x_i)} = \frac{T_B(x_i)}{T_C(x_i)} + \frac{T_A(x_i)-T_B(x_i)}{T_C(x_i)} \leq \frac{T_B(x_i)}{T_C(x_i)} \quad (54)$$

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(since $T_A(x_i) \leq T_B(x_i)$)

Furthermore, for all x_i in X

$$\frac{\min(T_A(x_i),T_B(x_i))}{Max(T_A(x_i),T_B(x_i))} \geq \frac{\min(T_A(x_i),T_C(x_i))}{Max(T_A(x_i),T_C(x_i))} \quad (55)$$

Or

$$\frac{T_A(x_i)}{T_B(x_i)} \geq \frac{T_A(x_i)}{T_C(x_i)} \text{ or } T_B(x_i) \leq T_C(x_i) \quad (56)$$

(since $T_C(x_i) \geq T_B(x_i)$)

Inequality (53) implies that, for all x_i in X

$$\frac{T_A(x_i)}{T_C(x_i)} \leq \frac{T_A(x_i)}{T_B(x_i)} \quad (57)$$

From the inequalities (54) and (57), the property (P4) for $S_T(A,B) \geq S_T(A,C)$ is proven.

In a similar way we can prove that $S_I(A,B)$ and $S_F(A,B)$.

We will to prove that $S_I(A,C) \geq S_I(A,B)$. For all x_i∈ X we have:

$$S_I(A,C) = 1 - \frac{\min(I_A(x_i),I_C(x_i))}{Max(I_A(x_i),I_C(x_i))} = 1 - \frac{I_C(x_i)}{I_A(x_i)} \geq 1 - \frac{I_B(x_i)}{I_A(x_i)} \quad (58)$$

Since $I_C(x_i) \leq I_B(x_i)$

Similarly we prove $S_F(A,C) \geq S_F(A,B)$ for all x_i in X

(59)

$$S_F(A,C) = 1 - \frac{min(F_A(x_i),F_C(x_i))}{Max(F_A(x_i),F_C(x_i))} =$$
$$1 - \frac{F_C(x_i)}{F_A(x_i)} \geq 1 - \frac{F_B(x_i)}{F_A(x_i)}$$

(60)

Since $F_C(x_i) \leq F_B(x_i)$

Then S(A, C)≤S(A, B) where

S(A,C)=($S_T(A,C)$, $S_I(A,C)$, $S_F(A,C)$) and S (A, B) = ($S_T(A,B)$, $S_I(A,B)$, $S_F(A,B)$).

(61)

In a similar way we can prove that S (B, C) ≥ S (A, C). If A⊆B⊆C therefore S (A, B) satisfies (P4) of definition 4.1.

By applying (50), the degree of similarity between the neutrosophic sets (A, B), (A, C) and (B, C) are:

S(A, B) =$(S_T(A,B),\ S_I(A,B),\ S_F(A,B))$= (0.75, 0.35, 0.30)

S (A, C) = $(S_T(A,C),\ S_I(A,C),\ S_F(A,C))$= (0.53, 0.7, 0.30)

S (B, C) = $(S_T(B,C),\ S_I(B,C),\ S_F(B,C))$= (0.73, 0.63, 0)

Then (49) satisfies property P4: S(A, C) ≤ S(A, B) and S(A, C) ≤ S(B, C).

Usually, the weight of the element x_i ∈ X should be taken into account, then we present the following weighted similarity between NS. Assume that the weight of x_i ∈ X={1,2,…,n} is w_i (i=1,2,…, n) when $w_i \in [0,1]$, $\sum_1^n w_i = 1$.

Denote $S_w^T(A,B) = (\sum_1^N w_i \left[\frac{min(T_A(x_i),T_B(x_i))}{Max(T_A(x_i),T_B(x_i))}\right])/n$

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(62)

$$S_w^I(A,B) = 1 - (\sum_1^N w_i \left[\frac{min(I_A(x_i),I_B(x_i))}{Max(I_A(x_i),I_B(x_i))}\right])/n$$

(63)

$$S_w^F(A,B) = 1 - (\sum_1^N w_i \left[\frac{min(F_A(x_i),F_B(x_i))}{Max(F_A(x_i),F_B(x_i))}\right])/n$$

(64)

and $S_w(A,B) = (\ S_w^T((A,B),\ S_w^I((A,B),\ S_w^F((A,B))$

(65)

It is easy to check that $S_w(A,B)$ satisfies the four properties P1-P4 defined above.

4.3 Similarity Measure Based on the Type1 Geometric Distance Model

In the following, we express the definition of similarity measure between fuzzy sets based on the model of geometric distance proposed by Pappis and Karacapilidis in [10] to similarity of neutrosophic set.

4.3.Definition: Let A,B be two neutrosophic sets in X={x_1, x_2,..., x_n}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {< x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of X in A and B respectively, then the similarity measure between the neutrosophic sets A and B can be evaluated by the function

For all x_i in X

$$L_T(A,B) = 1 - \frac{\sum_1^n |T_A(x_i) - T_B(x_i)|}{\sum_1^n (T_A(x_i) + T_B(x_i))}$$

(66)

$$L_I(A,B) = \frac{\sum_1^n |I_A(x_i) - I_B(x_i)|}{\sum_1^n (I_A(x_i) + I_B(x_i))}$$

(67)

$$L_F(A,B) = \frac{\sum_1^n |F_A(x_i) - F_B(x_i)|}{\sum_1^n (F_A(x_i) + F_B(x_i))}$$

(68)

and

$$L(A,B) = (L_T(A,B),\ L_I(A,B),\ L_F(A,B))$$

(69)

We will prove this similarity measure satisfies the properties 1-4 as above. The property (P1) for the similarity measure (69) is obtained directly from the definition 4.1.

Proof: obviously, (68) satisfies P1-P3-P4 of definition 4.1. In the following L (A, B) will be proved to satisfy (P2) and (P4).

Proof of (P2) for the (69)

For all x_i in X

First of all, $L_T(A,B) = 1 \leftrightarrow \frac{\sum_1^n |T_A(x_i) - T_B(x_i)|}{\sum_1^n (T_A(x_i) + T_B(x_i))} = 0$

(70)

$$\leftrightarrow |T_A(x_i) - T_B(x_i)| = 0$$
$$\leftrightarrow T_A(x_i) = T_B(x_i)$$

$$L_I(A,B) = 0 \leftrightarrow \frac{\sum_1^n |I_A(x_i) - I_B(x_i)|}{\sum_1^n (I_A(x_i) + I_B(x_i))} = 0$$

(71)

$$\leftrightarrow |I_A(x_i) - I_B(x_i)| = 0 \quad \leftrightarrow I_A(x_i) = I_B(x_i)$$

$$L_F(A,B) = 0 \leftrightarrow \frac{\sum_1^n |F_A(x_i) - F_B(x_i)|}{\sum_1^n (F_A(x_i) + F_B(x_i))} = 0$$

(72)

$$\leftrightarrow |F_A(x_i) - F_B(x_i)| = 0 \leftrightarrow F_A(x_i) = F_B(x_i)$$

Then $L(A,B) = (L_T(A,B),\ L_I(A,B),\ L_F(A,B))$ = (1, 0, 0) if A=B for all A, B ∈ NS.

(73)

Proof of P3 for the (69) is obvious.

By applying (69) the degree of similarity between the neutrosophic sets (A, B), (A, C) and (B, C) are:

L (A, B) =$(L_T(A,B), L_I(A,B), L_F(A,B))$= (0.8, 0.2, 0.17).
L (A, C) = $(L_T(A,C), L_I(A,C), L_F(A,C))$= (0.67, 0.5, 0.17).
L (B, C) = $(L_T(B,C), L_I(B,C), L_F(B,C))$= (0.85, 0.33, 0).

The result indicates that the degree of similarity between neutrosophic sets A and B ∈ [0, 1]. Then (69) satisfies property P4: L(A, C) ≤ L(A, B) and L(A, C) ≤ L(B, C).

4.4 Similarity Measure Based on the Type 2 Geometric Distance model

In this section we extend the similarity measure proposed by Yang and Hang [16] to neutrosophic set as follow:

4.4.Definition: Let A, B be two neutrosophic set in X={$x_1, x_2, .., x_n$}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {<x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of X in A and B respectively, then the similarity measure between the neutrosophic set A and B can be evaluated by the function:

For all x_i in X

$$M_T(A, B) = \frac{1}{n}\sum_1^n \left(1 - \frac{|T_A(x_i) - T_B(x_i)|}{2}\right). \quad (74)$$

$$M_I(A, B) = \frac{1}{n}\sum_1^n \left(\frac{|I_A(x_i) - I_B(x_i)|}{2}\right). \quad (75)$$

$$M_F(A, B) = \frac{1}{n}\sum_1^n \left(\frac{|F_A(x_i) - F_B(x_i)|}{2}\right).$$

And

$$M_{T,I,F} = (M_T(A,B), M_I(A,B), M_F(A,B)) \quad \text{for}$$

all i={$x_1, x_2, ..., x_n$} (76)

The proofs of the properties P1-P2-P3 in definition 4.1 (Axioms of a Similarity Measure) of the similarity measure in definition 4.4 are obvious.

Proof of (P4) for the (76).
Since for all x_i in X

$$T_A(x_i) \le T_B(x_i) \le T_C(x_i), I_A(x_i) \ge I_B(x_i) \ge I_C(x_i), F_A(x_i) \ge F_B(x_i) \ge F_C(x_i)$$

Then for all x_i in X

$$1 - \frac{|T_C(x_i) - T_A(x_i)|}{2} = 1 - \frac{(T_C(x_i) - T_A(x_i))}{2} \quad (77)$$

$$= 1 - \left(\frac{(T_C(x_i) - T_B(x_i))}{2} + \frac{(T_B(x_i) - T_A(x_i))}{2}\right)$$

$$\le 1 - \left(\frac{(T_C(x_i) - T_B(x_i))}{2}\right)$$

$$= 1 - \frac{|T_C(x_i) - T_B(x_i)|}{2}$$

Then M_T(A, C) ≤ M_T(B, C). (78)

Similarly, M_T(A, C) ≤ M_T(A, B) can be proved easily.

For M_I(A, C) ≥ M_I(B, C) and M_F(A, C) ≥ M_F(B, C) the proof is easy.

Then by the definition 4.4, (P4) for definition 4.1, is satisfied as well.

By applying (76), the degree of similarity between the neutrosophic sets (A, B), (A, C) and (B, C) are:

M(A, B)=(M_T (A,B), M_I (A,B), M_F (A,B))=(0.95 , 0.075 , 0.075)
M(A, C)= (M_T (A,C), M_I (A,C), M_F (A,C))=(0.9, 0.15 , 0.075)
M(B, C)= (M_T (B,C), M_I (B,C), M_F (B,C))=(0.9, 0.075 , 0)

Then (76) satisfies property P4**:**
M (A, C) ≤ M (A, B) and M (A, C) ≤ M (B, C). (79)

Another way of calculating similarity (degree) of neutrosophic sets is based on their distance. There are more approaches on how the relation between the two notions in form of a function can be expressed. Two of them are presented below (in section 4.5 and 4.6).

4.5 Similarity Measure Based on the Type3 Geometric Distance Model.

In the following we extended the similarity measure proposed by Koczy in [15] to neutrosophic set (NS).

4.5.Definition: Let A, B be two neutrosophic sets in X={$x_1, x_2, .., x_n$}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {< x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of x in A and B respectively, then the similarity measure between the neutrosophic sets A and B can be evaluated by the function

$\mathbf{H_T(A,B) = \frac{1}{1+d_\infty^T(AB)}}$ denotes the degree of similarity. (80)

$\mathbf{H_I(A,B) = 1 - \frac{1}{1+d_\infty^I(AB)}}$ denotes the degree of indeterminate similarity. (81)

$d_{\infty}^{T}(A,B) = \max\{|T_A(x_i) - T_B(x_i)|\}$.
(83)

$d_{\infty}^{I}(A,B) = \max\{|I_A(x_i) - I_B(x_i)|\}$.
(84)

$d_{\infty}^{F}(A,B) = \max\{|F_A(x_i) - F_B(x_i)|\}$.
(85)

and H (A, B) = ($H_T(A,B), H_I(A,B), H_F(A,B)$).
(86)

By applying the (86) in numerical example we obtain:

$d_{\infty}(A,B)$= (0.2, 0.2, 0.2), then H (A, B) = (0.83, 0.17, 0.17).

$d_{\infty}(A,C)$= (0.3, 0.4, 0.1), then H (A, C) = (0.76, 0.29, 0.17).

$d_{\infty}(B,C)$= (0.1, 0.2, 0), then H (B, C) = (0.90, 0.17, 0).

It can be verified that H (A, B) also has the properties (P1)-(P4).

4.6 Similarity Measure Based on Extended Hausdorff Distance

It is well known that similarity measures can be generated from distance measures. Therefore, we may use the proposed distance measure based on extended Hausdorff distance to define similarity measures. Based on the relationship of similarity measures and distance measures, we can define a new similarity measure between NS A and B as follows:

$$N(A,B) = 1 - d_H(A,B) \qquad (87)$$

Where $d_H(A,B)$ represent the extended Hausdorff distance between neutrosophic sets (NS) A and B.

According to the above distance properties (D1-D4).It is easy to check that the similarity measure (87) satisfies the four properties of axiom similarity defined in 4.1

By applying the (87) in numerical example we obtain:

$N(A,B)$ =0.8

$N(A,C)$ =0.7

$N(B,C)$ =0.85

Then (5) satisfies property P4:

N(A, C) ≤ N(A, B) and N(A, C) ≤ N(B, C)

Remark: It is clear that the larger the value of N(A, B), the more the similarity between NS A and B.

Next we define similarity measure between NS A and B using a matching function.

4.7 Similarity Measure of two Neutrosophic Sets Based on Matching Function.

Chen [11] and Chen et al. [12] introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, we extend the matching function to deal with the similarity measure of NS.

4.7 Definition Let F and E be two neutrosophic sets over U. Then the similarity between them, denoted by K (F, G) or $K_{F,G}$ has been defined based on the matching function as:

For all x_i in X

$$K(F,G) = K_{F,G} = \frac{\sum_1^n (T_F(x_i)\cdot T_G(x_i) + I_F(x_i)\cdot I_G(x_i) + F_F(x_i)\cdot F_G(x_i))}{\max(\sum_1^n ((T_F(x_i))^2 + (I_F(x_i))^2 + (F_F(x_i))^2), \sum_1^n ((T_G(x_i))^2 + (I_G(x_i))^2 + (F_G(x_i))^2))} \qquad (88)$$

Considering the weight $w_j \in [0, 1]$ of each element $x_i \in X$, we get the weighting similarity measure between NS as:

For all x_i in X

$$K_w(F,G) = \frac{\sum_1^n w_i (T_F(x_i)\cdot T_G(x_i) + I_F(x_i)\cdot I_G(x_i) + F_F(x_i)\cdot F_G(x_i))}{\max(\sum_1^n w_i ((T_F(x_i))^2 + (I_F(x_i))^2 + (F_F(x_i))^2), \sum_1^n w_i ((T_G(x_i))^2 + (I_G(x_i))^2 + (F_G(x_i))^2))} \qquad (89)$$

If each element $x_i \in$ X has the same importance, then (89) is reduced to (88). The larger the value of $K(F,G)$ the more the similarity between F and G. Here $K(F,G)$ has all the properties described as listed in the definition 4.1.

By applying the (88) in numerical example we obtain:

$K(A,B)$ = 0.75, $K(A,C)$ = 0.66, and $K(B,C)$ = 0.92

Then (87) satisfies property P4: K(A, C) ≤ K(A, B) and K(A, C) ≤ K(B, C)

2 Comparision of various similarity measures

In this section, we make a comparison among similarity measures proposed in the paper. Table 1 show the comparison of various similarity measures between two neutrosophic sets respectively.

	A, B	A, C	B, C
(50)	(0.75, 0.35, 0.3)	(0.53, 0.7, 0.3)	(0.73, 0.63, 0)
(69)	(0.8, 0.2,0.17)	(0.67, 0.5, 0.17)	(0.85, 0.33, 0)
(76)	(0.95, 0.075, 0.075)	(0.9, 0.15, 0.075)	(0.9, 0.075, 0)
(86)	(0.83, 0.17, 0.17)	(0.76, 0.29, 0.17)	(0.9, 0.17, 0)
(87)	0.8	0.7	0.85
(88)	0.75	0.66	0.92

Table 1: Example results obtained from the similarity measures between neutrosophic sets A , B and C.

Each similarity measure expression has its own measuring. They all evaluate the similarities in neutrosophic sets, and they can meet all or most of the properties of similarity measure.

(87)	0.8	0.7	0.85
(88)	0.75	0.66	0.92

Table 1: Example results obtained from the similarity measures between neutrosophic sets A , B and C.

Each similarity measure expression has its own measuring. They all evaluate the similarities in neutrosophic sets, and they can meet all or most of the properties of similarity measure.

In definition 4.1, that is P1-P4. It seems from the table above that from the results of similarity measures between neutrosophic sets can be classified in two type of similarity measures: the first type which we called “crisp similarity measure” is illustrated by similarity measures (N and K) and the second type called “neutrosophic similarity measures” illustrated by similarity measures (S, L, M and H). The computation of measure **H , N** and **S** are much simpler than that of **L, M** and **K.**

Conclusions

In this paper we have presented a new distance called "extended Hausdorff distance for neutrosophic sets" or "neutrosophic Hausdorff distance". Then, we defined a new series of similarity measures to calculate the similarity between neutrosophic sets. It’s hoped that our findings will help enhancing this study on neutrosophic set for researchers.

Acknowledgements

The authors are thankful to the anonymous referee for his valuable and constructive remarks that helped to improve the clarity and the completeness of this paper.

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Received: October 25th, 2013. Accepted: November 3rd, 2013.

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An Extended TOPSIS Method for the Multiple Attribute Decision Making Problems Based on Interval Neutrosophic Set

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Abstract. The interval neutrosophic set (INS) can be easier to express the incomplete, indeterminate and inconsistent information, and TOPSIS is one of the most commonly used and effective method for multiple attribute decision making, however, in general, it can only process the attribute values with crisp numbers. In this paper, we have extended TOPSIS to INS, and with respect to the multiple attribute decision making problems in which the attribute weights are unknown and the attribute values take the form of INSs, we proposed an expanded TOPSIS method. Firstly, the definition of INS and the operational laws are given, and distance between INSs is defined. Then, the attribute weights are determined based on the Maximizing deviation method and an extended TOPSIS method is developed to rank the alternatives. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: interval neutrosophic set; TOPSIS; multiple attribute decision making; Maximizing deviation method; Hamming distance.

1 Introduction

In real decision making, there exist many multi-criteria decision-making (MCDM) problems. Because of the ambiguity of people's thinking and the complexity of objective things, the attribute values of the MCDM problems cannot always be expressed by crisp numbers, and it maybe is easier to be described by fuzzy information. The fuzzy set (FS) theory, which is proposed by Zadeh [1], is one of the most effective tools for processing fuzzy information; however, its disadvantage is that it only has a membership, and is unable to express non-membership. On the basis of FS, Atanassov [2,3] proposed the intuitionistic fuzzy set (IFS) by adding a non-membership function, i.e., there are membership (or called truth-membership) $T_A(x)$ and non-membership (or called falsity-membership) $F_A(x)$ in intuitionistic fuzzy sets, and they satisfy the conditions $T_A(x), F_A(x) \in [0,1]$ and $0 \le T_A(x)+F_A(x) \le 1$. Further, Atanassov and Gargov [4], Atanassov [5] proposed the interval-valued intuitionistic fuzzy set (IVIFS) by extending the truth-membership function and falsity-membership function to interval numbers. IFSs and IVIFSs can only handle incomplete information not the indeterminate information and inconsistent information. In IFSs, the indeterminacy is $1\text{-}T_A(x)\text{-}F_A(x)$ by default. However, in practice, the decision information is often incomplete, indeterminate and inconsistent information. In order to process this kind of information, Smarandache [6] further proposed the neutrosophic set (NS) by adding an independent indeterminacy-membership on the basis of IFS, which is a generalization of fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and so on. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent.

Recently, NSs have become an interesting research topic and attracted widely attentions. Wang et al. [7] proposed a single valued neutrosophic set (SVNS) from scientific or engineering point of view, which is an instance of the neutrosophic set. Ye [8] proposed the correlation coefficient and weighted correlation coefficient for SVNSs, and he have proved that the cosine similarity degree is a special case of the correlation coefficient in SVNS. Ye [8a] proposed Single valued neutrosophic cross-entropy for multicriteria decision making problems. Similar to IVIFS, Wang et al. [9] proposed interval neutrosophic sets (INSs) in which the truth-membership, indeterminacy-membership, and false-membership were extended to interval numbers, and discussed some properties and comparing method of INSs. Ye [10] proposed the similarity measures between INSs based on the Hamming and Euclidean distances, and developed a multicriteria decision-making method based on the similarity degree. However, so far, there has been no research on extending TOPSIS for INSs.

TOPSIS (The Order Performance technique based on Similarity to Ideal Solution), which was proposed by Hwang and Yoon [11] is one of popular decision making methods. In last 20 years, many researchers have extended this method and proposed different modifications, and it has been applied usefully in the practice to solve many problems in different fields for decision makers.

Chen [12] extended the TOPSIS for group decision making problems in which the importance weights of various criteria and ratings of alternatives with respect to these criteria take the form of linguistic variables. The key of the proposed method is that these variables are transformed into triangular fuzzy numbers. Jin et al. [13] extended TOPSIS method to MADM problems in which the attribute values are the intuitionistic fuzzy sets, and applied it to the evaluation of human resources. Wei and Liu [14] extended TOPSIS method to the uncertain linguistic variables, and applied it to the risk evaluation of High-technology. Liu [15] proposed an extended TOPSIS method to resolve the multi-attribute decision-making problems in which the attribute weights and attribute values are all interval vague value. Firstly, the ideal and negative ideal solutions are calculated based on the score function. Then the distance between the interval Vague values is defined, and the distances between each alternative and the ideal and negative ideal solutions are calculated. The relative closeness degree is calculated by TOPSIS method, and then the ordering of the alternatives is confirmed according to the relative closeness degree. Liu and Su [16] proposed an extended TOPSIS based on trapezoid fuzzy linguistic variables, and gave the method for determining attribute weights. Liu [17] proposed an extended TOPSIS method for multiple attribute group decision making based on generalized interval-valued trapezoidal fuzzy numbers. Mohammadi et al. [18] used fuzzy group TOPSIS method for selecting adequate security mechanisms in e-business processes. Verma et al. [19] proposed an interval-valued intuitionistic fuzzy TOPSIS method for solving a facility location problem.

Obviously, because TOPSIS is an important decision making method, and the interval neutrosophic set can be easier to express the incomplete, indeterminate and inconsistent information, it is important to establish an extended TOPSIS method based on INS. In this paper, we will establish an extended TOPSIS method for the multiple attribute decision making problems in which the attribute weights are unknown and attribute values take the form of INSs. In order to do so, the remainder of this paper is shown as follows. In section 2, we briefly review some basic concepts and operational rules of INS and propose the Hamming distance and the Euclidian distance between interval neutrosophic values (INVs) or interval neutrosophic sets, and give a proof of Hamming distance and a calcualtion example. In Section 3, we propose a method for determining the attribute weights based on the Maximizing deviation method and extend the TOPSIS method to rank the alternatives, and give the detail decision steps. In Section 4, we give an example to illustrate the application of proposed method, and compare the developed method with the existing method. In Section 5, we conclude the paper.

2 The Interval Neutrosophic Set

2.1 The Definition of the Interval Neutrosophic Set

Definition 1 [6]. Let X be a universe of discourse, with a generic element in X denoted by x. A neutrosophic set (NS) A in X is

$$A = \{x(T_A(x), I_A(x), F_A(x)) \mid x \in X\} \tag{1}$$

where, T_A, I_A and F_A are the truth-membership function, indeterminacy-membership function, and the falsity-membership function, respectively. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $\left]0^-, 1^+\right[$.

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

The NS was presented from philosophical point of view. Obviously, it was difficult to use in the actual applications. Wang [7] further proposed the single valued neutrosophic set (SVNS) from scientific or engineering point of view, which is a generalization of the existing fuzzy sets, such as classical set, fuzzy set, intuitionistic fuzzy set and paraconsistent sets etc., and it was defined as follows.

Definition 2 [7]. Let X be a universe of discourse, with a generic element in X denoted by x. A single valued neutrosophic set A in X is

$$A = \{x(T_A(x), I_A(x), F_A(x)) \mid x \in X\} \tag{2}$$

where, T_A, I_A and F_A are the truth-membership function, indeterminacy-membership function, and the falsity-membership function, respectively. For each point x in X, we have $T_A(x), I_A(x), F_A(x) \in [0,1]$, and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

In the actual applications, sometimes, it is not easy to express the truth-membership, indeterminacy-membership and falsity-membership by crisp values, and they may be easier to be expressed by interval numbers. Wang et al. [9] further defined interval neutrosophic sets (INSs) shown as follows.

Definition 3 [7]. Let X be a universe of discourse, with a generic element in X denoted by x. A interval neutrosophic set A in X is

$$A = \{x\langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \quad (3)$$

where, T_A, I_A and F_A are the truth-membership function, indeterminacy-membership function, and the falsity-membership function, respectively. For each point x in X, we have $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$, and $0 \le \sup(T_A(x)) + \sup(I_A(x)) + \sup(F_A(x)) \le 3$.

For convenience, we can use $x = ([T^L, T^U],[I^L, I^U],[F^L, F^U])$ to represent a value in INS, and call interval neutrosophic value (INV).

2.2 The Operational Rules of the Interval Neutrosophic Values

Definition 4. Let $x = ([T_1^L, T_1^U],[I_1^L, I_1^U],[F_1^L, F_1^U])$ and $y = ([T_2^L, T_2^U],[I_2^L, I_2^U],[F_2^L, F_2^U])$ be two INVs, then the operational rules are defined as follows.

(1) The complement of x is

$$\overline{x} = ([F_1^L, F_1^U],[1 - I_1^U, 1 - I_1^L],[T_1^L, T_1^U]) \quad (4)$$

(2)
$$x \oplus y = \left(\left[T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U\right], \left[I_1^L I_2^L, I_1^U I_2^U\right], \left[F_1^L F_2^L, F_1^U F_2^U\right]\right) \quad (5)$$

(3)
$$x \otimes y = \left([T_1^L T_2^L, T_1^U T_2^U],[I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U],\left[F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U\right]\right) \quad (6)$$

(4)
$$nx = \left(\left[1 - (1 - T_1^L)^n, 1 - (1 - T_1^U)^n\right], \left[(I_1^L)^n, (I_1^U)^n\right], \left[(F_1^L)^n, (F_1^U)^n\right]\right) \; n > 0 \quad (7)$$

(5)
$$x^n = \left(\left[(T_1^L)^n, (T_1^U)^n\right], \left[1 - (1 - I_1^L)^n, 1 - (1 - I_1^U)^n\right], \left[1 - (1 - F_1^L)^n, 1 - (1 - F_1^U)^n\right]\right) \; n > 0 \quad (8)$$

2.2 The Distance between two INSs

In the following, we will discuss the distance between two INSs.

Definition 5. Let $x = ([T_1^L, T_1^U],[I_1^L, I_1^U],[F_1^L, F_1^U])$, $y = ([T_2^L, T_2^U],[I_2^L, I_2^U],[F_2^L, F_2^U])$ and $z = ([T_3^L, T_3^U],[I_3^L, I_3^U],[F_3^L, F_3^U])$ be three INVs, S be a collection of all INVs, and f be a mapping with $f : \hat{S} \times \hat{S} \to R$. If $d(x, y)$ meets the following conditions.

(1) $0 \le d(x, y) \le 1$, $d(x, x) = 0$

(2) $d(x, y) = d(y, x)$

(3) $d(x, y) + d(y, z) \ge d(x, z)$

Then we can call $d(x, y)$ a distance between twoINVs x and y.

Definition 6. Let $x = ([T_1^L, T_1^U],[I_1^L, I_1^U],[F_1^L, F_1^U])$, and $y = ([T_2^L, T_2^U],[I_2^L, I_2^U],[F_2^L, F_2^U])$ be two INVs, then

(1) The Hamming distance between x and y is defined as follows

$$d_H(x, y) = \frac{1}{6}\left(\left|T_1^L - T_2^L\right| + \left|T_1^U - T_2^U\right| + \left|I_1^L - I_2^L\right| + \left|I_1^U - I_2^U\right| + \left|F_1^L - F_2^L\right| + \left|F_1^U - F_2^U\right|\right) \quad (9)$$

Proof.

Obviously, (9) can meet the above conditions (1) and (2) in Definiation 5.

In the following, we will prove (9) meets condition (3).

For any an INV $z = ([T_3^L, T_3^U],[I_3^L, I_3^U],[F_3^L, F_3^U])$, we have

$$\begin{aligned} d_H(x, z) &= \frac{1}{6}\left(\left|T_1^L - T_3^L\right| + \left|T_1^U - T_3^U\right| + \left|I_1^L - I_3^L\right| + \left|I_1^U - I_3^U\right| + \left|F_1^L - F_3^L\right| + \left|F_1^U - F_3^U\right|\right) \\ &= \frac{1}{6}\left(\left|T_1^L - T_2^L + T_2^L - T_3^L\right| + \left|T_1^U - T_2^U + T_2^U - T_3^U\right| + \left|I_1^L - I_2^L + I_2^L - I_3^L\right| + \left|I_1^U - I_2^U + I_2^U - I_3^U\right| \right. \\ &\quad \left. + \left|F_1^L - F_2^L + F_2^L - F_3^L\right| + \left|F_1^U - F_2^U + F_2^U - F_3^U\right|\right) \\ &\le \frac{1}{6}\left(\left|T_1^L - T_2^L\right| + \left|T_2^L - T_3^L\right| + \left|T_1^U - T_2^U\right| + \left|T_2^U - T_3^U\right| + \left|I_1^L - I_2^L\right| + \left|I_2^L - I_3^L\right| + \left|I_1^U - I_2^U\right| + \left|I_2^U - I_3^U\right| \right. \\ &\quad \left. + \left|F_1^L - F_2^L\right| + \left|F_2^L - F_3^L\right| + \left|F_1^U - F_2^U\right| + \left|F_2^U - F_3^U\right|\right) \end{aligned}$$

and

$$\frac{1}{6}\Big(\left|T_1^L-T_2^L\right|+\left|T_2^L-T_3^L\right|+\left|T_1^U-T_2^U\right|+\left|T_2^U-T_3^U\right|$$
$$+\left|I_1^L-I_2^L\right|+\left|I_2^L-I_3^L\right|+\left|I_1^U-I_2^U\right|+\left|I_2^U-I_3^U\right|$$
$$+\left|F_1^L-F_2^L\right|+\left|F_2^L-F_3^L\right|+\left|F_1^U-F_2^U\right|+\left|F_2^U-F_3^U\right|\Big)$$
$$=\frac{1}{6}\Big(\left|T_1^L-T_2^L\right|+\left|T_1^U-T_2^U\right|+\left|I_1^L-I_2^L\right|+\left|I_1^U-I_2^U\right|$$
$$+\left|F_1^L-F_2^L\right|+\left|F_1^U-F_2^U\right|\Big)$$
$$+\frac{1}{6}\Big(\left|T_2^L-T_3^L\right|+\left|T_2^U-T_3^U\right|+\left|I_2^L-I_3^L\right|+\left|I_2^U-I_3^U\right|$$
$$+\left|F_2^L-F_3^L\right|+\left|F_2^U-F_3^U\right|\Big)$$
$$=d_H(x,y)+d_H(y,z)$$

i.e., $d_H(x,y)+d_H(y,z)\ge d_H(x,z)$.

(2) The Euclidian distance between x and is defined as follows.

$$d_E(x,y)=\sqrt{\frac{1}{6}\Big(\left(T_1^L-T_2^L\right)^2+\left(T_1^U-T_2^U\right)^2+\left(I_1^L-I_2^L\right)^2+\left(I_1^U-I_2^U\right)^2+\left(F_1^L-F_2^L\right)^2+\left(F_1^U-F_2^U\right)^2\Big)} \quad (10)$$

The proof is similar to that of (9), it is omitted here.

Further, we extend the distance between two INVs x and y to two INSs.

Definition 7 Let $X=([T_i^L,T_i^U],[I_i^L,I_i^U],[F_i^L,F_i^U])$ $(i=1,2,\cdots,n)$ and $Y=([\dot{T}_i^L,\dot{T}_i^U],[\dot{I}_i^L,\dot{I}_i^U],[\dot{F}_i^L,\dot{F}_i^U])$ $(i=1,2,\cdots,n)$ be two INSs, then

(1) The Hamming distance between X and Y is defined as follows

$$d_H(X,Y)=\frac{1}{6n}\sum_{i=1}^{n}\Big(\left|T_i^L-\dot{T}_i^L\right|+\left|T_i^U-\dot{T}_i^U\right|+\left|I_i^L-\dot{I}_i^L\right|+\left|I_i^U-\dot{I}_i^U\right|+\left|F_i^L-\dot{F}_i^L\right|+\left|F_i^U-\dot{F}_i^U\right|\Big)$$

(11)

(2) The Euclidian distance between X and Y is defined as follows

$$d_E(X,Y)=\sqrt{\frac{1}{6n}\sum_{i=1}^{n}\Big(\left(T_i^L-\dot{T}_i^L\right)^2+\left(T_i^U-\dot{T}_i^U\right)^2+\left(I_i^L-\dot{I}_i^L\right)^2+\left(I_i^U-\dot{I}_i^U\right)^2+\left(F_i^L-\dot{F}_i^L\right)^2+\left(F_i^U-\dot{F}_i^U\right)^2\Big)} \quad (12)$$

For example, if two INSs X and Y are (([0.5,0.6],[0.2,0.3],[0.9,0.9]), ([0.8,0.9],[0.4,0.4],[0.2,0.3]), ([0.3,0.4],[0.8,0.9],[0.7,0.8])) and (([0.7,0.8],[0.4,0.5],[0.2,0.3]),([0.5,0.6],[0.5,0.5],[0.3,0.4]),([0.1,0.2],[0.2,0.4],[0.3,0.4])), then the distances of Hamming and Euclidian between X and Y can be calculated as follows.

$$d_H(X,Y)=\frac{1}{6\times3}\big(|0.5-0.7|+|0.6-0.8|+|0.2-0.4|$$
$$+|0.3-0.5|+|0.9-0.2|+|0.9-0.3|+|0.8-0.5|+|0.9-0.6|$$
$$+|0.4-0.5|+|0.4-0.5|+|0.2-0.3|+|0.3-0.4|+|0.3-0.1|$$
$$+|0.4-0.2|+|0.8-0.2|+|0.9-0.4|+|0.7-0.3|+|0.8-0.4|\big)$$
$$=0.3$$

$$d_E(X,Y)=SQRT\bigg(\frac{1}{6\times3}\big((0.5-0.7)^2+(0.6-0.8)^2$$
$$+(0.2-0.4)^2+(0.3-0.5)^2+(0.9-0.2)^2+(0.9-0.3)^2$$
$$+(0.8-0.5)^2+(0.9-0.6)^2+(0.4-0.5)^2+(0.4-0.5)^2$$
$$+(0.2-0.3)^2+(0.3-0.4)^2+(0.3-0.1)^2+(0.4-0.2)^2$$
$$+(0.8-0.2)^2+(0.9-0.4)^2+(0.7-0.3)^2+(0.8-0.4)^2\big)$$
$$=0.26$$

3 An extended TOPSIS Method for multiple attribute decision making based on INSs

For a multiple attribute decision problem, let $A=(A_1,A_2,\cdots,A_m)$ be a discrete set of alternatives, $C=(C_1,C_2,\cdots,C_n)$ be the set of attributes, $W=(w_1,w_2,\cdots,w_n)^T$ be the weighting vector of the attributes, and meents $\sum_{j=1}^{n}w_j=1, w_j\ge 0$. where w_j is unknown. Suppose that $X=\left[x_{ij}\right]_{m\times n}$ is the decision matrix, where $x_{ij}=([T_{ij}^L,T_{ij}^U],[I_{ij}^L,I_{ij}^U],[F_{ij}^L,F_{ij}^U])$ takes the form of the INVs for alternative A_i with respect to attribute C_j.

The steps of the ranking the alternatives based on these conditions are shown as follows

Step 1. Standardized decision matrix

In general, there are two types in attributes, the more the attibute value is, the better the alternative is, this type is called benifit type; on the contrary, the more the attibute value is, the worse the alternative is, this type is called cost type.

In order to to eliminate the influence of the attribute types, we need convert the cost type to benifit type. Suppose the standardized matrix is expressed by $R = \left[r_{ij} \right]_{m\times n}$, where $r_{ij} = ([\dot{T}_{ij}^{L}, \dot{T}_{ij}^{U}], [\dot{I}_{ij}^{L}, \dot{I}_{ij}^{U}], [\dot{F}_{ij}^{L}, \dot{F}_{ij}^{U}])$, then we have

$$\begin{cases} r_{ij} = x_{ij} & if\ the\ attrbute\ j\ is\ benifit\ type \\ r_{ij} = \overline{x}_{ij} & if\ the\ attrbute\ j\ is\ \cos t\ type \end{cases} \quad (13)$$

Where, $\overline{x}$ is the complement of x.

Step 2. Calculate attribute weights

Because the attribute weights are completely unkown, we need to determine the attribute weights. The maximizing deviation method, which is proposed by Wang [20], is a good tool to calculate the attribute weights for MADM problems with numerical information. The principle of this method is described as follows.

For a MADM problem, if the attribute values for all alternatives have little differences, such an attribute will play a small important role in ranking the alternatives, especially, for an attribute, if the attribute values for all alternatives are equal, the attribute has no effect on the rankng results. Contrariwise, if attribute values for all alternatives under an attribute have obvious differences, such an attribute will play an important role in ranking the alternatives. Based on this view, if the attribute values of all alternatives for a given attribute have a little deviations, we can assign a little weight for thsi attribute; otherwise, the attribute which makes larger deviations should be set a bigger weight. Especially, if the attribute values of all alternatives are all equal with respect to a given attribute, then the weight of such an attribute may be set to 0.

For a MADM problem, the deviation values of alternative A_i to all the other alternatives under the attribute C_j can be defined as $D_{ij}(w_j) = \sum_{l=1}^{m} d(r_{ij}, r_{lj}) w_j$, then $D_j(w_j) = \sum_{i=1}^{m} D_{ij}(w_j) = \sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij}, r_{lj}) w_j$ represents the total deviation values of all alternatives to the other alternatives for the attribute C_j. $D(w_j) = \sum_{j=1}^{n} D_j(w_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij}, r_{lj}) w_j$ represents the deviation of all attributes for all alternatives to the other alternatives. The optimize model is constructed as follows:

$$\begin{cases} \max D(w_j) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij}, r_{lj}) w_j \\ s.t \sum_{j=1}^{n} w_j^2 = 1, w_j \ge 0, j = 1, 2 \ldots\ldots n \end{cases} \quad (14)$$

Then we can get

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij}, r_{lj})}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d^2(r_{ij}, r_{lj})}} \quad (15)$$

Furthermore, we can get the normalized attribute weight based on this model:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij}, r_{lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(r_{ij}, r_{lj})} \quad (16)$$

Step 3. Use the extended TOPSIS method to rank the alternatives

The basic principle of TOPSIS is that the best alternative should have the shortest distance to the positive ideal solution and the farthest distance to the negative ideal solution. The positive ideal solution (marked as V^+) is a best solution in which each attribute value is the best one of all alternatives, and the negative ideal solution (marked as V^-) is another worst solution in which each attribute value is the worst value of all alternatives. The steps of ranking the alternatives by the extended TOPSIS are shown as follows.

(1) calculate the weighted matrix

$$Y = (y_{ij})_{m\times n} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \ldots & w_n r_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix} \quad (17)$$

Where $y_{ij} = ([\ddot{T}_{ij}^{L}, \ddot{T}_{ij}^{U}], [\ddot{I}_{ij}^{L}, \ddot{I}_{ij}^{U}], [\ddot{F}_{ij}^{L}, \ddot{F}_{ij}^{U}])$

(2) Determine the positive ideal solution and negative ideal solution:

According to the definition of INV, we can define the absolute positive ideal solution and negative ideal solution shown as follows.

$$\begin{cases} y_j^+ = ([1,1],[0,0],[0,0] \\ y_j^- = ([0,0],[1,1],[1,1] \end{cases} \quad j = 1, 2, \cdots, n \quad (18)$$

or we can select the virtual positive ideal solution and negative ideal solution by selecting the best values for each attribute from all alternatives.

$$\begin{cases} y_j^+ = ([\max_i \ddot{T}_{ij}^L, \max_i \ddot{T}_{ij}^U], [\min_i \ddot{I}_{ij}^L, \min_i \ddot{I}_{ij}^U], \\ \quad [\min_i \ddot{F}_{ij}^L, \min_i \ddot{F}_{ij}^U]) \\ y_j^- = ([\min_i \ddot{T}_{ij}^L, \min_i \ddot{T}_{ij}^U], [\max_i \ddot{I}_{ij}^L, \max_i \ddot{I}_{ij}^U], \\ \quad [\max_i \ddot{F}_{ij}^L, \max_i \ddot{F}_{ij}^U]) \end{cases} \tag{19}$$

$$j = 1, 2, \cdots, n$$

(3) Calculate the distance between the alternative A_i and positive ideal solution/ Negative ideal solution

The distance between the alternative A_i and positive ideal solution/ negative ideal solution is:

$$\begin{cases} d_i^+ = \sum_{j=1}^{n} d(y_{ij}, y_j^+) \\ d_i^- = \sum_{j=1}^{n} d(y_{ij}, y_j^-) \end{cases} \quad i = 1, 2, \cdots, m \tag{20}$$

(4) Calculate the relative closeness coefficient

$$RCC_i = \frac{d_i^+}{d_i^+ + d_i^-}.(i = 1, 2, \cdots, m) \tag{21}$$

(5) Rank the alternatives

Utilize the relative closeness coefficient to rank the alternatives. The smaller RCC_i is, the better alternative A_i is.

4 An application example

In order to demonstrate the application of the proposed method, we will cite an example about the investment selection of a company (adapted from [10]). There is a company, which wants to invest a sum of money to an industry. There are 4 alternatives which can be considered by a panel, including: (1) A1 is a car company; (2) A2 is a food company; (3) A3 is a computer company; (4) A4 is an arms company. The evaluation on the alternatives is based on three criteria: (1) C1 is the risk; (2) C2 is the growth; (3) C3 is the environmental impact. where C1 and C2 are benefit criteria, and C3 is a cost criterion. Suppose the criteria weights are unkown. The final decision information can be obtained by the INVs, and shown in table 1.

Table 1 The evaluation values of four possible alternatives with respect to the three criteria

	C_1	C_2	C_3
A_1	([0.4,0.5],[0.2,0.3],[0.3,0.4])	([0.4,0.6],[0.1,0.3],[0.2,0.4])	([0.7,0.9],[0.2,0.3],[0.4,0.5])
A_2	([0.6,0.7],[0.1,0.2],[0.2,0.3])	([0.6,0.7],[0.1,0.2],[0.2,0.3])	([0.3,0.6],[0.3,0.5],[0.8,0.9])
A_3	([0.3,0.6],[0.2,0.3],[0.3,0.4])	([0.5,0.6],[0.2,0.3],[0.3,0.4])	([0.4,0.5],[0.2,0.4],[0.7,0.9])
A_4	([0.7,0.8],[0.0,0.1],[0.1,0.2])	([0.6,0.7],[0.1,0.2],[0.1,0.3])	([0.6,0.7],[0.3,0.4],[0.8,0.9])

4.1 Ranking the alternatives in this example

We adopt the proposed method to rank the alternatives.

To get the best alternative(s), the following steps are involved:

(1) Convert the cost criterion to benefit criterion. Since C3 is a cost criterion, we can replace x_{i3} $(i = 1, 2, 3, 4)$ with $\overline{x}_{i3}$ $(i = 1, 2, 3, 4)$, and get the decision matrix R:

$$R = \begin{bmatrix} ([0.4,0.5],[0.2,0.3],[0.3,0.4]) & ([0.4,0.6],[0.1,0.3],[0.2,0.4]) & ([0.4,0.5],[0.7,0.8],[0.7,0.9]) \\ ([0.6,0.7],[0.1,0.2],[0.2,0.3]) & ([0.6,0.7],[0.1,0.2],[0.2,0.3]) & ([0.8,0.9],[0.5,0.7],[0.3,0.6]) \\ ([0.3,0.6],[0.2,0.3],[0.3,0.4]) & ([0.5,0.6],[0.2,0.3],[0.3,0.4]) & ([0.7,0.9],[0.6,0.8],[0.4,0.5]) \\ ([0.7,0.8],[0.0,0.1],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2],[0.1,0.3]) & ([0.8,0.9],[0.6,0.7],[0.6,0.7]) \end{bmatrix}$$

(2) Calculate attribute weights

About the distance in formula (16), we can use the Hamming distance defined in (9), and get $d(r_{ij}, r_{lj})$ $i, l = 1, 2, 3, 4; j = 1, 2, 3.$

$$d(r_{11}, r_{11}) = d(r_{12}, r_{12}) = d(r_{13}, r_{13}) = 0$$
$$d(r_{21}, r_{11}) = 0.133, d(r_{22}, r_{12}) = 0.083, d(r_{23}, r_{13}) = 0.300$$
$$d(r_{31}, r_{11}) = 0.033, d(r_{32}, r_{12}) = 0.050, d(r_{33}, r_{13}) = 0.250$$
$$d(r_{41}, r_{11}) = 0.233, d(r_{42}, r_{12}) = 0.100, d(r_{43}, r_{13}) = 0.217$$
$$d(r_{11}, r_{21}) = 0.133, d(r_{12}, r_{22}) = 0.083, d(r_{13}, r_{23}) = 0.300$$
$$d(r_{21}, r_{21}) = d(r_{22}, r_{22}) = d(r_{23}, r_{23}) = 0$$

$$d(r_{31},r_{21})=0.133, d(r_{32},r_{22})=0.100, d(r_{33},r_{23})=0.083$$
$$d(r_{41},r_{21})=0.100, d(r_{42},r_{22})=0.017, d(r_{43},r_{23})=0.083$$
$$d(r_{11},r_{31})=0.033, d(r_{12},r_{32})=0.050, d(r_{13},r_{33})=0.250$$
$$d(r_{21},r_{31})=0.133, d(r_{22},r_{32})=0.100, d(r_{23},r_{33})=0.083$$
$$d(r_{31},r_{31})=d(r_{32},r_{32})=d(r_{33},r_{33})=0$$
$$d(r_{41},r_{31})=0.233, d(r_{42},r_{32})=0.117, d(r_{43},r_{33})=0.100$$
$$d(r_{11},r_{41})=0.233, d(r_{12},r_{42})=0.100, d(r_{13},r_{43})=0.217$$
$$d(r_{21},r_{41})=0.100, d(r_{22},r_{42})=0.017, d(r_{23},r_{43})=0.083$$
$$d(r_{31},r_{41})=0.233, d(r_{32},r_{42})=0.117, d(r_{33},r_{43})=0.100$$
$$d(r_{41},r_{41})=d(r_{42},r_{42})=d(r_{43},r_{43})=0$$

Then according to (16), we can get the attribute weights shown as follows.

$$w_1=0.366,\ w_2=0.197, w_3=0.437$$

(3) Use the extended TOPSIS method to rank the alternatives

(i) calculate the weighted matrix

In formula (17), we can calculate $w_j r_{ij}$ $(i=1,2,3,4;\ j=1,2,3)$ by formula (7). For example, we can calculate

$$w_1r_{11}=([1-(1-0.4)^{0.366},1-(1-0.5)^{0.366}],[0.2^{0.366},0.3^{0.366}],[0.3^{0.366},0.4^{0.366}])$$
$$=([0.171,0.224],[0.555,0.643],[0.643,0.715])$$

Then we can get the weighted matrix Y

$$Y=\begin{bmatrix}
([0.171,0.224],[0.555,0.643],[0.643,0.715]) & ([0.096,0.165],[0.635,0.789],[0.728,0.835]) & ([0.200,0.261],[0.856,0.907],[0.856,0.955]) \\
([0.285,0.357],[0.430,0.555],[0.555,0.643]) & ([0.165,0.211],[0.635,0.728],[0.728,0.789]) & ([0.505,0.634],[0.739,0.856],[0.591,0.800]) \\
([0.122,0.285],[0.555,0.643],[0.643,0.715]) & ([0.128,0.165],[0.728,0.789],[0.789,0.835]) & ([0.409,0.634],[0.800,0.907],[0.670,0.739]) \\
([0.357,0.445],[0.000,0.430],[0.430,0.555]) & ([0.165,0.211],[0.635,0.728],[0.635,0.789]) & ([0.505,0.634],[0.800,0.856],[0.800,0.856])
\end{bmatrix}$$

(ii) Determine the positive ideal solution and negative ideal solution.

According to (19), we can get the virtual positive ideal solution and negative ideal solution shown asa follows.

$$y^{+}=(([0.357,0.445],[0.000,0.430],[0.430,0.555])\ ([0.165,0.211],[0.635,0.728],[0.635,0.789])\ ([0.505,0.634],[0.739,0.856],[0.591,0.739]))$$

$$y^{-}=(([0.122,0.224],[0.555,0.643],[0.643,0.715])\ ([0.096,0.165],[0.728,0.789],[0.789,0.835])\ ([0.200,0.261],[0.856,0.907],[0.856,0.955]))$$

(iii) Calculate the distance between the alternative A_i and positive ideal solution/ Negative ideal solution

According to (19), we can get the distance between the alternative A_i and positive ideal solution/ negative ideal solution shown as follows.

$$d_1^{+}=0.532,\ d_2^{+}=0.180,\ d_3^{+}=0.377, d_4^{+}=0.065$$

$$d_1^{-}=0.034,\ d_2^{-}=0.385,\ d_3^{-}=0.189, d_4^{-}=0.501$$

(iv) Calculate the relative closeness coefficient

According to (21), we can calculate the the relative closeness coefficient shown as follows.

$$RCC_1=0.941,\ RCC_2=0.319, RCC_3=0.666, RCC_4=0.114$$

(v) Rank the alternatives

According to the relative closeness coefficient, we can get the ranking from the best to worst.

$$A_4 \succ A_2 \succ A_3 \succ A_1$$

4.2 Compare with the existing method

In order to further illustrate the effectiveness of the proposed method in this paper, we compare with method proposed by Ye [10]. However, because the attribute weights and positive ideal solution/ Negative ideal solution are different from Ye [10], the ranking result is different; in addition, Ye [10] only consider the similarity measure between each alternative and positive ideal solution. If we adopt the same attribute weights and positive ideal solution ideal solution, and only consider the distance between each alternative and positive ideal solution, we can get the same ranking result from these two methods. Comparing with the method proposed by Ye [10], the method proposed in this paper can solve the multiple attribute problems with unknown weights, and can provide a compromise solution which considers the distances to positive ideal solution and Negative ideal solution. In addition, it is simpler in calculation process than Ye [10].

5 Conclusions

The interval neutrosophic set can be easier to express the incomplete, indeterminate and inconsistent information, and it is a generalization of fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and so on. This paper proposed the operational laws of the interval neutrosophic set, and defined the Hamming distance and the Euclidian distance. Then Maximizing deviation method is used to determine the attribute weights and the TOPSIS method is extended to interval neutrosophic set. Finally, an illustrative example has been given to show the steps of the developed method. It shows that this method is simple and easy to use and it constantly enriches and develops the theory and method of multiple attribute decision making, and proposed a new idea for solving the MADM problems. In the future, we shall continue working in the extension and application of the proposed method.

Acknowledgements

This paper is supported by the National Natural Science Foundation of China (No. 71271124), the Humanities and Social Sciences Research Project of Ministry of Education of China (No. 13YJC630104), the Natural Science Foundation of Shandong Province (No. ZR2013GQ011, No. ZR2011FM036), Social Science Planning Project of Shandong Province (No. 13BGLJ10), and Graduate education innovation projects in Shandong Province (SDYY12065).

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Received: November 30, 2013. Accepted: December 8, 2013.

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