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Methodology for Analyzing and Characterizing Error Generation in Presence of Autocorrelated Demands in Stochastic Inventory Models

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METHODOLOGY FOR ANALYZING AND CHARACTERIZING ERROR GENERATION IN PRESENCE OF AUTOCORRELATED DEMANDS IN STOCHASTIC INVENTORY MODELS

by

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ABSTRACT

METHODOLOGY FOR ANALYZING AND CHARACTERIZING ERROR GENERATION IN PRESENCE OF AUTOCORRELATED DEMANDS IN STOCHASTIC INVENTORY MODELS

Rafael Diaz
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Most techniques that describe and solve stochastic inventory problems rely upon the assumption of identically and independently distributed (IID) demands. Stochastic inventory formulations that fail to capture serially-correlated components in the demand lead to serious errors. This dissertation provides a robust method that approximates solutions to the stochastic inventory problem where the control review system is continuous, the demand contains autocorrelated components, and the lost sales case is considered. A simulation optimization technique based on simulated annealing (SA), pattern search (PS), and ranking and selection (R&S) is developed and used to generate near-optimal solutions. The proposed method accounts for the randomness and dependency of the demand as well as for the inherent constraints of the inventory model.

The impact of serially-correlated demand is investigated for discrete and continuous dependent input models. For the discrete dependent model, the autocorrelated demand is assumed to behave as a discrete Markov-modulated chain (DMC), while a first-order autoregressive AR(1) process is assumed for describing the continuous demand. The effects of these demand patterns combined with structural cost variations on estimating both total costs and control policy parameters were examined.

Results demonstrated that formulations that ignore the serially-correlated component performed worse than those that considered it. In this setting, the effect of holding cost and its interaction with penalty cost become stronger and more significant as the serially-correlated component increases. The growth rate in the error generated in total costs by formulations that ignore dependency components is significant and fits exponential models.

To verify the effectiveness of the proposed simulation optimization method for finding the near-optimal inventory policy at different levels of autocorrelation factors, total costs, and stockout rates were estimated. The results provide additional evidence that serially-correlated components in the demand have a relevant impact on determining inventory control policies and estimating measurement of performance.

To my wonderful wife and son, Kate and Alex, and in memory of my grandmother, Ramona

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NOMENCLATURE

General

μ	Mean
σ	Standard deviation
k	Order process for any positive integer
n	Length of a Discrete time series
$\mathbb{R}^d$	Euclidean space, composed by d-dimensional vector space over $\mathbb{R}$
f	Objective function
Ξ	A subset of the Euclidean space $\mathbb{R}^d$
ξ_d	Random components
$\vec{\xi}$	A finite collection of d random components $(\xi_1, \xi_2, ..., \xi_d)$
$F(\xi)$	Joint cumulative distribution function (CDF) of a d-dimensional random vector $\vec{\xi}$.
$\vec{\xi}_t$	Multivariate time series $\{\vec{\xi}_t; t = 1, 2, ...\}$
p_{ij}	Transition probability matrix
π_i	Stationary probability
ϕ	Autocorrelation factor
ρ	Correlation factor
$f(\xi)$	Expected value of the system performance measure
$\Psi_j(\xi, \omega)$	Performance of a simulation model observed (output response)
ω	Vector of the stochastic effects of the system

Inventory model

i	Period
n	Period planned horizon
s	Minimum reorder point for inventory
S	Maximum inventory level
x_i	Initial inventory level at the beginning of period i
z_i	Quantity of items ordered to resupply inventory at period i
w_i	The amount of inventory on-hand at period i
y_i	Inventory level after ordering ($x_i + z_i$) at period i
d_i	Demand at period i
c	Cost of ordering per inventory unit
h	Holding cost per inventory unit
$C(0)$	Minimal holding cost
p	Penalty cost per inventory unit
$\varphi_C(\xi)$	Probability density function of the stochastic demand
$\varphi_D(\xi)$	Probability mass function of the stochastic demand
$\Phi(a)$	Cumulative distribution function of the demand
λ	Delivery lag
$f(x_i)$	The expected total costs for periods i through n if the amount of inventory on-hand at the beginning of period i is x.
CR	Critical Ratio.
$\beta(C)$	Error generated in the costs function.

$\beta(s,S)$	Error generated in the inventory policy

Simulated Annealing, Pattern Search, and Ranking and Selection

j	Iteration
x_j	Accepted state (accepted candidate solution)
y	Nominated state (proposed candidate solution)
$H(x_j)$:	Objective function evaluating state x_j.
T	Temperature
α	Acceptance function
χ	Decision Space
$\tau_1, \tau_2, \ldots$	Temperatures from a cooling schedule
l_k	Stage length k^{th} stage
Z	Accepted candidate solution
δ_j	Step length.
n_0	Initial sample size,
h	A constant that depends on the number of alternatives
A	Number of alternatives
$1-\theta$	Desired confidence level,
S_i^2	Sample variance of the n_0 observations,
d^*	Significant difference specified by the user
N_i	Additional replications

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1 INTRODUCTION

1.1 Thesis Statement

A simulation optimization technique based on Simulating Annealing enhanced with Pattern Search and Ranking and Selection can be used to approximate solutions to stochastic inventory models that consider autocorrelated demands. Failing to capture the probabilistic properties of input processes that exhibit autocorrelated components generates errors that can be characterized using regression analysis.

1.2 Problem Statement

In the enterprise, control and inventory management have been recognized as a critical area that can significantly affect a firm's performance (Silver, 1985)[1]. Failing to properly characterize its inventory systems can lead a firm to poor inventory management. As a result, the enterprise may be reporting results far below optimal performance. When characterizing an inventory system where demands are uncertain, stochastic inventory modeling provides techniques to characterize, analyze, and solve problems associated with the optimal distribution of scarce resources. A key risk factor that can be hidden or ignored is the presence of certain types of dependency in the stochastic demands. In solving stochastic inventory problems, a variety of methodological and analytical tools are available (Silver, 1985; Bernard, 1999; Stadtler & Kilger, 2002). The effectiveness of many of these techniques depends upon their assumptions. In the stochastic inventory setting, some simplifying assumptions are critical to the efficacy of a given technique. Most techniques rely upon the assumption of identically and independently distributed

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[1] Citation and reference list format for this manuscript are taken from the American Psychological Association.

(IID) demands when describing and solving stochastic problems (Biller & Soumyadip, 2004). As demonstrated in the literature, techniques that attempt to solve stochastic problems reliant upon IID data can be misleading in estimating measures of performance (Melamed, Hill, & Goldsman, 1992; Ware, Page, & Nelson, 1998). Further, in the enterprise, these miscalculations may have a significant impact on critical issues such as facility planning or policy making[2].

Many frameworks have been created to aid inventory managers in finding the optimal inventory policy. The IID assumption predominates in most analyzed inventory models. Deriving analytical solutions to stochastic inventory problems that present dependency components can be very difficult due to complicated multivariate time series integration. In addition, inventory managers may face challenges in recognizing and correctly modeling discrete or continuous autocorrelated demand. Inventory systems are characterized either as lost sales systems, where unmet demand results in the customer seeking the goods elsewhere, or backlog systems, where the fulfillment of the demand is simply delayed. In inventory planning and control, policy-making is a critical factor that directly impacts the operation of the enterprise (Silver, 1985).

In this dissertation, the impact of ignoring this demand dependency component is quantified and analyzed for the lost sales case. A simulation optimization technique is developed and used to generate near-optimal solutions to the described complex problem. Specifically, the inventory problem is characterized as a stochastic Dynamic Programming (DP) problem. The solution technique employed finds approximately near-

[2] Inventory policy involves deciding appropriate stock levels, reorder points and quantities. It has a direct effect on planning and resource distribution (Silver, 1985).

optimal inventory control policies using an extension of Simulated Annealing (SA) that combines Pattern Search (PS) and Ranking and Selection (R&S).

1.3 Motivation

This part of the dissertation provides details as to why considering dependency issues in a certain class of inventory problems is relevant to the literature. Further, this section explains the importance of considering these dependency issues in terms of policy-making in continuous control systems that consider the lost sales case.

1.3.1 Inventory control and the stochastic demand

The significant impact of inventories on the balance sheet is well known in the enterprise. In general, senior management perceives inventory as a large potential risk. Silver (1985) indicates that diverse factors that include merchandise stocked in excess, obsolescence, inflation, technological changes, fluctuations in the demands, and business cycles support this view. Thus, corporate management is constantly challenged by the rewards and inconveniences of carrying inventory. Corporate strategy relates to decision making in planning and inventory control in the sense of distributing resources and interacting with multiple functional areas. For example, while production and sales management forces toward keeping higher inventory, finance and accounting management pressure downward inventory levels. In inventory settings, complexities can be associated with the type of items to be produced, the nature of the demand, and the multiple interactions with other functional areas. Silver (1985) states:

"...inventory management is therefore a problem of coping with large numbers and with a diversity of factors external and internal to the organization".

As a result, decision systems and rules must be designed to rationalize, coordinate, and control such physical and conceptual issues. Thus, any inventory manager must be able to provide answers to the following questions (Taha, 2002; Hillier & Lieberman, 2001; Silver, 1985):

1. When an item should be ordered;

2. How much of the item should be requested on any particular order;

3. How often the inventory status should be determined.

In this sense, many inventory models and frameworks have been designed to provide answers to these questions. These techniques vary according to the types of conditions and interactions present in the inventory system. When the manager has relatively little or no uncertainty regarding the demand, order quantity decision systems prevail. Order quantity decisions answer the question of how large a replenishment quantity should be under rather stable conditions. When the manager recognizes the uncertain nature of the demand, additional factors have to be considered. These factors include deciding between lost sales versus backorders and continuous versus periodic review. Periodic review specifies the review interval, which is defined as the time that elapses between two consecutive moments at which the stock level is known. In continuous review systems, inventory level is always known.

This dissertation describes a situation where the inventory model contains dependent stochastic components in the demand and considers an order-up-to-level (s, S) control system, where s is the inventory level that triggers ordering and S is the target inventory for a reorder action.

1.3.2 Stochastic autocorrelated demand

The autocorrelation function of a random process describes the correlation between successive random observations of the process. Effects of autocorrelation have been extensively studied by the research community in a large variety of settings. In inventory models, autocorrelation components in demands, lead time, and a combination of the two have also been investigated (Johnson & Thompson, 1975; Ray, 1980; Ray, 1981; Ray, 1982; Zinn, Marmorstein, & Charnes, 1992; Marmorstein & Zinn, 1993; Charnes, Marmorstein, & Zinn, 1995; Urban, 2000; Urban, 2005). These authors have presented significant evidence that autocorrelation has a significant impact when estimating inventory control parameters. Moreover, autocorrelated demand and service processes are critical features of modern failure-prone manufacturing systems (Bertsimas & Paschalidis, 2001). As a result, a diverse collection of techniques and considerations have been developed to mitigate the negative effect of this type of dependency in specific inventory settings. Presence of autocorrelation components in stochastic demand can be positive or negative. On the one hand, positive autocorrelation implies that if the current demand is above (or below) the expected demand, the next demand will also tend to be above (or below) the expected demand. In other words, demand exhibits runs of above and below average levels. On the other hand, although less frequently encountered, negative autocorrelation means that the current demand will be followed by a demand on the opposite side of the expected demand. Positive autocorrelated demands were reported in the work of Erkip and Hausman (1994), who examined the inventory/warehouse of a major national supplier of consumer products and discovered autocorrelations of about 0.7. More recently, Lee, So, and Tang (2000) analyzed the effects of grocery store weekly sales and found autocorrelations from 0.26 to 0.89. In the same work, they asserted that high serial correlated demand is observed in the electronics retail industry as well. Although negative autocorrelated demands have been reported as "extremely rare in practice" (Zinn et al., 1992), they have been found and studied. Examples of negative autocorrelated demand include the work of Magson (1979), in which spare parts are considered, and the work of Ray (1980), who analyzed actual monthly sales quantities of a specific product manufactured for the food industry with serial correlation of -0.33.

Nonetheless, dealing with autocorrelation components in inventory systems is very difficult and sometimes intractable due to complicated multivariate integration. Generally, stochastic inventory models assume that the random variables involved follow some specific continuous distribution with IID observations (Charnes et al., 1995). Considering and analyzing the effect of autocorrelations in the stochastic demand in continuous inventory control systems that consider lost sales is an open research question. In this sense, studying and analyzing the errors generated by models that ignore dependency benefits practitioners and the research community. Thus, a method that considers the complex multivariate component of the demand in an inventory system controlled by a continuous *(s, S)* review method and in the view of the lost sales case, while approximating near-optimal solutions, is the subject of this dissertation.

1.4 Filling the literature gap, research goals and questions

Sections 1.1 to 1.3 provided a brief overview in which a problem in the inventory control area is described. Specifically, considering a stochastic autocorrelated demand and the inventory setting described above, there exists some indication that there is an open research issue concerning the methods and techniques used to derive solutions to these problems. In this sense, an overview of the research questions related to this topic is presented in this section. Chapter 2 explores the literature review that substantiates the evidence of this research gap. Thus, in this section a general idea of the research goals, questions, and how this gap is filled is presented below.

The goal of this research is to investigate the effects of autocorrelation components on demands for a certain class of stochastic inventory problems. Thus, errors caused by ignoring the dependency components can be characterized. A simulation optimization technique capable of generating near-optimal solutions considering dependent autocorrelated input data is developed. This framework can be applied to a given complex inventory problem that presents certain characteristics as indicated in section 3.2. The motivations and potential benefits cited in section 1.2 resulted in the primary research questions of this dissertation. These research questions include: (1) To what extent can a method that allows handling and solving inventory models that pose autocorrelated demands be built using a simulation optimization approach? (2) To what extent is the difference between results obtained by stochastic inventory methods that assume IID demands and those that do not significant? (3) What is the structural effect of the dependency issue on the cost and the inventory control policy as the autocorrelation amplifies? (4) What is the behavior (characterization) and significance of the error generated between solution methods that assume IID demands and those that do not? (5) How can these results be validated?

From these questions, more focused objectives are developed and are used to guide this research effort. These objectives include:

1. Developing sampling techniques that allow one to represent and generate autocorrelated and correlation-free sample input data.
2. Developing a simulation optimization heuristic in which near-optimal inventory policies and measure of performance can be determined considering autocorrelated components.
3. Analyzing and discussing the potential effects of autocorrelation factors in measuring performance and deriving near-optimal control inventory policies.
4. Characterizing the errors generated by those methods that ignore dependencies in a certain class of inventory problems.
5. Providing a validation mechanism to verify that the inventory policy obtained using the aforementioned heuristic corresponds to a near-optimal solution that considers dependency issues.

1.5 Research Approach Overview

The method used to characterize errors generated by dependency-ignoring methods consists of four fundamental parts that include: model formulation, dependency representation and sampling, simulation optimization technique, data generation, and error characterization. Each part is subdivided into sections that form the methodological framework of this research. Figure 1 summarizes the aforementioned strategy and its main components.

This research presents a stochastic searching technique based on Markov Chain Monte Carlo[3] (MCMC), namely, SA combined with PS and R&S to investigate the effects of autocorrelation components on a certain class of stochastic inventory problems.

When probabilistic distributions are intractable, MCMC overcomes this limitation by generating a sample sequence where each decision point has the desired distribution (Fishman, 2005). In this research, the impact of serially-correlated demand is investigated for discrete and continuous dependent input models. For the discrete dependent model, the autocorrelated demand is assumed to behave as a DMC, while an AR(1) process is assumed for describing the continuous demand.

To generate correlated and correlation-free demands for the DMC model, the transition and the invariant probability distribution were considered respectively. A set of transition probabilities was assumed for the DMC stationary model. From each transition matrix, autocorrelation components were quantified. By using well-known properties of stationary Markov chains, the invariant distribution, which represents the correlation-free case, was obtained. For the AR(1), demands were generated using a first-order autoregressive AR(1) process in which errors are distributed normally. The autocorrelation factor equal to zero represented the IID case, while autocorrelation factors other than zero represented the correlated case.

In the approach used in this research, the stochastic inventory problem with dependency is stated in terms of a stochastic DP formulation. The DP formulation allows one to represent and evaluate the objective function in terms of the multivariate component of the dependent demand.

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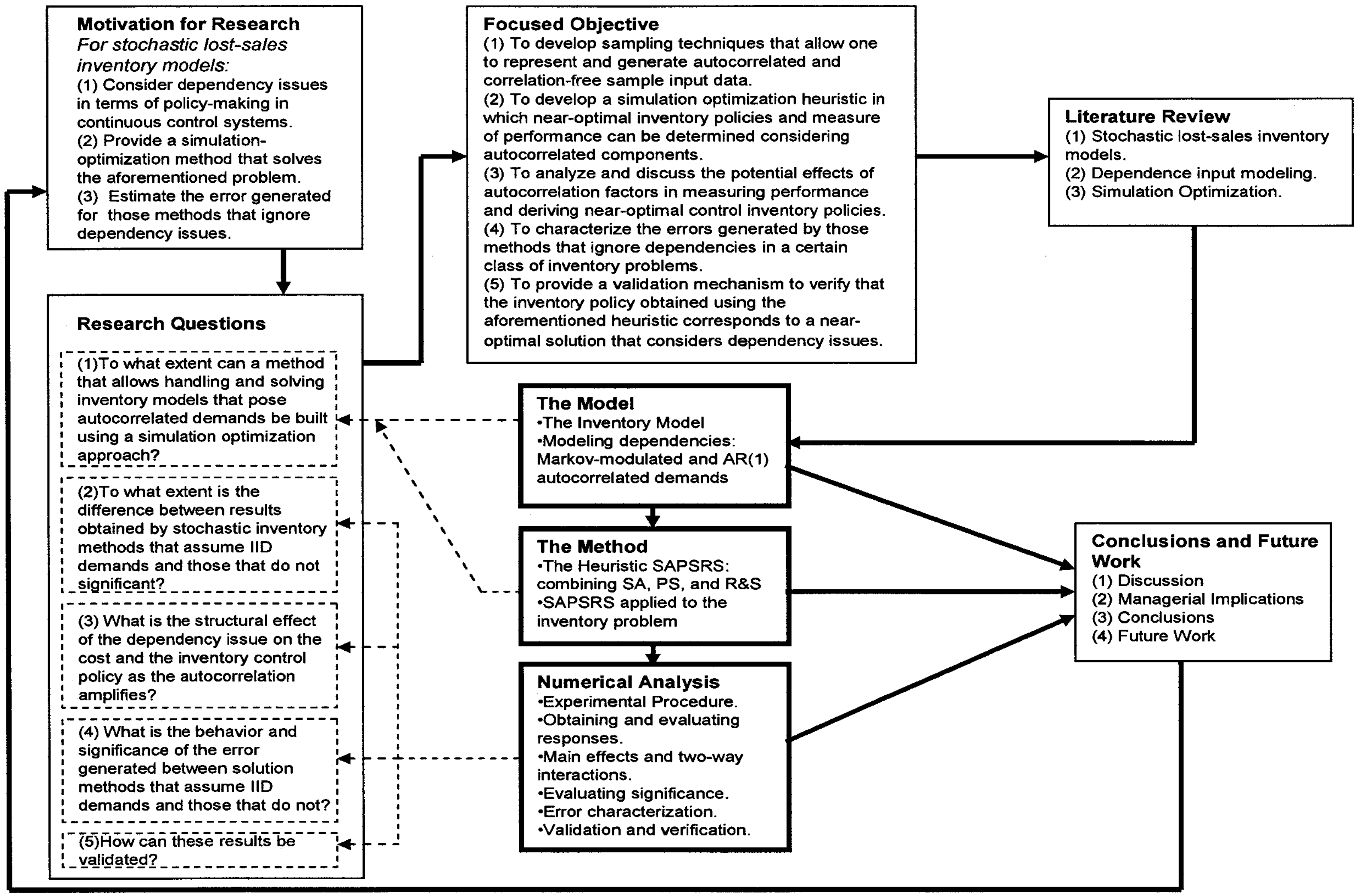
[3] Convergence properties are discussed on Fishman (2005)

Figure 1 Overview of the Research Approach



The process of searching for a near-optimal solution was based on the exploratory mechanics of SA. This probabilistic local search technique permits direct sampling of tractable and intractable probabilistic distributions. Furthermore, it estimates solutions to the objective function by randomly generating a location in the feasible space and applying randomized (SA) and deterministic (PS and R&S) rules to decide whether to move to a new location on the path to a solution. A combination of PS and R&S enhances the search process by deterministically proposing and evaluating additional locations and possible solutions distributed around the neighborhood of the proposed solution.

To investigate and compare the effects of serially-correlated demands in the inventory setting described above, eight experiments were designed in terms of varying cost factors. The average total costs of the system and the near-optimal (s, S) policy were defined as the response of each experiment. Main effects and two-way interaction per cost factor were determined. In addition, each experiment was evaluated in terms of the effect of the autocorrelation factor.

To test the significance of the difference between the correlated and correlation-free cases, ANOVA tests were conducted. To find the errors generated by dependency-ignoring methods in the average total costs and the (s, S) policy, the absolute differences were calculated between the correlated and correlation-free cases as the autocorrelation component varied. Finally, the error characterization between the two cases was accomplished by applying regression analysis.

To validate and verify that the proposed algorithm produced near-optimal solutions, total costs, stockouts, and replenishment rates were analyzed.

- An analysis and a suggested set of actions for dealing with autocorrelated components in the discrete and continuous stochastic demands from the inventory management perspective.

1.6 Dissertation Organization

This dissertation is organized as follows.

- **Chapter 2. Background and Literature Review.** In this section relevant research in inventory theory, dependence input modeling, and simulation optimization is presented.
- **Chapter 3. Model.** In this section, a detailed description and the considerations used to develop the stochastic inventory model are presented. It includes the stochastic DP formulation of the problem and the models and algorithms used to derive the correlated and correlation-free cases.
- **Chapter 4. Method.** In this chapter, a comprehensive description of the heuristic used to approximate solutions to the described problem is provided.
- **Chapter 5. Numerical Analysis.** The experimental design is discussed and presented. Experimental results obtained from applying the heuristic to the problem are presented and summarized.
- **Chapter 6. Conclusions and Future Work.** In this section, results are generalized, and managerial implications and conclusions are stated. In addition, future work to extend the present research effort is described.

1.5 Contributions

This dissertation provides a methodological framework that identifies, characterizes, and analyzes the error generated by dependency-ignoring techniques in stochastic inventory problems that considers a lost sales case in a continuous review control system and presents serially correlated demands. While the particular application in this dissertation is inventory problems, the approach can also be applied to other stochastic problems in which the input probability distribution presents relevant autocorrelation components.

This research study not only builds upon the existing inventory and optimization literature, but also introduces methods and models not used before to solve complex inventory problems. Specifically, the primary contributions of the reported work include:

- A methodological framework capable of recognizing, analyzing, and approximating solutions to a certain class of inventory problems with autocorrelated demands.
- A novel stochastic local search technique, based on SA combined with PS and R&S, capable of deriving results for stochastic problems that present probability distributions that contain dependencies in their input data.
- A characterization of the bias generated by estimations obtained between methods that assume IID and those that consider structural dependency in a certain class of stochastic inventory problems.
- An experimental analysis of the effects of ignoring autocorrelated components on the demands of stochastic lost sales inventory problems.

2 BACKGROUND AND LITERTATURE REVIEW

This chapter provides a review of past research relevant to this study. First, it considers an overview of the essentials of the inventory theory. Next, the models that consider stochastic demand and those that have considered autocorrelated demands in their formulation are presented. Then, available simulation optimization techniques to implement near-optimal solutions are provided.

2.1 Stochastic inventory models

Inventory models are representations that allow one to determine answers to the essential questions presented in section 1.3.1 that include when and how much of an item should be ordered and how often the inventory status should be determined.

Several factors influence the decision of using a specific model. Most critical factors include: the nature of the demand; the type of item (A-B-C)[4] ; and the interaction with other areas. As indicated, in the stochastic settings, backlogging and periodicity of the review of the stock level determine the type of models to be considered. In this section, background information regarding inventory control whose demand is probabilistic is presented. First, the concept of backlogging is presented. Next, an idea of review control systems is offered. Afterward, relevant aspects of the Order-point and order-up-to level (s, S) system are provided.

[4] A-B-C classification refers to a categorization of items into three classes according to the dollar usage (Krajewski & Ritzman, 2004)

2.1.1 Lost sales and related inventory costs

Consider an organization that supplies a single item and needs to make decisions about how many items to keep in inventory for each of n time periods. The number of periods for which the company would like to schedule its inventory is known as the planning horizon.

Dreyfus & Law (1977) defined the dynamic inventory system as a probabilistic inventory control model that poses certain specific features. The system is essentially characterized by:

- The demand for the item in period i denoted by D_i .
- The probability that $D_i = d$ represented by $p_i(d)$.
- The on-hand stock x_i before the ordering period i.
- The amount of item ordered z_i in time i, which arrives in $i+\lambda$; λ is the delivery lag;
- The amount of inventory on-hand and on order y_i (inventory position) after ordering period i. Therefore, $y_i = x_i + z_i$. The amount of inventory on-hand w_i , or safety stock, without including what is on order in period i after the order from period $i-\lambda, z_{i-\lambda}$ has been delivered but before the demand occurs .
- w_i is the real amount of inventory available to satisfy the demand in period i. If delivery lag $\lambda = 0$, then $w_i = y_i$.

Figure 2 illustrates the sequence of these events.

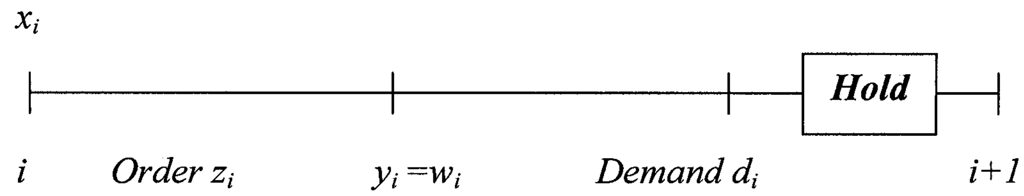


Figure 2 Inventory Events

Many authors have used similar representations to study inventory with backlogged assumptions. Several authors have emphasized the limited research available on lost sales inventory models (Feng & Suresh, 1999; Johansen & Hill, 2000; Toktay, Wein, & Zenios, 2000; Urban, 2005).

As indicated by Silver (1985), in most practical situations, one finds a combination of backlogging and lost sales cases. However, theoretical models provide reasonable approximations. Thus, both cases determine how the safety stock is configured. In this regard, in a period i when a stockout takes place, the value of the net stock, defined by what is on-hand less backorders, depends on the value that the backorder assumes. Thus, independently of the situation, when the demand is stochastic, there is a probability of stockout. Several authors have considered this situation and have researched different criteria to establish safety stock levels (Silver, 1985; Tersine, 1988; Banks, Carson, & Nelson, 1996; Bernard, 1999). Most popular criteria for establishing safety stock include the use of common factor such as using common time supply; the costing of shortages; introducing service level parameters; and the effect of disservice on future demand. Thus, in order to manage the opportunity costs of stockouts, firms must maintain a level of safety stock that balances the loss of sales and customer goodwill (Zinn et al., 1992).

2.1.2 Review of the stock level: Continuous or periodic

The frequency of review of the stock level is one of the factors that defines the selection of determined control system. Moreover, if the stock level is always known, the system is categorized as continuous; if the stock level is reviewed at certain time intervals, the system can be classified as periodic. Silver (1985) summarizes advantages and disadvantages of both review systems indicating that periodic reviews allow coordination of replenishment, reasonable prediction of the workload to issue replenishment orders, and accurate prediction of spoilage in slow-moving perishable items. In a continuous review system, since orders may occur anytime, workload prediction is less accurate. Also, a continuous review model is considered more expensive in terms of updating costs and reviewing errors. This is true, since there are more transactions to record and, in case of errors, to review. Nonetheless, the major advantage of continuous over periodic review systems is that the former requires less safety stock to provide the same service level. Since the review systems are periodically monitored, the inventory system requires more safety stock as protection, since inventory levels may drop significantly between two consecutives periods.

Dreyfus and Law (1977) point out that there are three costs related to operating a given inventory system. Cost of ordering $c_i(z)$ include those costs of ordering z items in period i and incurred at the time of delivery $i+\lambda$. There is a holding cost h_i, if net inventory on-hand after demand has occurred or $w_i - d_i$ is higher than zero, thus

$h_i(w_i - d_i)$. A Penalty cost, $p(d_i - w_i)$ and a minimal holding cost $C(0)$ are incurred if there is a shortage. Other costs related to those where the length of a period is sufficiently long and the interest rate of the invested capital is sufficiently large include discount costs. In this dissertation discount factors are not considered. Details of the cost components will be explained in section 3.2.2.

2.1.3 The order-point, order-up-to-level (s, S) control system

As indicated by (Silver, 1985; Tersine, 1988; Banks et al., 1996; Bernard, 1999), the (s,S) control system is one of the most commonly found continuous stochastic control systems where replenishment is made whenever the inventory position drops to the order point s or lower. Furthermore, the replenishment quantity is variable and is used to raise the inventory position to the order-up-to-level S. Figure 3 depicts the behavior of a (s,S) control system. Notice that inventory level is initialized at the maximum inventory level S. Then, the stochastic demand depletes the inventory until it reaches the reorder point s. An order z_1 is placed up-to-S level. Then, the depleting cycle begins again. Notice that before period 2, the inventory level reaches the reorder point and uses safety stock to satisfy the demand. If the demand exceeds the inventory level, the unfilled portion is not backlogged, but is lost.

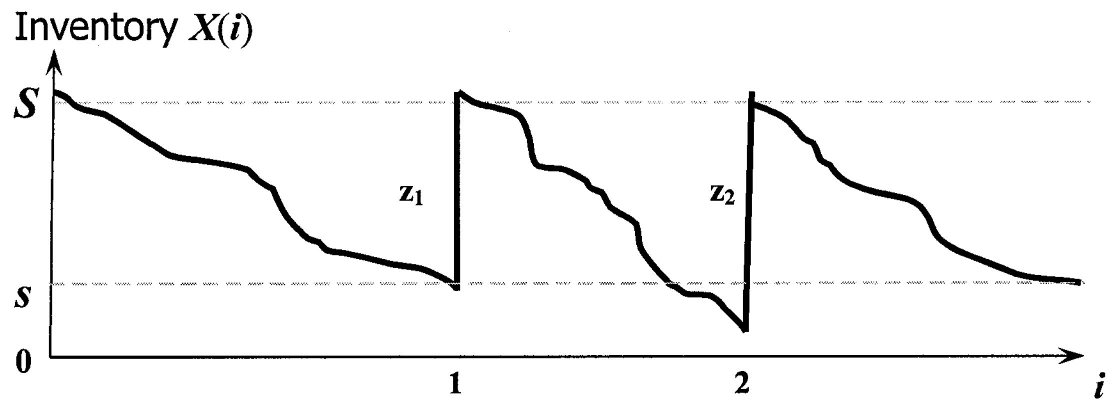


Figure 3 Continuous inventory control system (s, S)

2.2 Inventory models and the autocorrelated demand

The autocorrelation function of a random process describes the correlation between the processes at different points in time. Consider d_i as the value of the demand process at time i (where i may be an integer for a discrete-time process). For a discrete time series of length n $\{d_1, d_2, ..., d_n\}$ with known mean and variance, an estimate of the autocorrelation may be expressed as

$$\phi = \frac{1}{(n-k)\sigma^2} \sum_{i=1}^{n-k} (d_i - \mu)(d_{i+k} - \mu)$$

where μ is the mean, σ is the standard deviation, and k is the order process for any positive integer k < n. For example, if the autocorrelation is calculated for a first-order process, then $k = 1$.

In essence, autocorrelation components may be present in a positive or negative fashion. On the one hand, positive autocorrelation implies that there is a sequence of unit times that the current demand is above the expected demand. As a result, variability of the demand increases systematically. On the other hand, negative autocorrelation means current demand above average in a time unit is followed by a demand that is below the average per time unit. An example of positive autocorrelated demand can be found in Erkip & Hausman (1994) where the effects of sales incentives for an item were considered in an actual multi-echelon inventory system of a major national producer and supplier of consumer products. In this regard, the sales incentive produced an increase in the current demand above the expected demand. Zinn et al. (1992) considered a positive autocorrelated demand for sweaters after a spell of cold weather that caused sales to be above average for several days. A negative serially correlated demand example can be found in Magson (1979) where the author describes a situation in ordering engineering spares that present highly negatively correlated monthly demands.

Figures 4, 5, and 6 show a set of IID, negative, and positive autocorrelation demands for an AR(1) process whose expected demand is 2,500 units, an error normally distributed mean 0, and standard distribution of 300 units or $N(0,300^2)$.

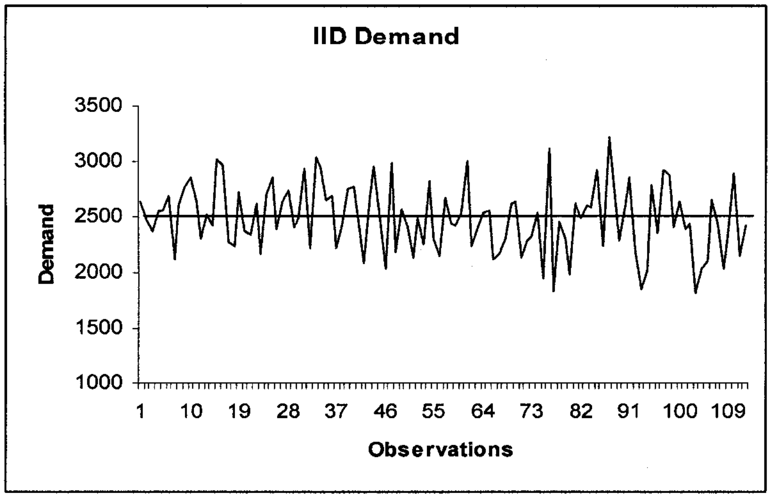


Figure 4. IID demand

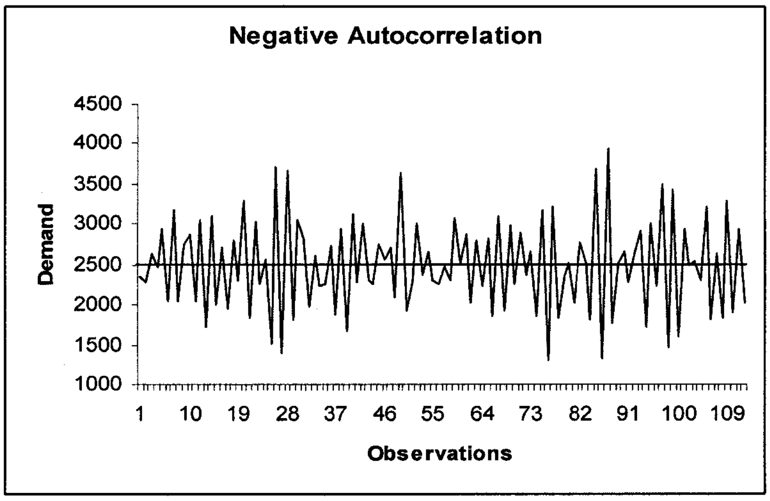


Figure 5. Negative Autocorrelated demand

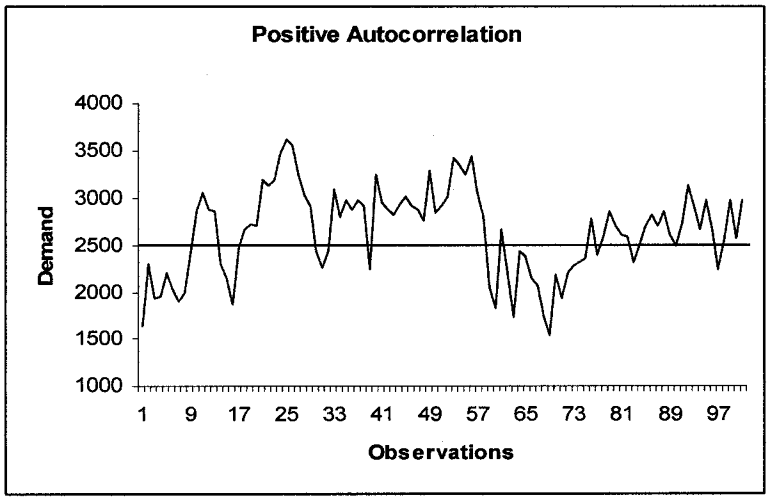


Figure 6. Positive Autocorrelated demand

When autocorrelated demand exists, positive cases have been commonly found in inventory settings. As mentioned, negative autocorrelated demand, although theoretically possible and found in some cases (Magson, 1979; Ray, 1980), is considered to be extremely rare in practice (Zinn et al., 1992).

Early work in analyzing inventory demand that is not IID can be attributed to Veinott (1965) and Veinott & Wagner (1965). First, Veinott et al. (1965) considered a multi-period single product nonstationary review inventory problem in which the demands in different periods are independent but not necessarily identically distributed. Later, Veinott (1965) removed the independency assumption to provide conditions that ensured that the base stock ordering policy was optimal and that the base stock levels in each period were easy to calculate.

Later on, Johnson et al. (1975), based on Veinott (1965), proved optimal policies for stationary and nonstationary demand for an ARMA processes.

Ray (1980) studied the case of first-order autoregressive demand patterns and three different distributions of lead time. As a result, the author concluded that in the presence of the negative autocorrelation, assumptions of independence might lead to over provision of stock, in cases of positive autocorrelation the under provision will be very significant. Furthermore, the author stated that this condition worsens as the expected lead time increases. Later on, Ray (1981) derived a method for calculating the reorder level (ROL) of a stock control system when the demands are correlated and the lead time is random. In this sense, the proposed method requires determining the first four moments of the total demand in the lead time. Then, these moments are used to find approximate percentiles of the distribution. Finally, the author uses both of these to evaluate the ROL corresponding to a required service level. Afterwards, Ray (1982) considers the modifications required when the ARIMA model.

An, Fotopoulos, & Wang (1989) derived solutions for calculating reorder points (ROPs) of an inventory system based on the Pearson system. This system was derived from the exact first four moments of lead-time demand for an AR(1) and an MA(1) demand structure where the arbitrary lead-time is independent of the demand.

Zinn et al. (1992) analyzed the effect of autocorrelated demand on the level of customer service provided by a firm. They found that observed stockouts are appreciably more frequent and larger; the effect of autocorrelation on stockouts is directly related to the variability of customer demand and inversely related to the variability of lead time from suppliers. Thus, they quantified the effect of autocorrelated demand on stockouts. Later, Marmostein et al. (1993) investigated and found a relevant impact of the effect of autocorrelation on the safety stock required to achieve a managerially prescribed level of customer service. Thus, they quantified the conditional effect of autocorrelation on safety stock requirements.

Charnes et al. (1995) considered a periodic-review inventory for deterministic lead times and a covariance-stationary stochastic demand process. They derived a method for setting the inventory safety stock to achieve an exact desired stockout probability when the autocovariance function for Gaussian demand is known.

Inderfurth (1995) demonstrated that serial and cross-correlation in demand product have contrary effects on the distribution of safety stocks over the manufacturing stages and that overlooking it can lead to significant divergence from the optimal buffer policy. He presented a procedure for integrated multilevel safety stock optimization that was applied to arbitrary serial and divergent systems.

Urban (2000) analyzed the effect of serially-correlated demand on the determination of appropriate reorder levels. The author argues that previous research has investigated this effect on the required levels of safety stock while ignoring its effect on the expected demand during lead time. In his paper, the author investigated the determination of accurate reorder levels for first-order autoregressive and moving average demand processes. Finally, the author concluded that experimentation indicates that existing approaches of managing serially-correlated demand can result in both excessive inventories and shortages for high levels of autocorrelation.

Urban (2005) developed a periodic-review model that considers two types of dependencies that influence the demand, serial correlation, and the amount of inventory displayed to the customer. As a result, the author developed a methodology based on an adaptive, base-stock policy founded on the critical fractile.

The methods described above are exact or bound approximations developed to provide solutions to the inventory problem. As a result, when faced by more complex situations, these methods contain restrictive assumptions. Thus, as recommended by Silver (1985), near-optimal methods can be used to solve these representations with a high probability of converging to reliable solutions. In the next section, the simulation heuristics for generating near-optimal solution are explored.

2.3 Optimum-seeking Heuristic

This section is subdivided into two parts, dependence-input-modeling techniques and simulation optimization methods. In the dependence input modeling section, methods for constructing fully and partially specified joint distributions are provided. In the second part, an overview of simulation optimization is given while emphasizing in stochastic local search heuristics. Then, fundamentals for SA, PS, and R&S are presented.

2.3.1 Introduction

To illustrate the context of the optimum-seeking process, a summary of the work of Avriel (2003), Boyd Stephen & Vandenberghe Lieven, (2004), and Papalambros & Wilde (2000) is presented as follows.

In an optimization problem, one seeks to minimize or maximize a real function by choosing values of real or integer variables from an allowed set.

In general, a function f is called an objective function from some set Ξ, which is generally a subset of the Euclidean space $\mathbb{R}^n$. This space is often limited by of a set of constraints. These constraints are usually expressed as equalities or inequalities that have to be satisfied. The elements of Ξ are known as feasible solutions (candidate solutions). In a set of feasible solutions, an optimal solution is a solution that can minimize or maximize the objective function. When the feasible region or the objective function of the problem does not present convexity[5], there may be several local minima and maxima. Most heuristics proposed for solving non-convex problems cannot make the distinction between local optimal solutions and global optimal solutions. The existence of

[5] The feasibility solution space is said to form a convex set if the line segment joining any two distinct feasible points also falls in the set (Taha, 2002).

derivatives is not always assumed and many methods were devised for specific situations. The basic classes of optimization methods include combinatorial methods, derivative-free methods, first-order methods, and second-order methods. The most popular methods include: Linear programming, Integer programming, Stochastic programming, and Dynamic programming. The most common methods that include those that do not assume the existence of derivatives are Gradient descent steepest descent or steepest ascent, Nelder-Mead method, and Interior point methods.

2.3.2 Dependence Input modeling

2.3.2.1 Introduction

Biller et al. (2004) provide an overview of dependency modeling for stochastic simulation in which essential elements, along with associated techniques, are discussed. In the next paragraphs, a summary of basics and general approaches presented in their tutorial, along with additional references, is provided.

2.3.2.2 Essentials

- $\vec{\xi} = (\xi_1, \xi_2, \ldots, \xi_d)$ is a finite collection of d random components in which each component has a distribution function in $\mathbb{R}$.
- The joint or multivariate distribution is the random vector associated to a probability distribution in the $\mathbb{R}^d$.
- The joint cumulative distribution of a n-dimensional random vector $\vec{\xi}$ is defined as $F(\xi') = P(\xi \le \xi') = P(\xi_1 \le \xi_1', \ldots, \xi_d \le \xi_d')$ for any fixed n-vector $\xi' = (\xi_1', \xi_2', \ldots, \xi_d')'$.

- The joint distribution determines the behavior of $\vec{\xi}$. It describes the distribution of its stochastic component, termed the marginal distributions. Also, it determines their stochastic relationships. If and only if the random variables of the joint (cumulative density function) CDF is the product of the marginal CDFs, they are identically independent; otherwise, they are dependent.
- The two most popular measures of dependency in input data include the product-moment correlation and the fractile correlation, whose sample analog is the Spearman or Rank correlation.
- The multivariate time series $\vec{\xi}_t$, where $\{\vec{\xi}_t; t = 1, 2, ...\}$ is a joint distribution, can be expressed in terms of the stochastic distributions of the individual stochastic variables of ξ_t . This form relies on the concept of autocorrelation or the correlation between observations contained by the series. Section 2.2 defines and describes the demand autocorrelation for an inventory model.

2.3.2.3 Dependence-input-modeling techniques

In general, techniques for dependence input modeling can be categorized into two families: those where the joint distribution function available (fully specified) and those where the distribution has been partially specified. The type of multivariate process that captures dependencies among a certain number of random variables is jointly called a random vector and is composed of independent samples of identically distributed stochastic vectors. The other type captures the temporal dependence that occurs over time and is conventionally analyzed as time series.

In this dissertation, multivariate time series expressed as the autocorrelated demand in the inventory system is analyzed.

Biller et al. (2004) present a summary of common methods for constructing partially specified joint distributions, which are usually devised by providing the marginal distributions and their dependence measures. Thus, they assert that there are two main groups, those methods that are based on transformation-based univariate generation procedures and those that include mixture models. The Transformation Based Methods include ARTA, NORTA, and VARTA Processes, Chessboard Distributions, Vine Copula Method, and TES Processes. The Autoregressive-To-Anything (ARTA) processes designed by Cario (1996), define a time series with marginals via the transformation, where the base process is a stationary and Gaussian autoregressive of order with the representation. Normal-To-Anything (NORTA) is a related method for obtaining random vectors with arbitrary marginal distributions and its correlation matrix is described in Cario & Nelson (1997). The purpose is to transform a multivariate normal vector into the specified random vector. The NORTA method can be seen as expanding the ARTA process further than an ordinary marginal distribution. The VARTA framework created by Biller & Nelson (2003) integrate the theory behind the ARTA and NORTA processes. The reader is referred to the aforementioned authors to obtain detailed descriptions for such procedures.

The Chessboard Distributions, Vine Copula Method, TES Processes family can also be used to represent stochastic vectors with arbitrary marginals and a certain rank correlation values by component-wise transformations of the random vector. Ghosh & Henderson (2002) proposed a class of copulas called the chessboard distributions. Ghosh et al. (2002) showed that the constraints on the probabilities that ensure that a given function f is a bivariate density function and matches a given rank correlation are linear. Melamed et al. (1992) described the Transfer-Expand-Sample (TES) as a sequence of serially-correlated uniforms generated using an autoregressive process to be used as a base process. The TES process can reach the full range of possible lag-1 serial correlations for a certain marginal distribution and can regularly match serial correlation factors at higher lags. Hill & Reilly (1994) provided mixture models where the essential idea was to mix the distributions that correspond to zero correlation and an extremal correlation.

2.3.3 Simulation Optimization

2.3.3.1 Introduction

Tekin & Sabuncuoglu (2004) assert that a simulation model is typically a descriptive model of the system, i.e. it describes the behavior of the system under consideration, which assists in understanding the dynamics and complex interactions among the elements of a given system. They argued that simulation by itself lacks optimization capability. Simulation results have typically been replications with a set of observations, rather than an optimal solution to the problem. Optimal solutions are usually developed by prescriptive or normative models (i.e., linear programming, and dynamic programming). Thus, incorporating optimization features in simulation systems removes its major limitation and, therefore, makes it more a prescriptive tool. Since the problems that can be solved by simulation optimization vary in terms of the number and structure (i.e. discrete or continuous, quantitative or qualitative) of decision variables and shape of the response function, there is no single method to solve all of these problems. Consequently, researchers are forced to develop more robust techniques that can handle a larger class of problems. Figure 7 provides a classification scheme for simulation optimization from Tekin et al. (2004). In this representation, Tekin et al. (2004) distinguish two types of optimization methods categorized in terms of the response surface. Specifically, they stated local methods for unimodal surface and global search for multi-modal response surface. Notice, however, that many authors assume a different perspective when describing the optimum-seeking method. For example, Fishman (2005), Aarts et al. (2003), and Hoos et al. (2005) present optimization techniques expressed in terms of local and global moves throughout the decision space.

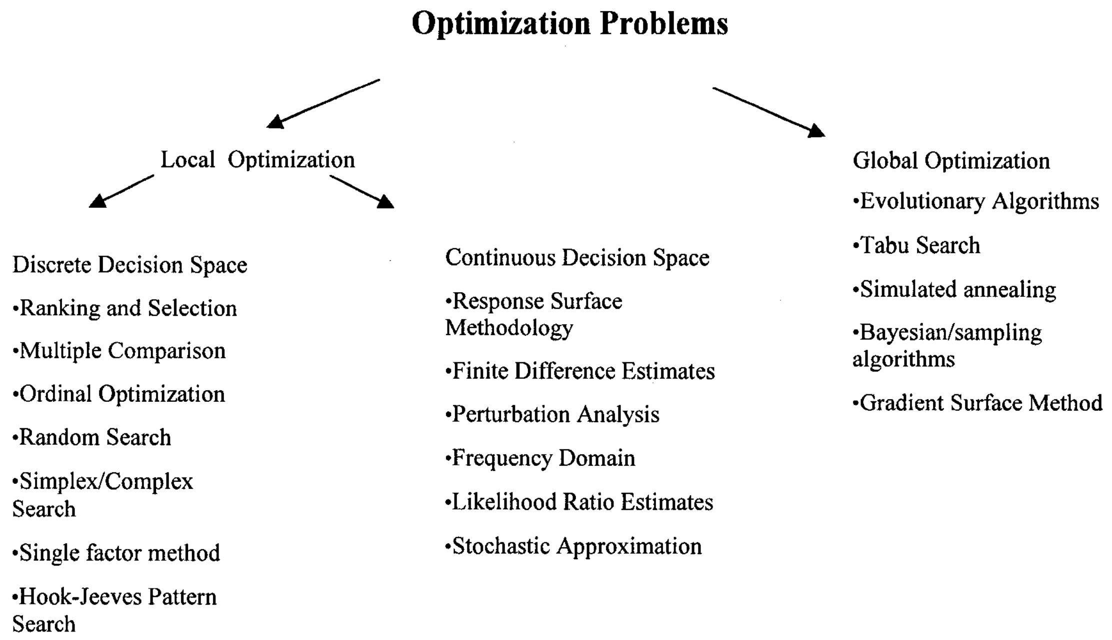


Figure 7 Simulation Optimization - Classification scheme (Tekin & Sabuncuoglu, 2004).

2.3.3.2 Searching techniques and stochastic local search

Combinatorial optimization is a branch of optimization that solves instances of problems that are believed to be hard in general by exploring the usually large solution space of these instances. Combinatorial optimization algorithms achieve this by reducing the effective size of the space and by exploring the space efficiently. Combinatorial optimization algorithms are typically concerned with problems that are NP-hard[6]. These problems involve finding groups or assignments of a discrete, finite set of objects that satisfy certain conditions or constraints. Combinations of these solution components form the potential solution. Thus, combinatorial problems can be regarded as optimization

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[6] NP-Hard (Nondeterministic Polynomial-time hard) which are known to be at least as hard as problems in NP." (Hoos & Stutzle, 2005) provided a brief discussion regarding NP-Hard and NP-complete problems.

problems where solutions of a given instance are specified by a set of logical conditions. These solutions can be evaluated according to an optimal objective function whose goal can be to minimize or maximize a function value.

A well-accepted way of solving most optimization problems involves searching for solutions in the decision space of its candidate solutions. However, the set of candidate solutions for a given process is usually very vast. Thus, efficiently searching candidate solutions in a vast decision space becomes an issue. Hoos et al. (2005) summarize three common ways of dealing with NP-hard problems: (1) find a relevant subclass that can be solved efficiently; (2) use efficient approximation algorithms; (3) use stochastic approaches. They also added:

> "for many problems where exponential time complexity is unavoidable, or even incomplete, can still be dramatically more efficient than others and hence make it feasible to solve the problem for practically interesting instance sizes. This is where heuristic guidance, combined with randomization and probabilistic decisions can make the difference."

Based on these strategies and depending upon the degree of complexity of the problem, many authors have created or combined searching techniques to efficiently explore such decision space. The essential idea is to iteratively produce and evaluate candidate solutions in the terms of the objective function. The way in which candidate solutions are generated determines the searching technique. As a result, two fundamental approaches have been developed for searching algorithms, local search and global search (Aarts & Korts, 1989; Osman & Kelly, 1996; Fishman, 2005; Hoos et al., 2005). Global search techniques traverse the search space to eventually find a solution; local search starts at one location and moves to a new location and does not guarantee the optimal solution.

The searching mechanisms and the rules in generating and selecting candidate solutions are the two most important features that define local search techniques.

- The two main search mechanisms to explore the decision space include perturbative or constructive searching[7] (Hoos et al., 2005).
- Rules for generating or selecting candidate solutions can be deterministic, random, or a combination of the two. Stochastic rules can be subdivided into random-deterministic and random-random (Fishman, 2005).

Many well-known local search methods use random rules to generate or select candidate solutions for a given problem. These search methods are called stochastic local search (SLS) (Hoos et al., 2005). Typical SLS components include the search space, solution set, and neighborhood. In addition, an initialization and step function from the underlying process may exist. The evaluation function maps each search position in such a way that an optimum corresponding to the solution is determined. Often, the objective function is used as an evaluation function such that the values of the evaluation function correspond directly to the quantity to be optimized. Other components associated with SLS methods include dealings with local minima[8] and intensification and diversification[9]

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[7] Perturbative is referred to the process of transforming current candidate solution into a new one by modifying one or more of the corresponding solution component, which is perturbing a candidate solution on the search space. Constructive is referred to the task of generating candidate solutions by iteratively extending partial candidates solutions, which can be formulated as search where the goal is to obtain a good candidate solution (Hoos et al., 2005).

[8] Local minima are position in search space from which no single search step can achieve an improvement with the evaluation function (Fishman, 2005).

[9] Intensification is referred to search strategies that aim to greedily improve solution quality or the chances of finding a solution in the near future by exploiting the guidance by the evaluation function. Diversification strategies attempt to prevent search stagnation by making sure that the search process

of the technique (Hoos et al., 2005). They categorized stochastic local search techniques in four groups: simple SLS, hybrid local search, population-based search, and evolutionary algorithms.

- **Simple SLS.** In a Simple SLS the step function is modified such that the search process can perform worsening steps that help it to escape from local minima. Simple SLS considers randomized and probabilistic improvement and techniques, including the hill-climbing algorithm, simulated annealing (Kirkpatrick, Gelatt, & Vecchi, 1983), simulated tempering (Marinari & Parisi, 1992), and stochastic tunneling (Wenzel & Hamacher, 1999). In dynamic local search techniques, the evaluation function is modified whenever local minima are found. This modification is performed in such a way that further improvement steps become possible by assigning penalty weights with individual solution components. Thus, the penalties of some solution components are increased. Techniques include guided local search by Voudouris & Tsang (1999).
- **Hybrid SLS**. Hybrid SLS methods refer to combinations of simpler SLS. These methods include iterated local search, greedy randomized adaptive search procedures, and adaptive iterated construction search. The hybrid SLS - Iterated local search techniques (Loureno, Martin, & Stutzle, 2003) combines procedures for local searching, perturbing, and accepting solutions. Iterated local search techniques include Large Step Markov chains (Martin, Otto, & Feltern, 1991), and Chained local search (Martin & Otto, 1996).

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achieves a reasonable coverage when exploring the search space and does not get stuck in relatively confined regions that do not contain (sufficiently high-quality) solutions (Hoos et al., 2005).

- **Population-based SLS**. These methods are characterized by simultaneously exploring potential solutions rather than one candidate per search step. Population-based SLS methods include the Ant colony optimization technique. The Ant colony optimization is based on aspects of the pheromone-based trial-following behavior. In essence, population agents indirectly communicate via distributed, dynamically changing information known as pheromone trails. These pheromone trails reflect the collective search experienced by the ants in their attempts to solve a given problem instance. In other words, the interaction among individual elements of potential solutions is through the indirect modification of a common memory (pheromone trails). Dorigo & Stutzl (2004) provide detailed metaheuristics for Ant colony optimization.
- **Evolutionary algorithms**. They transfer the principle of evolution through mutation, recombination, and selection of the fittest, which leads to the development of species that are better adapted for survival in a given environment to combinatorial optimization. They are iterative, population-based approaches that start with a set of candidate solutions and repeatedly apply a series of three genetic operators, namely selection, mutation, and recombination. Thus, the current population is replaced by a new set of candidate solutions are known as generations. Evolutionary algorithms include the entire family of genetic algorithms (Holland, 1975; Goldberg, 1989) and evolutionary strategies (Fogel, Owens, & Walsh, 1966; Schwefel, 1981). For discussion of evolutionary algorithms, the reader is referred to (Back, 1996).

In Table 1 the most popular local search approaches are compared (Aarts et al., 1989; Aarts & Lenstra, 2003; Osman et al., 1996; Hoos et al., 2005).

Heuristic	Basic Description	Advantage/Disadvantage	Basic algorithm
SA	Annealing means to heat and then cool. Its search technique incorporates processes analogous to heating and cooling to coerce a sample path to converge to a solution in the optimal solution set. A good example includes an ingot, which is a solid block whose atoms have arranged themselves in a crystalline structure x with corresponding energy $H(x)$. Heating the ingot sufficiently converts it to a molten state, energizing its atoms (increasing $H(x)$) so that all possible crystalline configurations are equally likely. As the molten ingot cools, its atoms lose energy. If cooling proceeds at a sufficiently slow rate, the atoms combine into a configuration that gives the solidified ingot its greatest structural strength. If cooled too rapidly, the atoms combine to a crystalline configuration x corresponding to one of the local minima in $H(x)$, leaving the solidified ingot with less structural strength.	• Successfully applied to a wide range of problems. • Randomized nature enables asymptotic convergence to optimal solution. • Easy to implement and capable of handling almost any optimization problem. • It is able to improve upon the relatively poor performance of local search by replacing the deterministic selection by probabilistic choices. • Theoretical knowledge makes a robust approach. Convergence has been easily proved by describing the ergodic properties of the sample path. • Convergence typically requires exponential time, which implies high costs of running time.	• Determine initial candidate solution • Set initial temperature according to annealing schedule • While termination condition is not satisfied: - Probabilistically select a new neighbor -If new neighbor satisfies probabilistic acceptance criterion (depending on the temperature): accept -Update temperature according to annealing schedule. -Repeat.
Tabu Search	To escape from local minima, tabu search systematically uses memory for guiding the search process. The simplest method consists of an iterative improvement algorithm enhanced with a form of short-term memory that enables it to escape from local minima. It uses a best improvement strategy to select the best neighbor of the current candidate solution in each search step. It forbids steps to recently visited search positions by memorizing previous candidate solutions and ruling out any step that would lead back to these steps.	• It must be tailored to the details of the problem. • There is little theoretical knowledge that guides the tailoring process. No clean proof of convergence. • Remarkable efficiency for many problems.	• Determine initial candidate solution • While termination criteria is not satisfied: -Determine set of non-tabu neighbors of candidate solution -Choose a best improving solution in the neighborhood -Update tabu attributes based on new accepted solution. -Repeat.
Genetic Algorithm	Genetic algorithms are implemented as a simulation in which a population of abstract representations (called chromosomes) of candidate solutions to an optimization problem evolves toward better solutions. The evolution usually starts from a population of randomly generated individuals and occurs in generations. In each generation, the fitness of every individual in the population is evaluated; multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly mutated) to form a new population. The new population is then used in the next iteration of the algorithm.	• It can be intuitively understood. • It may fail in finding satisfactory solutions. • However, the algorithm combines two different strategies: a random search by mutation and a biased search by recombination of the string contained in the population.	• Determine initial population • Assign a fitness value to each string in the population - Pick a pair of strings for breeding - Put offspring produced in temporary population (mating pool) -If the temporary population is not full, then repeat last step. - Replace the current population with the temporary population and portion of the current population - If the termination criterion is not met, repeat the fitness assignment process.

Table 1 Comparison of most popular local search techniques

2.3.4 Fundamentals for Simulated annealing (SA), Pattern Search (PS), and Ranking and Selection (R&S)

This section provides an overview and basic elements of the heuristic used in this dissertation. The essential concepts and components of SA, PS, and R&S are provided. As a result, the foundations of the SAPSRS algorithm (named using the initials of each method) will be set down.

2.3.4.1 Simulated Annealing (SA)

Annealing is a search technique that incorporates processes analogous to heating and cooling to coerce a sample path to converge to a solution in the optimal solution set.

To conduct a search for a near-optimal solution, SA employs a specialized form of Hastings-Metroplis (HM) sampling (Fishman, 2005), which is a rejection sampling algorithm used to generate a sequence of samples. The HM sampling can generate samples from any probability distribution and requires only that a function proportional to the density can be calculated at the incumbent. The most frequently encountered form of SA states that a candidate solution depends on the previous state to generate a proposed sample according to $U \le e^{-[H(y)-H(x_{j-1})]/T}$. Thus, the assessment process evaluates a potential candidate solution in accordance with:

$$x_j = \begin{cases} y & \text{if } U \le e^{-[H(y)-H(x_{j-1})]/T} \\ x_{j-1} & \text{Otherwise} \end{cases}$$

Where x_{j-1} is the previous state, y is the potential candidate solution, T is a scheduled temperature, and x_j is the accepted candidate solution.

Fishman (2005) summarizes the most essential components of SA as follows.

2.3.4.1.1 Nominating candidate solutions

SA introduces the concept of nominating candidate solutions. Nominating candidate solutions is referred to as the process of randomly generating new states x_j (candidate solutions) in the decision space per iteration. In general, it frequently produces a new candidate solution x_j in the neighborhood of the previous neighbor x_{j-1}. When exploring the decision spaces, at a given temperature T, small moves are more likely to be accepted than large ones if $H(x_j)$ is close to a minimum. Thus, convergence to a solution is more probable.

2.3.4.1.2 Cooling, terminating, and other components

As indicated previously, in SLS there is no guarantee of finding an optimal solution, regardless of the length of the sample path. However, by progressively reducing T, convergence can be accomplished. In SA this is achieved by the cooling process. Many studies have been devoted to analyzing and proposing cooling schedules that exploit and speed up the convergence properties.

In general, a large value of T increases the probability that $x_j = y$ in the acceptance function above. Conversely, small values of T decrease that probability. The temperature T influences the speed of convergence to equilibrium. Specifically, the speed of convergence tends to directly increase with an increase in T. A small T induces slow convergence to an equilibrium distribution concentrated in a small region in χ that includes $\chi_{\min}$. Overall, the cooling process in conventional SA is composed of four components: initial temperature; temperature gradient; stage length; and stopping rule.

- The initial temperature is referred to when determining that T is large enough to initiate the searching process. One approach relies on the empirical acceptance rate computed from sample-path data. As T increases, the probability distribution becomes more uniform, and the acceptance rate increases. In other words, proposed candidate solutions have a high probability of being accepted due the large temperature.
- Temperature gradient is related to a variable quantity that describes in which direction and at what rate the temperature changes given a particular value. One practice for choosing $\tau_1, \tau_2, \ldots$ is the assignment $\tau_{k+1} = \alpha \tau_k$ where $0 < \alpha < 1$ and $0.80 \leq \alpha \leq 0.99$.
- Stage length refers to the issue of how long sampling can be executed given a certain temperature. Since each successive lowering temperature leads to the convergence of a new equilibrium, l_k should be taken so that $l_{k+1} - l_k < l_{k+2} - l_{k+1}$, $k = 1, 2, \ldots$. After the kth stage, an inspection of the sample path $H(x_{l_k+1}), \ldots, H(x_{l_k})$ provides insights for assessing the adequacy of the warm-up interval $l_{k+1} - l_k$.
- Stopping rule refers to the termination criteria for ending the SA process. Monitoring the obtained sample path data $H(x_1), \ldots, H(x_{l_k})$ allows visualizing rules to stop the sampling process. Some termination rules include: observing certain propensity to concentrate in a relatively small neighborhood (Kolinski & Skolnick, 1994); no change in the objective function value during a given number of stages (Aarts et al.,

1989); and the number of acceptances having fallen below a specific value (Kirkpatrick et al., 1983).

In this dissertation, SA is used to stochastically generate, propose, and evaluate potential solutions throughout the search process. In addition, a combination of the termination mechanism described above is used to stop the search process. For further information about SA, the reader is referred to (Davis, 1987; van Laarhoven & Aarts, 1987; Otten & van Ginnenken, 1989; Aarts et al., 1989; Aarts et al., 2003; Pham & Karaboga, 2000).

2.3.4.2 Pattern Search (PS)

Pattern search is a direct search technique that considers the direct evaluation of objective function values and does not require the use of derivatives. As indicated by Sriver & Chrissis (2004), the basic idea is to explore the decision space using a finite set of directions defined per iteration. The step length parameter and the direction set define a mesh centered in relation to the current iterate (the candidate solution). From the mesh, test points are selected, assessed, and contrasted to the candidate solution in order to decide on the next iterate. If an enhancement is found in the resultant values of the objective function, the iteration is confirmed successful and the mesh is preserved. If no improvement is found, the mesh is redefined and a new set of candidate solutions is built. They assert that a critical aspect of generating the mesh is related to the direction set. This allows any vector creating from the incumbent to be shaped as a nonnegative linear combination. Lewis & Torczon (1996) provide more details of the direction set where a positive spanning set concept is defined. Thus, if the gradient of the objective function is nonzero, at least one component of the direction set is assumed to descent direction (Sriver & Chrissis, 2004). In Figure 8, a candidate solution x_j and the test points $\{a,b,c,d\}$ are shown in two dimensions with the search directions defined by the step length δ_j.

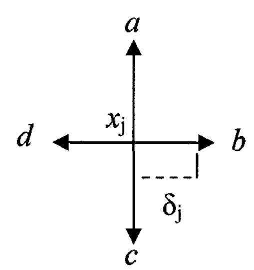

Figure 8 Example of Pattern Search

Torczon (1997) formally asserted that to define a pattern two components are necessary: the basis matrix and the generating matrix. The basis matrix can be any nonsingular matrix $A \in \mathbb{R}^{n\times n}$ while the generating matrix is $C_k \in Z^{n\times p}$, where $p > 2n$. Thus, the generating matrix can be decomposed into the following components.

$$C_k = [Q_K \quad -Q_K \quad L_k] = [\Upsilon_k \quad L_k]$$

This requires that $Q_K \in Q \subset Z^{n\times n}$ where Q is a finite set of nonsingular matrices. In addition, it requires that $L_k \in Z^{n\times(p-2n)}$ and contains at least one column that can have zeros. Thus, Torczon (1997) defined a pattern P_K by the columns of the matrix $P_K = AC_k$.

Since both the basis matrix A and the generating matrix C_K have rank n, the columns of P_K span $\mathbb{R}^n$. Then, the partition of the generating matrix C_K can be used to partition P_K as follows:

$$P_K = AC_k = [AQ_K \quad -AQ_K \quad AL_k] = [A\Upsilon_k \quad AL_k]$$

Finally, the author defined a trial step s_k^i as:.

$$s_k^i = \delta_k A c_k^i$$

where c_k^i represents a column of $C_K = \begin{bmatrix} c_k^1 & \cdots & c_k^p \end{bmatrix}$, the component Ac_k^i determines the direction of the step, δ_k is the step length parameter at iteration k. Thus, considering a given incumbent x_k, a trial point is simply defined as any point of the form:

$$x_k^i = x_k + s_k^i.$$

Proofs of convergence for this algorithm, commonly called coordinate search, can be found in Torczon (1997). In this dissertation, PS is used to deterministically generate the mesh according to a gradient (step-size) and a given set of rules. In addition, the coordinate search with fixed step lengths is considered for the PS piece of the SAPSRS procedure. Variations of this fundamental algorithmic strategy can be found under numerous names, including compass search, alternating directions, alternating variable search, axial relaxation, coordinate search, and local variation (Kolda, Lewis, & Torczon, 2004).

2.3.4.3 Ranking and selection (R&S)

R&S was introduced by describing a problem in which the objective is to select the population containing the largest mean for some population statistic from a set of k normal populations (Bechhofer, Dunnett, & Sobel, 1954). This population was referred to as the "best" (Law & Kelton, 2000).

When mean performance is investigated, the typical indifference zone (IZ) procedure is commonly used. Nelson, Swann, Goldsman, & Song (2001) summarize this IZ procedure as follows. First, for each alternative, obtain a number of observations of the performance measures of interest and calculate measures of variability of the observations. Next, based on the measure of variability, the numbers of options, and the desired confidence level, calculate the total number of observations required from each option to guarantee that a user-specified significant difference in performance can be revealed at the desired confidence level. Finally, obtain the prescribed number of additional observations from each option and decide on the one with the best performance (Nelson, Swann, Goldsman, & Song, 2001). Let n_0 be the initial sample size, h a constant that depends on the number of alternatives A, $1-\theta$ be the desired confidence level, S_i^2 be the sample variance of the n_0 observations, and d^* the practically significant difference specified by the user. Then, the procedure provided by Rinott (1978a) defines the number of additional replications to compare performance measures and to reach a decision, assuming that the number of observations is independent and normally distributed, as given by:

$$N_i = \max\left\{n_0, \left\lceil\left(\frac{hS_i}{d^*}\right)^2\right\rceil\right\}$$

Goldsman (1985) explored the use of standardized time series theory to determine variance estimators for R&S methodologies; the R&S problem was formulated as an multi-stage optimization problem in which clearly inferior designs are discarded in the earlier stages (Chen, 1995; Chen, Chen, & and Dai, 1996; Chen, Chen, and Dai, & Yucesan, 1997; Chen, Yuan, Chen, Yucesan, & Dai, 1998). A Bayesian analysis for selecting the best simulated system was provided by (Chick, 1997). A comparison between Bayesian and other approaches for selecting the best system was given by Inoue & Chick (1998). Chick and Inoue (1998) extended Chick's (1997) work to the study of sampling costs and value of information arguments to improve the computational efficiency of identifying the best system. In this research study, IZ procedures are used.

In this dissertation, R&S is used to evaluate and select the best candidate solution amongst the proposed candidate generated by the PS step.

In this chapter, relevant work from the literature has been discussed and summarized. There are many differences between the methods traditionally used to solve inventory models and methods used in this research. In general, traditional methods lack the capability of incorporating and handling autocorrelated demand. This dissertation solves the stochastic inventory problem, where the control review system is continuous, the demand contains autocorrelated components, and the lost sales case is considered. The proposed simulation optimization method accounts for the randomness and dependency of the demand as well as the inherent constraint of the inventory model. Although some work has been done in stochastic inventory models that considers autocorrelated demands, none of the research considered a continuous review model with DMC and AR(1) demand process along with the lost sales case. This dissertation considers these situations and provides a validated method to solve them.

3 THE INVENTORY MODEL

3.1 Introduction

In this chapter, a mathematical representation of the stochastic inventory model that presents autocorrelated components is provided. This mathematical model is based on the continuous review control policy (s, S) and the lost sales case. In addition, the sampling mechanisms to represent and obtain the discrete and continuous probability distribution are presented.

3.2 Mathematical model

3.2.1 Introduction

In this subsection, the mathematical model is described. First the general assumptions are stated. Then, the problem is formulated as DP characterization. Finally, the DP formulation that deals with discrete and continuous random variables is presented.

3.2.2 Assumptions

Consider an inventory problem over an infinite horizon, where

1. Each application involves a single item.
2. The inventory level is under continuous review, so its current value is always known.
3. A (s, S) policy is used. As a result, the only decision to be made is to choose s and S.
4. The demand for withdrawing units from inventory to sell them is uncertain ξ. The probability distribution for the continuous case is known to be first-order autoregressive AR(1) while in the discrete case it is Markovian-modulated according to a given transitional probability p_{ij}.

5. If a stockout occurs before the order is received, the excess of demand is lost. In other words, given a certain demand, demand is partially covered with existing inventory while the unfilled portion is lost.
6. Since transportation can be assumed to be provided by the same company, i.e. in a grocery company that provides the item, no setup cost is incurred each time that an order is placed.
7. The cost of the order c is proportional to the order quantity z_i.
8. No discount costs are considered.
9. A certain holding cost h is incurred for each unit in inventory per unit time.
10. When a stockout occurs, a certain shortage cost p is incurred for each unit lost per unit time. In addition, a minimal holding cost $C(0)$ is incurred every time that the system reaches the zero level.
11. Replenishment occurs immediately.
12. Demand occurs instantaneously at the start of the period immediately after the order is received.
13. Setup costs are assumed only for the model that considers AR(1) demands.

3.2.3 Deterministic formulation

In general, finding a solution to the problem can be formulated as solving a DP model where the minimization of the total expected cost is given by the interaction generated by the demand component over the ordering, shortage, and holding costs. The deterministic formulation assumes that the demand values are known. In the next section, this assumption is relaxed, allowing the component demand to be unknown but described according to a probabilistic distribution. The deterministic formulation is presented as follows.

$$\text{Min}\left[c(y_i - x_i) + L_i^0(y, D)\right] \tag{1}$$

subject to

$$z_i = \begin{cases} S - x_i & \text{If } x_i \le s \\ 0 & \text{Elsewhere} \end{cases} \tag{2}$$

$$L_i^0(y, d) = \begin{cases} h*(y_i - D_i) & \text{If } (y_i - D_i) \ge 0 \\ p*(D_i - y_i) + C(0) & \text{If } (D_i - y_i) \le 0 \end{cases} \tag{3}$$

$$\Pr(y_i \le 0) \ge \mathit{Critical\ Ratio} \tag{4}$$

$$s \ge 1{,}000 \tag{5}$$

$$S \le 7{,}000 \tag{6}$$

$$S \ge 1.10s \tag{7}$$

$$D \ge 0 \tag{8}$$

The model determines the optimal value of order quantity y that minimizes the sum of the expected purchasing c, holding h, and shortage costs p plus an administrative cost associated to operating the inventory system without any item $C(0)$. In general, given the demand D_i for period i and the optimal y $(= y^*)$, the inventory policy calls for ordering $y^* - x$ if $x > y$; otherwise, no order is placed.

In terms of the (s, S) control policy, the constraints presented in (2) – (8) shape the course of actions in evaluating the objective function where s indicates the inventory level that triggers ordering, and S is the target inventory for a reorder action, and CR represents the critical ratio as a constraint for calculating the required service level (see section 3.2.5.3).

3.2.4 Formulating the problem as a Dynamic Programming model

A policy is a rule that specifies which action to take at each point in time. In general, the decisions specified by a policy depend on the current state of the system. A policy is defined as a function that assigns an action to each state, independent of previous states, previous actions, and a given time. In the dynamic programming framework, it is a policy that is independent of time. The DP formulation of an inventory problem allows one to iteratively find solutions that improve the optimal policy search process. It can capture the simulation optimization process and allows a flexible representation of the various elements that compose the inventory formulation. For example, DP incorporates the penalization constraint in its recursive equation. In the stochastic case, it allows an intuitive representation of both discrete and continuous random parameters. For these reasons, many authors prefer to formulate their inventory models using DP formulations.

In this research study, the stochastic lost sales inventory problem is formulated as a DP problem. In the following paragraphs, the deterministic and stochastic formulations are presented in detail.

3.2.5 Detailed DP formulations

The amount of inventory to acquire depends on the probability distribution of demand D. A balance is needed between the risk of being short, and thereby incurring shortage costs, and the risk of having an excess, thereby incurring the wasted costs of ordering and holding excess units (Hillier et al., 2001). This is accomplished by minimizing the expected value of the sum of these costs. As a result, from Equations (1) – (8), in terms of DP, the cost incurred if the demand is D is given by

$$C(D_i, y_i) = c(y_i - x_i) + p\max\{0, D_i - y_i\} + h\max\{0, y_i - D_i\} \qquad (9)$$

Since the demand is a stochastic variable with probability distribution $P_D(d)$, the cost is also a stochastic variable. As a result, the expected cost variable for a single period is given and represented by $C(y)$ as follows.

$$C(y_i) = E[C(D_i, y_i)] = \sum_{d=0}^{\infty}(c(y_i - x_i) + p\max\{0, d_i - y_i\} + h\max\{0, y_i - d_i\}P_D(d_i)) \quad (10)$$

$$C(y_i) = E[C(D_i, y_i)] = c(y_i - x_i) + \sum_{d=0}^{\infty} p(d_i - y_i)P_D(d_i) + \sum_{d}^{y-1} h(y_i - d_i)P_D(d_i) \qquad (11)$$

The function $C(y)$ depends upon the probability distribution of d. Normally, a representation of this probability distribution is difficult to find (Hillier et al., 2001; Taha, 2002). Thus, for the continuous random variable D, let

$\varphi_{CD}(\xi)$: probability density function of the stochastic demand

$\Phi(a)$: cumulative distribution function of the demand (CDF)

So, $\Phi(a) = \int_0^a \varphi_{CD}(\xi)d\xi$

For the discrete random variable D, let

$\varphi_{DD}(\xi)$: probability mass function of the stochastic demand

$\Phi(a)$: cumulative mass function of the demand (CMF)

$$\Phi(a) = \sum_0^a \varphi_{DD}(\xi)$$

Thus, the optimal service level is obtained by minimizing the CDF/CMF of $C(y)$ in the continuous and discrete cases respectively. This value can be found either by solving its mathematical expression or by finding the area under the curve by simulation optimization. In this research, the simulation optimization approach is used to approximate the CDF/CMF function.

3.2.5.1 Continuous formulation case

The CDF is the probability that a shortage will not occur before the period ends. This probability is referred to as the service level being provided by the order quantity. Thus, the corresponding expected cost is given by:

$$C(y_i) = E(C(D_i, y_i)) = \int_{y_i}^{\infty} C(\xi_i, y_i)\varphi_{CD}(\xi_i)d\xi \tag{12}$$

$$C(y_i) = E(C(D_i, y_i)) = \int_{y_i}^{\infty} (c(y_i - x_i) + (p(\xi_i - y_i) + c(0))\max\{0, \xi_i - y_i\} + h\max\{0, y_i - \xi_i\}\varphi_{CD}(\xi_i)d\xi \tag{13}$$

$$C(y_i) = c(y_i - x_i) + \int_{y_i}^{\infty} (p(\xi_i - y_i) + c(0))\varphi_{CD}(\xi_i)d\xi + \int_{0}^{y_i} h(y_i - \xi_i)\varphi_{CD}(\xi_i)d\xi \tag{14}$$

3.2.5.2 Discrete formulation case

Accordingly, the CMF represents the probability that a shortage will not occur before the period ends in the discrete case where discrete demands can take place. It is expressed as follows.

$$f(x_i) = \underset{y \ge x}{\text{Min}} \left[c(y_i - x_i) + \sum_{\xi=0}^{y_i} h(y_i - \xi_i)\varphi_{DD}(\xi_i) + \sum_{y_i}^{\infty} (p(\xi_i - y_i) + h(0))\varphi_{DD}(\xi_i) \right] \tag{15}$$

3.2.5.3 Service level based on critical ratio

Service level in the constraints can be expressed in many different ways. It can be formulated either by minimizing the ordering cost subject to satisfying some customer service-level criterion or by assuming a penalty cost for each stockout and unsatisfied demand (Silver, 1985). In this research study, the penalty costs assumption is considered and is defined by Treharne & Sox (2002):

$$p = \frac{h * CR}{(1 - CR)} \tag{16}$$

When the holding cost is held constant at $h = 1$ and the penalty cost is calculated to match the critical ratio CR, a CR close to 1 indicates a high penalty cost or a high service level. In essence, the function of the penalty criterion is defined in such a way that at the end of every period, the net inventory will be not negative. For a discussion of different service constraints, the reader is referred to (Silver, 1985).

3.3 Modeling autocorrelated demand

In this section, specific probabilistic representations of autocorrelated data in the forms of DMC and AR(1) processes are presented. Further, an arbitrary Markovian-modulated demand and specific parameters for the first-order autoregressive AR(1) is given. Sampling algorithms for both cases are also provided.

3.3.1 Generating discrete Markov-modulated demand

A DMC is a model that allows the representation of discrete values according to a transition and invariant probability distributions. Markov-modulated demand modeling is a popular method for representing demands. Many applications have used it in acquisition sequences analysis (Prinzie & Van den Poel, 2006), in describing priority demands (Cohen, Kleindorfer, & Lee, 1988), and in considering the effects of promotions in a periodic inventory model. (Cheng & Sethi, 1999) .

In this research, a DMC demand is considered to describe the discrete case of the stochastic demand where autocorrelations can be modeled from the transition probability matrix. From this matrix and by using well-known properties of Markov chains, the stationary (invariant) distribution is derived. This stationary distribution is commonly referred to as the correlation-free case. Thus, for each given transitional distribution matrix that represents the autocorrelated case, invariant distributions are derived.

3.3.1.1 Transition probability matrix

Consider the inventory formulation from section 3.2.5.2 where four types of arbitrary discrete demands can occur and be represented according to a given transition probability matrix as illustrated in Figures 9 and 10 respectively.

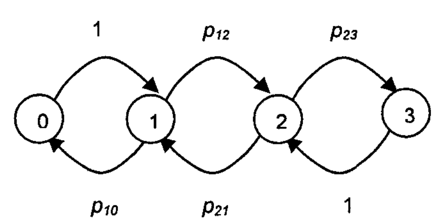

Figure 9 Transition probability graphical representation

$$p_{ij} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ p_{10} & p_{12} & 0 & 0 \\ 0 & p_{21} & p_{23} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Figure 10 Matrix 1- Probabilistic transition distribution

In this stationary case, the inventory demand can be represented according to the aforementioned arbitrary transitional probability distribution. The form of this transitional probability expresses the degree of autocorrelation among their states. Basawa (1972) studied the asymptotic behavior of estimating autocorrelation in a simple Markov chain. In this research study, serial correlation is calculated using the traditional form provided in section 2.2. This autocorrelation is determined from the given DMC and according to different combinations of the values presented in the transitional probabilities.

3.3.1.2 Deriving invariant probability distribution

The transition probability of a given Markov Chain is presented as a way to represent serially-correlated demand. However, as indicated previously, in order to analyze whether ignoring dependency has an effect on estimating the minimal costs and the *(s, S)* policy, the correlation-free representation of the DMC must be derived. The stationary distribution is obtained from the transition probability matrix and by using common properties of ergodic[10] Markov chains. The reader is referred to Durret (1999), who provides a succinct discussion of these concepts and properties.

Consider

$$0 \le p_{ij} \le 1 \qquad (18)$$

$$\sum_{i=1}^{j} p_{ij} = 1 \qquad (19)$$

[10] For a succinct discussion of ergodicity (Behrends, 2000)

$$\begin{aligned}
\pi_0 &= 0 + \pi_1 p_{10} + 0 + 0 \\
\pi_1 &= \pi_0 p_{01} + 0 + \pi_2 p_{21} + 0 \\
\pi_2 &= 0 + \pi_1 p_{12} + 0 + \pi_3 p_{32} \\
\pi_3 &= 0 + 0 + \pi_2 p_{23} + 0
\end{aligned} \tag{20}$$

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \tag{21}$$

$$\begin{aligned}
p_{01} &= 1 \\
p_{10} + p_{12} &= 1 \\
p_{21} + p_{23} &= 1 \\
p_{32} &= 1
\end{aligned} \tag{22}$$

Assuming $p_{12} = p_{21} = 0.1$, $p_{10} = p_{23} = 0.9$, and $p_{01} = p_{32} = 1$, then

$$\begin{aligned}
\pi_0 &= p_{10} \\
\pi_1 &= \pi_0 p_{01} + \pi_2 p_{21} \\
\pi_1 &= (\pi_1 p_{10}) p_{01} + \pi_2 p_{21}
\end{aligned} \tag{23}$$

Assuming that $p_{10} = 0.9$ and $p_{01} = 1$, then

$$\pi_1 = (1 - 0.9) = \pi_2 p_{21}$$

Considering that

$$\begin{aligned} p_{21} &= 1 - p_{23} \\ p_{21} &= 1 - 0.9 = 1 \end{aligned} \quad \text{and} \quad \begin{aligned} \pi_3 &= \pi_2 p_{23} \\ \pi_3 &= 0.9\pi_2 \end{aligned}$$

Substituting

$$\pi_1 = \frac{\pi_2 p_{21}}{0.1} \Rightarrow \pi_1 = \pi_2 \tag{25}$$

then

$$1 = \pi_0 p_{01} + \pi_1 (p_{10} + p_{12}) + \pi_2 (p_{21} - p_{23}) + \pi_3 p_{32}$$

$$1 = \pi_0 p_{01} + \pi_1 (p_{10} + p_{12}) + \pi_1 (p_{21} + p_{23}) + \pi_3 p_{32}$$

$$1 = \pi_0 p_{01} + 2\pi_1 + \pi_3 p_{32}$$

$$2\pi_1 = 1 - \pi_0 p_{01} - \pi_3 p_{32}$$

$$2\pi_1 = 1 - \pi_1 - \pi_2 p_{21} - 0.9\pi_2$$

$$2\pi_1 = 1 - \pi_1 + 0.1\pi_1 - 0.9\pi_1$$

$$\pi_1 = \frac{1}{3.8} \qquad (26)$$

By following this method, the next invariant distribution values, shown in Tables 2 and 3 were obtained.

	Transition Probabilities							
$p_{12} = p_{21}$	0.1		0.2		0.3		0.4	
$p_{23} = p_{10}$	0.9		0.8		0.7		0.6	
	Invariant Probabilities							
	Individual	*Cumulative*	*Individual*	*Cumulative*	*Individual*	*Cumulative*	*Individual*	*Cumulative*
π_0	0.237		0.222		0.206		0.188	
π_1	0.263	0.500	0.278	0.500	0.294	0.500	0.313	0.500
π_2	0.263	0.763	0.278	0.778	0.294	0.794	0.313	0.813
π_3	0.237	1.000	0.222	1.000	0.206	1.000	0.188	1.000

Table 2 Invariant distribution values derived from given Transition probability distribution values. Part I: p_{12}=0.1-0.4.

	Transition Probabilities									
$p_{12} = p_{21}$	0.5		0.6		0.7		0.8		0.9	
$p_{23} = p_{10}$	0.5		0.4		0.3		0.2		0.1	
	Invariant Probabilities									
	Individual	*Cumulative*	*Individual*	*Cumulative*	*Individual*	*Cumulative*	*Individual*	*Cumulative*	*Individual*	*Cumulative*
π_0	0.167		0.143		0.115		0.083		0.045	
π_1	0.333	0.500	0.357	0.500	0.385	0.500	0.417	0.500	0.455	0.500
π_2	0.333	0.833	0.357	0.857	0.385	0.885	0.417	0.917	0.455	0.955
π_3	0.167	1.000	0.143	1.000	0.115	1.000	0.083	1.000	0.045	1.000

Table 3 Invariant distribution values derived from given Transition probability distribution values. Part II: p_{12}=0.5-0.9.

3.3.1.3 Algorithm to generate correlation-free DMC

The following algorithm generates stationary time series demand given the values of the invariant probability distribution π_D and discrete demand ξ_i.

Given the values of the number of period i, the invariant probability distribution π_D, and the discrete values of the stochastic demand $\xi_D = \{a,b,c,d\}$.

1. Initialization: Generate arbitrary $\xi_{i-1} \leftarrow \xi_i$ from $\xi_D = \{a,b,c,d\}$
2. Generate π from a uniform distribution.
3. Generate new ξ_i according to:

$$\xi_i = \begin{cases} a & \text{for } [0,\pi_0) \\ b & \text{for } [\pi_0,\pi_0+\pi_1) \\ c & \text{for } [\pi_0+\pi_1,\pi_0+\pi_1+\pi_2) \\ d & \text{elsewhere} \end{cases}$$

4. Repeat steps 2 and 3 until a given number of iteration i is completed.

Since the probability of a PMF has to add up to one, the unit interval $[0,1]$ can be divided into subintervals with widths equal to the individual values given by the PMF. In this case the subintervals can be generalized as $[0,\pi_0)$, $[\pi_0,\pi_0+\pi_1)$, $[\pi_0+\pi_1,\pi_0+\pi_1+\pi_2)$, $[\pi_0+\pi_1+\pi_2,1]$. These values are shown in Tables 2 and 3 where both the individual and cumulative probabilities are presented. When a random number U is generated from a uniform distribution, it will be uniformly distributed over the interval $[0,1]$. As a result, when $U[0,1]$ is generated, it will fall in the first subinterval with probability $\pi_0 - 0 = \pi_0$, in the second subinterval with probability $\pi_0+\pi_1-\pi_0 = \pi_1$, in the third subinterval with probability $\pi_0 + \pi_1 + \pi_2 - \pi_0 - \pi_1 = \pi_2$, and in the fourth $1 - \pi_0 + \pi_1 + \pi_2 = \pi_3$.

To illustrate the workings of sampling the correlation-free DMC, consider the case from Table 2 when $p_{23} = p_{10} = 0.6$. The probability mass function (PMF) was derived according equations (18) – (26), so the PMF took the form of:

$$p(\Xi = \xi) = \begin{cases} 0.188 & \text{for } \xi = a = 1{,}000 \\ 0.313 & \text{for } \xi = b = 2{,}000 \\ 0.313 & \text{for } \xi = b = 3{,}000 \\ 0.188 & \text{for } \xi = c = 4{,}000 \end{cases}$$

Once again, because the probabilities of PMF have to add up to 1 and subdividing the unit interval into subintervals using the values of PMF, the obtained subintervals are: $[0,0.188),[0.188,0.5),[0.5,0.813),[0.813,1]$. Thus, when a $U[0,1]$ is generated, it will fall in the first subinterval with probability 0.188-0=0.188, second subinterval with probability 0.5-0.188 = 0.313, third subinterval with probability 0.813-0.5 = 0.313, and in the fourth subinterval with probability 1 - 0.813 = 0.188.

3.3.1.4 Algorithm to generate correlated DMC Demand

The following algorithm generates stationary time series demand given the values of the transition probability distribution p_{ij} and discrete stochastic demand ξ_i.

Given the values of the number of period i, the transition probability distribution p_{ij}, and the discrete values of the stochastic demand $\xi_D = \{a,b,c,d\}$,

Initialization: Generate arbitrary $\xi_{i-1} \leftarrow \xi_i$ from $\xi_D = \{a,b,c,d\}$

1. Generate p from a uniform distribution.

2. Generate new ξ_i according to:

   a. If $\xi_{i-1} = a$ then $\xi_i = b$;

   b. If $\xi_{i-1} = b$, then $\xi_i = \begin{cases} a & \text{for } [0, p_{01}) \\ c & \text{elsewhere} \end{cases}$

   c. If $\xi_{i-1} = c$, then $\xi_i = \begin{cases} b & \text{for } [0, p_{21}) \\ d & \text{elsewhere} \end{cases}$

   d. If $\xi_{i-1} = d$ then $\xi_i = c$;

4. Repeat steps 2 and 3 until a given number of periods i are completed.

Similar to the rationale of DMC stationary distribution sampling, the transition distribution sampling uses the concept of generating $U[0,1]$ from a uniform distribution and dividing the unit interval $[0,1]$ into subintervals with widths equal to the individual values given by the probability mass function. However, given the transitional matrix, the associated probabilities to a known state conforms its PMF. Indeed, per each state this PMF adds up to one. In other words, depending on the current state there are a given number of probabilities that add up to one and are assumed to be the PMF. These probabilities are shown in Tables 2 and 3.

To illustrate the mechanics of this algorithm, consider the case that the current state is $b = 2{,}000$. Given the transition structure used in section 3.3.1.1, the future state is either a or c with associated probabilities of $p_{ba} = 0.6$ and $p_{bc} = 0.4$ respectively. Then, the subintervals are $[0, p_{ba})$ and $[p_{ba}, 1]$. In other words, for the selected example,

$[0, 0.6)$ and $[0.6, 1]$. As a result, when $U[0,1]$ is generated, it will fall in the first subinterval with probability $0.6 - 0 = 0.6$ and in the second subinterval with probability $1 - 0.6 = 0.4$.

3.3.2 Generating Autoregressive AR (1) demand

3.3.2.1 Introduction

An autoregressive AR(1) model is a representation that allows one to model continuous demand. AR(1) has been used by most authors that have considered autocorrelated components in the demand (Ray, 1980; Ray, 1981; Erkip & Hausman, 1994; Inderfurth, 1995; Lee, So, & Tang, 2000; Urban, 2000; Urban, 2005). Based on real data, these authors argue that AR(1) is representative of the autocorrelated process exhibited in a certain class of serially-correlated demand streams.

The autoregressive order-1 AR(1) considers a sequence of identically distributed, but autocorrelated data (Banks et al., 1996).

$$\xi_i = \mu + \phi(\xi_{i-1} - \mu) + \varepsilon_i \qquad (27)$$

For $i = 1, 2, \ldots$, where ε_i are IID with $\mu = 0$ and variance σ_ε^2, and $-1 \le \phi \le 1$.

The autocorrelation takes the form of:

$$\rho_h = corr(\xi_i, \xi_{i+h}) = \rho^h \text{ , while } \rho_h = \phi^h \qquad (28)$$

Estimation of the parameter ϕ is given by the fact that $\phi = \rho^1 = corr(d_i, d_{i+1})$ the lag-1 autocorrelation; as result the lag-1 correlation is given by the autocovariance.

$$\mathrm{cov}(\xi_i, \xi_{i+1}) = \frac{1}{n-1}\sum_{i=1}^{n-1}(\xi_i - E[\xi])(\xi_{i+1} - E[\xi]) \qquad (29)$$

If $E[\xi] = 0$,

$$\hat{\phi} = \frac{\hat{\mathrm{cov}}(\xi_i, \xi_{i+1})}{\hat{\sigma}} \qquad (30)$$

Finally, μ and variance σ_ε^2 are given by $\hat{\mu} = \bar{\xi}$ and $\hat{\sigma}_\varepsilon^2 = \hat{\sigma}^2(1 - \hat{\phi}^2)$ respectively

The next algorithm presents a method of sampling from a given AR(1) distribution (Banks et al., 1996):

3.3.2.2 Algorithm to generate stationary AR(1)

Given the values of the parameter autocorrelation ϕ, the mean μ, and variance of the error σ_ε^2.

1. Generate the stochastic ε_i from the normal distribution with mean 0 and σ_ε^2.
2. Set $\xi_i = \mu + \phi(\xi_{i-1} - \mu) + \varepsilon_i$
3. Set $i = i + 1$ and go to 2

Where the error ε_i is assumed to be normally distributed. The algorithm used to generate normally distributed observations was the Polar method taken from Law et al. (2000).

4 METHOD

4.1 Introduction

This chapter provides a description of a methodology for solving the stochastic inventory problem introduced in Chapter 3. First, preliminaries for the SAPSRS methodology are provided. Then, details of the SAPSRS based methodology that is used for solving the stochastic inventory problem are presented. Recall that SA is one of the most widely used probabilistic heuristics for approximating solutions to complex problems. Once a potential candidate is proposed by SA, PS and R&S executes a near-neighborhood exploration. Benefits of exploring a near-neighborhood can be two-fold: a. Obtaining and evaluating candidate solutions that may report a better performance, and b. Improving the process of avoiding local minima. By using PS, one can deterministically define additional candidate solutions. R&S is used as a statistical method to evaluate and select the best performance from a pool of potential candidate solutions.

4.2 Preliminaries

Consider the following optimization problem

$$\min f(\xi) \qquad (31)$$

$$\text{s.t. } \xi \in \Xi \qquad (32)$$

where $f(\xi) = E[\Psi(\xi,\omega)]$ is the expected value of the system performance measure estimated by $\hat{f}(\xi)$, which is obtained from j sample performance $\Psi_j(\xi,\omega)$ of a simulation model observed under an instance of discrete or continuous feasible input parameters ξ constrained within a set of feasible $\Xi \in \mathbb{R}^d$. ω is a vector of random elements that represents the stochastic effects of the system.

The complexity in this problem is that $f(\xi)$ is an implicit function of the decision parameters in which an observation of the random variable can only be obtained by simulation. As a result, the simulation model is considered as a black-box process, which takes an input vector ξ and produces the output response $\Psi(\xi,\omega)$. In the research community, a vast number of search methods that uses black-box simulation are available. Among these approaches, one can find combinatorial optimization techniques.

In this dissertation, a new iterative method for simulation optimization based on SA, PS, and R&S is presented. The method combines SA, to randomly generate and accept potential candidate solutions, with PS, to systematically explore the candidate solutions neighborhood, and R&S, to select the best of the proposed neighbors. This method resembles simulated tempering (Marinari et al., 1992) in the sense that once a candidate solution is obtained, it executes another exploration of its neighborhood with an additional iteration. A similar work that combines SA and R&S was developed by Ahmed & Alkhamis (2002) in which the neighborhood exploration is performed according to a customized function. The R&S method used in their simulation optimization algorithm is based on Dudewicz and Dalal's technique (Law et al., 2000) .

4.3 The Heuristic

4.3.1 Approximating solutions by using the SAPSRS algorithm

Whereas SA candidate solutions are simply accepted or rejected according to an evaluation function in each iteration, SAPSRS explores and evaluates the neighborhood of an accepted solution. In this regard, SAPSRS incorporates statistical knowledge of the response of generated candidate solutions and their associated neighbors by including Patten Search combined with Ranking and Selection.

Fundamentally, SA consists of two steps. The first step randomly produces a candidate solution in the decision space. The second step randomly chooses to accept the nominated location. SAPSRS adds two additional steps. Given the selected candidate solution at the end of the second step, the third deterministically produces and assesses the candidate's neighbors using a pattern search procedure. The fourth step picks the best candidate solution by using a Ranking and Selection procedure.

While solutions generated by SA can be trapped in a local minimum, SAPSRS increases the probability of an escape from it. As in simulated tempering algorithms (Marinari et al., 1992; Fishman, 2005), by generating additional neighbors to potentially accepted candidate solutions, SAPSRS generates a sample path that presents more variations and accuracy than using traditional SA alone. In addition, by including R&S as an evaluation tool, the probability of selecting the best candidate solution, considering the inherent randomness of the observed demand, is guaranteed.

4.3.2 The SAPSRS Algorithm

The proposed approach integrates PS and R&S in a typical SA procedure to enhance the quality of obtained candidate solutions. A detailed description of each step is provided in the following paragraphs.

As in traditional SA, an initial experimentation can be performed to obtain the maximum temperature parameter that maximizes a given acceptance function. At the same time, stage length, temperature gradient, number of stages, and parameters for the IZ procedure must be provided.

The algorithm initiates the SA step whereby the procedure randomly samples a probability distribution and a uniform distribution to proceed with an evaluation step. The evaluation step consists of accepting or rejecting a candidate solution. The assessment function is reliant on the theory of the Boltzmann probability density function in which for every high temperature, each candidate solution (state) has an equal probability of being the current state while for low temperature values, only states with low energies have a high probability of being the current state. In other words, if the temperature is high enough, acceptance of candidate solutions is high and if the temperature is low, acceptance is low. Up to this point, traditional SA would basically accept or reject a proposed state (nominated candidate solution) and a new iteration would take place. However, in this procedure, after a candidate solution is probabilistically accepted, the evaluation process is refined by an extended procedure whose goal is to explore and evaluate the nearest-neighborhood of the potential candidate solution.

The PS step deterministically generates a given number of neighbors, producing a mesh that is composed of test points that pose a distance of step length δ from the potential candidate solution. These neighbors have a response mean sufficiently close to the response mean of the candidate solution. Subsequently, a R&S step is used to evaluate such neighbors. In this sense, the response of each point is initially evaluated according a given n_0 sample size. In addition, a measure of variability per test point is obtained. The number of additional replications N_i is then calculated. As a result, each test point is replicated and responses are determined. Finally, the batched mean of each response is sorted. The candidate solution with the lowest response is assumed to be the best.

Because SAPSRS swap between displacement and temperature, the induced Markov chain is not reversible. Nonetheless, adjusting the algorithm to include the scheme: temperature, displacement, temperature, displacement, etc. would ensure reversibility. Thus, as in simulated tempering, the reversibility can facilitate the analysis of convergence. The setup for SAPSRS is greater than for SA, requiring the user to also specify the step-size and indifferent zone parameters.

As indicated in section 4.2. (Ahmed & Alkhamis, 2002) used SA combined with R&S. However, the main differences between SAPSR&S and SARS (Ahmed et al., 2002) are how each algorithm generates and evaluates candidate solutions. In SARS, a candidate solution is generated from an accepted solution (incumbent) and according to certain specific function. Then, measures of performance based on R&S are obtained for both the incumbent and a candidate solution. Notice that since R&S is applied, the obtained measures inherently contain user-specified IZ parameters (Dudewicz & Dalal, 1975). Finally, these measures are used to accept or reject such a candidate solution based on the Boltzmann probability distribution. In contrast, in the SAPSRS algorithm, at each iteration, candidate solutions are generated in two ways: randomly and deterministically. First, as traditional SA, a candidate solution is selected uniformly at random, and then pre-accepted according to Metropolis condition (see section 2.3.4.1). Next, the neighborhood of the pre-accepted candidate solution (incumbent) is systemically explored by generating a finite set of neighbors using a Pattern Search algorithm. Finally, both the incumbent and neighbors are evaluated using the IZ procedure by Rinott (1978b). Thus, the best candidate solution is obtained from the alternative set presented during the neighborhood exploration. In summary, SARS uses R&S to improve the quality of the measures of performance to be evaluated by SA, while SAPSRS improves candidate solutions obtained from applying traditional SA by using a PS and R&S step.

SAPSRS Algorithm

Objective: To estimate $f_{\min}$ and an element in $\chi_{\min}$ given: acceptance function α, stage length $\{l_1 < l_2 < ...\}$, k-stages, maximum temperature τ_1, step length δ, h, n_0.

1. $i = 1$ and $k = 1$

2. Assigns an initial state x_0, and $\hat{f}_{\min} = f(x_0)$

3. Repeat:

SA:

a. while $k \le r$:

b. while $i \le l_k$

c. Randomly sample y from the given distribution.

d. Randomly sample U from $U(0,1)$

e. If $U \le \min\left\{1, e^{-[f(y)-f(x_{i-1})]/T}\right\}, x_i = y$

PS:

f. Deterministically generate n additional neighbors (test points) to x^* using step length δ.

g. Calculate $f(x^*)$ per potential neighbor.

R&S:

h. Select x^* pairs within $\pm 5\%$ (IZ)

i. Determine additional replications N_i per test point.

j. Execute replications per each competing alternative.

k. Select the best x^* pair.

l. If $f(x_j) < \hat{f}_{\min}, \hat{f}_{\min} = H(x_j)$, and $Z = x_j$

m. $j = j + 1$,

n. $k = k + 1$,

o. $\tau_k = \alpha\tau_{k-1}$ or until termination criteria is satisfied or $k > r$

4. $(H_{\min}, Z)$ is the estimated solution.

4.3.3 Applying the heuristic to the stochastic inventory problem

This section presents the SAPSRS algorithm to approximate near-optimal solutions to stochastic inventory systems. The steps of the proposed approach are as follows.

1. Perform an initial experiment to obtain the maximum temperature parameter that maximizes a given acceptance function.
2. Given constraints from Equations (2) – (8), generate arbitrary initial (s_i, S_i) candidate solution.
3. Determine costs $f(x_i)$ Equation (1) using arbitrary (s, S).
4. Generate U from $U(0,1)$.
5. Evaluate objective function according to:

   $$U \le \min\left\{1, e^{-[f(s_i,S_i)-f(s_{i-1},S_{i-1})]/T}\right\}, (s_i, S_i) = (s^*, S^*)$$

6. If a candidate solution is accepted, explore a local neighborhood of the candidate solution by generating a given pairs of (s, S) (policy solutions) with step-size of $\pm\delta$.
7. Determine costs $f(x_i)$ using each new neighbor pair (s, S) and select pairs that have costs of $\pm 5\%$ compared to the original candidate solution (Indifference Zone).
8. If all costs are above the original costs generated by candidate solution, this is accepted as a final solution. Otherwise, determine additional replications and replicate each pair that reported lower costs.
9. After replications, if obtained costs are below cost from original (s_{i-1}, S_{i-1}), then select the lowest cost and corresponding pair as a final solution. Otherwise, select the original (s_i, S_i) as final.

10. Repeat until stopping criterion is met.

Figure 11 illustrates the SAPSRS algorithm applied to the stochastic inventory problem. Then the SAPSRS algorithm is presented for the inventory problem.

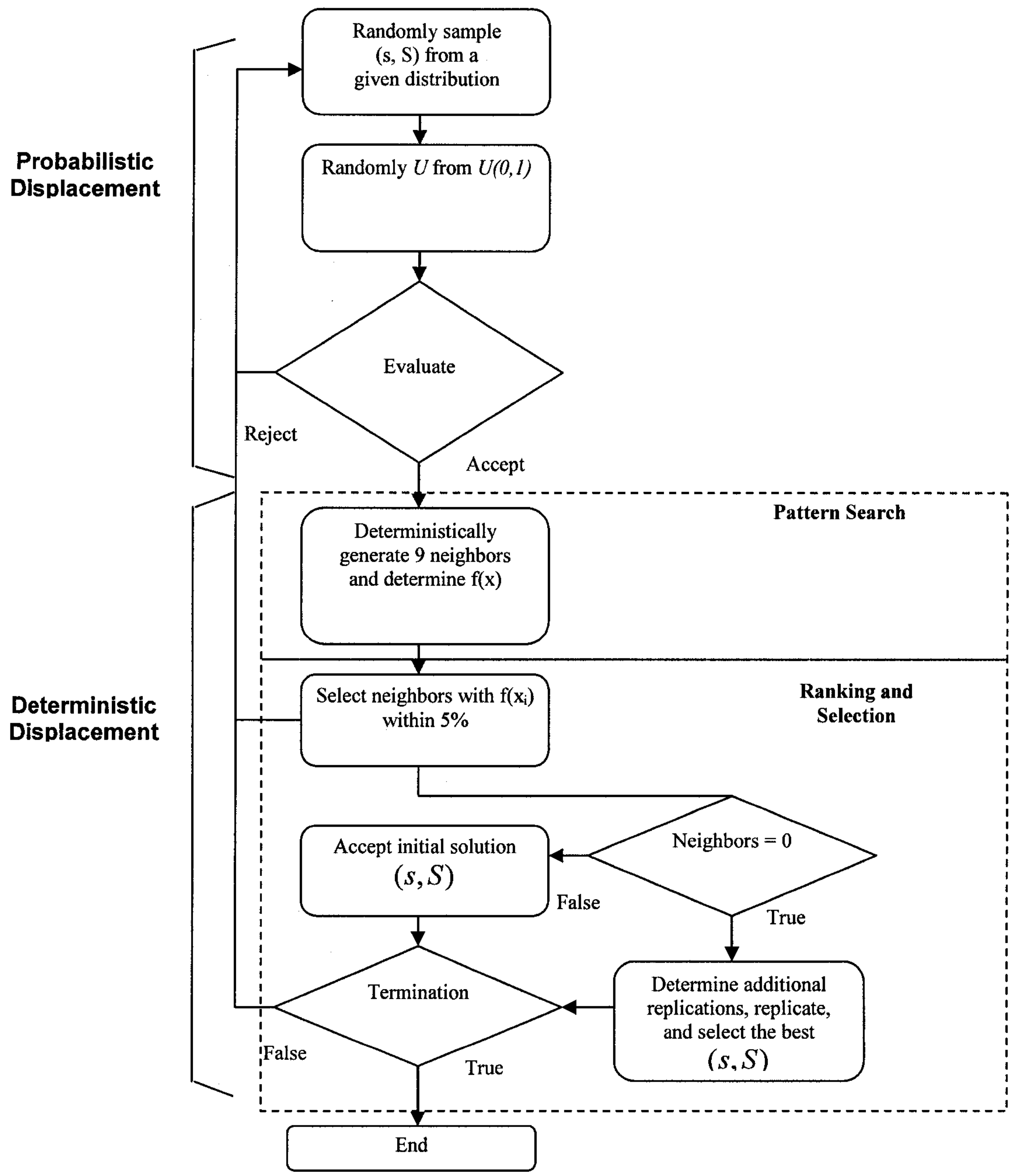

Figure 11 Flow Diagram for the SAPSRS Algorithm

5 NUMERICAL ANALYSIS

5.1 Introduction

In this chapter, experimental results and numerical analyses are presented. The main purpose of this chapter is to present and conduct experimental procedures that allow one to answer the research questions presented in Chapter 1. To achieve this purpose, a set of experiments was designed and performed according to the experimental procedures that are presented in the next section. The experimental procedure is designed in terms of the experimental design, analysis and evaluation of results, and characterization of the error generated between the correlated and correlation-free cases. Obtained results and statistical analysis are presented and analyzed in sections 5.3 – 5.7.

5.2 Experimental procedure

In this section, the analysis strategy and techniques to analyze and test a series of hypotheses related to the effect of serially-correlated components on demands are provided.

5.2.1 Overview of the Experimental procedure

To examine and evaluate the effects of ignoring autocorrelated components on the demand for inventory control models, the analysis process was subdivided into five stages. Figure 12 illustrates and details the aforementioned procedure. This process can be summarized as follows.

1. **Experimental design**. Experimental design provides guidance to determine the importance and behavior of factors and interactions on the studied inventory system.

In this stage, dependent and independent variables are selected as well as levels and responses that will be analyzed. In terms of the inventory system, the varying factors and potential interactions are defined in terms of the cost structure and autocorrelation factors while the responses are quantified in terms of average total cost and near-optimal control policies.

2. **Analyzing and evaluating responses**. Responses are obtained by applying the SAPSRS algorithm to the inventory problem. In addition, assessment of the level of significance is conducted in order to evaluate whether any relevant differences exist between correlated and correlation-free cases.
3. **Main effects and two-way interactions.** The main effects and two-way interactions of the costs structure (ordering, shortage, and holding) are determined. Notice that main effects and two-way interactions are calculated for each autocorrelation level. Then, ANOVA tests are conducted to evaluate the significance of each interaction and effect.
4. **Error characterization**. The errors generated in the selected measures of performance are determined, described, and evaluated in this stage. Regression Analysis is used to describe the behavior of error by experiment. ANOVA is used to evaluate the significance level of these errors.

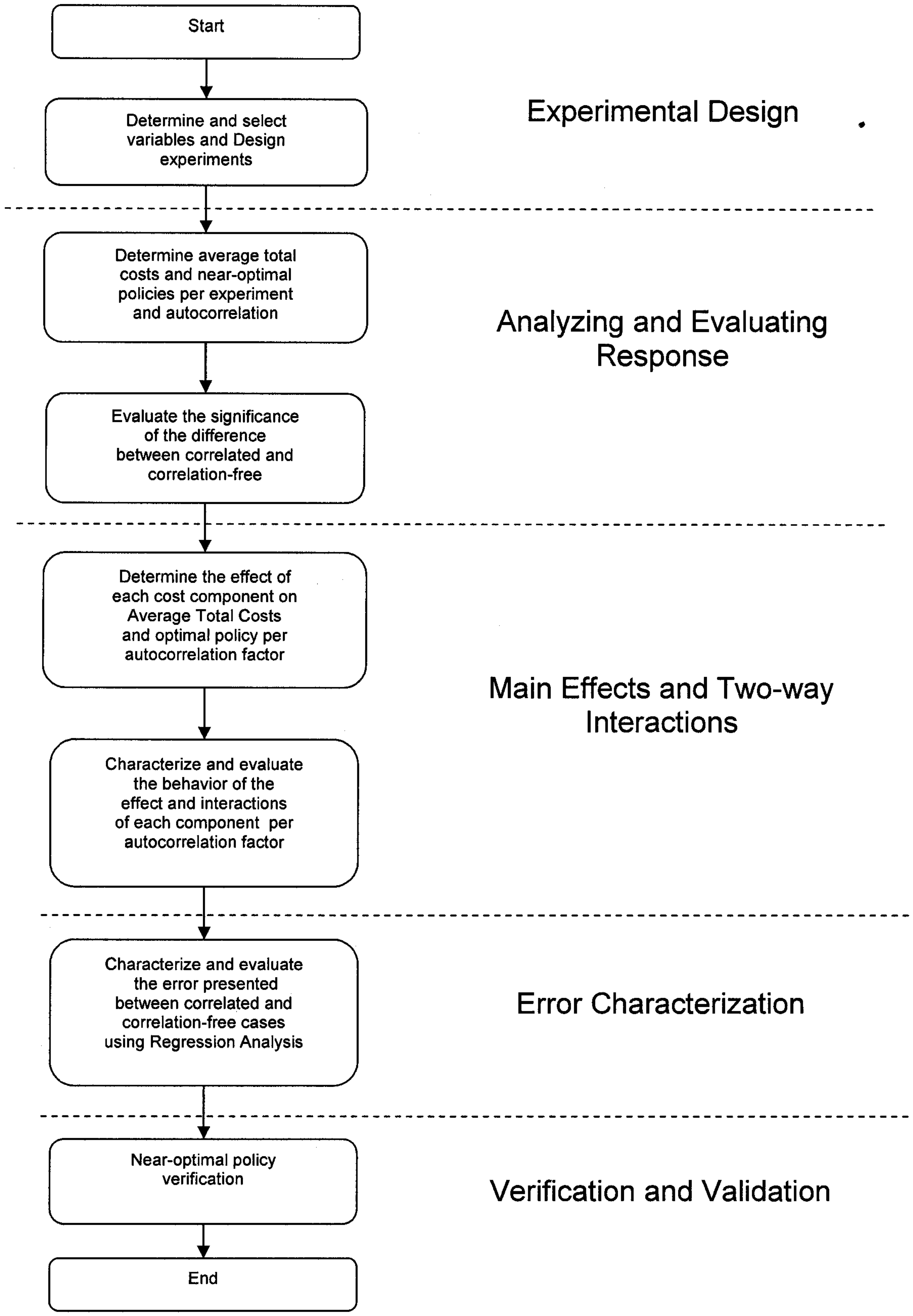

Figure 12 Detailed Numerical Analysis Procedure

5. **Verification and validation.** In this stage, the results obtained using the SAPSRS algorithm are validated by quantifying and comparing total costs, stockout, and replenishment rates. To verify that a model implementation accurately represents its conceptual description, the number of times that a solution improved upon the new evaluation process is measured and compared.

5.3 Experimental design

5.3.1 Determining dependent and independent variables

Three independent variables and three dependent variables are adopted in a series of simulation experiments. The independent variables include ordering costs, shortage cost, and holding cost. The dependent variables include the average total cost of the inventory system and the near-optimal policy that minimizes the average total cost. The near-optimal policy is composed of two dimensions: the reorder point "s" and the maximum inventory level "S." However, as pointed out by Law et al. (2000), in order to conveniently analyze the inventory system, instead of considering ordering "up-to-S" it is convenient to reparameterize the decision in terms of order quantity D. The order quantity is defined as the difference between the "up-to-S" level and the reorder point. In other words $D = S - s$. Each experiment is evaluated in terms of autocorrelation level.

5.3.2 Design of Experiments

In addition to the three design factors identified above, two classes of stochastic autocorrelated demands have been considered to design these experiments (see section 3.3). First, an AR(1) process is considered for representing stochastic continuous demand in which mean, autocorrelation factor, and error distribution are known. Secondly, a

DMC is considered for modeling a stochastic discrete demand where each discrete value is represented by a state.

Based on the demand and probabilistically generated control policies (s, S) using the algorithm SAPSRS, near-optimal policies and average total cost are calculated. The inventory level at the reorder points act as a trigger for adjusting the ordered quantities in order to minimize the total inventory costs. Further, the experiments are designed to evaluate the effects of each cost component per autocorrelation factor. As a result, along with the autocorrelation parameters, the three selected factors and corresponding levels are presented in Tables 4 and 5. A full-factorial design for these factors and levels required a 2^3 design which implies a 48 trial experiment per correlation factor. Each simulation run was of 20,000 periods conditional upon the autocorrelation level and the reaching of the termination criteria. The simulation run length corresponded to the stage length of the SAPSRS algorithm. The runs included five replications of all combinations: Ordering costs (2 levels), penalty costs (2 levels), holding costs (2 levels), autocorrelation levels (10 levels for AR(1) and 18 levels for MC subdivided into correlated (9 levels) and correlation-free cases (9 levels)).

Factor	Name	-	+
1	Ordering Cost (c)	1	2
2.1.	Penalty (p)	5	19
2.2.	Cost @ Invent 0	100	200
3	Holding Cost (h)	0.5	2.5

Table 4 Design factors

The value for h was selected based on Lee et al. (2000) where a retail inventory is analyzed assuming a continuous autocorrelated demand AR(1). The magnitude of the penalty p value was derived using the approach of service level based on critical ratio discussed in section 3.2.5.3. From assuming a service level with a critical ratio near one led the system to provide a service level of about 97% while a penalty cost of $19 per unit. Then, a relaxation of this condition, which portrays a situation where an inventory system selects a critical ratio that is not close to one, derived in a lower penalty cost of $5 per unit. Taha (2002) asserted that is nonsensical to purchase an item whose penalty is higher than the ordering cost. As a result, based on literature statements, ordering costs c was assumed to be lower than p, with its lowest level at $0.5 per unit and its highest level at $ 2.5 per unit.

ϕ	Factors					
Experiment	**c**	**p+C**	**h**	**c x (p+C)**	**c x h**	**(p+C) x h**
A	-	-	-	+	+	+
B	+	-	-	-	-	+
C	-	+	-	-	-	-
D	+	+	-	+	+	-
E	-	-	+	+	-	-
F	+	-	+	-	+	-
G	-	+	+	-	-	+
H	+	+	+	+	+	+

Table 5 Experiment Design

In addition to the factors and levels described above, the inventory system that considered the AR(1) model included an ordering fee or a setup cost of $24 each time that an order was placed.

5.3.3 Input and parameter data

The specific input variables integrated in the simulation model are specified in table 6 as follows. Notice that Table 4 provides two levels of input data, specifically, for the inventory model and the SAPSRS algorithm such that given information is processed and output data is generated. Figure 13 provides a view of the diverse points where input data are included and processed.

Input Type	Description
1. Inventory model	1.1. Demand distribution: 1.1.1 Discrete demand modeled as Markov Chain according to section 3.3.1. 1.1.2 Continuous demand modeled as AR(1) process described in section 3.3.2.
	1.2 Costs (Table 4): 1.2.1 Ordering 1.2.2 Holding 1.2.3 Penalty
	1.3 Maximum / minimum inventory level allowed in the system (s = 1,000; S = 8,000).
2. SAPSR&S algorithm	2.1 SA 2.1.1 Maximum temperature (based in acceptance $\geq 98\%$) 2.1.2 Temperature Gradient $\tau_i = 0.85 * \tau_{i-1}$ 2.1.3 Length of the stage (20,000 periods) 2.1.4 Stopping criteria (combination of (s,S) $\pm 10\%$, average costs $\pm 5\%$, and $\tau_i < 100$ units)
	2.2 Pattern search 2.2.1 Step Size for reorder $\delta_s \pm 15\%$ and resupply level $\delta_S \pm 15\%$ 2.2.2 Number of neighbors to explore per iteration = 9
	2.3 Ranking and selection. 2.3.1 Indifference zone value 5%. 2.3.2 h based on the indifference value and the number of neighbor to explore 3.619 2.3.3 Initial number of replications $n_0 = 20$

Table 6 Input data

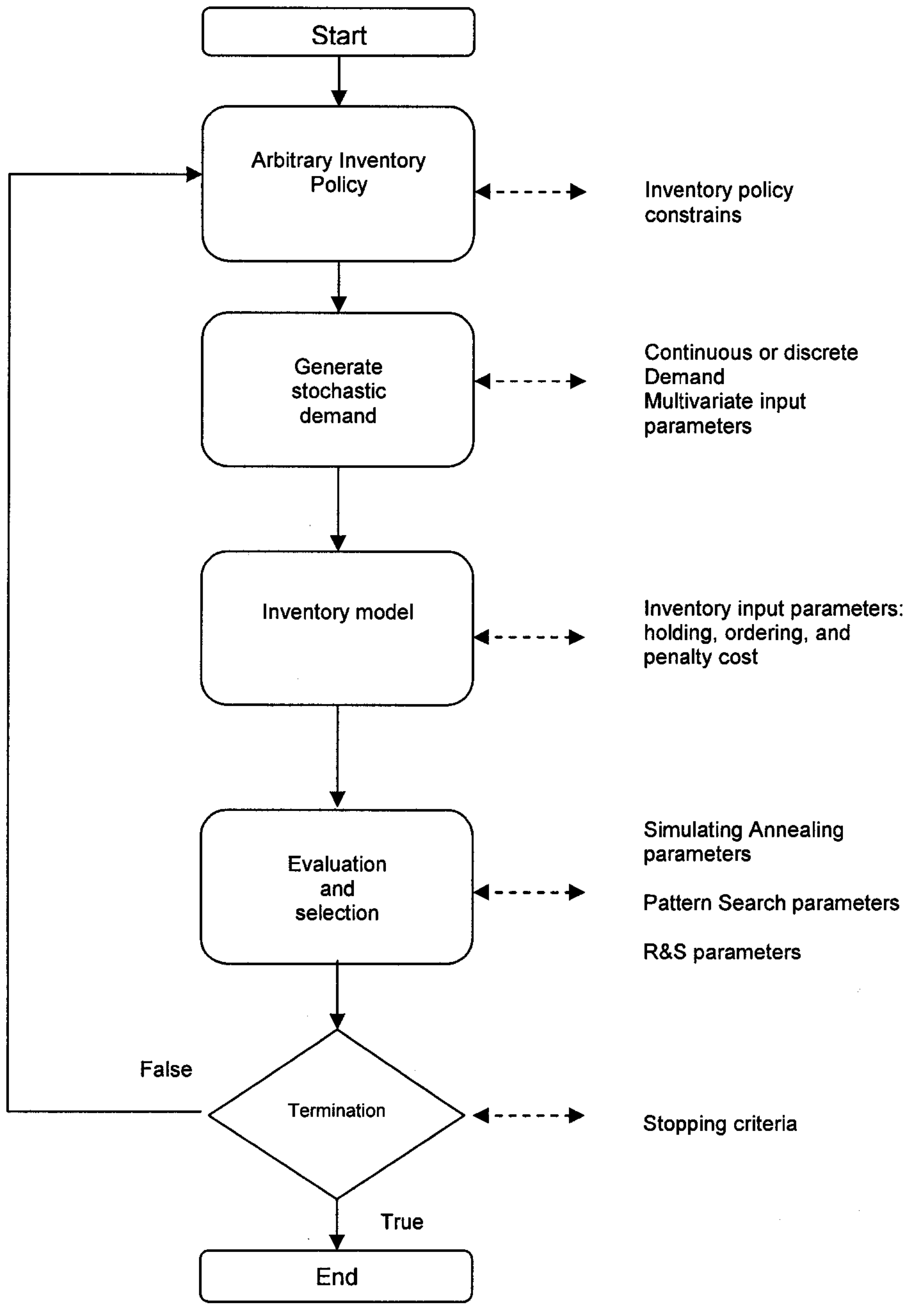

Figure 13 Input data processing

In this model, the arbitrary policy is generated considering inventory constraints that include maximum and minimum reorder and inventory level allowed. Then, considering the given probabilistic distribution parameters, the correlated and correlation-free demands are generated. Based upon the demand, the inventory control is determined. Inventory levels fluctuate with the values of demand in each period, therefore, making it a stochastic function. Specific input data for the stochastic distribution include:

a) AR(1) Model

- Parameters for the $AR(1)$ function included the mean of the demand, $\mu = 2500$, and the error ε is normally distributed according to $N_{\varepsilon}(0, 300^2)$.
- Positive autocorrelation includes the 0.1. to 0.9 increments in intervals of 0.1 units.

b) DMC model

- The discrete values for the Markovian demands include $\xi_D = \{1000, 2000, 3000, 4000\}$. It behaves according to a given p_{ij}, where $p_{10} = p_{23}$, $p_{12} = p_{21}$, and $p_{01} = p_{32} = 1$.
- Invariant distributions were derived from their transition probability distribution. Autocorrelation values were obtained from each given transition probability distribution.
- For the values used in the transition probability distribution and the derived invariant distribution, the reader is referred to section 3.3.1.2.

c) Simulated Annealing

To apply SA to the stochastic inventory problem, the following parameters must be specified: the state space, the objective function, the candidate generator procedure, the acceptance probability function, and the annealing schedule.

Section 3.2.3 provides the objective function and constraints that restrict the decisions space of the problem. The candidate generator procedure assumes that (s, S) candidate solutions are drawn from a uniform distribution restricted by the constraints stated in section 3.2.3. The acceptance of a candidate solution is dictated by the Boltzmann probability density function presented in section 2.3.41.

In general, parameters of the annealing schedule were derived from extensive preliminary experimentation. The rationale behind each value is presented as follows.

The maximum temperature was determined from a preliminary simulated annealing sampling experiment in which the acceptance rate shows values above 99%. The approach of the empirical acceptance rate is based on the Hasting-Metropolis sampling method in which the acceptance of candidate solutions is computed from sample-path data. As the temperature increases, the probability distribution of the sample path grows to be more uniform. As a result, acceptance increases. Thus, acceptance values close to 100% implies that the unobservable probability distribution of the sample path is close to uniform (Fishman, 2005). A temperature gradient of $\alpha = 85\%$ from tested ranges between $0.80 \leq \alpha \leq 0.99$ yielded to a smooth cooling schedule that progressively minimized the acceptance function. The stage length was selected to be 20,000 periods so output data was collected when the system reached the steady state. Finally, the stopping rule was derived from a combination of established well-known rules (Kirkpatrick et al., 1983; Aarts et al., 1989; Kolinski et al., 1994) that include observing certain propensity of accepted candidate solutions (s, S) to concentrate in a relatively small neighborhood $\pm 5\%$; no change in the objective function, specifically, the total average cost value during 15 consecutive stages did not improve more than $\pm 5\%$; and the number of acceptances having fallen below a specific value 10^{-6}.

d) Pattern Search

The method of coordinate search with fixed step lengths is perhaps the simplest and most intuitive of all the pattern search methods. According to Torczon (1997), a PS algorithm for a minimization problem with two variables can be summarized as follows: Try steps to the East, West, North, and South. The final coordinates of these steps compose the basis matrix. If one of these steps lead to a reduction in the function, the improved incumbent becomes the new iterate. If none of these steps yields improvement, try again. Typically, the coordinate search is defined so that the basis matrix is the identity matrix I. Nonetheless, knowledge of the problem led to a different choice for the basis matrix and the magnitude of step size δ.

In the context considered in this dissertation, a different coordinate system is studied. In this sense, the variables (s,S) that define the incumbent are known to differ by several orders of magnitude. Thus, the generating matrix $C_k = C$ contains in its column all possible combinations of (s,S). As a result, C has $p = 3^n$ columns. In particular, the columns of C contain the equivalent of what corresponds to the identity matrix I and $-I$ as well as a column of basic (s,S). Since C is fixed across all interactions of the method, there is no need for an update algorithm. For $n = 2$, the total neighbors to be explored are $3^2 = 9$. In other words, 9 trial points constitute the neighbors to be explored. In this sense, the C matrix with all combination of (s,S) is given by:

$$C = \begin{vmatrix} s & s^- & s^+ & s & s & s^- & s^+ & s^- & s^+ \\ S & S & S & S^- & S^+ & S^+ & S^- & S^- & S^+ \end{vmatrix}$$

where the positive and negative sign indicate an increase and decrease in the magnitude of the incumbent for the reorder point s and maximum inventory S respectively. The step size was selected on extensive preliminary runs where different step lengths were evaluated. As a result, the magnitude of 15% for both the reorder point and maximum inventory level step sizes provided the best echelon that shows significant improvement in the evaluation of the measures of performance (see section 5.7.3). Moreover, these lengths demonstrated the best enhancement in terms of the times that a candidate solution improved upon the ranking and selection evaluation. The issue of avoiding a poor choices of step lengths is discussed in Kolda et al. (2004).

e) Ranking and Selection

As indicated, in this study the IZ procedure (Rinott, 1978b) was selected to evaluate the neighbors of a potential candidate solution. The number of alternatives k is determined using a PS step. In this case, the number of alternatives to be compared is $k = 9$. As mentioned in section 2.3.4.2, the d^* parameter represents the value where a user is indifferent. In this problem, the probability of correct selection was assumed to be 95%, therefore, the magnitude of d^* is 5%. The magnitude of the initial number of replications n_0 was based on various assertions of the literature including Law et al. (2000) in which at least 20 replications are required to be able to calculate the additional number of necessary replications to compare them. Considering the parameters of k, d^*, n_0, and the table for constants for selection procedures from Law et al. (2000), the required $h = 3.619$ value was determined.

5.4 Analyzing and evaluating response

One of the main objectives of this research is to determine and evaluate the effects of autocorrelated demands on average cost and near-optimal (s,S) policy for the given stochastic inventory model. This is accomplished by:

1. Determining the near-optimal policy per autocorrelation factor via SAPSRS algorithm using the input data according to the experiments described in section 5.3.
2. Determining the response of the system to the different levels of input data per autocorrelation factor.
3. Testing whether there is a significant difference in the minimum average total cost and the near-optimal inventory policy between the correlated and correlation-free cases.

In this subsection, the responses and effects for the DMC and AR(1) cases per correlation factor are presented. Then, hypotheses for evaluating the significant difference are stated. Finally, results of the ANOVA tests are presented.

5.4.1 Response of the inventory system DMC Case

To illustrate the response of the system, an experiment is selected and presented. In Appendix A.1, Tables A.1.1 - A.1.7 summarize the rest of the responses produced per experiment. Column P01 corresponds to the values of the transition probability described in section 3.3.1.1 and presented in Tables 2 and 3. Notice that as discussed in section 3.3.1.1 the autocorrelation value of ϕ was obtained from each transition probabilities distribution presented on Tables 2 and 3. Table 7 summarizes the computational results obtained for the experiment coded as F in section 5.3.2. The table reports the minimum average cost and near-optimal policy obtained for the correlated and correlation-free cases considering DMC demands. It is apparent that differences between the average total costs "Cost_Dep" and "Cost_CF" are substantial. Furthermore, the difference between the ordered quantities "D_Dep" and "D_CF" for both cases is considerable as well. As the autocorrelation increases, the differences between costs and reorder points for the correlated and correlation-free case also increase. Similar results were obtained for the rest of the experiments. Notice that the order quantity "D_Dep" increases by reducing the reorder point s, and keeping the up-to level S quantity to similar levels for both the correlation-free and the dependency cases. The significance of these differences is tested in the section 5.4.3.

P01	ϕ	*Cost_Dep*	*s_Dep*	*Sdep*	*Cost_CF*	*s_CF*	*S CF*	*D_Dep*	*D_CF*
0.10	-0.15	6,394.22	1626	3002	6,505.39	2501	3001	1376	500
0.20	0.13	6,509.96	1562	3002	6,716.83	2468	3001	1440	533
0.30	0.29	6,608.50	1507	3002	6,895.95	2501	3001	1494	500
0.40	0.38	6,690.88	1528	3002	7,050.32	2349	3001	1474	652
0.50	0.45	6,766.83	1480	3003	7,184.79	2502	3002	1523	500
0.60	0.49	6,831.02	1425	3003	7,299.11	2418	3001	1578	584
0.70	0.53	6,887.74	1485	3004	7,403.02	2400	3003	1519	603
0.80	0.56	6,713.82	805	3004	7,492.90	2502	3002	2199	500
0.90	0.64	6,541.69	595	3011	7,574.97	2444	3001	2415	557

Table 7 Effects of autocorrelated DMC demands

5.4.2 Response of the inventory system AR(1) Case

Table 8 summarizes the computational results obtained for the experiment labeled G in section 5.3.2. It reports the average total cost and near-optimal inventory policy for correlated ϕ and IID cases but considers AR (1) demands instead. In Appendix A.2, responses obtained for the rest of the experiments are reported. Compared to the DMC demand case, it is clear that the differences between the average total costs and the ordered quantity are substantial for both cases. However, notice that unlike the discrete cases, the order quantity increases by reducing the reorder point s, and increasing the up-to level S quantity. As the autocorrelation increases, the differences also increase. These differences are tested in the next subsection. In addition, observe that even though the autocorrelated component is considered, the obtained total costs are higher as the autocorrelation increases. In section 5.7.2, the effect of ignoring the autocorrelation components and using the inventory policy designed for the IID situation is determined as the serial correlation increases.

ϕ	Cost	s	S	D
IID = 0	3,789.09	2202	2854	651
0.1	3,795.28	2219	2856	637
0.2	3,815.09	1970	2866	896
0.3	3,845.03	2030	2871	841
0.4	3,901.62	2390	2888	498
0.5	3,979.33	2221	2907	686
0.6	4,096.55	2169	2949	780
0.7	4,280.87	1968	2992	1024
0.8	4,591.31	1626	3104	1477
0.9	5,098.00	1807	3290	1483

Table 8 Effects of autocorrelated AR(1) demands

5.4.3 Testing the significance for the DMC and AR(1)

The underlying assumptions for a one-way analysis of the variance (ANOVA) include:

- The data set consists of k random samples from k populations.
- Each population has a normal distribution and the standard deviation is identical, so that $\sigma_1 = \sigma_2 = ... = \sigma_k$.

The null hypothesis for the test in one-way analysis claims that the k populations (represented by the k samples) all have the same mean value while the alternative hypothesis claims that are not all the same. Thus, the following hypotheses are tested:

Hypothesis	Statement	Expression
1	The autocorrelated demand does not change the average total cost of the inventory system.	$H_0 : \mu_{IID} = \mu_{Autocorrelated}$
	The autocorrelated demand does change the average total cost of the inventory system	$H_a : \mu_{IID} \neq \mu_{Autocorrelated}$
2	The autocorrelated demand does not change the s reorder point for the inventory system	$H_0 : \mu_{s_{IID}} = \mu_{s_{Autocorrelated}}$
	The autocorrelated demand does change the s reorder point for the inventory system	$H_a : \mu_{s_{IID}} \neq \mu_{s_{Autocorrelated}}$
3	The autocorrelated demand does not change the D quantity of items reordered per period for the inventory system	$H_0 : \mu_{D_{IID}} = \mu_{D_{Autocorrelated}}$
	The autocorrelated demand does change the D quantity of items reordered per period for the inventory system	$H_0 : \mu_{D_{IID}} \neq \mu_{D_{Autocorrelated}}$

Table 9 Hypothesis testing autocorrelation

The ANOVA analyses were conducted for each response. Results are summarized in Tables B.1 and B.2 in appendix B. To illustrate the ANOVA test results, significance values from two experiments, F for the DMC, and G, for the AR(1) are presented in Tables 10 and 11 respectively.

		P-value	*Hypothesis*		*P-value*	*Hypothesis*		*P-value*	*Hypothesis*	
P01	ϕ	***Cost***	***Ho***	***Ha***	s	***Ho***	***Ha***	***D***	***Ho***	***Ha***
0.10	-0.15	6.26E-07	Reject	Accept	0.00066	Reject	Accept	0.000594	Reject	Accept
0.20	0.13	4.65E-11	Reject	Accept	0.000263	Reject	Accept	0.000243	Reject	Accept
0.30	0.29	3.06E-16	Reject	Accept	1.89E-08	Reject	Accept	2E-08	Reject	Accept
0.40	0.38	2.8E-13	Reject	Accept	0.003486	Reject	Accept	0.003612	Reject	Accept
0.50	0.45	1.75E-16	Reject	Accept	0.001825	Reject	Accept	0.001844	Reject	Accept
0.60	0.49	2.5E-16	Reject	Accept	6.17E-06	Reject	Accept	6.17E-06	Reject	Accept
0.70	0.53	5.25E-15	Reject	Accept	0.003887	Reject	Accept	0.003768	Reject	Accept
0.80	0.56	4.4E-13	Reject	Accept	3.84E-05	Reject	Accept	4.26E-05	Reject	Accept
0.90	0.64	6.65E-13	Reject	Accept	1.75E-10	Reject	Accept	1.77E-10	Reject	Accept

Table 10 P-values MC demand

	P-value	*Hypothesis*		*P-value*	*Hypothesis*		*P-value*	*Hypothesis*	
Autoc.	*Cost*	*Ho*	*Ha*	*s*	*Ho*	*Ha*	*D*	*Ho*	*Ha*
0.10	1.43E-10	Reject	Accept	0.846	Accept	Not Accepted	0.849	Accept	Not Accepted
0.20	1.90E-15	Reject	Accept	0.668	Accept	Not Accepted	0.709	Accept	Not Accepted
0.30	8.89E-19	Reject	Accept	0.610	Accept	Not Accepted	0.702	Accept	Not Accepted
0.40	1.78E-21	Reject	Accept	0.317	Accept	Not Accepted	0.416	Accept	Not Accepted
0.50	6.36E-24	Reject	Accept	0.720	Accept	Not Accepted	0.942	Accept	Not Accepted
0.60	5.87E-21	Reject	Accept	0.252	Accept	Not Accepted	0.466	Accept	Not Accepted
0.70	1.09E-24	Reject	Accept	6.65E-01	Reject	Accept	2.83E-01	Reject	Accept
0.80	3.38E-23	Reject	Accept	2.54E-02	Reject	Accept	1.12E-03	Reject	Accept
0.90	3.71E-24	Reject	Accept	4.02E-02	Reject	Accept	1.08E-04	Reject	Accept

Table 11 P-values and hypothesis testing AR(1) demand

5.4.4 Output Analysis

Based upon the p-values obtained from the ANOVA test at a significance level of 0.05, and represented in Tables 10 and 11, the following conclusions are drawn.

Regarding the inventory model that considered DMC demand, the existing difference between the correlated and correlation-free case for the total cost, near-optimal inventory policy, and order quantity are all highly significant. As a result, the alternative hypotheses that claim that these differences are relevant are accepted. Concerning the AR(1) case, the obtained p-values for evaluating the differences in average cost are significant while the degree of significance increases as the autocorrelation component increases. As a result, all the hypotheses that claim differences between the IID and the dependency case are accepted. The p-values obtained for the reorder point and quantity order measures for lower to medium autocorrelation levels (0.1- 0.6) indicate that there is no relevant difference. However, for the same measures, highly significant differences were obtained for high levels of autocorrelation factors (0.7 – 0.9). As a result, the alternative hypotheses that claim such differences are accepted for the medium-high to high levels of autocorrelation factors.

As demonstrated in Appendix B, similar values were obtained per experiment. As a result, comparable conclusions can be drawn by observing such values. In-depth exploration and analysis is addressed in the rest of this chapter.

5.5 Main Effects and Two-way Interactions

In this section, the effects of individual design factors on the experiments and the interactions among the various factors for both correlated and correlation-free cases are determined and analyzed. The view used to analyze these main effects and interactions follows the traditional analysis approach, in which the main effects are determined first followed by determining two-way interactions (Kleijnen, 1987; Law et al., 2000). Test hypotheses are presented for DMC and AR(1) cases. A total of eight experiments were conducted per correlation factor for two levels of variable changes in ordering, shortage, and holding costs as mentioned in section 5.3.2. Each experiment was performed five times per correlation factor. As indicated previously, the response variables are the average of the total costs of the inventory system, the reorder point s, and order quantities D.

5.5.1 Determining main effects and two-way interaction of the experiment per correlation factor

The main effects assess the average change in the response as a result of a change in an individual factor, with this average calculated over all possible combinations. However, the effect of a given factor may depend in some way on the level of some other factor. Changes in these factors and their interactions may be significant and have an effect on the average cost and the selected (s, S) policy.

Table 12 and 13 report the main effects and two-way interaction values obtained for the DMC and AR(1) case. Further, these effects and interactions are stated in terms of the average total cost, the reorder points, and the order quantities per autocorrelation factor.

P_01	ϕ	Effect	CostDep	sDep	Ddep	CostCF	sCF	DCF
0.1	**-0.149**	**1**	2453.90	109.80	-0.55	2453.98	-3.55	-0.10
		2	449.51	516.00	499.15	449.43	441.05	500.10
		3	4646.70	-2105.60	-2008.20	5004.14	-1493.80	-2003.60
		1X2	21.32	-3.90	0.25	22.79	41.05	0.40
		1X3	1226.33	88.50	0.55	1226.66	78.25	0.20
		2X3	178.31	-631.30	-496.65	178.48	-421.65	-498.00
0.2	**0.13**	**1**	2416.16	109.20	-2.55	2414.79	12.55	-5.25
		2	613.90	592.20	493.65	613.77	463.95	492.65
		3	6001.35	-1757.20	-2029.00	6659.59	-1503.80	-2035.00
		1X2	41.82	-14.80	2.45	40.25	-41.55	5.65
		1X3	1207.97	69.70	2.65	1206.41	-34.45	5.65
		2X3	534.25	-443.70	-489.85	535.37	-449.65	-489.65
0.3	**0.29**	**1**	2411.51	105.60	1.00	2414.21	-70.30	2.40
		2	794.32	81.90	7.30	793.09	37.30	6.20
		3	7305.62	-3588.80	-3982.40	8222.55	-2959.60	-3980.00
		1X2	28.16	10.10	0.40	29.91	15.70	2.40
		1X3	1162.44	-5.60	0.30	1164.38	15.20	1.90
		2X3	793.33	-16.90	4.60	794.34	-28.70	6.20
0.4	**0.38**	**1**	2426.17	-96.10	0.95	2427.70	-13.05	-0.45
		2	780.62	459.80	499.65	779.79	476.95	500.05
		3	7758.03	-1603.60	-2007.80	8896.77	-1910.60	-2001.40
		1X2	72.81	-55.10	-0.15	71.57	15.65	0.45
		1X3	1177.95	8.80	-0.15	1178.90	-4.45	-0.65
		2X3	780.54	533.10	500.15	780.54	479.65	499.75
0.5	**0.45**	**1**	2414.04	-75.75	1.55	2427.92	66.85	6.35
		2	701.10	527.15	498.75	689.60	350.85	494.45
		3	7883.03	-1682.60	-2009.80	9163.88	-1845.40	-2025.80
		1X2	85.02	-90.75	-0.15	94.63	61.85	4.55
		1X3	1167.31	-114.05	-0.15	1178.79	61.95	4.65
		2X3	699.51	489.15	499.45	689.49	319.05	494.95
0.6	**0.49**	**1**	2405.29	-66.20	2.10	2405.57	-30.00	0.00
		2	632.79	538.50	498.10	633.45	438.00	500.10
		3	7987.97	-2275.20	-2011.60	9481.70	-1716.40	-2003.20
		1X2	93.70	-260.50	-0.60	93.20	-27.80	0.20
		1X3	1156.84	-214.00	0.10	1156.17	-60.20	-0.70
		2X3	632.65	541.10	500.80	633.71	407.90	499.80
0.7	**0.53**	**1**	2396.18	27.70	0.60	2396.88	-27.30	0.20
		2	572.68	551.10	499.10	572.69	461.20	500.30
		3	8084.41	-1954.80	-2002.80	9725.73	-1714.40	-2002.40
		1X2	101.95	63.70	0.20	102.49	18.50	0.60
		1X3	1147.72	95.70	0.00	1147.51	-13.30	0.50
		2X3	573.09	571.30	498.50	572.98	404.60	498.80
0.8	**0.56**	**1**	2330.04	-196.35	-6.50	2387.84	-24.85	0.05
		2	576.42	901.35	504.80	519.30	391.65	500.05
		3	7955.18	-2564.60	-1980.40	9938.72	-1771.00	-2005.40
		1X2	167.25	81.55	-9.40	110.88	-24.65	0.25
		1X3	1085.67	-134.35	-9.30	1139.10	-24.65	0.25
		2X3	574.31	683.35	508.60	519.13	391.45	499.85
0.9	**0.64**	**1**	2336.33	-197.45	3.60	2380.78	-12.35	0.10
		2	579.05	871.55	492.60	471.36	402.95	500.00
		3	7140.06	-3898.20	-2008.00	10133.48	-1837.80	-2003.60
		1X2	144.81	106.25	-3.60	118.47	8.75	-0.30
		1X3	1106.43	45.45	-3.20	1132.44	-20.05	-0.60
		2X3	562.01	67.95	498.80	471.43	371.45	500.10

Table 12 Obtained values MC main effect and two-way interaction

ϕ	Effect/Interaction	CostDep	s	D
0	1	2472.59	90.80	-104.80
	2	319.98	229.00	21.60
	3	2709.43	-989.20	-102.80
	1X2	16.57	-57.90	66.60
	1X3	1221.20	-20.10	3.20
	2X3	229.97	-38.40	78.30
0.1	1	2472.53	-170.70	154.10
	2	321.26	337.00	-81.70
	3	2721.46	-744.40	-336.40
	1X2	16.67	97.80	-90.60
	1X3	1221.09	-68.80	47.90
	2X3	230.92	86.20	-51.20
0.2	1	2472.20	178.60	-197.40
	2	326.88	264.60	-3.90
	3	2764.23	-423.20	-686.00
	1X2	16.91	245.30	-234.90
	1X3	1220.86	218.80	-233.70
	2X3	234.41	-71.90	108.00
0.3	1	2472.00	-57.50	37.50
	2	334.00	187.00	77.70
	3	2832.51	-1780.80	643.60
	1X2	18.14	116.20	-98.90
	1X3	1220.77	-35.50	19.00
	2X3	239.49	-36.90	82.70
0.4	1	2469.67	40.45	-60.70
	2	348.69	268.65	2.20
	3	2953.24	-1591.40	408.80
	1X2	18.48	-181.55	188.30
	1X3	1219.06	-42.05	24.10
	2X3	249.66	70.75	-33.60
0.5	1	2467.59	111.85	-131.30
	2	368.49	279.05	14.50
	3	3121.33	-857.40	-380.40
	1X2	19.79	-169.15	185.20
	1X3	1217.74	-10.85	-3.70
	2X3	263.44	69.45	-24.00
0.6	1	2463.74	-97.75	81.55
	2	398.74	443.15	-128.55
	3	3375.93	-2374.60	1096.20
	1X2	21.67	21.15	-10.75
	1X3	1215.13	-167.85	149.25
	2X3	283.70	23.25	18.15
0.7	1	2458.93	122.95	-152.35
	2	445.30	431.35	-72.85
	3	3774.25	-1814.20	317.80
	1X2	24.55	-172.25	198.65
	1X3	1211.47	-19.25	3.65
	2X3	317.47	73.65	-30.95
0.8	1	2449.79	3.70	-45.90
	2	527.10	357.60	80.70
	3	4439.79	-1915.60	169.60
	1X2	30.01	-49.80	86.90
	1X3	1205.17	-112.50	97.60
	2X3	374.11	108.00	-64.00
0.9	1	2445.76	-32.40	21.60
	2	671.71	528.00	37.30
	3	5615.01	-1883.20	-539.20
	1X2	85.03	-22.10	63.50
	1X3	1205.46	-35.70	27.80
	2X3	471.35	104.30	-42.50

Table 13 Obtained values AR main effect and two-way interaction

5.5 Evaluating significance of main effects and two-way interactions

In this section, hypothesis tests are conducted to determine the significance of the levels of the individual factors and their interactions. In essence, the null hypothesis for the one-way analysis test claims that the effects of individual factors and their interactions have the same mean value while the alternative hypothesis claims that not all are the same.

5.5.1 Main Effect Hypotheses

Hypothesis	Statement	Expression
1	H_0: Levels of factor 'Ordering cost' per correlation factor does not change the average total cost of the inventory system.	$H_0 : \mu_{OC_{IID}} = \mu_{OC_{Autocorrelated}}$
	H_a: levels of factor 'Ordering cost' per correlation factor does change the average total cost of the inventory system	$H_a : \mu_{OC_{IID}} \neq \mu_{OC_{Autocorrelated}}$
2	H_0: Levels of factor 'Shortage cost' per correlation factor does not change the average total cost of the inventory system.	$H_0 : \mu_{SC_{IID}} = \mu_{SC_{Autocorrelated}}$
	H_a: Levels of factor 'Shortage cost' per correlation factor does change the average total cost of the inventory system	$H_a : \mu_{SC_{IID}} \neq \mu_{SC_{Autocorrelated}}$
3	H_0: Levels of factor 'Holding cost' per correlation factor does not change the average total cost of the inventory system.	$H_0 : \mu_{HC_{IID}} = \mu_{HC_{Autocorrelated}}$
	H_a: Levels of factor 'Holding cost' per correlation factor does change the average total cost of the inventory system	$H_a : \mu_{HC_{IID}} \neq \mu_{HC_{Autocorrelated}}$

Table 14 Hypotheses simple effects

5.5.2 Two-way Interaction hypotheses

Hypothesis	Statement	Expression
1	H_0: Levels of interaction between 'Ordering cost' and 'Shortage cost' per correlation factor does not change the average total cost of the inventory system.	$H_0 : \mu_{1X2_{IID}} = \mu_{1X2_{Autocorrelated}}$
	H_a: Levels of interaction between 'Ordering cost' and 'Shortage cost' per correlation factor does change the average total cost of the inventory system.	$H_a : \mu_{1X2_{IID}} \neq \mu_{1X2_{Autocorrelated}}$
2	H_0: Levels of interaction between 'Ordering cost' and 'Holding cost' per correlation factor does not change the average total cost of the inventory system.	$H_0 : \mu_{1X3_{IID}} = \mu_{1X3_{Autocorrelated}}$
	H_a: Levels of interaction between 'Ordering cost' and 'Holding cost' per correlation factor does change the average total cost of the inventory system.	$H_a : \mu_{1X3_{IID}} \neq \mu_{1X3_{Autocorrelated}}$
3	H_0: Levels of interaction between 'Shortage cost' and 'Holding cost' per correlation factor does not change the average total cost of the inventory system.	$H_0 : \mu_{2X3_{IID}} = \mu_{2X3_{Autocorrelated}}$
	H_a: Levels of interaction between 'Shortage cost' and 'Holding cost' per correlation factor does change the average total cost of the inventory system	$H_a : \mu_{2X3_{IID}} \neq \mu_{2X3_{Autocorrelated}}$

Table 15 Hypotheses two-way interactions

Results of the ANOVA tests conducted for each factor and their interactions are presented in Table 16 for the DMC case and Table 17 for the AR(1) case.

P_01	Effect/Interaction	COST	REORDER	D
0.1	1	0.99402	0.65782	0.65569
	2	0.99317	0.76959	0.76615
	3	1.40E-90	0.01752	0.01807
	1X2	0.72385	0.67464	0.67433
	1X3	0.93641	0.92372	0.92600
	2X3	0.96751	0.05121	0.04876
0.2	1	0.89366	0.70566	0.71278
	2	0.98940	0.61629	0.61811
	3	1.34E-142	0.32252	0.30994
	1X2	0.70542	0.80270	0.77874
	1X3	0.70757	0.33115	0.31534
	2X3	0.78729	0.95568	0.95399
0.3	1	0.79285	0.49200	0.48741
	2	0.90437	0.86162	0.86464
	3	1.50E-172	0.01460	0.01471
	1X2	0.67365	0.95828	0.97306
	1X3	0.64091	0.84595	0.85706
	2X3	0.80888	0.91223	0.89996
0.4	1	0.88163	0.74551	0.74071
	2	0.93600	0.94656	0.94766
	3	1.94E-192	0.23095	0.22019
	1X2	0.76413	0.50890	0.51066
	1X3	0.81832	0.90150	0.90479
	2X3	0.99942	0.61768	0.61879
0.5	1	0.17756	0.57743	0.58931
	2	0.26349	0.49102	0.50051
	3	2.66E-203	0.52477	0.56524
	1X2	0.02138	0.15500	0.16621
	1X3	0.00607	0.10123	0.10929
	2X3	0.01646	0.11313	0.12131
0.6	1	0.97818	0.88749	0.88069
	2	0.94917	0.69452	0.68797
	3	1.48E-217	2.98E-02	3.19E-02
	1X2	0.90503	0.03063	0.03048
	1X3	0.87151	0.15179	0.14792
	2X3	0.79835	0.21424	0.21569
0.7	1	0.94553	0.82980	0.83058
	2	0.99946	0.72535	0.72112
	3	2.59E-226	0.34792	0.34744
	1X2	0.89701	0.67293	0.66885
	1X3	0.96001	0.30918	0.30487
	2X3	0.97942	0.12047	0.11821
0.8	1	5.51E-08	0.50289	0.51821
	2	7.70E-08	0.04736	0.04885
	3	5.90E-244	2.16E-03	1.52E-03
	1X2	7.52E-31	0.32174	0.27775
	1X3	1.39E-28	0.30609	0.34792
	2X3	6.29E-30	0.00686	0.00841
0.9	1	2.28E-05	0.46967	0.46013
	2	4.40E-21	0.06808	0.06318
	3	2.07E-282	5.08E-14	4.95E-14
	1X2	1.21E-09	0.36289	0.34481
	1X3	1.87E-09	0.54082	0.52308
	2X3	1.20E-56	4.97E-03	4.97E-03

Table 16 P-values MC main effect and two-way interaction

ϕ	Effect/Interaction	COST	REORDER	D
0.1	1	0.9989	0.4379	0.4505
	2	0.9778	0.7484	0.7631
	3	0.7938	0.4676	0.4958
	1X2	0.9937	0.2105	0.2071
	1X3	0.9930	0.6944	0.3281
	2X3	0.9405	0.3157	0.2888
0.2	1	0.9932	0.7943	0.7870
	2	0.8807	0.9158	0.9407
	3	0.2349	0.0946	0.0907
	1X2	0.9790	0.0157	0.7190
	1X3	0.9783	0.0557	0.8664
	2X3	0.7263	0.7869	0.9971
0.3	1	0.9896	0.6597	0.6781
	2	0.7606	0.9007	0.8700
	3	0.0084	0.0201	0.0310
	1X2	0.9014	0.1618	0.2982
	1X3	0.9731	0.9011	0.3684
	2X3	0.4537	0.9903	0.3798
0.4	1	0.9494	0.8811	0.8976
	2	0.5329	0.9063	0.9549
	3	0.0000	0.0756	0.1373
	1X2	0.8807	0.3194	0.0165
	1X3	0.8659	0.8594	0.3405
	2X3	0.1227	0.3792	0.8702
0.5	1	0.9134	0.9501	0.9384
	2	0.2927	0.8818	0.9835
	3	4.76E-15	0.6955	0.4185
	1X2	0.8000	0.3701	0.0583
	1X3	0.7850	0.9405	0.9557
	2X3	9.34E-03	0.3849	0.4477
0.6	1	0.8474	0.5757	0.5868
	2	0.0888	0.5251	0.6614
	3	3.43E-28	6.90E-05	6.40E-04
	1X2	0.6878	0.5239	0.8110
	1X3	0.6328	0.2345	0.4108
	2X3	4.41E-05	0.6190	0.2532
0.7	1	0.7665	0.9239	0.8896
	2	0.0073	0.5481	0.7829
	3	3.65E-46	1.55E-02	0.2211
	1X2	0.5301	0.3570	0.1843
	1X3	0.4438	0.9945	0.5337
	2X3	2.55E-10	0.3667	0.9801
0.8	1	0.6203	0.7959	0.8635
	2	0.0000	0.7026	0.8631
	3	2.32E-68	6.74E-03	0.4273
	1X2	0.2909	0.9479	0.8988
	1X3	0.2083	0.4564	0.2410
	2X3	8.61E-21	0.2388	0.8430
0.9	1	0.5600	0.7145	0.7123
	2	5.24E-12	0.3753	0.9635
	3	4.87E-94	8.86E-03	0.2044
	1X2	3.37E-07	0.7727	0.9717
	1X3	0.2164	0.8998	0.6283
	2X3	4.36E-38	2.51E-01	0.3317

Table 17 P-values AR main effect and two-way interaction

For each of the observed main effects and two-way interaction, it is important to understand the direction and the magnitude of the effect.

For the DMC and the AR(1) cases, the magnitude of the effects of the cost structure on the average total cost in descending order was the holding costs (3), the ordering cost (1), and the penalty cost (2). The magnitude of the interactions, in descending order, was ordering and holding costs (1X3), penalty and holding costs (2X3), and ordering and penalty costs (1X2). However, in general, as the autocorrelation increased, from the perspective of main effects, the effect of holding costs (3) became not only stronger but also very significant. From the point of view of two-way interactions, the penalty and holding costs (2X3) interaction become stronger and significant as the autocorrelation increases. Ordering and holding costs (1X3) and ordering and penalty costs (1X2) demonstrated high level of significance at higher levels of autocorrelation factors. A detailed explanation is presented in the next sections.

5.5.3 Analysis of the main effects inventory with DMC and AR(1) demands

Based on an overall assessment of the factors that produce the major effect on the system performance (Tables 19 and 20), results indicate that holding cost (3) is the individual factor that has the highest impact on the inventory system for all measures of performance. Tables 18 and 19 provide the values obtained for the effect of holding costs on cost average, reorder point, and order quantities.

φ	CostDep	s	D
-0.149	4646.7	-2105.6	97.4
0.13	6001.348	-1757.2	-271.8
0.29	7305.624	-3588.8	-393.6
0.38	7758.03	-1603.6	-404.2
0.45	7883.03	-1682.6	-327.2
0.49	7987.972	-2275.2	263.6
0.53	8084.414	-1954.8	-48
0.56	7955.176	-2564.6	584.2
0.64	7140.058	-3898.2	1890.2

Table 18 Factor 3 - Holding costs simple effect on DMC demands

φ	CostDep	s	D
0	2709.43	-989.20	-102.80
0.1	2721.46	-744.40	-336.40
0.2	2764.23	-423.20	-686.00
0.3	2832.51	-1780.80	643.60
0.4	2953.24	-1591.40	408.80
0.5	3121.33	-857.40	-380.40
0.6	3375.93	-2374.60	1096.20
0.7	3774.25	-1814.20	317.80
0.8	4439.79	-1915.60	169.60
0.9	5615.01	-1883.20	-539.20

Table 19 Factor 3 - Holding costs effect on AR(1)

From the viewpoint of individual measure of performance, in average costs, the major effect of holding cost (3) is followed by the ordering (1) and penalty (2) cost respectively. However, when the autocorrelation component is considered and progressively increases, the behavior of these costs is modified. In this sense, the effect of holding costs (3) becomes stronger as the autocorrelation increases, the impact of ordering (1) costs becomes weaker, and the effects of penalty cost (2) become stronger. In addition, when the results of significance were incorporated in the analysis, statistical results demonstrated that holding costs were significant for all autocorrelation levels in the MC chain case, while relevant from lower to higher values of autocorrelation in the

AR(1) case. Figures 14 and 15 presents the values obtained for the behavior of the effect of holding cost on total costs.

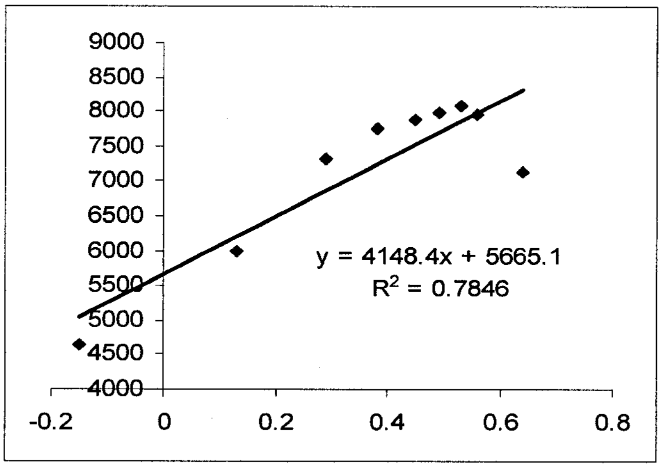

Figure 14 Behavior of holding costs per autocorrelation factor. MC case

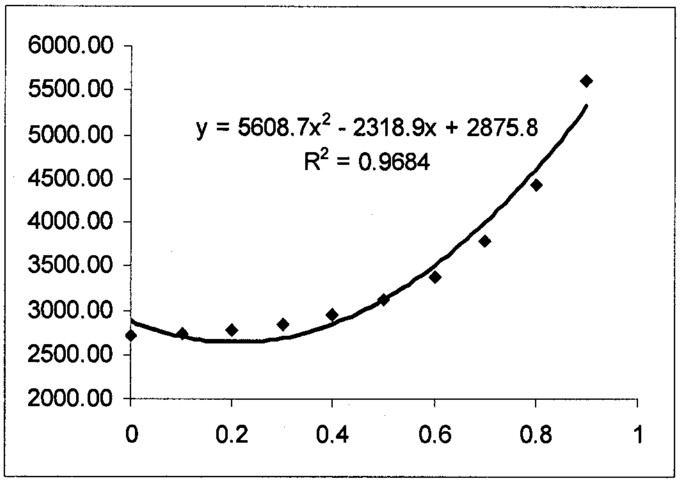

Figure 15 Behavior of holding costs per autocorrelation factor. AR(1) Case

From the reorder point perspective, the main effect values did not present a clear behavior. Nonetheless, trends in the behavior of the individual effect allow one to identify the tendency of the reorder point as the autocorrelation increases. This can be summarized as follows:

- The effect of the ordering cost on the reorder point shows a decreasing behavior. As the autocorrelation increases, the effect becomes weaker.
- The effects of penalty costs on reorder point shows an increasing behavior. As the autocorrelation increases, the effect becomes stronger. The higher the penalty costs, the higher the reorder points. Magnitude of the effect was relatively low for most autocorrelation levels. However, it became more relevant as the correlation increased.
- The effect of holding costs on reorder point is strongly negative. Results reported that such magnitude was the highest. As the autocorrelation increases, the negative effects of holding cost on reorder points force the system to select lower reorder points.

In addition, the significance test indicates that the holding cost is significant for almost all levels of serially-correlated factors for both AR(1) and DMC cases respectively. Figures 16 and 17 provide the values obtained for the behavior of the effect of holding cost on reorder points.

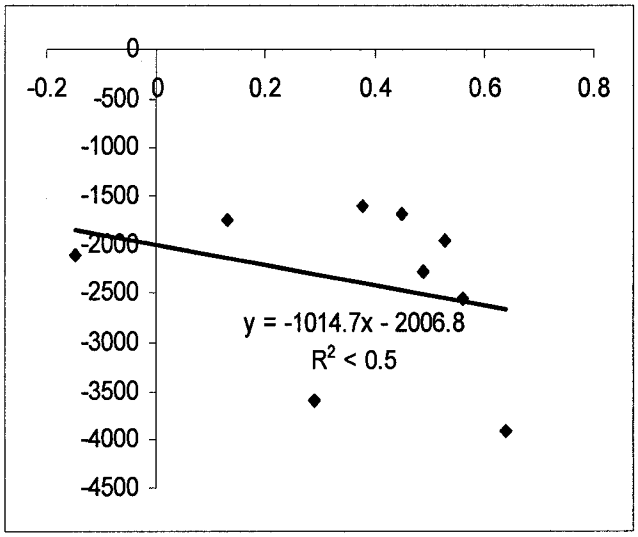

Figure 16 Behavior of the holding costs effect on Reorder point "s" MC

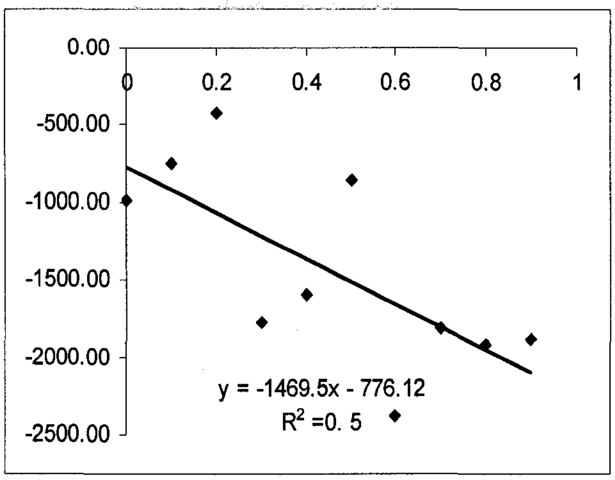

Figure 17 Behavior of the holding costs effect on Reorder point "s" AR(1)

From the order quantity point of view, the main effects of ordering and penalty costs were erratic and chaotic. In addition, the magnitude of such effects was relatively not significant.

The effects of holding costs on the order quantity posed a diffuse tendency to increase. However, at higher levels of autocorrelation, the magnitude became relevant. This suggests that an increase in holding costs leads to increased order quantity. The analysis of the variance revealed that the effect of holding costs on the order quantity was significant for almost all the levels of autocorrelation factor. The two-way interaction analysis provides more proof that supports these findings.

5.5.4 Analysis of two-way interactions for the inventory with DMC and AR(1) demands

Based upon the results obtained from the two-way interactions, the interaction between factors was present for all measures of performance.

In the average costs performance, the interaction between ordering and holding cost (1X3) has the highest magnitude and becomes weaker as the autocorrelation increases. As the autocorrelation increases, the inventory system is enforced to hold fewer items. As a result, the magnitude of the interaction becomes weaker.

The interaction penalty and holding costs (2X3) is positive and becomes stronger as the autocorrelation increases. As the autocorrelation increases, variability increases, therefore, increasing the probabilities of being penalized. Thus, the effects of holding costs depend upon the levels of penalties. As the correlated component increases, not only does this interaction lead the costs higher, but it is also very significant for almost all levels of autocorrelation.

Finally, the ordering and penalty costs (1X2) interaction is positive and becomes stronger as the autocorrelation increases. However, the magnitude of this interaction is relatively low. As the autocorrelation increases, this interaction drives total costs higher. Statistical tests showed that that this interaction was highly significant for higher levels of autocorrelation values.

Tables 20 and 21 present the interaction between penalty and holding costs for the DMC and AR(1) cases. Figures 18 and 19 provide the scatter plots for such interactions.

2X3	**Two-factor**		
φ	**Cost_Dep**	**s**	**D**
-0.149	178.312	-631.3	134.65
0.13	534.246	-443.7	-46.15
0.29	793.334	-16.9	21.5
0.38	780.5445	533.1	-32.95
0.45	699.5115	489.15	10.3
0.49	632.653	541.1	-40.3
0.53	573.0855	571.3	-72.8
0.56	574.306	683.35	-174.75
0.64	562.0085	67.95	430.85

Table 20 Two-way interaction shortage and holding cost – MC Case

2X3	Two-factor		
φ	Cost_Dep	s	D
0	229.97	-38.40	78.30
0.1	230.92	86.20	-51.20
0.2	234.41	-71.90	108.00
0.3	239.49	-36.90	82.70
0.4	249.66	70.75	-33.60
0.5	263.44	69.45	-24.00
0.6	283.70	23.25	18.15
0.7	317.47	73.65	-30.95
0.8	374.11	108.00	-64.00
0.9	471.35	104.30	-42.50

Table 21 Two way interaction shortage and holding costs – AR(1) Case

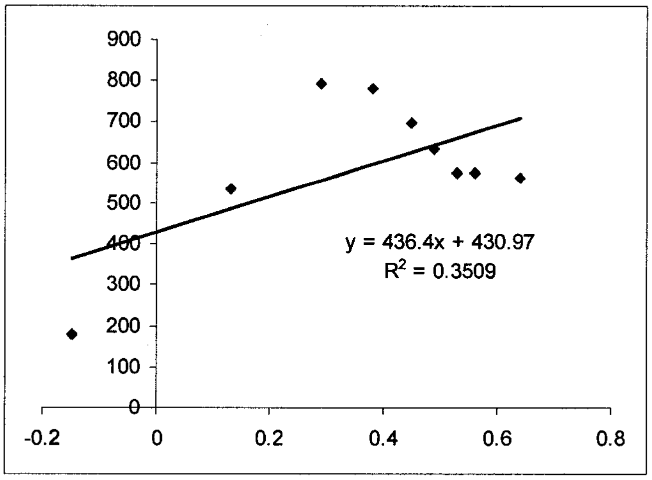

Figure 18 Behavior of two-way interaction 2X3 on average cost per autocorrelation factor – MC Case

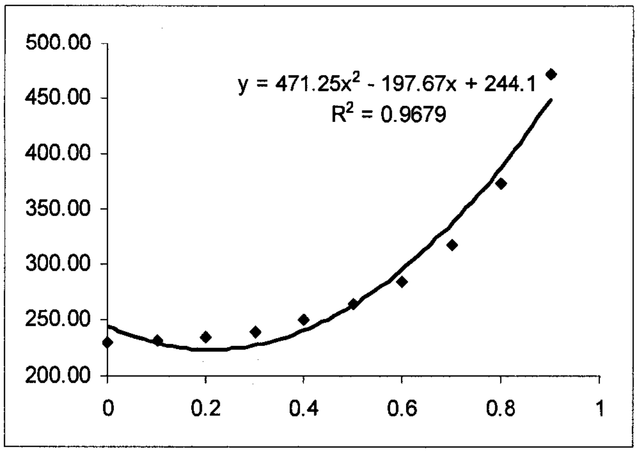

Figure 19 Behavior of two-way interaction 2X3 on average cost per autocorrelation factor –AR(1) Case

From the reorder point viewpoint, the obtained values for the ordering and penalty costs (1X2) and ordering and holding cost (1X3) interactions did show an erratic behavior. Nonetheless, for the penalty and holding costs (2X3) interaction, a positive trend as the autocorrelation increased was identified. Statistical tests show significance for this interaction at higher levels of autocorrelations. Figures 20 and 21 provide the scatter plots for such interactions.

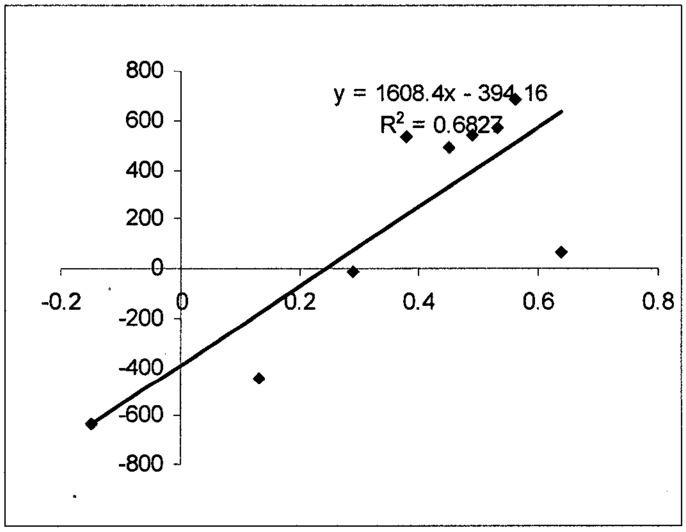

Figure 20 Behavior of two-way interaction 2X3 on average reorder point per autocorrelation factor. MC Case

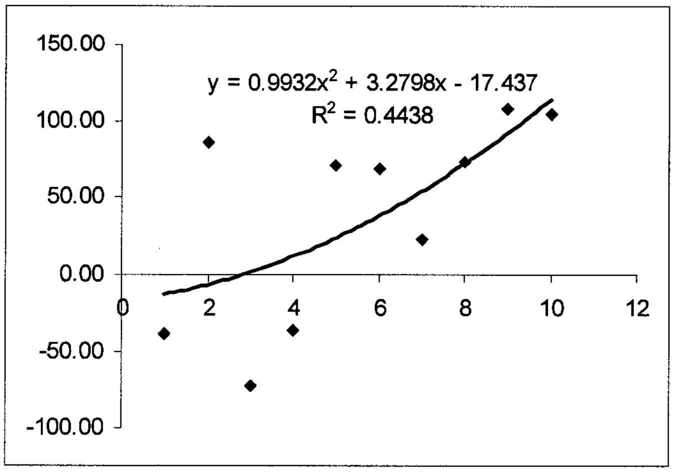

Figure 21 Behavior of two-way interaction 2X3 on average reorder point per autocorrelation factor. AR(1) Case

From the quantity order dimension, the behavior of ordering and penalty costs (1X2) and penalty and holding costs (2X3) interactions was not clear. However, the ordering and holding cost (1X3) interactions presented a tendency to increase as the autocorrelation increased. According to results obtained in the statistical tests, neither the ordering and penalty costs interaction (1X2) nor the ordering and holding cost (1X3) were significant. Only the interaction between penalty and holding costs (2X3) for the DMC case was significant for high autocorrelation values.

Main effects and two way interaction for the rest of the factors are presented in Appendices C.1 and C.2. In addition to this analysis, a characterization of the effects and interactions that presented a uniform behavior is presented in Appendices D.1 and D.2.

5.6 Error characterization

Errors between the stochastic inventory for the IID and autocorrelated cases were determined using the minimum average cost obtained and the near-optimal inventory policy. In this section, the error characterizations are derived from the output data and presented for the DMC and AR(1) demand cases.

5.6.1 Error characterization using regression analysis

The errors generated in the cost function $\beta(C)$ were derived by the following formula:

$$\beta(C) = |C_{CF} - C_{Autocor}| \tag{33}$$

where C_{CF} represents the cost assuming correlation-free demand while $C_{Autocor}$ corresponds to the costs generated considering dependencies.

The errors generated between inventory near-optimal policies $\beta(s^*, S^*)$ were determined using the Euclidean distance stated in terms of the continuous review policy:

$$\beta(s^*, S^*) = \left\| \sqrt{(s_{CF} - s_{Autocor})^2 - (S_{CF} - S_{Autocor})^2} \right\| \tag{34}$$

where (s_{CF}, S_{CF}) represents the inventory control policy that ignores dependency while $(s_{Autocor}, S_{Autocor})$ corresponds to the inventory policy that considers dependencies.

Regression analysis was used to characterize the errors between the two situations.

5.6.1.1 Error characterization - DMC Analysis

For the stochastic inventory model that presented DMC demands, performance measures and error characterization were investigated. To illustrate the analysis, the experiment G was investigated. Notice that similar analyses were performed on the rest of the experiments presented in Appendix E.1. Table 23 summarizes the computational results of applying the SAPSRS algorithm to the stochastic inventory problem. The table reports the near-optimal (s^*, S^*) policy and the associated cost found for the correlation-free case and for the autocorrelated cases. In addition, the table reports values for the cost and inventory policy errors generated, $\beta(C)$ and $\beta(s^*, S^*)$, between the correlation-free and dependent cases using Equations 33 and 34 respectively.

P01	ϕ	*CostDep*	*sDep*	*Sdep*	*CostIID*	*sIID*	*SIID*	$\beta(C)$	$\beta(s^*,S^*)$
0.10	-0.15	4,591.43	1296	3005	4,702.28	2503	3004	110.84	1207
0.20	0.13	5,282.78	1622	3006	5,493.86	2504	3004	211.09	882
0.30	0.29	5,871.43	1615	3013	6,153.65	2508	3010	282.22	893
0.40	0.38	5,896.59	2572	4002	6,252.40	3335	4002	355.81	763
0.50	0.45	5,834.04	2627	4001	6,206.26	3022	3981	372.22	395
0.60	0.49	5,782.42	2534	4001	6,253.31	3335	4002	470.89	802
0.70	0.53	5,736.96	2643	4002	6,253.16	3322	4002	516.20	679
0.80	0.56	5,695.75	2665	4034	6,253.12	3335	4002	557.37	670
0.90	0.64	5,485.48	1643	4001	6,253.48	3251	4002	768.00	1608

Table 22 Costs and Policy Error MC - Experiment G

Notice that in Table 22, as the autocorrelation increases, the near-optimal reorder point s^* tends to decrease while the near-optimal level of the items to be ordered S^* is kept to the same level. As a result, as the autocorrelation increases, the order quantity increases. Figure 22 shows the scatter plot, the trend line, and the equation for the characterization of the $\beta(C)$ error generated by ignoring autocorrelation components in determining (s^*, S^*) inventory control policy. The error $\beta(C)$ behaves as an exponential function and, as indicated by the R^2 factor, the characterization has the capability of including up to 98% of the errors generated per autocorrelation factor.

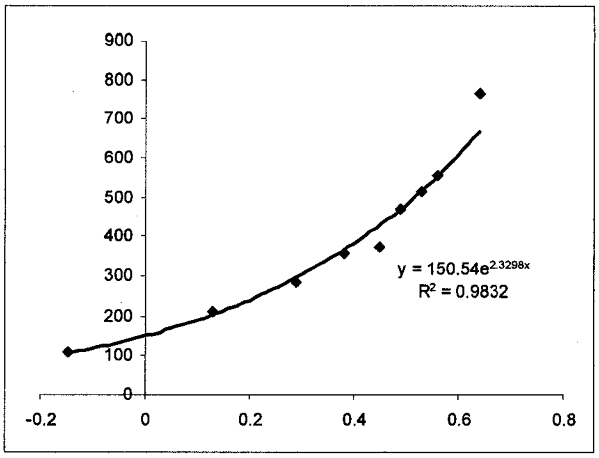

Figure 22 Costs Error Behavior MC

Consistent with other research, the autocorrelation factor has a significant impact on performance measures. As the autocorrelation amplifies, the cost of the inventory system increases as well. Moreover, as the autocorrelation increases, the error in the costs component $\beta(C)$ also increases. The predictive equation shows that errors generated in the cost component grow approximately exponentially in ϕ . This equation can explain up to 98% of the cost differences between ignoring and considering the dependency component.

Regarding the error generated in the near-optimal inventory policy (s^{*},S^{*}), empirical data indicate that its behavior is chaotic. As a result, characterization or description of the error was not viable.

5.6.1.2 Error characterization - AR(1) Analysis

Similar to the inventory model that considered DMC demands, error characterization were obtained, in the AR(1) case. To illustrate this investigation, the experiment 'G' was selected. Results for the rest of the experiments are presented in

Appendix E.2. Table 23 reports the computational results of applying the SAPSRS algorithm to the stochastic inventory model using the selected setting.

ϕ	Cost	s*	S*	$\beta(C)$	$\beta(s^*,S^*)$	D
IID = 0	3,789.09	2202	2854	0.00	0.00	651
0.1	3,795.28	2219	2856	6.19	15.62	637
0.2	3,815.09	1970	2866	26.00	230.54	896
0.3	3,845.03	2030	2871	55.94	162.86	841
0.4	3,901.62	2390	2888	112.53	150.60	498
0.5	3,979.33	2221	2907	190.24	189.37	686
0.6	4,096.55	2169	2949	307.46	305.69	780
0.7	4,280.87	1968	2992	491.78	432.54	1024
0.8	4,591.31	1626	3104	802.22	558.16	1477
0.9	5,098.00	1807	3290	1,308.91	1,247.88	1483

Table 23 Costs and Policy Error AR(1) - Experiement G

Notice that from Table 23, as the autocorrelation increases, the near-optimal reorder point s^* tends to decrease while the level S^* increases. Observe that the order quantity increases as the autocorrelation increases.

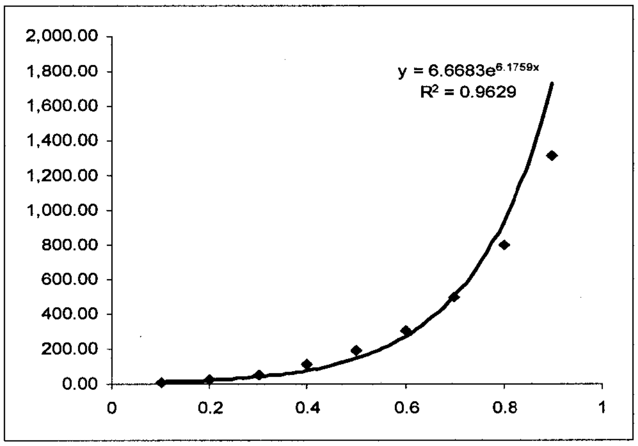

Figure 23 Costs Error Behavior AR(1)

Figure 23 shows the scatter plot, trend line, and the equation for the characterization of the $\beta(C)$ error generated by ignoring autocorrelation components in determining near-optimal (s^*, S^*) inventory control policy. The $\beta(C)$ error behaves as an exponential function and, as indicated by the R^2 factor, the characterization has the capability of explaining up to 96% of the errors generated per autocorrelation factor. As the autocorrelation amplifies, the cost of the inventory system increases as well.

Tables E.1.1 – E.2.8 in Appendix E show the experimental results obtained for error characterization of the near-optimal inventory policy. As in the DMC case, the behavior of the inventory policy error can be described as chaotic and ambiguous. However, the policy error obtained from experiment G presented a uniform behavior that made it possible to represent and obtain its description. Figure 24 illustrates the scatter plot and the equation for the characterization of the error generated by ignoring autocorrelation components $\beta(s^*,S^*)$. The error $\beta(s^*,S^*)$ behaves as an exponential function and, as indicated by the R^2 factor, the characterization has the capability of fitting up to 74% of the errors generated per autocorrelation factor.

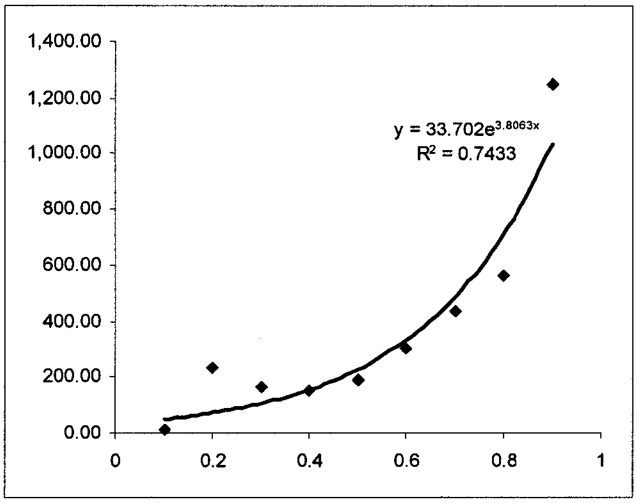

Figure 24 Inventory Policy Error Behavior

Similar values were obtained for $\beta(C)$ and $\beta(s^*,S^*)$ in the rest of the experiment presented in Appendix E. Aside from experiment G in the DMC demand case, most of the policy errors $\beta(s^*, S^*)$ presented a chaotic behavior. As a result, they could not be characterized.

In the next section, significance tests are conducted to validate the relevance of the obtained errors.

5.6.2 Error Evaluation - ANOVA test

To evaluate the significance of the error obtained in the costs and in the near-optimal inventory policy, a set of hypotheses are stated and tested using ANOVA. The hypotheses are presented in Table 24 as follows.

Hypothesis	Statement	Expression
1	H_0: The Error presented in the cost $\beta(C)$ per experiment is not significant.	$H_0 : \mu_{\beta(C)\_CF} = \mu_{\beta(C)\_Autocorrelated}$
	H_a: The Error presented in the cost $\beta(C)$ per experiment is significant.	$H_a : \mu_{\beta(C)\_CF} \neq \mu_{\beta(C)\_Autocorrelated}$
2	H_0: The Error presented in the near-optimal inventory policy $\beta(s^*, S^*)$ per experiment is not significant.	$H_0 : \mu_{\beta(s^*,S^*)_{CF}} = \mu_{\beta(s^*,S^*)_{Autocorrelated}}$
	H_a: The Error presented in the near-optimal inventory policy $\beta(s^*, S^*)$ experiment is significant.	$H_0 : \mu_{\beta(s^*,S^*)_{CF}} \neq \mu_{\beta(s^*,S^*)_{Autocorrelated}}$

Table 24 Hypothesis for error evaluation

Each hypothesis was tested per experiment and autocorrelation factor. Relevant interactions between autocorrelation factors and experiments were revealed. Results presented in Tables 25 and 26 indicated that there were significance interaction between these two factors.

Dependent variable	Interaction	P-value
$\beta(C)$	Autocorrelation*Experiment	<0.0001
$\beta(s^*,S^*)$	Autocorrelation*Experiment	<0.0001

Table 25 Interaction between autocorrelation and experiment for costs and policy errors - DMC

Dependent variable	Interaction	P-value
$\beta(C)$	Autocorrelation*Experiment	<0.0001
$\beta(s^*,S^*)$	Autocorrelation*Experiment	<0.0001

Table 26 Interaction between autocorrelation and experiment for costs and policy errors - AR(1)

Both inventory models with AR(1) and DMC Demands were evaluated. Tables F.1 – F.4 in Appendix F show the p-values obtained for the rest of the experiments. To illustrate the significance of these errors, the IID for the AR(1) case and $\phi = 0.1$ for the DMC demand case are tested against the rest of the autocorrelation levels per experiment (pairwise comparison). Tables 27 and 28 show the resultant p-values for the cost and policy errors for the AR(1) case. Tables 29 and 30 provide the obtained p-values for the cost and policy errors for the DMC case.

Ref	ϕ	Experiment							
		A	B	C	D	E	F	G	H
IID	0.1	0.931304	0.927706	0.920897	0.912796	0.843177	0.859183	0.765565	0.770789
	0.2	0.798992	0.802639	0.698999	0.707429	0.465108	0.506025	0.211113	0.216559
	0.3	0.545247	0.556107	0.408821	0.423709	0.086795	0.12826	0.007386	0.005162
	0.4	0.268883	0.302496	0.12908	0.137779	0.00126	0.004466	1.15E-07	1.6E-07
	0.5	0.058904	0.081574	0.010147	0.012692	8.07E-08	1.94E-06	5.96E-18	1.18E-17
	0.6	0.00291	0.006649	4.68E-05	7.33E-05	6.26E-17	1.09E-13	3.43E-38	3.15E-37
	0.7	2.11E-06	1.18E-05	2.33E-10	6.17E-10	1.77E-35	4.21E-29	2.05E-72	5.52E-71
	0.8	7.45E-14	5.15E-12	7.03E-23	4.52E-22	4.21E-69	3.17E-58	1.3E-122	1.2E-120
	0.9	2.38E-34	2.35E-30	2.08E-53	2.85E-52	1.4E-128	1.1E-103	1.4E-182	4E-191

Table 27 P-values for Error in costs per Experiment and autocorrelation factor AR(1)

Ref	ϕ	Experiment							
		A	B	C	D	E	F	G	H
IID	0.1	0.205544	0.039003	0.163647	0.08902	0.047609	0.000731	0.268425	0.015704
	0.2	0.009642	0.001282	0.102242	0.775973	0.016218	0.052341	0.057414	0.028596
	0.3	0.007843	0.189082	0.006177	0.696552	0.347581	0.000547	0.000925	0.00479
	0.4	0.149683	0.445137	0.065602	0.544169	0.019006	0.123852	0.20922	0.000334
	0.5	0.040629	0.555708	0.049749	0.131707	0.38221	0.522482	0.511119	0.116185
	0.6	0.011717	0.075626	0.049729	0.529342	0.410175	6.51E-05	0.004047	0.386404
	0.7	0.061316	0.11824	0.029174	0.044016	0.00101	0.055978	4.51E-05	0.023823
	0.8	0.002131	0.006046	0.083449	0.021239	5.42E-05	1.51E-07	0.000229	0.016973
	0.9	0.006913	0.021201	1.08E-05	7.12E-05	7.19E-06	2.15E-07	0.006509	0.004961

Table 28 P-values for Error in inventory policy per Experiment and autocorrelation factor AR(1)

Ref	P_01	Experiment A	B	C	D	E	F	G	H
0.1	0.2	0.242676	7.6E-06	0.401112	0.217905	5.2E-131	1.4E-92	2.06E-75	2E-87
	0.3	0.023658	3.59E-10	0.038607	0.011487	4E-143	2.3E-169	1.14E-91	1.2E-102
	0.4	0.000648	2.83E-13	0.001474	0.000133	1.1E-155	1.4E-179	4.9E-108	2.1E-118
	0.5	2.19E-06	2.66E-16	6.14E-06	4.64E-07	1.5E-168	1.2E-189	1.1E-138	2.5E-134
	0.6	1.8E-09	4.16E-22	7.74E-09	4.29E-11	2.6E-181	1.7E-200	3.2E-143	1.1E-150
	0.7	7.37E-14	1.29E-27	1.62E-13	3.38E-16	1.3E-194	1.1E-212	6.4E-162	1.6E-166
	0.8	2.32E-19	1.34E-34	4.48E-19	1.56E-21	9.7E-209	3.9E-225	7.6E-178	3.3E-185
	0.9	2.01E-27	8.7E-44	9.55E-27	1.14E-29	1.8E-223	1.7E-238	1.7E-197	3.6E-202

Table 29 P-values for Error in costs per Experiment and autocorrelation factor MC

Ref	P_01	Experiment A	B	C	D	E	F	G	H
0.1	0.2	0.021658	1.87E-05	0.68816	0.796806	0.005171	0.409411	5.27E-07	1.02E-07
	0.3	0.000436	3.17E-08	0.32519	0.299867	7.38E-05	7.5E-07	8.58E-07	8.19E-07
	0.4	0.018047	7.67E-10	0.298386	0.311597	0.000669	5.29E-06	1.77E-05	1.19E-05
	0.5	0.014895	7.39E-08	0.852337	0.894194	1.62E-05	1.08E-05	8.45E-06	2.73E-05
	0.6	0.004986	9.86E-06	0.383444	0.356937	2.25E-05	5.99E-08	7.14E-06	0.000117
	0.7	0.059942	3.27E-10	0.314695	0.105792	1.23E-05	5.5E-06	0.000134	7E-05
	0.8	0.015797	7.52E-07	0.614156	0.341949	0.000111	6.02E-07	0.000106	3.07E-06
	0.9	0.152384	2.75E-07	0.3823	0.585639	2.06E-05	2.61E-07	0.030801	3.73E-05

Table 30 P-values for Error in inventory policy per Experiment and autocorrelation factor MC

From the results obtained in Tables 27-30, one can conclude that most of the errors found in the costs and inventory policy are significantly different. These results are in accordance with the results obtained in evaluating significant difference between ignoring and considering correlated demands. In general, as the autocorrelation level increases, the errors become more significant. For lower levels of autocorrelation, results report a mix of cases, where certain experiments show no relevant difference while other experiments demonstrate highly significant differences. In all cases, middle and high levels of autocorrelation reported high levels of significant differences. Table 31 summarizes major findings in the outcome of the ANOVA test evaluations.

Demand	Error	Experiments	Description
AR(1)	$\beta(C)$	A,B,C, F, D, H	Becomes significant at medium levels of ϕ
		E, G	Becomes significant from lower levels of ϕ
	$\beta(s^*,S^*)$	A,B,D,E,F	Most of the ϕ are significant but erratic
		C, G	Becomes significant from lower levels of ϕ
MC	$\beta(C)$	All	Becomes significant from lower levels of ϕ
	$\beta(s^*,S^*)$	C,D	Do not become significant
		A, B,E,F,G,H	Becomes significant from lower levels of ϕ

Table 31 Significance test summary

5.6.3 Error characterization - Description

Once the errors have been determined and significance tests have been conducted, attempts to describe such behavior can be performed using traditional statistical tools.

Visual inspection of the scatter plots from Figures 22 – 24 considering both types of demands indicate that:

1. Cost Errors $\beta(C)$ present a uniform behavior that can be characterized. The behavior of the error can be described by a linear model using regression analysis. However, an exponential model seems to fit better with data behavior. Therefore, a nonlinear regression method is used.

2. Most of the policy errors $\beta(s^*, S^*)$ show an increasing erratic behavior. However, only results from two experiments, more specifically E and G from the AR(1) case, presented uniform behaviors that can be described.

As mentioned in remark 1, nonlinear regression techniques to characterize these errors are advisable. Devore (2004) provides a regression method with transformed variables in which the obtained value is transformed by a linearization method. In this case, the transforming method is described as follows.

Function	**Transformation to linearize**	**Lineal form**
$y = \alpha e^{\beta x}$	$y' = \ln(y)$	$y' = \ln(\alpha) + \beta x$

Where y is dependent variable, α is the interception point, β is the slope (y increases if $\beta > 0$, or decreases if $\beta < 0$), and y' is the dependent variable transformed.

Tables G.1.1 – G.1.6 in Appendix G show the linearization process and the regression analysis for the errors generated in estimating the average total cost $\beta(C)$. Parameters for the exponential equations were automatically generated using the trend line function from MS Excel.

EXP	EQUATION	R^2
A	$y = 32.142e^{2.1705x}$	0.9966
B	$y = 31.306e^{2.3793x}$	0.9913
C	$y = 31.432e^{2.2028x}$	0.9956
D	$y = 31.311e^{2.2582x}$	0.9949
E	$y = 149.9e^{2.4359x}$	0.9671
F	$y = 145.6e^{2.6463x}$	0.956
G	$y = 150.54e^{2.3298x}$	0.9832
H	$y = 152.49e^{2.3483x}$	0.9877

Table 32 Cost Error Description for DMC demand

EXP	EQUATION	R^2
A	$y = 1.3077e^{4.9471x}$	0.9907
B	$y = 1.6371e^{5.8101x}$	0.9800
C	$y = 2.0823e^{6.0337x}$	0.9690
D	$y = 2.1737e5.^{9358x}$	0.9744
E	$y = 4.1542e^{6.1395x}$	0.9680
F	$y = 3.8246e^{6.0407x}$	0.9560
G	$y = 6.6683e^{6.1759x}$	0.9629
H	$y = 6.5507e^{6.2118x}$	0.9629

Table 33 Cost Error Description for AR(1) demand

Similar reasoning was used to characterize the error generated by the inventory policy for experiments E and G from the inventory model that considered AR(1)

EXP	EQUATION	R^2
E	$y = 78.719e^{1.7661x}$	0.5405
G	$y = 33.702e^{3.8063x}$	0.7433

Table 34 Inventory Policy Error Description for AR(1) demand

Notice that values for R^2 from Tables 32 and 33 are above 95%, which indicate a good fit of the estimated original nonlinear model to the observed responses. Table 34 indicates that the linear model for experiment G can explain up to approximately 75% of the obtained errors.

5.7 Validation and verification

5.7.1 Introduction

In the preceding sections, the effects of ignoring the autocorrelation components have been determined through applying the SAPSRS algorithm, which accounts for this component. This section deals with validating and verifying the purpose and workings of the proposed heuristic.

5.7.2 Validation

Validation is the process of determining the degree to which a model (and data) is an accurate representation of the real world from the perspective of the model's intended usage (DOD, 1996). SAPSRS is intended to generate and provide near-optimal inventory policies that minimize total costs considering the autocorrelated DMC and AR(1) demands. To accomplish this goal, two additional variables, stockout and replenishment rates, are considered. The stockout rate is referred to as the average of the times that the system has not able to totally satisfy a given demand per period of time, in other words, when the inventory level reaches zero. The replenishment rate is referred to as the average of the times that the inventory level reaches the reorder point triggering an order. Some authors have demonstrated that for some continuous demand distributions, as the autocorrelation increases, the stockout increases as well (Zinn et al., 1992; Charnes et al., 1995; Urban, 2000). This is apparent since as the autocorrelation component increases the variability increases while decreasing the probability of facing certain demands (Zinn et al., 1992). To demonstrate this point, experiment D from the AR(1) case is considered.

Assuming that the inventory policy is near-optimal for the IID case, the stockout and the replenishment rates are determined. Thus, if autocorrelation factors are ignored, and the obtained policy is assumed to be near-optimal for the system, the increasing autocorrelation levels lead to an increasing variability that pushes total costs and stockout rates higher. Table 35 shows results obtained for assuming the IID inventory policy to be near-optimal (2521, 3082) at different levels of autocorrelation factors.

Auto	Cost	Stockout	Replenishment
IID	5,372.6	0.021	0.9999
0.1	5,380.5	0.027	0.9999
0.2	5,381.9	0.029	0.9999
0.3	5,395.2	0.033	0.9999
0.4	5,409.75	0.039	0.9999
0.5	5,447.62	0.050	0.9999
0.6	5,498.25	0.070	0.9999
0.7	5,615.38	0.090	0.9997
0.8	5,866.12	0.130	0.9890
0.9	6,687.92	0.200	0.9860
0.95	8,217.31	0.260	0.9850
0.99	18,196.85	0.454	0.9853

Table 35 Effects of using policy obtained assuming IID in stockout and replenishment rates

Notice how the autocorrelation factor leads to an increase in total costs and stockouts as the autocorrelation increases while replenishment is kept ordering almost all the time.

As a result, given the nature of the demand, the inventory model, and the behavior of the stockout and replenishment rates, the following proposition may prove that the policy found by the heuristic leads to a better approximation than would be obtained by ignoring the autocorrelation factors.

Proposition

If the proposed simulation optimization procedure finds a near-optimal inventory policy considering serially-correlated demands, stockouts in the inventory system should be kept controlled to a certain level as the autocorrelation increases.

Evidence

Table 36 shows the proposed near-optimal inventory policy with the associated stockout and replenishment rates by autocorrelation factor.

Auto	Cost	s	S	D	Stockout	Replan
IID	5,370.93	2521	3082	561	0.0210	0.9999
0.1	5,373.20	2246	3082	836	0.0210	0.9999
0.2	5,378.72	2572	3099	527	0.0270	0.9999
0.3	5,387.55	2571	3107	536	0.0250	0.9999
0.4	5,401.80	2576	3133	557	0.0220	0.9999
0.5	5,422.93	2308	3167	859	0.0230	0.9999
0.6	5,454.29	2663	3222	559	0.0240	0.9999
0.7	5,503.32	2299	3320	1021	0.0210	0.9997
0.8	5,587.05	1951	3491	1540	0.0210	0.989
0.9	5,753.69	2119	3865	1745	0.0270	0.986

Table 36 Stockout, ordering, and replenishment rate for obtained near-optimal policy - AR(1) case

From Table 36, notice how the stockout rate is kept fairly constant with variations <0.01 throughout the exposed autocorrelation factor level. The replenishment rate barely decreases as the autocorrelation factor increases suggesting that even though there is an improvement in the stockout rate, the highly noisy demand keeps the inventory system ordering most of the time.

5.7.3 Verification

Verification is the process of determining that a model implementation accurately represents the developer's conceptual description and specifications (DOD 1996). In this dissertation, it is argued that the algorithm SAPSRS enhanced traditional SA. This enhancement incorporates PS and R&S into the searching and evaluation process. The enhancement of this technique was defined and measured by the number of times that a candidate solution improved upon the evaluation process. Tables 38 and 39, and Figures 25 and 26 present the values and a pie chart representing such information.

	SAPSRS - AR(1)
Exp	**%**
A	0.3885
B	0.3453
C	0.3666
D	0.3408
E	0.4569
F	0.4277
G	0.4160
H	0.4098

Table 37 Portion of candidate solutions enhanced by SAPSRS algorithm per Experiment

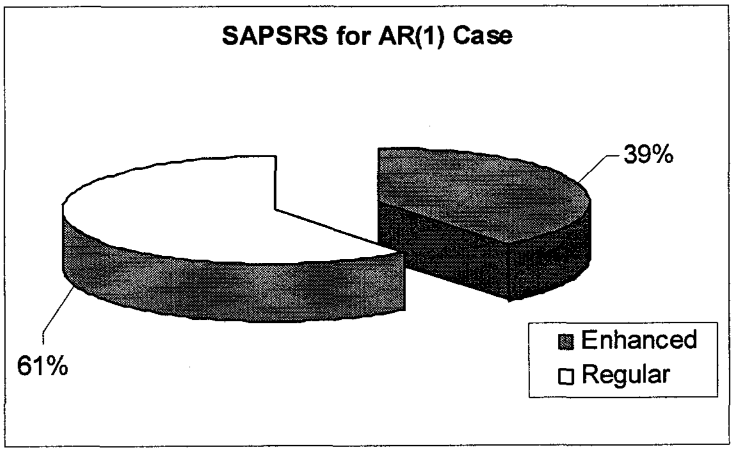

Figure 25 Overall Portion of candidate solutions enhanced by SAPSRS algorithm

	SAPSRS Correlated DMC
Exp	%
A	0.6053
B	0.5996
C	0.6218
D	0.6177
E	0.5963
F	0.5613
G	0.5564
H	0.5374

Table 38 Portion of candidate solutions enhanced by SAPSRS algorithm for DMC Case

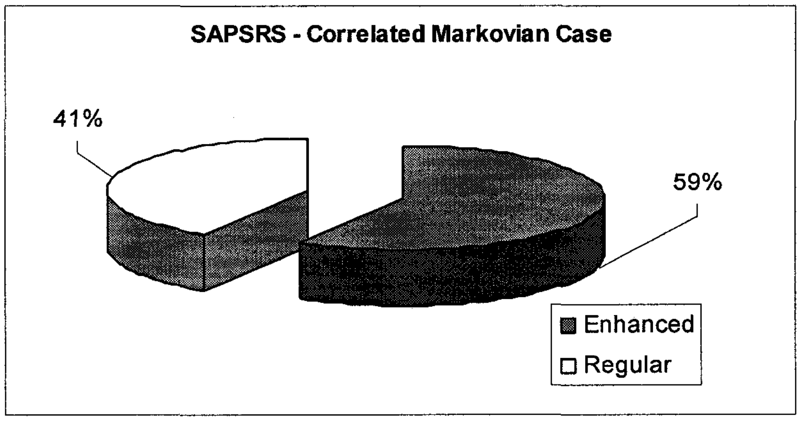

Figure 26 Overall Portion of candidate solutions enhanced by SAPSRS algorithm - correlated DMC demand

Notice that the new SAPSRS algorithm reported a 40%-60% improvement in evaluating and selecting candidate solutions. Further, given a candidate solution produced by SA, PS with R&S proposed, evaluated, and selected a candidate solution that reported a better performance.

6 CONCLUSIONS AND FUTURE WORK

6.1 Conclusions from the Proposed Research Questions

In accordance with the structure of the research questions proposed in this study summarized in section 1.4, the first research question addresses methodological frameworks that allow handling and solving stochastic inventory models that present dependency in their input variables. The second and third questions address the impact of considering dependency issues in traditional measures of performance in inventory models, namely total cost, reorder points, and order quantity. The fourth research question addresses the description of the error generated by methods that ignore dependency issues. The fifth question proposes a validation mechanism to verify that the obtained inventory policy represents a near-optimal policy.

Notice that these research questions have cumulative effects, moving from a general belief toward a contingent view. When the SAPSRS algorithm is designed and implemented in section 4.3.3, the first research question is addressed. When the algorithm is used and results are generated and presented in sections 5.4 and 5.5, the implications for the second, third, and fourth research questions become self-explanatory. Once the differences are obtained and effects are determined, the error is characterized in section 5.6. Finally, in section 5.7, the effectiveness of the proposed algorithm in obtaining near-optimal inventory policy is validated by comparing costs and stockout rates. In addition, statistical information concerning improvements in the evaluation step of SAPSRS algorithm using PS and R&S is presented.

6.2 Implications and Discussion

Through a detailed analysis of the cost and inventory policy impact upon stochastic inventory lost sales models that ignore continuous and discrete autocorrelated demands, this study suggests several points.

I. Inventory stochastic models that ignore AR(1) or DMC may lead to misleading inventory policies that result in higher total cost.

In other words, ignoring autocorrelation components leads to overestimating total costs in the inventory system. Tests of significance support these findings. Thus, methods that fail to consider autocorrelated demands lead to an erroneous inventory policy.

Regardless of the type of dependent demand considered, the error in the costs of ignoring this component is significantly relevant and increases as the autocorrelation increases. From the reorder level perspective, as the serial-correlated is incremented, the reorder point is lower compared to the one obtained by the correlation-free demand. This is true for both for both autocorrelated AR(1) and the DMC case. The order quantity also increases as the autocorrelation widens. Reorder points and order quantities present significant differences for all values between the correlated and correlation-free in the DMC case. For the AR(1) case, all costs were significantly different while the differences obtained in reorder points and order quantity were significant at higher levels of autocorrelation.

II. The effects of holding costs and the interaction of ordering and holding costs have the most relevant impact.

Another issue relates to the main effects and interactions observed in total costs, reorder points, and order quantity as the autocorrelation varies. From the analysis of the main effects, holding costs represent the factor that has a major impact on the studied inventory models. The obtained values reported that the effect becomes stronger as the correlation component increases. For the total costs, holding costs become stronger, driving it higher as the autocorrelation increases. In addition, the effect of holding costs is strongly negative on the reorder point while positive on the order quantities. In other words, the higher the autocorrelation, the lower the obtained reorder point and the higher the order quantity. On the one hand, as the autocorrelation increases, holding costs make the inventory system more expensive, which pushes reorder points lower in order to minimize total costs. On the other hand, the lower reorder point enforces the increase in order quantity. The effect of the holding cost factor on total costs, and reorder point is significant for almost all levels of autocorrelation components in both cases. The individual impact of the penalty costs and ordering cost increases and decreases respectively for both cases. These factors are only significant at higher levels of autocorrelation for the DMC case.

The analysis of two-way interactions indicates that in total costs, the ordering-holding cost interaction poses the highest magnitude and becomes weaker as the autocorrelation increases. In addition, penalty-ordering and penalty-holding costs become stronger and make total costs higher. However, only the interaction between penalty and holding costs is significant. The effect that holding cost has on total costs depends upon the levels of penalty costs and vice versa. As a result, there is further indication that in order to observe increase or decrease in total costs, both penalty and holding costs must be set at comparable levels. Additional significant interactions were observed at the highest levels of autocorrelation for ordering-penalty and ordering-holding costs only for the DMC case. This suggests that at such dependency levels, the effects of each factor on total costs depend upon the effect of one on another. For the reorder points and order quantities, the description of interactions shows an erratic behavior. Only the penalty-holding cost interaction consistently shows significance at higher levels of autocorrelation. This reinforces the view that the impact of such interaction is significant in the inventory model.

III. Errors generated in total cost, reorder, and order quantity can be quantified and characterized.

A third point relates to analyzing the error produced in the cost, reorder, and order quantity of the (s, S) policy. Above, it was mentioned that there are significant differences between methods that ignore dependency and those that acknowledge it; the produced errors were quantified and characterized. ANOVA tests were conducted to evaluate the significance of these errors. As a result, most experiments show a significant difference that increases as the serially-correlated component increases. As predicted, costs error behaved in an exponential fashion. Most of the errors obtained for near-optimal policies presented an erratic behavior. Only policy errors presented in two of eight experiments that presented AR(1) demands showed a uniform behavior that could be characterized as an exponential function. In the DMC case, differences between correlated and correlation-free cases were tested using ANOVA.

Most of the experiments showed significant differences at higher levels of autocorrelation while half of them posed significant difference from lower to higher levels. This supports findings that the (s, S) policy is not optimal for inventory systems that do not assume IID demands (Iyer & Schrage, 1992).

IV. Empirical validation and verification of SAPSRS heuristic.

The effectiveness of the algorithm was analyzed by demonstrating its ability to obtain near-optimal policies. To verify this, total costs and stockout rates were compared. The First-autoregressive AR(1) case was selected to validate and verify the algorithm.

Total costs and stockouts rates were obtained by ignoring the autocorrelation factor while using the near-optimal inventory policy found for the IID case. As expected and consistent with other research (Zinn et al., 1992; Charnes et al., 1995; Urban, 2000), the empirical evidence demonstrated that as the autocorrelation component increases and is ignored, total costs and stockouts increase. In addition, an increasing level of stockouts along with high reorder points and lower ordered quantities forces the inventory system to constantly order items. This keeps replenishment rates high as well.

When autocorrelation was ignored, service level in terms of stockouts was decreased by almost 50% from 98% while total cost increased nearly three times. By using the near-optimal policy suggested by SAPSRS and considering the autocorrelation factor, service level was kept around 98% while total cost increased only a little.

V. *The SAPSRS algorithm reported a 40%-60% improvement in evaluating and selecting candidate solutions.*

A final point involves the efficiency of the presented algorithm. The efficiency of this technique was defined and measured by the times that a candidate solution improved upon the evaluation process. As a result, the new SAPSRS algorithm reported a 40%-60% improvement in evaluating and selecting candidate solutions. Specifically, given a candidate solution generated by SA, PS combined with R&S proposed and selected a solution that reported a better performance between 40 and 60 % of the time.

6.3 Managerial Implications

This study suggests that modeling and solving stochastic inventory models that ignore serially-correlated components in the demands when they are present lead to serious and significant errors. From the obtained empirical data and the analysis described above, implications for a manager that suspects that the facing demand contains autocorrelated components include:

1. Observe the behavior of the demand and determine if it contains autocorrelation components. Visual inspection, Durbin-Watson statistics, and calculating the sample autocorrelation function are generally recommended techniques to detect autocorrelated components (Neter, Wasserman, & Kutner, 1990; Zinn et al., 1992; Pinder, 1996).
2. If autocorrelation components are identified, the manager may use the method presented in this study to mitigate the effects of autocorrelation. In general, the

empirical results of applying this method suggest that to mitigate the effects of autocorrelation, the manager may

a. Make the reorder point smaller.

b. Increase the order quantity.

c. Evaluate stockouts and decide whether to increase replenishment rates.

The rationale behind this reasoning can be explained as follows. From the experiments, for the analyzed lost sales inventory model, holding costs dominate and become stronger and highly significant as the autocorrelation increases. As a result, reducing minimum stock or reorder levels implies a reduction in holding cost. Increasing the order quantity leads to increasing ordering costs. However, the empirical data shows that the effects of ordering costs on total cost decreases as the autocorrelation increases. Therefore, as demonstrated in the results obtained in the DMC case, a reduction in the reorder point combined with an increase on the ordered quantity reported better performance of the inventory system. Nonetheless, in parallel, stockout rates should be monitored. If they are not significant, empirical evidence demonstrates that replenishment rates will decrease. However, if stockouts are present and are significant, the replenishment rate will be high as well.

Empirical results indicate that managers may obtain a better performance of their inventory system by acknowledging that autocorrelated components may be present in the inflow demand and by following the described actions.

6.4 Summary

Stochastic lost sales inventory models that present autocorrelated demands are very common in competitive markets. The error characterization presented in this research provides insights into how autocorrelated demand affects the estimation of total costs and control policies. Acknowledging these errors results in a requirement for a correction method for countering the effects of autocorrelated demand. As a result, inventory control policies that recognize and deal with dependency components need to be set efficiently in order to satisfy customer demands. In this dissertation, a method to approximately solve the inventory problem that presents autocorrelated demand has been developed. To model and solve this complex inventory problem, a simulation optimization technique called SAPSRS was developed and implemented. This approach combined and adapted three well-known heuristics that include SA, PS, and R&S.

The SAPSRS algorithm was implemented in a computer program that was developed using the C++ language. This program is subdivided into three parts. One piece models and generates autocorrelated demands from a Discrete Markov Chain distribution and from an AR(1) continuous process. A second part includes a simulation model that represents and mimics the behavior of a lost sales inventory model. Finally, the third part of the program includes the algorithm that explores a decision space and proposes, evaluates, and selects candidate solutions. Extensive numerical analysis to test the efficiency of the methodology has been used.

Managerial implications include recognizing the effects of autocorrelation in the stochastic demand and using the SAPSRS algorithm to obtain more realistic and reliable control policy settings.

6.5 Future Research Directions

In this section, specific research directions following from this dissertation are discussed:

1. **Studying other inventory models**

   The inventory model studied in this research includes the lost sales case and assumes immediate replenishment. The described situation can be found in food retailers in which replenishment of items occur overnight. If the backlogging case is considered and different replenishment rules are set, the mechanics of the inventory model must be changed. In this sense, other types of retailers, such as department store may experience a delay in receiving certain class of order. This delay in which lead times are larger than zero changes the mechanics of the inventory system.

2. **Studying the effect of autocorrelation demand in other inventory policies**

   This research was designed to study the effects of autocorrelations in terms of the continuous (s, S) policy. However, there are many inventories that use other types of policy, i.e. a combination of periodic and continuous review policy (Q, s, S). The effect of autocorrelation might be different and a different set of rules to face the autocorrelated demand may be required. As a result, the workings of setting and evaluating the proposed inventory policy must be changed.

3. **Studying other types of stochastic dependent demand**

   The studied autocorrelated demand is either DMC or continuous AR(1). However, in the inventory model, other types of dependent demand with seasonal effects might be present, i.e. ARIMA. If additional dependency is considered, the algorithms for data generation have to include such additional sources of dependency. Thus, the near-optimal policy and measures of performance could be more realistic.

4. **Investigating the effects of high autocorrelations and obtaining their characterizations**

   As demonstrated, in the DMC case, individual cost components and their interaction have a high impact on the performance of the system. These components are related in such a way that they affect each other with certain significant magnitude. Further research is required to describe the behavior of the system at high levels of the serially-correlated components. This would allow focusing and managing those factors that most influence the system.

5. **Applications of this methodology to other areas and fields**

   This study solves the problem of an inventory system that presents autocorrelated demands. However, other similar problems may be faced in different areas and fields where autocorrelation components could be present in the input processes.

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APPENDICES

A RESPONSES FOR MARKOVIAN-MODULATED AND AR(1) CASES

Appendix A.1. Responses: Markovian Case

a. Experiment description

Experiment	c	p+C	h
A	1	5*(#items) +100	0.5
B	2	5*(#items) +100	0.5
C	1	19*(#items) +200	0.5
D	2	19*(#items) +200	0.5
E	1	5*(#items) +100	2.5
F	2	5*(#items) +100	2.5
G	1	19*(#items) +200	2.5
H	2	19*(#items) +200	2.5

b. Response

P01	φ	*CostDep*	*sDep*	*Sdep*	*CostIID*	*sIID*	*SIID*	*ddep*	*diid*
0.10	-0.15	2,978.27	1407	3008	3,001.64	2497	3004	1601	507
0.20	0.13	3,169.52	1433	3024	3,212.75	2340	3031	1591	691
0.30	0.29	3,192.08	2292	4001	3,250.35	3298	4002	1709	704
0.40	0.38	3,178.56	2503	4003	3,250.54	3335	4002	1500	667
0.50	0.45	3,166.57	2375	4003	3,250.02	3277	4002	1629	724
0.60	0.49	3,155.09	2341	4004	3,250.04	3257	4002	1663	745
0.70	0.53	3,145.49	2581	4003	3,250.18	3279	4002	1421	723
0.80	0.56	3,136.55	2408	4005	3,250.40	3336	4003	1597	667
0.90	0.64	3,127.15	1941	4004	3,250.72	3296	4002	2063	706

Table A.1.1 Response Experiment A

P01	φ	*CostDep*	*sDep*	*Sdep*	*CostIID*	*sIID*	*SIID*	*ddep*	*diid*
0.10	-0.15	5,433.43	1450	3006	5,456.29	2334	3004	1556	670
0.20	0.13	5,585.91	1512	3014	5,629.51	2434	3009	1502	575
0.30	0.29	5,690.22	2515	4003	5,750.01	3127	4003	1488	876
0.40	0.38	5,675.00	2293	4005	5,748.13	3318	4002	1712	685
0.50	0.45	5,660.02	2451	4007	5,748.27	3287	4005	1555	718
0.60	0.49	5,652.00	2637	4008	5,748.84	3317	4003	1372	686
0.70	0.53	5,642.42	2445	4004	5,748.92	3251	4002	1559	751
0.80	0.56	5,625.28	2284	4010	5,747.87	3336	4003	1727	667
0.90	0.64	5,586.96	1456	4018	5,747.40	3312	4003	2562	692

Table A.1.2 Response Experiment B

P01	φ	CostDep	sDep	Sdep	CostIID	sIID	SIID	ddep	diid
0.10	-0.15	3,227.44	2592	4002	3,250.30	3221	4001	1410	781
0.20	0.13	3,208.57	2418	4003	3,250.43	3302	4002	1584	700
0.30	0.29	3,192.93	2359	4004	3,250.07	3335	4002	1645	667
0.40	0.38	3,178.19	2360	4003	3,250.24	3312	4002	1642	691
0.50	0.45	3,166.95	2511	4003	3,250.16	3335	4002	1492	667
0.60	0.49	3,155.60	2738	4002	3,250.40	3335	4002	1264	667
0.70	0.53	3,146.20	2334	4003	3,250.40	3306	4003	1669	697
0.80	0.56	3,136.12	2620	4003	3,250.59	3336	4003	1384	667
0.90	0.64	3,128.60	2546	4005	3,250.05	3336	4003	1459	667

Table A.1.3 Response Experiment C

P01	φ	CostDep	sDep	Sdep	CostIID	sIID	SIID	ddep	diid
0.10	-0.15	5,726.67	2560	4003	5,749.53	3336	4003	1443	667
0.20	0.13	5,706.17	2599	4003	5,748.64	3300	4003	1403	703
0.30	0.29	5,691.36	2645	4006	5,747.79	3222	4003	1360	781
0.40	0.38	5,675.52	2288	4004	5,746.94	3336	4003	1716	667
0.50	0.45	5,662.82	2392	4006	5,748.34	3293	4003	1614	710
0.60	0.49	5,651.76	2235	4005	5,747.95	3299	4004	1770	705
0.70	0.53	5,640.90	2652	4005	5,748.12	3337	4004	1353	667
0.80	0.56	5,629.94	2508	4004	5,748.03	3337	4004	1497	667
0.90	0.64	5,619.60	2459	4005	5,747.93	3335	4002	1546	667

Table A.1.4 Response Experiment D

P01	φ	CostDep	sDep	Sdep	CostIID	sIID	SIID	ddep	diid
0.10	-0.15	3,984.21	1441	3001	4,097.65	2427	3001	1560	575
0.20	0.13	4,177.65	1393	3002	4,384.50	2454	3001	1609	547
0.30	0.29	4,339.95	1539	3002	4,627.01	2502	3002	1463	500
0.40	0.38	4,480.61	1400	3002	4,835.65	2389	3003	1602	614
0.50	0.45	4,602.25	1527	3003	5,016.45	2502	3002	1477	500
0.60	0.49	4,704.75	1332	3002	5,173.19	2482	3003	1669	521
0.70	0.53	4,796.21	1421	3004	5,312.99	2464	3003	1583	539
0.80	0.56	4,876.98	1237	3004	5,436.46	2502	3002	1767	500
0.90	0.64	4,618.44	717	3010	5,547.03	2502	3002	2293	500

Table A.1.5 Response Experiment E

P01	φ	CostDep	sDep	Sdep	CostIID	sIID	SIID	ddep	diid
0.10	-0.15	4,591.43	1296	3005	4,702.28	2503	3004	1708	500
0.20	0.13	5,282.78	1622	3006	5,493.86	2504	3004	1384	500
0.30	0.29	5,871.43	1615	3013	6,153.65	2508	3010	1398	501
0.40	0.38	5,896.59	2572	4002	6,252.40	3335	4002	1430	667
0.50	0.45	5,834.04	2627	4001	6,206.26	3022	3981	1375	959
0.60	0.49	5,782.42	2534	4001	6,253.31	3335	4002	1468	667
0.70	0.53	5,736.96	2643	4002	6,253.16	3322	4002	1358	680
0.80	0.56	5,695.75	2665	4034	6,253.12	3335	4002	1369	667
0.90	0.64	5,485.48	1643	4001	6,253.48	3251	4002	2358	751

Table A.1.6 Response Experiment G

P01	*φ*	*CostDep*	*sDep*	*Sdep*	*CostIID*	*sIID*	*SIID*	*ddep*	*diid*
0.10	-0.15	7,042.65	1540	3003	7,156.58	2463	3003	1463	540
0.20	0.13	7,701.13	1630	3005	7,905.73	2447	3004	1376	557
0.30	0.29	8,252.34	1561	3014	8,544.17	2512	3018	1454	506
0.40	0.38	8,397.22	2341	4002	8,754.24	3316	4002	1660	685
0.50	0.45	8,336.28	2413	4001	8,753.17	3322	4002	1589	680
0.60	0.49	8,284.24	2383	4001	8,753.31	3256	4002	1618	745
0.70	0.53	8,238.52	2508	4001	8,754.18	3273	4002	1494	729
0.80	0.56	8,196.51	2547	4001	8,753.12	3235	4001	1454	767
0.90	0.64	7,956.76	1548	4002	8,754.09	3244	4002	2454	758

Table A.1.7 Response Experiment H

Appendix A.2. Response: AR(1)

φ	Cost	s	S	D
0	2,781.31	2018	2869	851
0.1	2,783.10	2223	2864	640
0.2	2,786.60	2054	2879	824
0.3	2,793.88	2277	2896	619
0.4	2,804.29	2204	2907	703
0.5	2,820.64	2065	2926	861
0.6	2,843.56	2155	2945	790
0.7	2,881.54	1879	3016	1136
0.8	2,943.68	1620	3123	1503
0.9	3,067.84	1673	3364	1690

A.2. 1 Response Experiment A

φ	Cost	s	S	D
IID	5,284.09	2240	2875	635
0.1	5,285.98	2020	2872	853
0.2	5,289.28	1974	2871	897
0.3	5,296.32	2233	2889	656
0.4	5,305.52	2369	2902	533
0.5	5,320.34	2310	2916	606
0.6	5,340.77	2295	2950	654
0.7	5,376.46	2164	2988	824
0.8	5,432.90	1852	3068	1216
0.9	5,548.45	1680	3358	1678

A.2. 2 Response Experiment B

φ	Cost	s	S	D
IID	2,874.50	2272	3083	812
0.1	2,876.56	2498	3095	596
0.2	2,882.10	2129	3100	970
0.3	2,891.66	2386	3115	729
0.4	2,906.07	2393	3144	751
0.5	2,928.15	2486	3171	685
0.6	2,960.14	2627	3219	592
0.7	3,010.34	2460	3315	855
0.8	3,095.51	2020	3488	1468
0.9	3,263.32	2081	3864	1783

A.2. 3 Response Experiment C

φ	Cost	s	S	D
IID	5,370.93	2521	3082	561
0.1	5,373.20	2246	3082	836
0.2	5,378.72	2572	3099	527
0.3	5,387.55	2571	3107	536
0.4	5,401.80	2576	3133	557
0.5	5,422.93	2308	3167	859
0.6	5,454.29	2663	3222	559
0.7	5,503.32	2299	3320	1021
0.8	5,587.05	1951	3491	1540
0.9	5,753.69	2119	3865	1745

A.2. 4 Response Experiment D

φ	Cost	s	S	D
IID	3,275.46	1882	2584	702
0.1	3,279.57	2016	2591	575
0.2	3,290.63	2007	2587	580
0.3	3,311.10	1998	2596	598
0.4	3,342.97	1679	2597	918
0.5	3,389.43	1745	2597	851
0.6	3,458.99	1797	2615	818
0.7	3,568.17	1341	2627	1286
0.8	3,748.96	1212	2667	1455
0.9	4,120.12	1115	2743	1628

A.2. 5 Response Experiment E

φ	Cost	s	S	D
IID	5,684.72	1958	2533	575
0.1	5,688.41	1683	2535	852
0.2	5,698.54	1954	2536	582
0.3	5,716.36	1695	2528	834
0.4	5,744.14	1958	2547	590
0.5	5,785.33	2062	2536	474
0.6	5,845.91	1419	2557	1138
0.7	5,942.01	1647	2543	896
0.8	6,099.29	1086	2563	1476
0.9	6,360.97	1088	2644	1556

A.2. 6 Response Experiment F

φ	Cost	s	S	D
IID	6,270.99	2019	2845	826
0.1	6,277.04	2326	2850	524
0.2	6,296.68	2376	2850	475
0.3	6,329.42	1963	2874	911
0.4	6,382.19	1925	2872	947
0.5	6,459.31	2284	2904	620
0.6	6,573.23	1980	2935	955
0.7	6,754.85	2030	2980	950
0.8	7,059.37	1603	3091	1487
0.9	7,669.22	1660	3351	1691

A.2. 7 Response Experiment H

B EVALUATION OF SIGNIFICANCE OF THE RESPONSES

B.1. P-values for Responses. Markovian Demand

			P-value	*Hypthesis*		*P-value*	*Hypthesis*		*P-value*	*Hypthesis*	
Experiment	***P01***	***Autocorrelation***	***Cost***	***Ho***	***Ha***	***s***	***Ho***	***Ha***	***d***	***Ho***	***Ha***
	0.10	-0.15	6.26E-07	Reject	Accept	0.00066	Reject	Accept	0.000594	Reject	Accept
	0.20	0.13	4.65E-11	Reject	Accept	0.000263	Reject	Accept	0.000243	Reject	Accept
	0.30	0.29	3.06E-16	Reject	Accept	1.89E-08	Reject	Accept	2E-08	Reject	Accept
	0.40	0.38	2.8E-13	Reject	Accept	0.003486	Reject	Accept	0.003612	Reject	Accept
A	0.50	0.45	1.75E-16	Reject	Accept	0.001825	Reject	Accept	0.001844	Reject	Accept
	0.60	0.49	2.5E-16	Reject	Accept	6.17E-06	Reject	Accept	6.17E-06	Reject	Accept
	0.70	0.53	5.25E-15	Reject	Accept	0.003887	Reject	Accept	0.003768	Reject	Accept
	0.80	0.56	4.4E-13	Reject	Accept	3.84E-05	Reject	Accept	4.26E-05	Reject	Accept
	0.90	0.64	6.65E-13	Reject	Accept	1.75E-10	Reject	Accept	1.77E-10	Reject	Accept
	0.10	-0.15	1.54E-07	Reject	Accept	6.11E-05	Reject	Accept	6E-05	Reject	Accept
	0.20	0.13	4.08E-10	Reject	Accept	8.57E-06	Reject	Accept	9.67E-06	Reject	Accept
	0.30	0.29	1.92E-10	Reject	Accept	5.65E-05	Reject	Accept	5.5E-05	Reject	Accept
	0.40	0.38	1.51E-11	Reject	Accept	0.001248	Reject	Accept	0.000127	Reject	Accept
B	0.50	0.45	2.62E-12	Reject	Accept	1.55E-07	Reject	Accept	2.48E-07	Reject	Accept
	0.60	0.49	1.68E-13	Reject	Accept	6.42E-05	Reject	Accept	6.58E-05	Reject	Accept
	0.70	0.53	3.99E-11	Reject	Accept	0.001883	Reject	Accept	0.002227	Reject	Accept
	0.80	0.56	1.91E-12	Reject	Accept	1.83E-05	Reject	Accept	1.53E-05	Reject	Accept
	0.90	0.64	7.93E-10	Reject	Accept	3.2E-07	Reject	Accept	3.26E-07	Reject	Accept
	0.10	-0.15	8.74E-11	Reject	Accept	1.84E-11	Reject	Accept	1.05E-07	Reject	Accept
	0.20	0.13	3.7E-13	Reject	Accept	0.000396	Reject	Accept	0.000373	Reject	Accept
	0.30	0.29	2.7E-15	Reject	Accept	0.000511	Reject	Accept	0.000498	Reject	Accept
	0.40	0.38	9.19E-13	Reject	Accept	0.000158	Reject	Accept	0.000164	Reject	Accept
C	0.50	0.45	1.28E-13	Reject	Accept	4.29E-06	Reject	Accept	4.54E-06	Reject	Accept
	0.60	0.49	1.65E-09	Reject	Accept	0.07219	Reject	Accept	0.133545	Reject	Accept
	0.70	0.53	2.11E-15	Reject	Accept	0.001601	Reject	Accept	0.00151	Reject	Accept
	0.80	0.56	3.13E-13	Reject	Accept	1.2E-05	Reject	Accept	1.25E-05	Reject	Accept
	0.90	0.64	2.35E-16	Reject	Accept	0.000156	Reject	Accept	0.000149	Reject	Accept
	0.10	-0.15	8.5E-08	Reject	Accept	0.014762	Reject	Accept	0.014277	Reject	Accept
	0.20	0.13	6.01E-10	Reject	Accept	0.000131	Reject	Accept	0.000137	Reject	Accept
	0.30	0.29	2.19E-10	Reject	Accept	0.000477	Reject	Accept	0.000469	Reject	Accept
	0.40	0.38	2.23E-12	Reject	Accept	1.8E-05	Reject	Accept	1.76E-05	Reject	Accept
D	0.50	0.45	4.22E-14	Reject	Accept	9.79E-05	Reject	Accept	9.48E-05	Reject	Accept
	0.60	0.49	1.64E-12	Reject	Accept	9.01E-05	Reject	Accept	8.78E-05	Reject	Accept
	0.70	0.53	1.89E-10	Reject	Accept	0.000162	Reject	Accept	0.000142	Reject	Accept
	0.80	0.56	1.44E-11	Reject	Accept	4.69E-07	Reject	Accept	3.77E-07	Reject	Accept
	0.90	0.64	1.33E-17	Reject	Accept	0.000644	Reject	Accept	0.000615	Reject	Accept

Table B.1 P-Values Markovian Demand

			P-value	Hypthesis		P-value	Hypthesis		P-value	Hypthesis	
Experiment	***P01***	***Autocorrelation***	***Cost***	***Ho***	***Ha***	***s***	***Ho***	***Ha***	***d***	***Ho***	***Ha***
	0.10	-0.15	3.75E-09	Reject	Accept	6.66E-06	Reject	Accept	6.58E-06	Reject	Accept
	0.20	0.13	8.66E-15	Reject	Accept	6.33E-07	Reject	Accept	6.29E-07	Reject	Accept
	0.30	0.29	1.07E-17	Reject	Accept	0.001069	Reject	Accept	0.001069	Reject	Accept
	0.40	0.38	8.18E-16	Reject	Accept	0.000123	Reject	Accept	0.000121	Reject	Accept
E	0.50	0.45	6.54E-17	Reject	Accept	0.000438	Reject	Accept	0.000443	Reject	Accept
	0.60	0.49	1.02E-18	Reject	Accept	9.5E-05	Reject	Accept	9.32E-05	Reject	Accept
	0.70	0.53	7.25E-18	Reject	Accept	0.000103	Reject	Accept	0.0001	Reject	Accept
	0.80	0.56	4.01E-18	Reject	Accept	4.57E-09	Reject	Accept	3.97E-09	Reject	Accept
	0.90	0.64	1.62E-16	Reject	Accept	2.01E-12	Reject	Accept	1.65E-12	Reject	Accept
	0.10	-0.15	1.09E-09	Reject	Accept	0.000141	Reject	Accept	0.000137	Reject	Accept
	0.20	0.13	2.55E-14	Reject	Accept	2.21E-05	Reject	Accept	2.27E-05	Reject	Accept
	0.30	0.29	2.08E-15	Reject	Accept	0.000111	Reject	Accept	0.000114	Reject	Accept
	0.40	0.38	4.61E-15	Reject	Accept	0.000141	Reject	Accept	0.000141	Reject	Accept
F	0.50	0.45	1.72E-15	Reject	Accept	3.62E-06	Reject	Accept	4.14E-06	Reject	Accept
	0.60	0.49	2.04E-17	Reject	Accept	4.34E-05	Reject	Accept	4.42E-05	Reject	Accept
	0.70	0.53	6.41E-18	Reject	Accept	0.000406	Reject	Accept	0.000375	Reject	Accept
	0.80	0.56	6.36E-17	Reject	Accept	5.48E-08	Reject	Accept	5.78E-08	Reject	Accept
	0.90	0.64	1.08E-15	Reject	Accept	1.67E-10	Reject	Accept	1.96E-10	Reject	Accept
	0.10	-0.15	9.68E-07	Reject	Accept	3.45E-05	Reject	Accept	3.14E-05	Reject	Accept
	0.20	0.13	1.08E-12	Reject	Accept	0.000256	Reject	Accept	0.00023	Reject	Accept
	0.30	0.29	2.79E-14	Reject	Accept	0.000187	Reject	Accept	0.000193	Reject	Accept
	0.40	0.38	4.01E-19	Reject	Accept	1.37E-06	Reject	Accept	1.44E-06	Reject	Accept
G	0.50	0.45	1.62E-19	Reject	Accept	0.000812	Reject	Accept	0.000785	Reject	Accept
	0.60	0.49	5.64E-17	Reject	Accept	4.11E-07	Reject	Accept	4.02E-07	Reject	Accept
	0.70	0.53	4.24E-17	Reject	Accept	0.002372	Reject	Accept	0.000926	Reject	Accept
	0.80	0.56	1.35E-17	Reject	Accept	0.000799	Reject	Accept	0.000808	Reject	Accept
	0.90	0.64	1.33E-17	Reject	Accept	6.46E-09	Reject	Accept	6.55E-09	Reject	Accept
	0.10	-0.15	8.13E-09	Reject	Accept	0.000338	Reject	Accept	0.000334	Reject	Accept
	0.20	0.13	1.08E-13	Reject	Accept	1.96E-09	Reject	Accept	2.02E-09	Reject	Accept
	0.30	0.29	1.39E-12	Reject	Accept	0.000155	Reject	Accept	0.000178	Reject	Accept
	0.40	0.38	5.88E-22	Reject	Accept	4.8E-06	Reject	Accept	4.5E-06	Reject	Accept
H	0.50	0.45	1.56E-22	Reject	Accept	3.72E-05	Reject	Accept	3.74E-05	Reject	Accept
	0.60	0.49	3.9E-20	Reject	Accept	0.000485	Reject	Accept	0.000479	Reject	Accept
	0.70	0.53	2.07E-19	Reject	Accept	0.000424	Reject	Accept	0.000431	Reject	Accept
	0.80	0.56	1.37E-19	Reject	Accept	2.92E-05	Reject	Accept	2.9E-05	Reject	Accept
	0.90	0.64	6.96E-19	Reject	Accept	3.85E-07	Reject	Accept	4.04E-07	Reject	Accept

Table B.1 P- Values Markovian Demand

B.2. P-values for Responses. AR(1) demand

		P-value	*Hypothesis*		*P-value*	*Hypothesis*		*P-value*	*Hypothesis*	
Exp	φ	*Cost*	*Ho*	*Ha*	*s*	*Ho*	*Ha*	*d*	*Ho*	*Ha*
	0.10	1.43E-10	Reject	Accept	0.846	Accept	Not Accepted	0.849	Accept	Not Accepted
	0.20	1.90E-15	Reject	Accept	0.668	Accept	Not Accepted	0.709	Accept	Not Accepted
	0.30	8.89E-19	Reject	Accept	0.610	Accept	Not Accepted	0.702	Accept	Not Accepted
	0.40	1.78E-21	Reject	Accept	0.317	Accept	Not Accepted	0.416	Accept	Not Accepted
A	0.50	6.36E-24	Reject	Accept	0.720	Accept	Not Accepted	0.942	Accept	Not Accepted
	0.60	5.87E-21	Reject	Accept	0.252	Accept	Not Accepted	0.466	Accept	Not Accepted
	0.70	1.09E-24	Reject	Accept	6.65E-01	Reject	Accept	2.83E-01	Reject	Accept
	0.80	3.38E-23	Reject	Accept	2.54E-02	Reject	Accept	1.12E-03	Reject	Accept
	0.90	3.71E-24	Reject	Accept	4.02E-02	Reject	Accept	1.08E-04	Reject	Accept
	0.10	4.80E-09	Reject	Accept	0.678	Accept	Not Accepted	0.674	Not Reject	Not Accepted
	0.20	1.34E-09	Reject	Accept	0.517	Accept	Not Accepted	0.495	Not Reject	Not Accepted
	0.30	3.05E-16	Reject	Accept	0.654	Accept	Not Accepted	0.720	Not Reject	Not Accepted
	0.40	2.00E-18	Reject	Accept	0.257	Accept	Not Accepted	0.356	Not Reject	Not Accepted
B	0.50	5.89E-25	Reject	Accept	0.281	Accept	Not Accepted	0.422	Not Reject	Not Accepted
	0.60	1.78E-20	Reject	Accept	0.232	Accept	Not Accepted	0.460	Not Reject	Not Accepted
	0.70	1.62E-21	Reject	Accept	4.50E-01	Reject	Accept	8.95E-01	Reject	Accept
	0.80	7.68E-23	Reject	Accept	1.25E-01	Reject	Accept	1.43E-02	Reject	Accept
	0.90	5.51E-20	Reject	Accept	5.34E-03	Reject	Accept	9.50E-06	Reject	Accept
	0.10	9.80E-08	Reject	Accept	0.189	Accept	Not Accepted	0.203	Accept	Not Accepted
	0.20	1.42E-13	Reject	Accept	0.515	Accept	Not Accepted	0.443	Accept	Not Accepted
	0.30	1.50E-15	Reject	Accept	0.930	Accept	Not Accepted	0.793	Accept	Not Accepted
C	0.40	3.63E-19	Reject	Accept	0.988	Accept	Not Accepted	0.727	Accept	Not Accepted
	0.50	2.10E-20	Reject	Accept	0.146	Accept	Not Accepted	0.443	Accept	Not Accepted
	0.60	3.41E-22	Reject	Accept	0.231	Accept	Not Accepted	0.230	Accept	Not Accepted
	0.70	5.36E-24	Reject	Accept	2.33E-01	Reject	Accept	7.82E-01	Reject	Accept
	0.80	7.42E-23	Reject	Accept	3.42E-03	Reject	Accept	4.61E-06	Reject	Accept
	0.90	1.85E-22	Reject	Accept	1.14E-02	Reject	Accept	6.96E-08	Reject	Accept
	0.10	2.95E-04	Reject	Accept	0.004	Accept	Not Accepted	0.004	Not Reject	Not Accepted
	0.20	4.12E-13	Reject	Accept	0.583	Accept	Not Accepted	0.499	Not Reject	Not Accepted
	0.30	7.46E-17	Reject	Accept	0.356	Accept	Not Accepted	0.252	Not Reject	Not Accepted
	0.40	4.25E-17	Reject	Accept	0.263	Accept	Not Accepted	0.981	Not Reject	Not Accepted
D	0.50	2.73E-18	Reject	Accept	0.220	Accept	Not Accepted	0.077	Not Reject	Not Accepted
	0.60	2.03E-19	Reject	Accept	0.883	Accept	Not Accepted	0.240	Not Reject	Not Accepted
	0.70	3.38E-25	Reject	Accept	0.044	Reject	Accept	0.001	Reject	Accept
	0.80	4.18E-24	Reject	Accept	2.30E-09	Reject	Accept	6.00E-12	Reject	Accept
	0.90	6.70E-22	Reject	Accept	2.50E-07	Reject	Accept	1.54E-12	Reject	Accept

Table B.2 P-Values AR(1) Demand

		P-value	*Hypothesis*		*P-value*	*Hypothesis*		*P-value*	*Hypothesis*	
Exp	***φ***	***Cost***	***Ho***	***Ha***	***s***	***Ho***	***Ha***	***d***	***Ho***	***Ha***
	0.10	1.44E-07	Reject	Accept	0.523	Accept	Not Accepted	0.546	Not Reject	Not Accepted
	0.20	3.43E-16	Reject	Accept	0.378	Accept	Not Accepted	0.388	Not Reject	Not Accepted
	0.30	3.60E-18	Reject	Accept	0.460	Accept	Not Accepted	0.503	Not Reject	Not Accepted
	0.40	1.33E-25	Reject	Accept	0.448	Accept	Not Accepted	0.410	Not Reject	Not Accepted
E	0.50	5.89E-25	Reject	Accept	0.108	Accept	Not Accepted	0.096	Not Reject	Not Accepted
	0.60	2.67E-25	Reject	Accept	0.398	Accept	Not Accepted	0.307	Not Reject	Not Accepted
	0.70	6.81E-27	Reject	Accept	1.69E-02	Reject	Accept	1.02E-02	Reject	Accept
	0.80	7.98E-29	Reject	Accept	2.15E-05	Reject	Accept	7.33E-06	Reject	Accept
	0.90	3.71E-24	Reject	Accept	9.26E-06	Reject	Accept	1.20E-06	Reject	Accept
	0.10	1.02E-07	Reject	Accept	0.330	Accept	Not Accepted	0.434	Accept	Not Accepted
	0.20	1.11E-12	Reject	Accept	0.611	Accept	Not Accepted	0.606	Accept	Not Accepted
	0.30	1.07E-15	Reject	Accept	0.156	Accept	Not Accepted	0.163	Accept	Not Accepted
	0.40	5.35E-19	Reject	Accept	0.850	Accept	Not Accepted	0.821	Accept	Not Accepted
F	0.50	3.15E-16	Reject	Accept	0.798	Accept	Not Accepted	0.774	Accept	Not Accepted
	0.60	1.06E-21	Reject	Accept	0.039	Accept	Not Accepted	0.037	Accept	Not Accepted
	0.70	2.85E-20	Reject	Accept	6.54E-02	Reject	Accept	6.18E-02	Reject	Accept
	0.80	3.45E-22	Reject	Accept	4.89E-05	Reject	Accept	4.16E-05	Reject	Accept
	0.90	4.66E-21	Reject	Accept	2.48E-05	Reject	Accept	8.15E-06	Reject	Accept
	0.10	7.05E-09	Reject	Accept	0.589	Accept	Not Accepted	0.583	Not Reject	Not Accepted
	0.20	1.07E-15	Reject	Accept	0.247	Accept	Not Accepted	0.223	Not Reject	Not Accepted
	0.30	1.83E-10	Reject	Accept	0.583	Accept	Not Accepted	0.529	Not Reject	Not Accepted
	0.40	6.43E-23	Reject	Accept	0.290	Accept	Not Accepted	0.405	Not Reject	Not Accepted
G	0.50	1.27E-24	Reject	Accept	0.897	Accept	Not Accepted	0.872	Not Reject	Not Accepted
	0.60	3.38E-27	Reject	Accept	0.919	Accept	Not Accepted	0.538	Not Reject	Not Accepted
	0.70	4.47E-24	Reject	Accept	9.57E-02	Reject	Accept	2.59E-02	Reject	Accept
	0.80	2.05E-27	Reject	Accept	4.05E-04	Reject	Accept	1.71E-05	Reject	Accept
	0.90	1.62E-09	Reject	Accept	1.15E-02	Reject	Accept	3.85E-04	Reject	Accept
	0.10	1.68E-06	Reject	Accept	0.049	Accept	Not Accepted	0.050	Accept	Not Accepted
	0.20	8.55E-16	Reject	Accept	0.111	Accept	Not Accepted	0.112	Accept	Not Accepted
	0.30	4.09E-19	Reject	Accept	0.718	Accept	Not Accepted	0.609	Accept	Not Accepted
	0.40	8.14E-24	Reject	Accept	0.527	Accept	Not Accepted	0.439	Accept	Not Accepted
H	0.50	4.65E-27	Reject	Accept	0.138	Accept	Not Accepted	0.261	Accept	Not Accepted
	0.60	4.41E-26	Reject	Accept	0.719	Accept	Not Accepted	0.385	Accept	Not Accepted
	0.70	2.07E-28	Reject	Accept	0.890	Accept	Not Accepted	0.393	Accept	Not Accepted
	0.80	3.42E-28	Reject	Accept	1.50E-03	Reject	Accept	4.47E-05	Reject	Accept
	0.90	2.68E-31	Reject	Accept	2.91E-03	Reject	Accept	2.85E-06	Reject	Accept

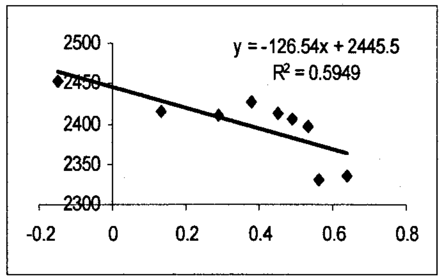
Table B.2 P-Values AR(1) Demand.

C MAIN EFFECTS AND TWO-WAY INTERACTIONS

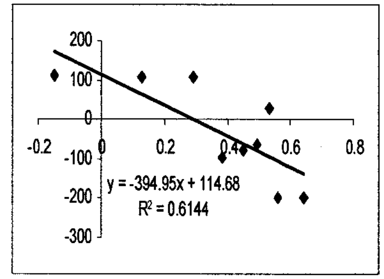
Appendix C.1 Main Effects and two-way interaction. Markovian demand

φ	CostDep	sDep	ddep
-0.149	2453.904	109.8	-110.35
0.13	2416.164	109.2	-111.75
0.29	2411.508	105.6	-104.6
0.38	2426.1695	-96.1	97.05
0.45	2414.0355	-75.75	77.3
0.49	2405.289	-66.2	68.3
0.53	2396.1805	27.7	-27.1
0.56	2330.039	-196.35	189.85
0.64	2336.3325	-197.45	201.05

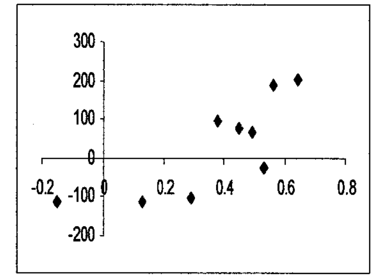
C.1.1 Main effect. Ordering



C.1.1. 1 Main effect. Ordering on Total costs



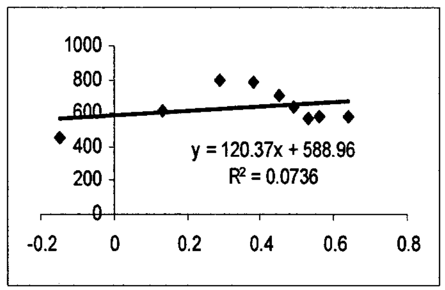
C.1.1. 2 Main effect. Ordering on Reorder



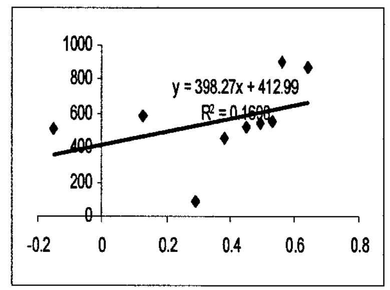
C.1.1. 3 Main effect. Ordg on order Qnt

φ	CostDep	sDep	ddep
-0.149	449.515	516	-16.85
0.13	613.904	592.2	-98.55
0.29	794.324	81.9	-74.6
0.38	780.6175	459.8	39.85
0.45	701.1045	527.15	-28.4
0.49	632.793	538.5	-40.4
0.53	572.6795	551.1	-52
0.56	576.422	901.35	-396.55
0.64	579.0485	871.55	-378.95

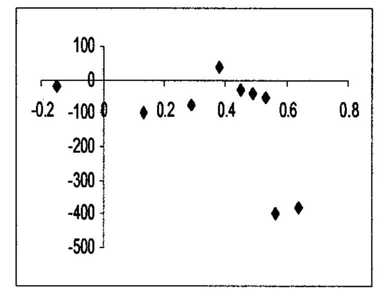
C.1.2 Main effect. Penalty



C.1.2. 4 Main effect. Penalty on Total costs



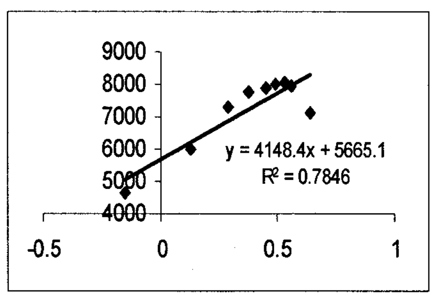
C.1.2. 5 Main effect. Penalty g on Reorder



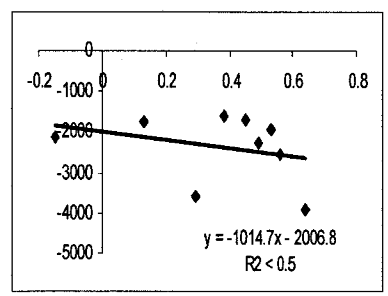
C.1.2. 6 Main effect. Penalty on order Qnt

φ	CostDep	sDep	ddep
-0.149	4646.7	-2105.6	97.4
0.13	6001.348	-1757.2	-271.8
0.29	7305.624	-3588.8	-393.6
0.38	7758.03	-1603.6	-404.2
0.45	7883.03	-1682.6	-327.2
0.49	7987.972	-2275.2	263.6
0.53	8084.414	-1954.8	-48
0.56	7955.176	-2564.6	584.2
0.64	7140.058	-3898.2	1890.2

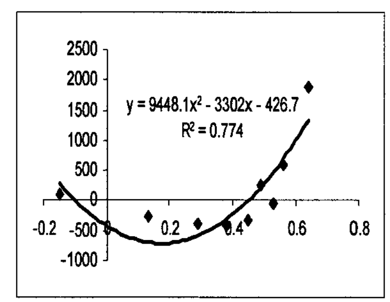
C.1. 3 Main effect. Holding



C.1.3.7 Main effect. holding on Total costs



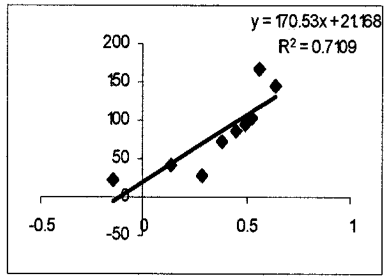
C.1.3.8 Main effect. holding on Reorder



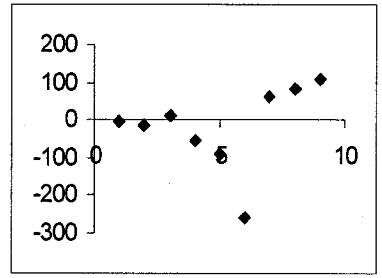
C.1.3.9 Main effect. holding on order Qnt

φ	CostDep	sDep	Ddep
-0.149	21.321	-3.9	4.15
0.13	41.815	-14.8	17.25
0.29	28.162	10.1	-9.7
0.38	72.8125	-55.1	54.95
0.45	85.0215	-90.75	90.6
0.49	93.699	-260.5	259.9
0.53	101.9545	63.7	-63.5
0.56	167.251	81.55	-90.95
0.64	144.8065	106.25	-109.85

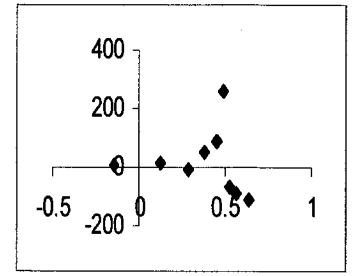
C.1.4 Two-way. Ordering and penalty 1X2



C.1.4.1 Two-way 1X2 on Total costs



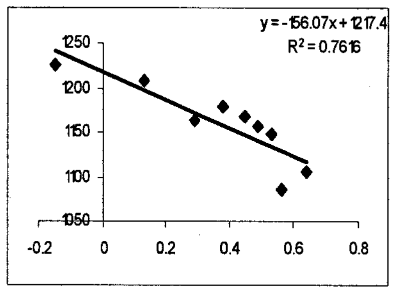
C.1.4.2 Two-way 1X2 on Reorder



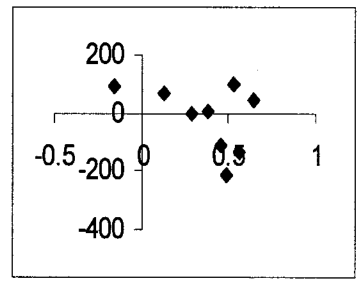
C.1.4.3 Two-way 1X2 on order Qnt

φ	CostDep	sDep	Ddep
-0.149	1226.325	88.5	-87.95
0.13	1207.967	69.7	-67.05
0.29	1162.44	-5.6	5.9
0.38	1177.9495	8.8	-8.95
0.45	1167.3095	-114.05	113.9
0.49	1156.837	-214	214.1
0.53	1147.7175	95.7	-95.7
0.56	1085.674	-134.35	125.05
0.64	1106.4305	45.45	-48.65

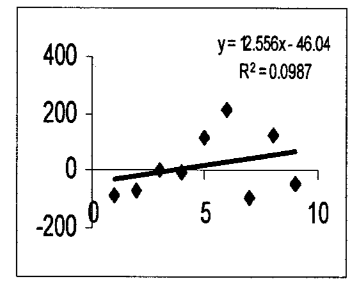
C.1.5 Two-way. Ordering and holding 1X3



C.1.5.1 Two-way 1X3 on Total costs



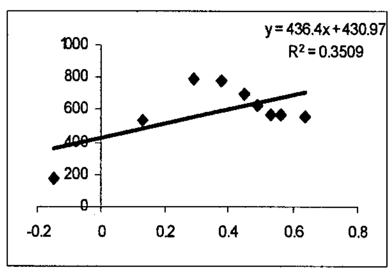
C.1.5.2 Two-way 1X3 on Reorder



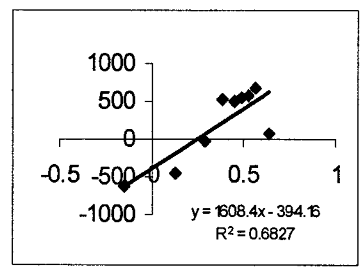
C.1.5.3 Two-way 1X3 on order Qnt

φ	CostDep	sDep	Ddep
-0.149	178.312	-631.3	134.65
0.13	534.246	-443.7	-46.15
0.29	793.334	-16.9	21.5
0.38	780.5445	533.1	-32.95
0.45	699.5115	489.15	10.3
0.49	632.653	541.1	-40.3
0.53	573.0855	571.3	-72.8
0.56	574.306	683.35	-174.75
0.64	562.0085	67.95	430.85

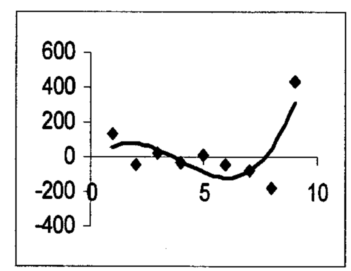
C.1.6 Two-way. Penalty and holding 2X3



C.1.6.1 Two-way 2X3 on Total costs



C.1.6.2 Two-way 2X3 on Reorder

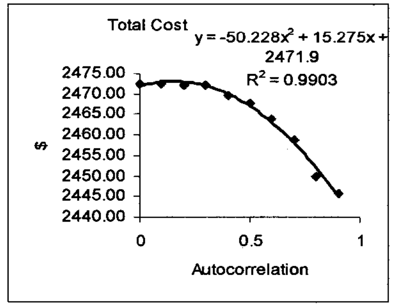


C.1.6.3 Two-way 2X3 on order Qnt

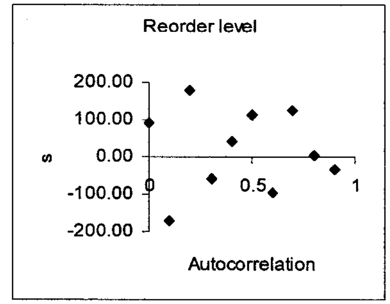
Appendix C.2 Main Effects and two-way interaction. AR(1) demands

φ	CostDep	s	d
0	2472.59	90.80	-104.80
0.1	2472.53	-170.70	154.10
0.2	2472.20	178.60	-197.40
0.3	2472.00	-57.50	37.50
0.4	2469.67	40.45	-60.70
0.5	2467.59	111.85	-131.30
0.6	2463.74	-97.75	81.55
0.7	2458.93	122.95	-152.35
0.8	2449.79	3.70	-45.90
0.9	2445.76	-32.40	21.60

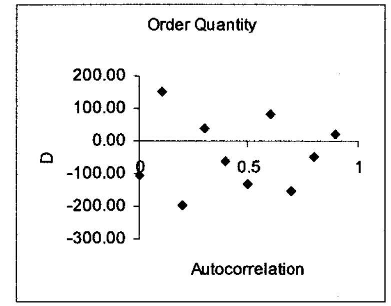
C.2.1 Main effect. Ordering



C.2.1. 1 Main effect. Ord. on Total costs



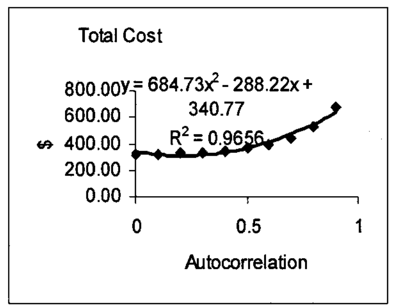
C.2.1. 2 Main effect. Ord. on Reorder



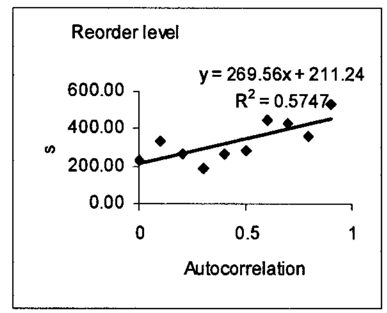
C.2.1. 3 Main effect. Ord. on order Qnt

φ	CostDep	s	d
0	319.98	229.00	21.60
0.1	321.26	337.00	-81.70
0.2	326.88	264.60	-3.90
0.3	334.00	187.00	77.70
0.4	348.69	268.65	2.20
0.5	368.49	279.05	14.50
0.6	398.74	443.15	-128.55
0.7	445.30	431.35	-72.85
0.8	527.10	357.60	80.70
0.9	671.71	528.00	37.30

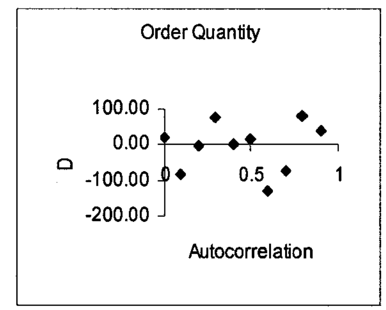
C.2.2 Main effect. Penalty



C.2.2.1 Main effect. Pen. on Total costs



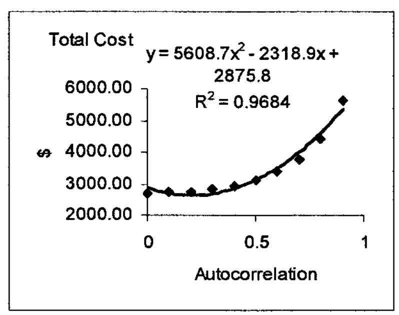
C.2.2.2 Main effect. Penalty g on Reorder



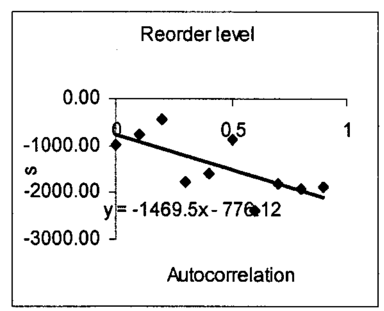
C.2.2.3 Main effect. Penalty on order Qnt

φ	CostDep	s	d
0	2709.43	-989.20	-102.80
0.1	2721.46	-744.40	-336.40
0.2	2764.23	-423.20	-686.00
0.3	2832.51	-1780.80	643.60
0.4	2953.24	-1591.40	408.80
0.5	3121.33	-857.40	-380.40
0.6	3375.93	-2374.60	1096.20
0.7	3774.25	-1814.20	317.80
0.8	4439.79	-1915.60	169.60
0.9	5615.01	-1883.20	-539.20

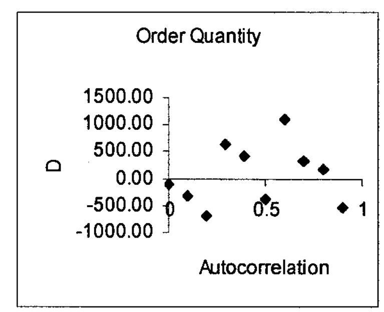
C.2.3 - Main effect. Holding



C.2.3.1 - Main effect. hold. on Total costs



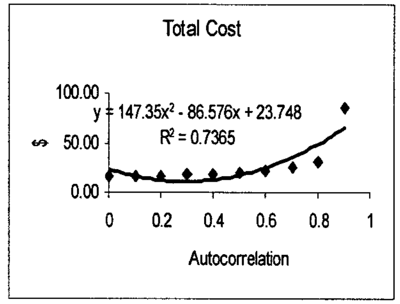
C.2.3.2 Main effect. holding on Reorder



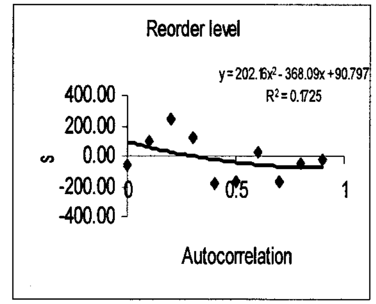
C.2.3.3 Main effect. holding on order Qnt

φ	CostDep	s	d
0	16.57	-57.90	66.60
0.1	16.67	97.80	-90.60
0.2	16.91	245.30	-234.90
0.3	18.14	116.20	-98.90
0.4	18.48	-181.55	188.30
0.5	19.79	-169.15	185.20
0.6	21.67	21.15	-10.75
0.7	24.55	-172.25	198.65
0.8	30.01	-49.80	86.90
0.9	85.03	-22.10	63.50

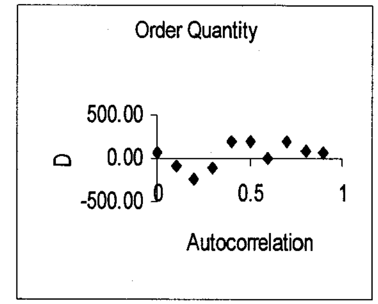
C.2.4 Two-way. Ordering and penalty 1X2



C.2.4.1 -Two-way 1X2 on Total costs



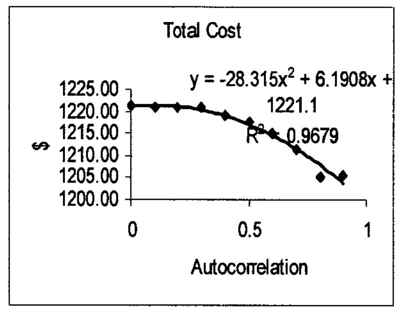
C.2.4.2 Two-way 1X2 on Reorder



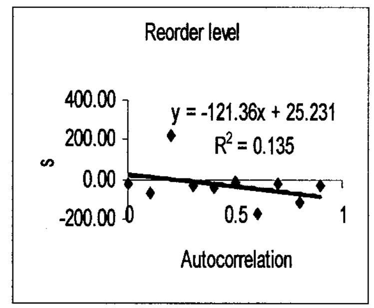
C.2.4.3 Two-way 1X2 on order Qnt

φ	CostDep	s	d
0	1221.20	-20.10	3.20
0.1	1221.09	-68.80	47.90
0.2	1220.86	218.80	-233.70
0.3	1220.77	-35.50	19.00
0.4	1219.06	-42.05	24.10
0.5	1217.74	-10.85	-3.70
0.6	1215.13	-167.85	149.25
0.7	1211.47	-19.25	3.65
0.8	1205.17	-112.50	97.60
0.9	1205.46	-35.70	27.80

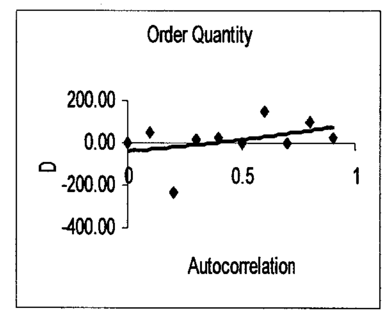
C.2.5 Two-way. Ordering and holding 1X3



C.2.5.1 -Two-way 1X3 on Total costs



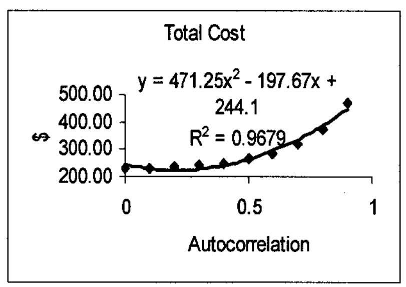
C.2.5.2 Two-way 1X3 on Reorder



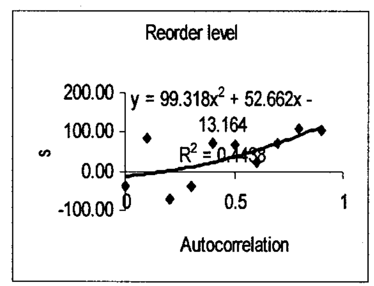
C.2.5.3 Two-way 1X3 on order Qnt

φ	CostDep	s	d
0	229.97	-38.40	78.30
0.1	230.92	86.20	-51.20
0.2	234.41	-71.90	108.00
0.3	239.49	-36.90	82.70
0.4	249.66	70.75	-33.60
0.5	263.44	69.45	-24.00
0.6	283.70	23.25	18.15
0.7	317.47	73.65	-30.95
0.8	374.11	108.00	-64.00
0.9	471.35	104.30	-42.50

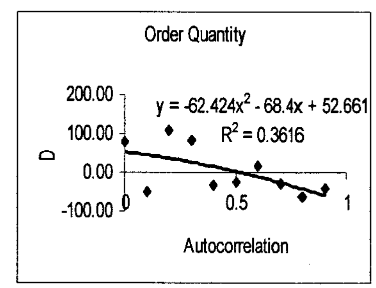
C.2.6 Two-way. Penalty and holding 2X3



C.2.6.1 -Two-way 2X3 on Total costs



C.2.6.2 Two-way 2X3 on Reorder



C.2.6.3 Two-way 2X3 on order Qnt

D CHARACTERIZATION OF THE BEHAVIOR OF MAIN EFFECTS AND TWO-WAY INTERACTIONS AS THE AUTOCORRELATION INCREASES

D.1 Markovian Case

Table C.1.1 – C.1.6 and charts C. 1.1.1 – C.1.6.3 illustrate how the main effects and the interaction between factors behaves as the autocorrelation increases. The behaviors are described by the equations presented in tables D.1 and D.2.

Factor	Effect	EQUATION / TREND	R^2
Cost	1	y = -126.54x + 2445.5	0.5949
	2	Increasing	Too chaotic
	3	y = 4148.4x + 5665.1	0.7846
s	1	y = -37.859x + 158.23	0.6929
	2	Increasing	<0.5
	3	Decreasing	<0.5
d	1	y = 37.944x - 158.64	0.6957
	2	Decreasing	<0.5
	3	y = 9448.1x2 - 3302x - 426.7	0.774

Table D. 1 - Equations and R^2 of the main effects MC

Factor	Interaction	EQUATION / TREND	R^2
Cost	1x2	y = 170.53x + 21.168	0.7109
	1x3	y = -156.07x + 1217.4	0.7616
	2x3	Increasing	Too chaotic
s	1x2	Diffuse	
	1x3	Diffuse	
	2x3	y = 1608.4x - 394.16	0.6827
d	1x2	Diffuse	
	1x3	Diffuse	
	2x3	y = 1608.4x - 394.16	0.6827

Table D. 2 - Equations and R^2 of the two-way interaction

D.2. AR(1) Case

Table C.1.1 – C.1.6 and charts C. 1.1.1 – C.1.6.3 show the behavior of main effects and two-way interaction for the AR(1) case. The behaviors are described by the equations presented in tables D.3 and D.4.

Factor	Effect	EQUATION / TREND	R^2
Cost	1	y = -50.228x2 + 15.275x + 2471.9	0.9903
	2	y = 684.73x2 - 288.22x + 340.77	0.9656
	3	y = 5608.7x2 - 2318.9x + 2875.8	0.9684
s	1	Diffuse	
	2	y = 269.56x + 211.24	0.5747
	3	y = -1469.5x - 776.12	0.5
d	1	Diffuse	
	2	Diffuse	
	3	Diffuse	

Table D. 3 - Equations and R^2 of the main effects

Factor	Interaction	EQUATION / TREND	R^2
Cost	1X2	y = 147.35x2 - 86.576x + 23.748	0.7365
	1X3	y = -28.315x2 + 6.1908x + 1221.1	0.9679
	2X3	y = 471.25x2 - 197.67x + 244.1	0.9679
s	1X2	Decreasing	<.5
	1X3	Decreasing	<.5
	2X3	Increasing	<.5
d	1X2	Diffuse	
	1X3	Increasing	<.5
	2X3	Decreasing	<.5

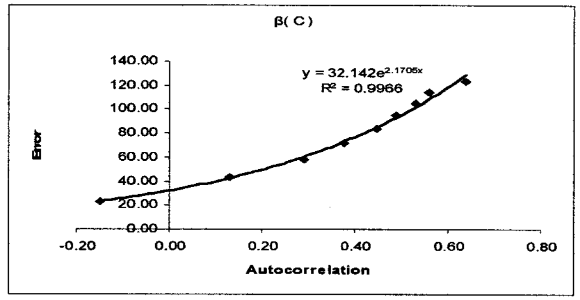
Table D. 4 - Equations and R^2 of the two-way interactions

E ERROR CHARACTERIZATION FOR MARKOVIAN-MODULATED AND AR(1) CASES

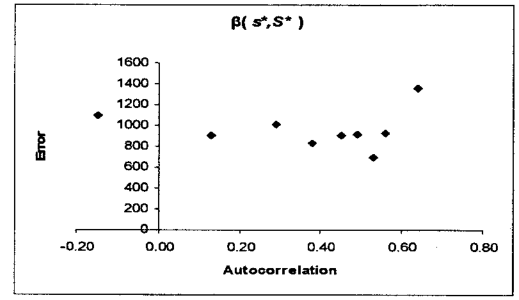
Appendix E.1. Errors – Markovian-modulated case

P01	ϕ	β(C)	β(s*,S*)
0.10	-0.15	23.37	1090
0.20	0.13	43.24	907
0.30	0.29	58.26	1006
0.40	0.38	71.98	832
0.50	0.45	83.45	903
0.60	0.49	94.94	916
0.70	0.53	104.69	698
0.80	0.56	113.85	929
0.90	0.64	123.57	1355

E.1.1 Errors generated in experiment A



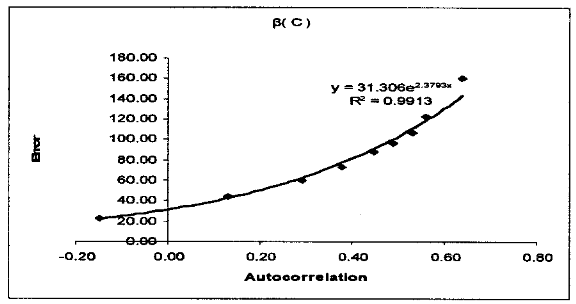
E.1.1.1 Errors generated in estimating costs. Experiment A



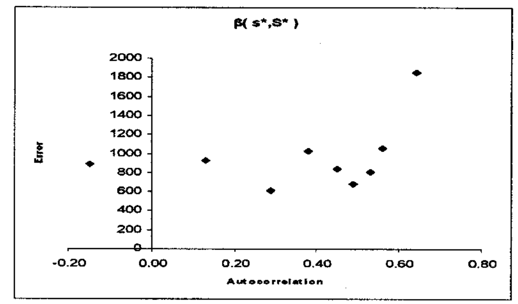
E.1.1.2 Errors generated in policy. Experiment A

P01	ϕ	β(C)	β(s*,S*)
0.10	-0.15	22.86	884
0.20	0.13	43.60	922
0.30	0.29	59.79	613
0.40	0.38	73.13	1025
0.50	0.45	88.25	836
0.60	0.49	96.85	681
0.70	0.53	106.51	806
0.80	0.56	122.59	1052
0.90	0.64	160.44	1856

E.1.2 Errors generated in experiment B



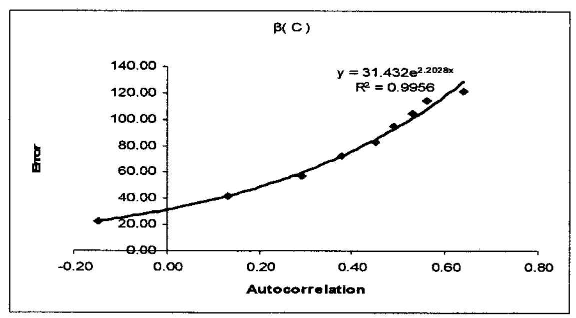
E.1.2.1 Errors generated in estimating costs. Experiment B



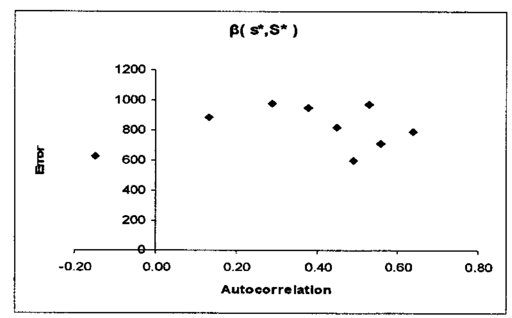
E.1.2.2 Errors generated in policy. Experiment B

P01	ϕ	β(C)	β(s*,S*)
0.10	-0.15	22.86	629
0.20	0.13	41.86	883
0.30	0.29	57.15	976
0.40	0.38	72.05	951
0.50	0.45	83.21	825
0.60	0.49	94.80	598
0.70	0.53	104.20	972
0.80	0.56	114.47	716
0.90	0.64	121.45	790

E.1.3 Errors generated in experiment C



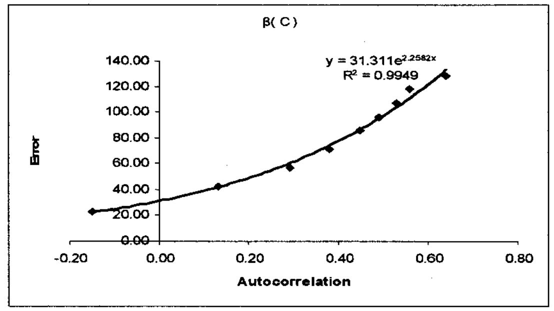
E.1.3.1 Errors generated in estimating costs. Experiment C



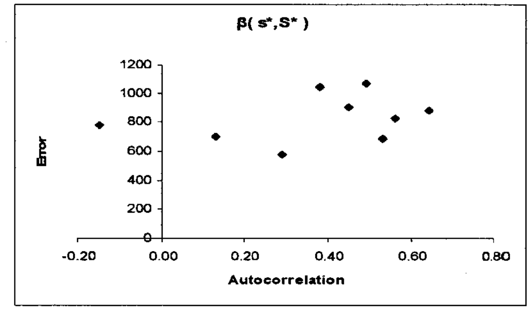
E.1.3.2 Errors generated in policy. Experiment C

P01	*ϕ*	*β(C)*	*β(s*,S*)*
0.10	-0.15	22.86	776
0.20	0.13	42.47	701
0.30	0.29	56.43	577
0.40	0.38	71.42	1047
0.50	0.45	85.51	901
0.60	0.49	96.18	1064
0.70	0.53	107.22	685
0.80	0.56	118.08	829
0.90	0.64	128.33	877

E.1.4 Errors generated in experiment D



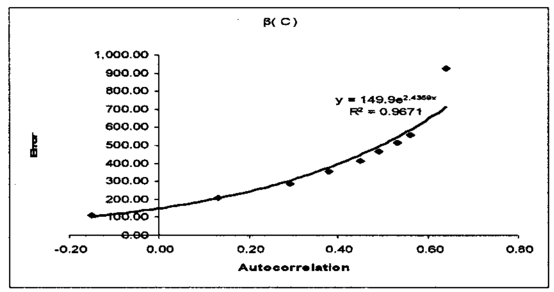
E.1.4.1 Errors generated in estimating costs. Experiment D



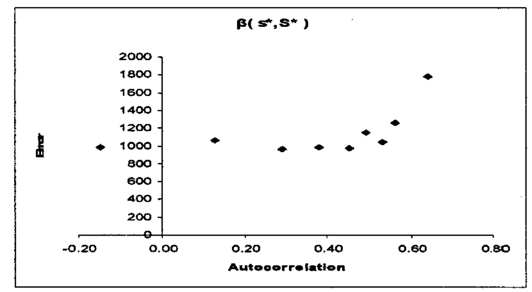
E.1.4.2 Errors generated in policy. Experiment D

P01	*ϕ*	*β(C)*	*β(s*,S*)*
0.10	-0.15	113.44	985
0.20	0.13	206.85	1061
0.30	0.29	287.07	963
0.40	0.38	355.05	989
0.50	0.45	414.20	975
0.60	0.49	468.44	1150
0.70	0.53	516.78	1042
0.80	0.56	559.48	1265
0.90	0.64	928.59	1785

E.1.5 Errors generated in experiment E



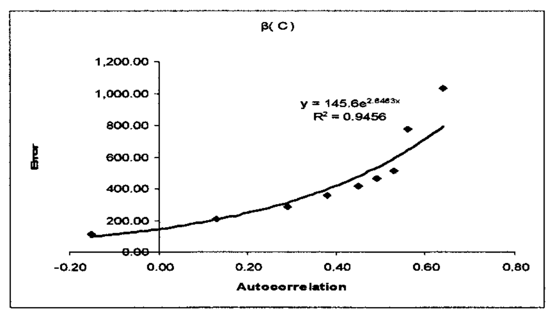
E.1.5.1 Errors generated in estimating costs. Experiment E



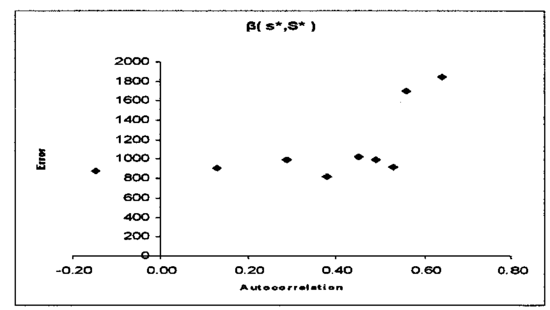
E.1.5.2 Errors generated in policy. Experiment E

P01	*ϕ*	*β(C)*	*β(s*,S*)*
0.10	-0.15	111.17	875
0.20	0.13	206.87	906
0.30	0.29	287.44	994
0.40	0.38	359.44	821
0.50	0.45	417.96	1022
0.60	0.49	468.09	992
0.70	0.53	515.29	915
0.80	0.56	779.08	1697
0.90	0.64	1,033.29	1849

E.1.6 Errors generated in experiment F



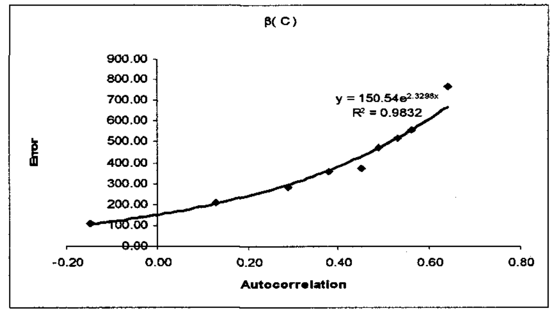
E.1.6.1 Errors generated in estimating costs. Experiment F



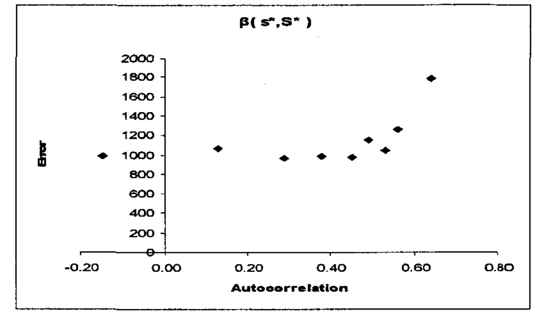
E.1.6.2 Errors generated in policy. Experiment F

P01	ϕ	*β(C)*	*β(s*,S*)*
0.10	-0.15	110.84	1207
0.20	0.13	211.09	882
0.30	0.29	282.22	893
0.40	0.38	355.81	763
0.50	0.45	372.22	395
0.60	0.49	470.89	802
0.70	0.53	516.20	679
0.80	0.56	557.37	670
0.90	0.64	768.00	1608

E.1.7 Errors generated in experiment G



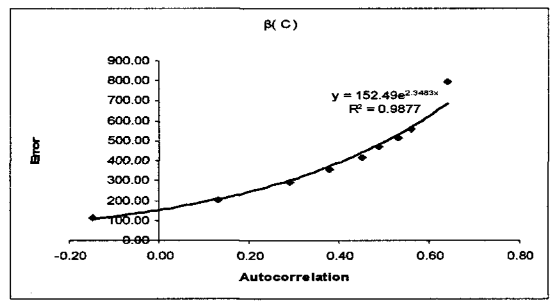
E.1.7.1 Errors generated in estimating costs. Experiment G



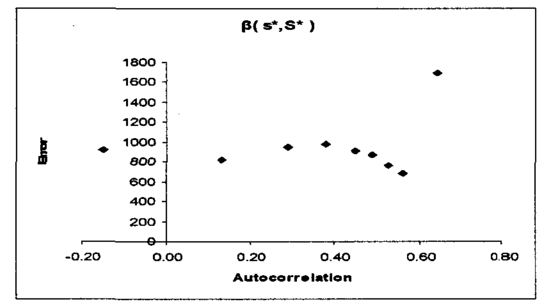
E.1.7.2 Errors generated in policy. Experiment G

P01	ϕ	*β(C)*	*β(s*,S*)*
0.10	-0.15	113.93	923
0.20	0.13	204.60	817
0.30	0.29	291.83	951
0.40	0.38	357.02	975
0.50	0.45	416.89	909
0.60	0.49	469.08	873
0.70	0.53	515.66	765
0.80	0.56	556.61	688
0.90	0.64	797.33	1696

E.1.8 Errors generated in experiment H



E.1.8.1 Errors generated in estimating costs. Experiment H

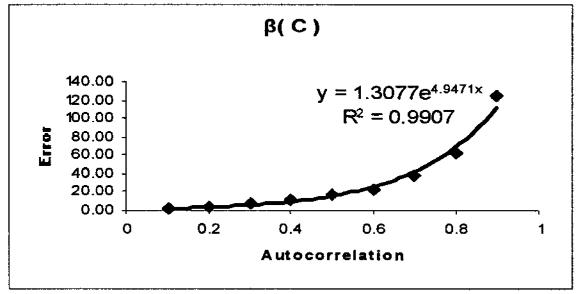


E.1.8.. Errors generated in policy. Experiment H

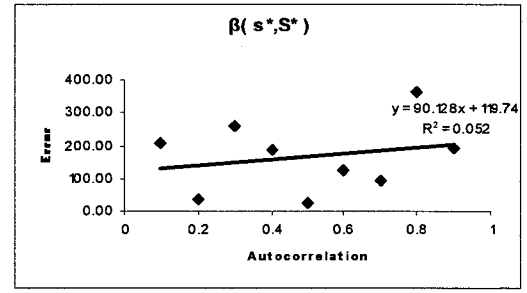
Appendix E.2. Errors – AR(1) demand case

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
0	0.00	0.00
0.1	1.79	205.39
0.2	3.50	36.01
0.3	7.28	258.29
0.4	10.42	184.78
0.5	16.35	25.36
0.6	22.92	122.27
0.7	37.98	95.73
0.8	62.14	363.59
0.9	124.16	191.81

E.2.1 Errors generated in experiment A



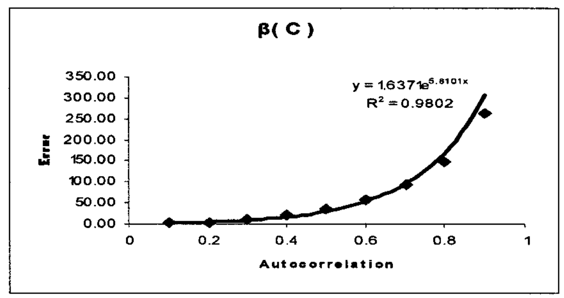
E.2.1.1 Errors generated in estimating costs. Experiment A



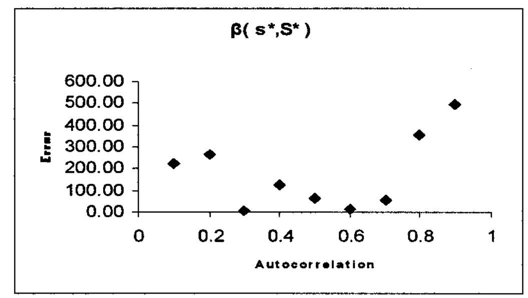
E.2.1.2 Errors generated in policy. Experiment A

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	1.88	220.19
0.2	5.19	265.75
0.3	12.23	9.88
0.4	21.43	127.61
0.5	36.25	60.35
0.6	56.68	11.01
0.7	92.37	52.49
0.8	148.81	357.89
0.9	264.36	493.67

E.2.2 Errors generated in experiment B



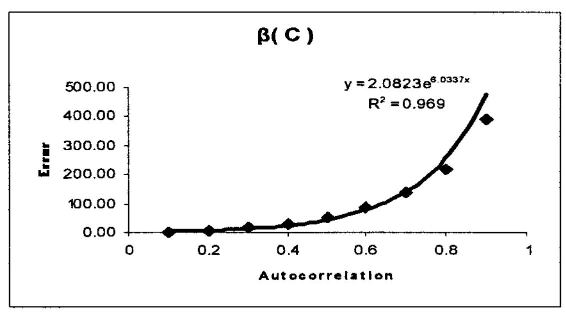
E.2.2.1 Errors generated in estimating costs. Experiment B



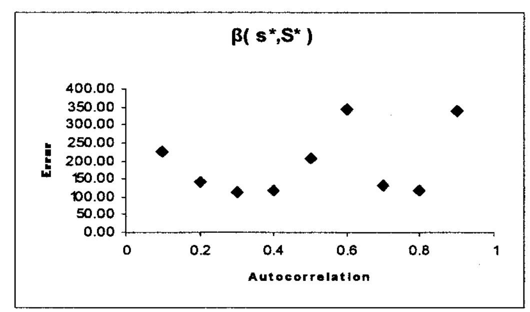
E.2.2.2 Errors generated in policy. Experiment B

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	2.06	226.79
0.2	7.60	142.00
0.3	17.16	113.11
0.4	31.57	117.02
0.5	53.66	207.58
0.6	85.64	345.13
0.7	135.85	129.96
0.8	221.01	119.81
0.9	388.82	338.90

E.2.3 Errors generated in experiment C



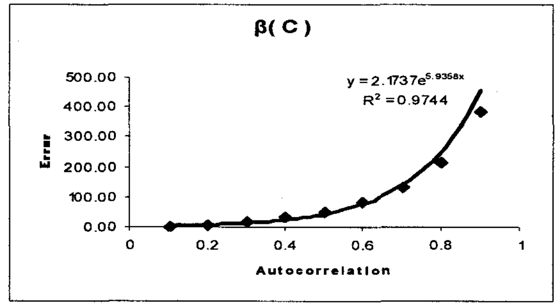
E.2.3.1 Errors generated in estimating costs. Experiment C



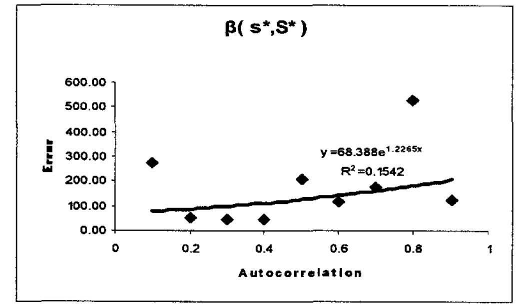
E.2.3.2 Errors generated in policy. Experiment C

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	2.27	274.79
0.2	7.79	50.40
0.3	16.62	47.16
0.4	30.87	46.00
0.5	52.00	206.35
0.6	83.36	115.20
0.7	132.39	178.20
0.8	216.12	527.44
0.9	382.76	121.57

E.2.4 Errors generated in experiment D



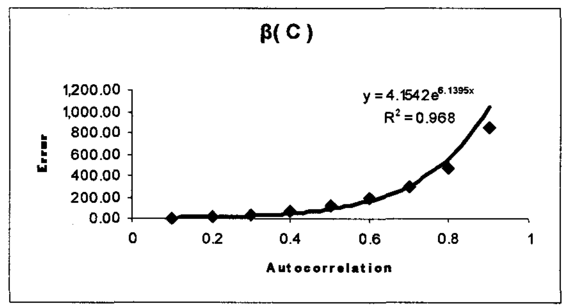
E.2.4.1 Errors generated in estimating costs. Experiment D



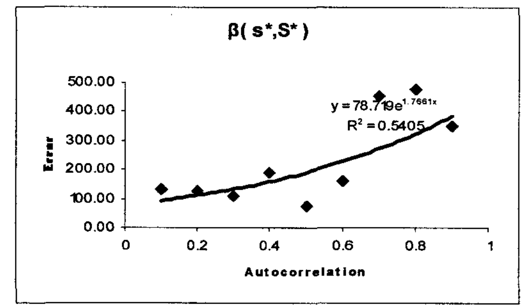
E.2.4.2 Errors generated in policy. Experiment D

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	4.11	133.54
0.2	15.17	123.67
0.3	35.64	110.18
0.4	67.51	192.08
0.5	113.97	75.66
0.6	183.53	162.77
0.7	292.71	454.74
0.8	473.50	474.58
0.9	844.66	353.35

E.2.5 Errors generated in experiment E



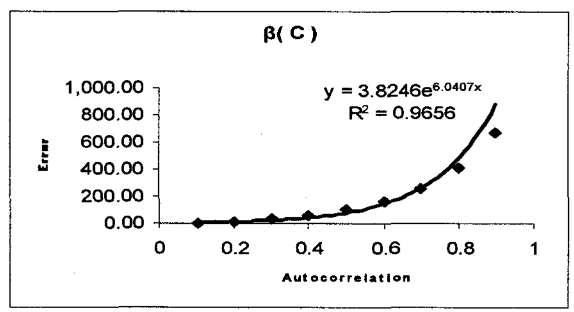
E.2.5.1 Errors generated in estimating costs. Experiment E



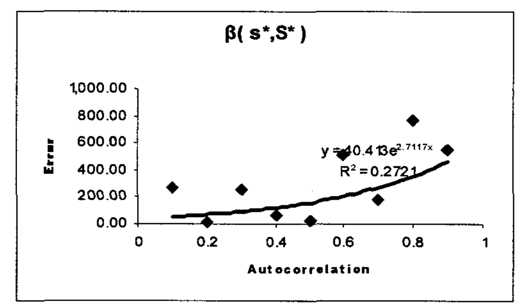
E.2.5.2 Errors generated in policy. Experiment E

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	3.68	275.18
0.2	13.81	13.22
0.3	31.64	261.29
0.4	59.41	59.41
0.5	100.61	27.12
0.6	161.19	513.71
0.7	257.29	173.64
0.8	414.56	766.47
0.9	676.25	547.34

E.2.6 Errors generated in experiment F



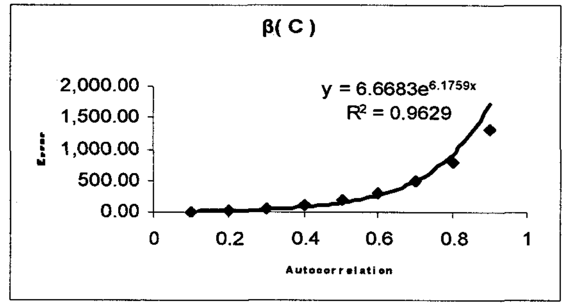
E.2.6.1 Errors generated in estimating costs. Experiment F



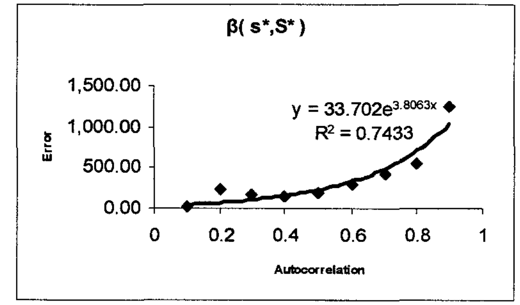
E.2.6.2 Errors generated in policy. Experiment F

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	6.19	15.62
0.2	26.00	230.54
0.3	55.94	162.86
0.4	112.53	150.60
0.5	190.24	189.37
0.6	307.46	305.69
0.7	491.78	432.54
0.8	802.22	558.16
0.9	1,308.91	1,247.88

E.2.7 Errors generated in experiment G



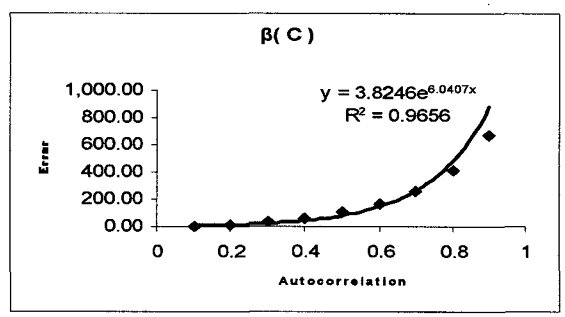
E.2.7.1 Errors generated in estimating costs. Experiment G



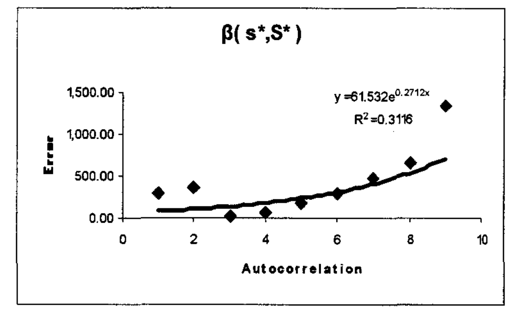
E.2.7.2 Errors generated in estimating policy. Experiment G

ϕ	$\beta(C)$	$\beta(s^*,S^*)$
IID	0.00	0.00
0.1	6.05	306.74
0.2	25.69	355.87
0.3	58.43	15.99
0.4	111.20	59.10
0.5	188.32	186.44
0.6	302.24	299.77
0.7	483.87	483.73
0.8	788.39	669.95
0.9	1,398.23	1,351.41

E.2.8 Errors generated in experiment H



E.2.8.. Errors generated in estimating costs. Experiment H



E.2.8.2 Errors generated in estimating policy. Experiment H

F Pairwise Comparison of Errors Generated Between Correlated and Correlation-free for Markovian-modulated and AR(1) Cases

F.1. Pairwise comparison of errors generated in estimating total costs considering Markov-modulated demands

		Experiment							
Ref	P_01	A	B	C	D	E	F	G	H
0.1	0.2	0.242676	7.6E-06	0.401112	0.217905	5.2E-131	1.4E-92	2.06E-75	2E-87
	0.3	0.023658	3.59E-10	0.038607	0.011487	4E-143	2.3E-169	1.14E-91	1.2E-102
	0.4	0.000648	2.83E-13	0.001474	0.000133	1.1E-155	1.4E-179	4.9E-108	2.1E-118
	0.5	2.19E-06	2.66E-16	6.14E-06	4.64E-07	1.5E-168	1.2E-189	1.1E-138	2.5E-134
	0.6	1.8E-09	4.16E-22	7.74E-09	4.29E-11	2.6E-181	1.7E-200	3.2E-143	1.1E-150
	0.7	7.37E-14	1.29E-27	1.62E-13	3.38E-16	1.3E-194	1.1E-212	6.4E-162	1.6E-166
	0.8	2.32E-19	1.34E-34	4.48E-19	1.56E-21	9.7E-209	3.9E-225	7.6E-178	3.3E-185
	0.9	2.01E-27	8.7E-44	9.55E-27	1.14E-29	1.8E-223	1.7E-238	1.7E-197	3.6E-202
0.2	0.3	0.27052	0.053585	0.2171	0.191568	4.96E-07	3.74E-96	1.21E-06	1.37E-06
	0.4	0.023483	0.002116	0.018452	0.008786	1.28E-23	9.8E-113	8.91E-22	3.35E-22
	0.5	0.000297	4.61E-05	0.000201	0.000109	3.21E-47	1.3E-128	1.05E-64	9.63E-45
	0.6	8.08E-07	7.42E-09	5.88E-07	4.46E-08	7.52E-73	3.1E-145	1.2E-71	7.91E-71
	0.7	1.12E-10	5.21E-13	3.19E-11	1.26E-12	2.6E-99	2.6E-163	2.7E-100	1.6E-96
	0.8	9.92E-16	7.59E-19	1.87E-16	1.44E-17	4.7E-126	4.9E-181	4.3E-124	7.3E-126
	0.9	2.16E-23	3.15E-27	7.5E-24	2.42E-25	3.9E-152	2E-199	2.9E-152	2.8E-151
0.3	0.4	0.241517	0.24559	0.258198	0.184812	1.54E-08	3.19E-08	1.04E-07	4.69E-08
	0.5	0.01103	0.028634	0.011968	0.009401	1.97E-28	3.21E-26	1.22E-46	7.98E-27
	0.6	0.000102	7.43E-05	0.000133	2.21E-05	1.64E-54	6.54E-52	6.4E-54	3.78E-53
	0.7	5.2E-08	4.33E-08	3.49E-08	3.08E-09	4.13E-83	2.27E-82	8.53E-85	9.05E-81
	0.8	1.48E-12	4.83E-13	7.47E-13	1.14E-13	2.2E-112	7.1E-112	9.2E-111	9.2E-113
	0.9	9.62E-20	1.18E-20	9.42E-20	6.17E-21	8.4E-141	5.1E-141	2.3E-141	2.3E-140
0.4	0.5	0.167221	0.301051	0.163715	0.19968	2.88E-10	4.75E-09	8.83E-27	1.2E-09
	0.6	0.006033	0.004591	0.006528	0.003091	4.28E-33	4.92E-31	7.9E-34	1.64E-32
	0.7	1.41E-05	1.15E-05	8.43E-06	2.69E-06	4.39E-63	9.05E-63	3.28E-66	2.67E-61
	0.8	1.66E-09	5.79E-10	7.17E-10	4.17E-10	2.43E-95	3.35E-95	1.12E-94	2.09E-96
	0.9	4.45E-16	5.81E-17	3.29E-16	1.02E-16	8.9E-127	2.3E-127	2.6E-128	8.1E-127
0.5	0.6	0.168106	0.069677	0.180096	0.090515	8.32E-12	1.34E-11	0.04906	4.88E-12
	0.7	0.002632	0.000696	0.001867	0.000532	3.84E-39	1.21E-40	3.4E-23	3.17E-38
	0.8	2.08E-06	1.56E-07	1.09E-06	4.06E-07	4.66E-74	1.35E-75	3.01E-54	4.33E-76
	0.9	4.16E-12	6.9E-14	3.4E-12	5.84E-13	2.8E-109	2.5E-111	2.92E-95	5E-110
0.6	0.7	0.099549	0.109029	0.073548	0.0721	8.61E-15	3.12E-16	8.12E-17	8.04E-14
	0.8	0.000616	0.000438	0.000327	0.000563	1.62E-48	1.85E-50	5.69E-47	2.16E-50
	0.9	1.29E-08	4.34E-09	8.84E-09	1.33E-08	9.08E-88	2.5E-90	4.58E-89	2.43E-88
0.7	0.8	0.071309	0.052155	0.066647	0.093559	2.56E-19	1.87E-19	6.43E-16	4.47E-22
	0.9	3.51E-05	1.24E-05	4.75E-05	6.73E-05	1E-59	7.09E-61	9.55E-59	1.4E-61
0.8	0.9	0.017329	0.013019	0.022793	0.018821	1.37E-24	1.54E-25	1.91E-27	1.81E-23

F. 1 - P-values for Error in costs per Experiment and autocorrelation factor MC

F.2. Pairwise comparison of errors generated in estimating inventory policy considering Markov-modulated demands

		Experiment							
Ref	P_01	A	B	C	D	E	F	G	H
0.1	0.2	0.021658	1.87E-05	0.68816	0.796806	0.005171	0.409411	5.27E-07	1.02E-07
0.1	0.3	0.000436	3.17E-08	0.32519	0.299867	7.38E-05	7.5E-07	8.58E-07	8.19E-07
0.1	0.4	0.018047	7.67E-10	0.298386	0.311597	0.000669	5.29E-06	1.77E-05	1.19E-05
0.1	0.5	0.014895	7.39E-08	0.852337	0.894194	1.62E-05	1.08E-05	8.45E-06	2.73E-05
0.1	0.6	0.004986	9.86E-06	0.383444	0.356937	2.25E-05	5.99E-08	7.14E-06	0.000117
0.1	0.7	0.059942	3.27E-10	0.314695	0.105792	1.23E-05	5.5E-06	0.000134	7E-05
0.1	0.8	0.015797	7.52E-07	0.614156	0.341949	0.000111	6.02E-07	0.000106	3.07E-06
0.1	0.9	0.152384	2.75E-07	0.3823	0.585639	2.06E-05	2.61E-07	0.030801	3.73E-05
0.2	0.3	0.21242	0.183287	0.166426	0.435522	0.229468	3.1E-05	0.9198	0.67466
0.2	0.4	0.944748	0.045156	0.522665	0.20462	0.534683	0.000167	0.443831	0.316266
0.2	0.5	0.888157	0.242381	0.556951	0.696205	0.117949	0.000308	0.550961	0.231995
0.2	0.6	0.60285	0.883213	0.203457	0.238807	0.136986	3.38E-06	0.576686	0.121012
0.2	0.7	0.674634	0.031582	0.159933	0.17339	0.103769	0.000173	0.208112	0.154905
0.2	0.8	0.905297	0.480868	0.365452	0.488134	0.271188	2.56E-05	0.230739	0.483411
0.2	0.9	0.382886	0.362717	0.636395	0.773509	0.131404	1.23E-05	0.003312	0.204237
0.3	0.4	0.238836	0.498293	0.043551	0.041049	0.560574	0.675422	0.505931	0.559868
0.3	0.5	0.268377	0.871066	0.424817	0.242283	0.716243	0.562019	0.620098	0.437461
0.3	0.6	0.466698	0.236252	0.910402	0.050809	0.774335	0.613458	0.64723	0.257343
0.3	0.7	0.096019	0.409323	0.982747	0.559742	0.669167	0.669096	0.246673	0.315275
0.3	0.8	0.25915	0.530445	0.631	0.930974	0.918954	0.963798	0.272154	0.778495
0.3	0.9	0.034553	0.673147	0.063829	0.622523	0.757847	0.829564	0.00453	0.394773
0.4	0.5	0.943131	0.401333	0.220417	0.379301	0.344684	0.871913	0.865308	0.846428
0.4	0.6	0.651957	0.06321	0.056523	0.927589	0.385219	0.355823	0.835433	0.581905
0.4	0.7	0.624797	0.882246	0.041372	0.00884	0.313154	0.993078	0.621113	0.673156
0.4	0.8	0.960385	0.192553	0.123086	0.050311	0.631006	0.642588	0.664467	0.762739
0.4	0.9	0.346323	0.27212	0.867662	0.119991	0.373529	0.526192	0.028951	0.788467
0.5	0.6	0.704137	0.30649	0.492889	0.430422	0.938786	0.278293	0.969591	0.721073
0.5	0.7	0.575243	0.32362	0.412398	0.080218	0.949102	0.87875	0.506851	0.819497
0.5	0.8	0.982715	0.641827	0.750417	0.278824	0.641795	0.531847	0.546336	0.620255
0.5	0.9	0.311106	0.795192	0.289438	0.497748	0.955987	0.426703	0.018697	0.940506
0.6	0.7	0.347439	0.045027	0.893279	0.011443	0.888164	0.351332	0.482789	0.897468
0.6	0.8	0.688127	0.576735	0.712992	0.061861	0.697697	0.645672	0.5213	0.394147
0.6	0.9	0.164149	0.445125	0.081546	0.143065	0.982763	0.771843	0.016891	0.777629
0.7	0.8	0.590095	0.146931	0.615716	0.503014	0.596817	0.636389	0.951716	0.469393
0.7	0.9	0.650385	0.212842	0.06082	0.28251	0.905257	0.520551	0.090147	0.877953
0.8	0.9	0.321541	0.837075	0.168748	0.684961	0.681786	0.865107	0.07931	0.568632

F. 2 P-values for Error in inventory policy per Experiment and autocorrelation factor MC

F.3. Pairwise comparison of errors generated in estimating total costs considering AR(1) demands

		Experiment							
Ref	ϕ	A	B	C	D	E	F	G	H
IID	0.1	0.931304	0.927706	0.920897	0.912796	0.843177	0.859183	0.765565	0.770789
IID	0.2	0.798992	0.802639	0.698999	0.707429	0.465108	0.506025	0.211113	0.216559
IID	0.3	0.545247	0.556107	0.408821	0.423709	0.086795	0.12826	0.007386	0.005162
IID	0.4	0.268883	0.302496	0.12908	0.137779	0.00126	0.004466	1.15E-07	1.6E-07
IID	0.5	0.058904	0.081574	0.010147	0.012692	8.07E-08	1.94E-06	5.96E-18	1.18E-17
IID	0.6	0.00291	0.006649	4.68E-05	7.33E-05	6.26E-17	1.09E-13	3.43E-38	3.15E-37
IID	0.7	2.11E-06	1.18E-05	2.33E-10	6.17E-10	1.77E-35	4.21E-29	2.05E-72	5.52E-71
IID	0.8	7.45E-14	5.15E-12	7.03E-23	4.52E-22	4.21E-69	3.17E-58	1.3E-122	1.2E-120
IID	0.9	2.38E-34	2.35E-30	2.08E-53	2.85E-52	1.4E-128	1.1E-103	1.4E-182	4E-191
0.1	0.2	0.866224	0.873503	0.77381	0.790375	0.594165	0.625719	0.340508	0.344565
0.1	0.3	0.60393	0.618509	0.467345	0.489796	0.129539	0.178808	0.017071	0.012068
0.1	0.4	0.307889	0.346925	0.155917	0.169094	0.002433	0.007609	5.15E-07	6.81E-07
0.1	0.5	0.071327	0.098635	0.013401	0.017111	2.22E-07	4.41E-06	5.23E-17	9.7E-17
0.1	0.6	0.003821	0.008672	7.03E-05	0.000114	2.58E-16	3.5E-13	4.76E-37	4.08E-36
0.1	0.7	3.16E-06	1.75E-05	4.18E-10	1.16E-09	9.97E-35	1.87E-28	2.69E-71	6.91E-70
0.1	0.8	1.32E-13	9.1E-12	1.54E-22	1.07E-21	2.36E-68	1.54E-57	9.6E-122	9.1E-120
0.1	0.9	5.05E-34	5.07E-30	5.06E-53	7.62E-52	5.1E-128	4.1E-103	5.6E-182	1.5E-190
0.2	0.3	0.726059	0.734747	0.660208	0.670842	0.324637	0.390898	0.149968	0.115562
0.2	0.4	0.394465	0.43443	0.257372	0.266892	0.012136	0.028681	3.92E-05	4.8E-05
0.2	0.5	0.101819	0.135381	0.028584	0.033869	2.91E-06	3.72E-05	4.06E-14	6.84E-14
0.2	0.6	0.006387	0.013591	0.000218	0.000315	1.06E-14	7.96E-12	1.98E-33	1.53E-32
0.2	0.7	6.8E-06	3.44E-05	2.18E-09	5.19E-09	1.02E-32	1.09E-26	1.08E-67	2.67E-66
0.2	0.8	4.01E-13	2.44E-11	1.48E-21	8.41E-21	2.5E-66	1.21E-55	7.1E-119	6.8E-117
0.2	0.9	2.18E-33	1.96E-29	6.67E-52	8.29E-51	1.8E-126	1.7E-101	5.2E-180	1E-188
0.3	0.4	0.616001	0.657699	0.487803	0.492704	0.125519	0.181646	0.006733	0.011444
0.3	0.5	0.197937	0.247777	0.079519	0.089091	0.00019	0.00099	3.61E-10	1.23E-09
0.3	0.6	0.017213	0.032904	0.001074	0.00143	6.8E-12	1.36E-09	4.33E-28	9.53E-27
0.3	0.7	3.12E-05	0.000136	2.44E-08	5.14E-08	4.59E-29	1.19E-23	3.51E-62	3E-60
0.3	0.8	3.81E-12	1.89E-10	4.39E-20	2.15E-19	1.48E-62	2.66E-52	1.9E-114	5.3E-112
0.3	0.9	4.48E-32	3.39E-28	3.46E-50	3.77E-49	1.3E-123	1.25E-98	5.5E-177	1.4E-185
0.4	0.5	0.431214	0.475518	0.287917	0.309167	0.025828	0.047941	0.000214	0.000238
0.4	0.6	0.059315	0.090285	0.009592	0.011891	4.82E-08	1.49E-06	1.12E-18	4.5E-18
0.4	0.7	0.000233	0.00071	8.36E-07	1.58E-06	1.41E-23	3.89E-19	1.3E-51	2.24E-50
0.4	0.8	8.44E-11	2.45E-09	7.92E-18	3.45E-17	1.27E-56	4.35E-47	8.3E-106	7.7E-104
0.4	0.9	3.27E-30	1.35E-26	1.76E-47	1.78E-46	5.2E-119	4.45E-94	4.2E-171	2E-180
0.5	0.6	0.270216	0.325535	0.124201	0.13171	0.000896	0.003753	3.56E-08	8.17E-08
0.5	0.7	0.003578	0.007204	9.19E-05	0.00013	3.26E-16	4.54E-13	4.25E-37	5.36E-36
0.5	0.8	7.86E-09	1.14E-07	1.47E-14	4.23E-14	6.55E-48	2.16E-39	2.85E-93	2.78E-91
0.5	0.9	2.47E-27	4.58E-24	2.42E-43	1.61E-42	4.1E-112	3.77E-87	1.2E-162	1.6E-172
0.6	0.7	0.068118	0.086408	0.016083	0.018721	2.62E-07	5.28E-06	4.77E-17	1.21E-16
0.6	0.8	2.17E-06	1.24E-05	2.67E-10	5.57E-10	5.62E-35	2.05E-28	5.96E-73	2.14E-71
0.6	0.9	2.01E-23	1.07E-20	2.12E-37	1.05E-36	2.7E-101	1.38E-76	6.2E-149	4.5E-160
0.7	0.8	0.00296	0.006874	5.14E-05	6.82E-05	1.61E-16	3.76E-13	9.67E-39	1.2E-37
0.7	0.9	2.39E-17	3.18E-15	2.41E-28	6.9E-28	4.54E-83	4.79E-59	9.6E-125	6.1E-138
0.8	0.9	5.82E-09	5.43E-08	1.27E-14	1.86E-14	4.3E-50	7E-30	4.3E-75	6.45E-93

F. 3 P-values for Error in costs per Experiment and autocorrelation factor AR(1)

F.4. Pairwise comparison of errors generated in estimating inventory policy considering AR(1) demands

		Experiment							
Ref	ϕ	A	B	C	D	E	F	G	H
IID	0.1	0.205544	0.039003	0.163647	0.08902	0.047609	0.000731	0.268425	0.015704
	0.2	0.009642	0.001282	0.102242	0.775973	0.016218	0.052341	0.057414	0.028596
	0.3	0.007843	0.189082	0.006177	0.696552	0.347581	0.000547	0.000925	0.00479
	0.4	0.149683	0.445137	0.065602	0.544169	0.019006	0.123852	0.20922	0.000334
	0.5	0.040629	0.555708	0.049749	0.131707	0.38221	0.522482	0.511119	0.116185
	0.6	0.011717	0.075626	0.049729	0.529342	0.410175	6.51E-05	0.004047	0.386404
	0.7	0.061316	0.11824	0.029174	0.044016	0.00101	0.055978	4.51E-05	0.023823
	0.8	0.002131	0.006046	0.083449	0.021239	5.42E-05	1.51E-07	0.000229	0.016973
	0.9	0.006913	0.021201	1.08E-05	7.12E-05	7.19E-06	2.15E-07	0.006509	0.004961
0.1	0.2	0.182642	0.240413	0.808423	0.156301	0.668747	0.144384	0.425236	0.818443
	0.3	0.160363	0.449861	0.174659	0.189301	0.295513	0.935377	0.026099	0.680613
	0.4	0.860697	0.19175	0.65204	0.272765	0.712457	0.062684	0.881155	0.231891
	0.5	0.431784	0.139104	0.566734	0.8459	0.266348	0.005923	0.652445	0.394051
	0.6	0.206214	0.771822	0.566619	0.282702	0.245361	0.525	0.074923	0.119428
	0.7	0.542743	0.613021	0.427278	0.752097	0.184545	0.13652	0.002661	0.87464
	0.8	0.068474	0.490043	0.733836	0.542738	0.03622	0.050811	0.009259	0.977002
	0.9	0.148007	0.808065	0.002278	0.021014	0.010483	0.059665	0.104016	0.689048
0.2	0.3	0.943	0.054153	0.264515	0.916031	0.140905	0.123484	0.151772	0.521502
	0.4	0.24694	0.013491	0.834821	0.747392	0.952688	0.685969	0.516955	0.154408
	0.5	0.583783	0.008232	0.741011	0.220929	0.124129	0.192038	0.212511	0.533263
	0.6	0.945075	0.143527	0.740883	0.730407	0.112351	0.036547	0.32372	0.183932
	0.7	0.468223	0.093466	0.581191	0.083329	0.368004	0.976574	0.026458	0.942771
	0.8	0.622718	0.627936	0.922261	0.043153	0.094904	0.000698	0.069718	0.840915
	0.9	0.908876	0.351515	0.004897	0.000218	0.032593	0.000895	0.406065	0.528982
0.3	0.4	0.219044	0.581602	0.364007	0.828472	0.157542	0.052159	0.037816	0.432582
	0.5	0.535712	0.468215	0.431898	0.263155	0.947707	0.004631	0.007611	0.206631
	0.6	0.888351	0.641291	0.431998	0.810974	0.907719	0.579139	0.654195	0.049315
	0.7	0.425581	0.802506	0.572107	0.103749	0.018014	0.116529	0.42836	0.569148
	0.8	0.674094	0.148704	0.308461	0.055143	0.00178	0.061112	0.701896	0.659617
	0.9	0.965736	0.318211	0.087181	0.000324	0.000339	0.071417	0.545745	0.990838
0.4	0.5	0.541296	0.86147	0.90289	0.366624	0.139233	0.367398	0.548654	0.041059
	0.6	0.276099	0.309473	0.902755	0.982041	0.126334	0.012767	0.102586	0.006129
	0.7	0.664802	0.423194	0.731435	0.158113	0.337342	0.707677	0.004265	0.176161
	0.8	0.099431	0.04643	0.911662	0.088618	0.083807	0.000155	0.014086	0.220865
	0.9	0.203417	0.121816	0.009087	0.000713	0.028111	0.000204	0.139669	0.425889
0.5	0.6	0.631926	0.233913	0.999864	0.378672	0.959859	0.000741	0.025938	0.479515
	0.7	0.858981	0.329522	0.824916	0.609942	0.015097	0.202194	0.000573	0.487214
	0.8	0.298706	0.030448	0.815803	0.422085	0.001429	3.36E-06	0.002331	0.410216
	0.9	0.507826	0.085321	0.012825	0.012438	0.000265	4.63E-06	0.038218	0.210773
0.6	0.7	0.511465	0.829106	0.825049	0.16482	0.013151	0.034019	0.215378	0.161448
	0.8	0.5749	0.327198	0.815671	0.092884	0.001204	0.186418	0.406329	0.126385
	0.9	0.854527	0.594153	0.012831	0.000772	0.000219	0.210897	0.875793	0.050644
0.7	0.8	0.223848	0.232075	0.649744	0.769572	0.439771	0.000629	0.68215	0.897405
	0.9	0.401095	0.454174	0.02319	0.046015	0.214081	0.000807	0.163141	0.576959
0.8	0.9	0.705731	0.654518	0.00657	0.08826	0.637711	0.94386	0.323949	0.667949

F. 4 P-values for Error in inventory policy per Experiment and autocorrelation factor AR(1)

G LINEARIZATION PROCESS FOR MARKOVIAN-MODULATED AND AR(1)

Appendix G.1 – Linearization process – Markovian demand case

P01	ϕ	$\beta(C)$	$\beta(s^*,S^*)$	Ln $\beta(C)$	Ln $\beta(s^*,S^*)$
0.10	-0.15	23.37	1090	3.151	6.994
0.20	0.13	43.24	907	3.767	6.810
0.30	0.29	58.26	1006	4.065	6.914
0.40	0.38	71.98	832	4.276	6.724
0.50	0.45	83.45	903	4.424	6.805
0.60	0.49	94.94	916	4.553	6.820
0.70	0.53	104.69	698	4.651	6.548
0.80	0.56	113.85	929	4.735	6.834
0.90	0.64	123.57	1355	4.817	7.212

G.1.1. Linearization. Experiment A

Regression Statistics	
Multiple R	0.998305421
R Square	0.996613713
Adjusted R Square	0.996129957
Standard Error	0.033434256
Observations	9

ANOVA

	df	SS	MS	F	Significance F
Regression	1	2.3029	2.3029	2060.1607	0.0000
Residual	7	0.0078	0.0011		
Total	8	2.3108			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%
Intercept	3.470176983	0.0209	166.2748	0.0000	3.4208
Autocorr	2.170468666	0.0478	45.3890	0.0000	2.0574

G.1.2. Regression Analysis. Experiment A

P01	ϕ	$\beta(C)$	$\beta(s^*,S^*)$	Ln $\beta(C)$	Ln $\beta(s^*,S^*)$
0.10	-0.15	22.86	884	3.129	6.784
0.20	0.13	43.60	922	3.775	6.827
0.30	0.29	59.79	613	4.091	6.418
0.40	0.38	73.13	1025	4.292	6.932
0.50	0.45	88.25	836	4.480	6.728
0.60	0.49	96.85	681	4.573	6.523
0.70	0.53	106.51	806	4.668	6.692
0.80	0.56	122.59	1052	4.809	6.959
0.90	0.64	160.44	1856	5.078	7.526

G.1.3. Linearization. Experiment B

Regression Statistics	
Multiple R	0.995627125
R Square	0.991273372
Adjusted R Square	0.990026711
Standard Error	0.058995827
Observations	9

ANOVA

	df	SS	MS	F	Significance F
Regression	1	2.7675	2.7675	795.1426	0.0000
Residual	7	0.0244	0.0035		
Total	8	2.7919			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%
Intercept	3.443808338	0.0368	93.5156	0.0000	3.3567
Autocorr	2.379332291	0.0844	28.1983	0.0000	2.1798

G.1.4. Regression Analysis. Experiment B

P01	ϕ	$\beta(C)$	$\beta(s^*,S^*)$	Ln $\beta(C)$	Ln $\beta(s^*,S^*)$
0.10	-0.15	22.86	629	3.129	6.443
0.20	0.13	41.86	883	3.734	6.784
0.30	0.29	57.15	976	4.046	6.884
0.40	0.38	72.05	951	4.277	6.858
0.50	0.45	83.21	825	4.421	6.715
0.60	0.49	94.80	598	4.552	6.393
0.70	0.53	104.20	972	4.646	6.880
0.80	0.56	114.47	716	4.740	6.574
0.90	0.64	121.45	790	4.800	6.672

G.1.5. Linearization. Experiment C

Regression Statistics	
Multiple R	0.997780492
R Square	0.995565911
Adjusted R Square	0.99493247
Standard Error	0.038850032
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.3722	2.3722	1571.6782	0.0000
Residual	7	0.0106	0.0015		
Total	8	2.3827			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	3.447834273	0.0243	142.1744	0.0000	3.3905
Autocorr	2.202847864	0.0556	39.6444	0.0000	2.0715

G.1.6. Regression Analysis. Experiment C

P01	ϕ	$\beta(C)$	$\beta(s^*,S^*)$	Ln $\beta(C)$	Ln $\beta(s^*,S^*)$
0.10	-0.15	22.86	776	3.129	6.654
0.20	0.13	42.47	701	3.749	6.553
0.30	0.29	56.43	577	4.033	6.358
0.40	0.38	71.42	1047	4.269	6.954
0.50	0.45	85.51	901	4.449	6.804
0.60	0.49	96.18	1064	4.566	6.970
0.70	0.53	107.22	685	4.675	6.529
0.80	0.56	118.08	829	4.771	6.720
0.90	0.64	128.33	877	4.855	6.776

G.1.7. Linearization. Experiment D

Regression Statistics	
Multiple R	0.959885864
R Square	0.921380872
Adjusted R Square	0.910149568
Standard Error	10.65857432
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	9319.8140	9319.8140	82.0369	0.0000
Residual	7	795.2364	113.6052		
Total	8	10115.0504			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	29.99522891	6.6532	4.5084	0.0028	14.2628
Autocorr	138.0749593	15.2444	9.0574	0.0000	102.0277

G.1.8. Regression Analysis. Experiment D

P01	ϕ	β(C)	β(s*,S*)	Ln β(C)	Ln β(s*,S*)
0.10	-0.15	113.44	985	4.731	6.893
0.20	0.13	206.85	1061	5.332	6.967
0.30	0.29	287.07	963	5.660	6.870
0.40	0.38	355.05	989	5.872	6.897
0.50	0.45	414.20	975	6.026	6.883
0.60	0.49	468.44	1150	6.149	7.048
0.70	0.53	516.78	1042	6.248	6.949
0.80	0.56	559.48	1265	6.327	7.143
0.90	0.64	928.59	1785	6.834	7.487

G.1.9. Linearization. Experiment E

Regression Statistics	
Multiple R	0.983393012
R Square	0.967061815
Adjusted R Square	0.96235636
Standard Error	0.118800584
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.9006	2.9006	205.5193	0.0000
Residual	7	0.0988	0.0141		
Total	8	2.9994			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	5.009973352	0.0742	67.5590	0.0000	4.8346
Autocorr	2.435881653	0.1699	14.3359	0.0000	2.0341

G.1.10. Regression Analysis. Experiment E

P01	ϕ	β(C)	β(s*,S*)	Ln β(C)	Ln β(s*,S*)
0.10	-0.15	111.17	875	4.711	6.774
0.20	0.13	206.87	906	5.332	6.809
0.30	0.29	287.44	994	5.661	6.902
0.40	0.38	359.44	821	5.885	6.711
0.50	0.45	417.96	1022	6.035	6.930
0.60	0.49	468.09	992	6.149	6.900
0.70	0.53	515.29	915	6.245	6.819
0.80	0.56	779.08	1697	6.658	7.436
0.90	0.64	1,033.29	1849	6.940	7.522

G.1.11. Linearization. Experiment F

Regression Statistics	
Multiple R	0.972426541
R Square	0.945613377
Adjusted R Square	0.93784386
Standard Error	0.167710349
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3.4233	3.4233	121.7081	0.0000
Residual	7	0.1969	0.0281		
Total	8	3.6201			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	4.980883568	0.1047	47.5787	0.0000	4.7333
Autocorr	2.646250433	0.2399	11.0321	0.0000	2.0791

G.1.12. Regression Analysis. Experiment F

P01	ϕ	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
0.10	-0.15	110.84	1207	4.708	7.096
0.20	0.13	211.09	882	5.352	6.782
0.30	0.29	282.22	893	5.643	6.795
0.40	0.38	355.81	763	5.874	6.638
0.50	0.45	372.22	395	5.919	5.979
0.60	0.49	470.89	802	6.155	6.687
0.70	0.53	516.20	679	6.246	6.520
0.80	0.56	557.37	670	6.323	6.507
0.90	0.64	768.00	1608	6.644	7.383

G.1.13. Linearization. Experiment G

Regression Statistics	
Multiple R	0.991539249
R Square	0.983150082
Adjusted R Square	0.980742951
Standard Error	0.080602534
Observations	9

ANOVA

	df	SS	MS	F	Significance F
Regression	1	2.6535	2.6535	408.4323	0.0000
Residual	7	0.0455	0.0065		
Total	8	2.6990			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%
Intercept	5.01420191	0.0503	99.6597	0.0000	4.8952
Autocorr	2.329807473	0.1153	20.2097	0.0000	2.0572

G.1.14. Regression Analysis. Experiment G

P01	ϕ	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
0.10	-0.15	113.93	923	4.736	6.827
0.20	0.13	204.60	817	5.321	6.706
0.30	0.29	291.83	951	5.676	6.858
0.40	0.38	357.02	975	5.878	6.882
0.50	0.45	416.89	909	6.033	6.812
0.60	0.49	469.08	873	6.151	6.772
0.70	0.53	515.66	765	6.245	6.640
0.80	0.56	556.61	688	6.322	6.533
0.90	0.64	797.33	1696	6.681	7.436

G.1.15. Linearization. Experiment H

Regression Statistics	
Multiple R	0.993824634
R Square	0.987687404
Adjusted R Square	0.985928461
Standard Error	0.069288169
Observations	9

ANOVA

	df	SS	MS	F	Significance F
Regression	1	2.6958	2.6958	561.5235	0.0000
Residual	7	0.0336	0.0048		
Total	8	2.7294			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%
Intercept	5.027123286	0.0433	116.2323	0.0000	4.9249
Autocorr	2.348304134	0.0991	23.6965	0.0000	2.1140

G.1.16. Regression Analysis. Experiment H

Appendix E.2. – Linearization process – AR(1) demand case

ϕ	Cost	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
0	2,781.31	0.00	0.00	0	0
0.1	2,783.10	1.79	205.39	0.582	5.325
0.2	2,786.60	3.50	36.01	1.252	3.584
0.3	2,793.88	7.28	258.29	1.985	5.554
0.4	2,804.29	10.42	184.78	2.343	5.219
0.5	2,820.64	16.35	25.36	2.794	3.233
0.6	2,843.56	22.92	122.27	3.132	4.806
0.7	2,881.54	37.98	95.73	3.637	4.562
0.8	2,943.68	62.14	363.59	4.129	5.896
0.9	3,067.84	124.16	191.81	4.822	5.256

G.2.1. Linearization. Experiment A

Regression Statistics	
Multiple R	0.990057895
R Square	0.980214636
Adjusted R Square	0.977388155
Standard Error	0.041181165
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.588128781	0.5881288	346.79687	3.19526E-07
Residual	7	0.011871219	0.0016959		
Total	8	0.6			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	-0.073272141	0.033705769	-2.1738754	0.066237	-0.152973619
X Variable 1	0.168708641	0.009059406	18.622483	3.195E-07	0.147286549

G.2.2. Regression Analysis. Experiment A

ϕ	Cost	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
IID	2,874.50	0.00	0.00	0	0
0.1	2,876.56	2.06	226.79	0.724	5.424
0.2	2,882.10	7.60	142.00	2.029	4.956
0.3	2,891.66	17.16	113.11	2.843	4.728
0.4	2,906.07	31.57	117.02	3.452	4.762
0.5	2,928.15	53.66	207.58	3.983	5.336
0.6	2,960.14	85.64	345.13	4.450	5.844
0.7	3,010.34	135.85	129.96	4.912	4.867
0.8	3,095.51	221.01	119.81	5.398	4.786
0.9	3,263.32	388.82	338.90	5.963	5.826

G.2.5. Linearization. Experiment C

Regression Statistics	
Multiple R	0.984388056
R Square	0.969019844
Adjusted R Square	0.964594107
Standard Error	0.315853817
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	21.84335617	21.843356	218.95109	1.54179E-06
Residual	7	0.698345435	0.0997636		
Total	8	22.5417016			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.733466479	0.229462478	3.196455	0.0151402	0.190874327
X Variable 1	6.0337048	0.407765524	14.796996	1.542E-06	5.069493243

G.2.6. Regression Analysis. Experiment C

ϕ	Cost	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
IID	5,370.93	0.00	0.00	0	0
0.1	5,373.20	2.27	274.79	0.822	5.616
0.2	5,378.72	7.79	50.40	2.053	3.920
0.3	5,387.55	16.62	47.16	2.811	3.853
0.4	5,401.80	30.87	46.00	3.430	3.829
0.5	5,422.93	52.00	206.35	3.951	5.330
0.6	5,454.29	83.36	115.20	4.423	4.747
0.7	5,503.32	132.39	178.20	4.886	5.183
0.8	5,587.05	216.12	527.44	5.376	6.268
0.9	5,753.69	382.76	121.57	5.947	4.800

G.2.7. Linearization. Experiment D

Regression Statistics	
Multiple R	0.987131364
R Square	0.974428331
Adjusted R Square	0.970775235
Standard Error	0.281518907
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	21.13995205	21.139952	266.74044	7.86058E-07
Residual	7	0.554770264	0.0792529		
Total	8	21.69472231			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.776426865	0.204518744	3.7963604	0.0067467	0.292817229
X Variable 1	5.935760559	0.363439346	16.33219	7.861E-07	5.076363683

G.2.8. Regression Analysis. Experiment D

ϕ	Cost	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
IID	3,275.46	0.00	0.00	0	0
0.1	3,279.57	4.11	133.54	1.413	4.894
0.2	3,290.63	15.17	123.67	2.720	4.818
0.3	3,311.10	35.64	110.18	3.574	4.702
0.4	3,342.97	67.51	192.08	4.212	5.258
0.5	3,389.43	113.97	75.66	4.736	4.326
0.6	3,458.99	183.53	162.77	5.212	5.092
0.7	3,568.17	292.71	454.74	5.679	6.120
0.8	3,748.96	473.50	474.58	6.160	6.162
0.9	4,120.12	844.66	353.35	6.739	5.867

G.2.9. Linearization. Experiment E

Regression Statistics	
Multiple R	0.983883237
R Square	0.968026225
Adjusted R Square	0.963458543
Standard Error	0.326674116
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	22.61625523	22.616255	211.92942	1.7226E-06
Residual	7	0.747011845	0.106716		
Total	8	23.36326708			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	1.424125211	0.237323243	6.0007827	0.0005418	0.862945316
X Variable 1	6.139524307	0.42173447	14.557796	1.723E-06	5.142281465

G.2.10 Regression Analysis. Experiment E

ϕ	Cost	β(C)	β(s*,S*)	Ln β(C)	Ln β(s*,S*)
IID	5,684.72	0.00	0.00	0	0
0.1	5,688.41	3.68	275.18	1.304	5.617
0.2	5,698.54	13.81	13.22	2.626	2.582
0.3	5,716.36	31.64	261.29	3.454	5.566
0.4	5,744.14	59.41	59.41	4.085	4.085
0.5	5,785.33	100.61	27.12	4.611	3.300
0.6	5,845.91	161.19	513.71	5.083	6.242
0.7	5,942.01	257.29	173.64	5.550	5.157
0.8	6,099.29	414.56	766.47	6.027	6.642
0.9	6,360.97	676.25	547.34	6.517	6.305

G.2.11. Linearization. Experiment F

Regression Statistics	
Multiple R	0.982632853
R Square	0.965567324
Adjusted R Square	0.960648371
Standard Error	0.333972934
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	21.894366	21.894366	196.29527	2.23476E-06
Residual	7	0.780765444	0.1115379		
Total	8	22.67513144			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	1.341453562	0.242625711	5.5289011	0.0008793	0.767735331
X Variable 1	6.040745814	0.431157204	14.010541	2.235E-06	5.021221764

G.2.12. Regression Analysis. Experiment F

ϕ	Cost	β(C)	β(s*,S*)	Ln β(C)	Ln β(s*,S*)
IID	3,789.09	0.00	0.00	0	0
0.1	3,795.28	6.19	15.62	1.823	2.748
0.2	3,815.09	26.00	230.54	3.258	5.440
0.3	3,845.03	55.94	162.86	4.024	5.093
0.4	3,901.62	112.53	150.60	4.723	5.015
0.5	3,979.33	190.24	189.37	5.248	5.244
0.6	4,096.55	307.46	305.69	5.728	5.723
0.7	4,280.87	491.78	432.54	6.198	6.070
0.8	4,591.31	802.22	558.16	6.687	6.325
0.9	5,098.00	1,308.91	1,247.88	7.177	7.129

G.2.13. Linearization. Experiment G

Regression Statistics	
Multiple R	0.981279012
R Square	0.9629085
Adjusted R Square	0.957609714
Standard Error	0.354872496
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	22.88512804	22.885128	181.72248	2.90239E-06
Residual	7	0.881541419	0.1259345		
Total	8	23.76666946			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	1.897357792	0.257808891	7.3595514	0.0001547	1.287737071
X Variable 1	6.175911274	0.458138422	13.480448	2.902E-06	5.092586825

G.2.14. Regression Analysis. Experiment G

ϕ	Cost	$\beta(C)$	$\beta(s^*,S^*)$	$Ln\ \beta(C)$	$Ln\ \beta(s^*,S^*)$
IID	6,270.99	0.00	0.00	0	0
0.1	6,277.04	6.05	306.74	1.800	5.726
0.2	6,296.68	25.69	355.87	3.246	5.875
0.3	6,329.42	58.43	15.99	4.068	2.772
0.4	6,382.19	111.20	59.10	4.711	4.079
0.5	6,459.31	188.32	186.44	5.238	5.228
0.6	6,573.23	302.24	299.77	5.711	5.703
0.7	6,754.85	483.87	483.73	6.182	6.182
0.8	7,059.37	788.39	669.95	6.670	6.507
0.9	7,669.22	1,398.23	1,351.41	7.243	7.209

G.2.15. Linearization. Experiment H

Regression Statistics	
Multiple R	0.981257752
R Square	0.962866777
Adjusted R Square	0.957562031
Standard Error	0.35714588
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	23.15223298	23.152233	181.51043	2.91389E-06
Residual	7	0.892872259	0.1275532		
Total	8	24.04510524			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	1.879574178	0.259460467	7.244164	0.0001708	1.266048105
X Variable 1	6.211847951	0.461073349	13.472581	2.914E-06	5.121583508

G.2.16. Regression Analysis. Experiment H

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