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An Integrated Risk Analysis Methodology in a Multidisciplinary Design Environment

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AN INTEGRATED RISK ANALYSIS METHODOLOGY IN A MULTIDISCIPLINARY DESIGN ENVIRONMENT

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ABSTRACT

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Design of complex, one-of-a-kind systems, such as space transportation systems, is characterized by high uncertainty and, consequently, high risk. It is necessary to account for these uncertainties in the design process to produce systems that are more reliable. Systems designed by including uncertainties and managing them, as well, are more robust and less prone to poor operations as a result of parameter variability.

The quantification, analysis and mitigation of uncertainties are challenging tasks as many systems lack historical data. In such an environment, risk or uncertainty quantification becomes subjective because input data is based on professional judgment. Additionally, there are uncertainties associated with the analysis tools and models. Both the input data and the model uncertainties must be considered for a multi disciplinary systems level risk analysis.

This research synthesizes an integrated approach for developing a method for risk analysis. Expert judgment methodology is employed to quantify external risk. This methodology is then combined with a Latin Hypercube Sampling – Monte Carlo simulation to propagate uncertainties across a multidisciplinary environment for the overall system. Finally, a robust design strategy is employed to mitigate risk during the optimization process. This type of approach to risk analysis is conducive to the examination of quantitative risk factors.

The core of this research methodology is the theoretical framework for uncertainty propagation. The research is divided into three stages or modules. The first two modules include the identification/quantification and propagation of uncertainties. The third module involves the management of uncertainties or response optimization. This final module also incorporates the integration of risk into program decision-making.

The risk analysis methodology, is applied to a launch vehicle conceptual design study at NASA Langley Research Center. The launch vehicle multidisciplinary environment consists of the interface between configuration and sizing analysis outputs and aerodynamic parameter computations. Uncertainties are analyzed for both simulation tools and their associated input parameters. Uncertainties are then propagated across the design environment and a robust design optimization is performed over the range of a critical input parameter.

The results of this research indicate that including uncertainties into design processes may require modification of design constraints previously considered acceptable in deterministic analyses.

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INTRODUCTION

Design for complex engineered systems is accomplished in a multidisciplinary environment where each one of the design disciplines have an element of risk associated with them. It is, therefore, a natural progression to couple the methodologies of multidisciplinary design optimization with the probabilistic estimation methods that are characteristic of risk analysis.

Most simulation design tools have been developed as single discipline analysis tools. Engineered systems having any level of complexity involves the integration of several disciplines. Examples of such interfaces include weight analysis inputs to structural analysis or computational fluid dynamic inputs into finite element analysis. Calculations used in single discipline analysis tools can be either simplistic or intricate in nature. Regardless of the level of difficulty, multidisciplinary design seeks to examine the interactions of several disciplines and their impact upon one another. Multidisciplinary design is gaining widespread attention in the engineering community. The seamless integration of single discipline tools into the design process promises savings in computational efficiencies, more effective update of the entire system design when one component changes, central location of design properties and specifications, and product cost savings from streamlining of the design process. Although seamless integration of discipline analysis may be a goal, in general, subsystem designs are not coupled together in one integrated algorithm or code. The design in different disciplines is handled in an iterative fashion as one discipline is updated based on the results of another discipline.

Traditionally, design analysis tools involve the computation of output variables based on point estimates of input variables. Such design processes leave out the very important element of risk. Risk consists of uncertainties associated with input variables and models. Input variable uncertainties can be the result of variations in processes, tolerances, material properties or other conditions subject to change. Model uncertainties represent errors between the actual system and the computer model as well as truncation errors associated with performing mathematical calculations.

Risk is inherent in the design of any engineered system, and until recently, the integration of risk analysis into design processes was often neglected. This omission could be attributed to the complexities encountered in quantifying the various elements of risk and the ultimate impact of the identified uncertainties on the decision making process. Risk analysis has now been incorporated into numerous design disciplines with varying degrees of fidelity or model accuracy. Risk analysis should also be part of multidisciplinary design. Solving a design problem using probabilistic elements requires additional effort. Consequently, the decision to use deterministic analysis or stochastic analysis is a tradeoff between the need for increased accuracy in design calculations and the increase in the computational endeavor.

Risk analysis can be divided into three distinct phases: 1) uncertainty identification/quantification, 2) uncertainty propagation and 3) uncertainty management. Each of these phases is briefly discussed below.

1.1 Risk Factor Identification/Quantification

There are essentially two sources of information used to acquire uncertainty for input variables. The first source is available data and the second is expert opinion. Available data can be obtained via scientific experiments, surveys, computerized databases, and computer simulations. Each of these forms of uncertainty data acquisition are very common. Scientific experiments generally require time, manpower and finances that are not readily available. This acquisition methodology can produce very accurate uncertainty distributions if a sufficient number of iterations can economically be performed. Surveys are best used when soliciting specific information from individuals and typically require numerous man-hours to tabulate. This acquisition methodology can produce accurate percentile data, but can present problems in structuring questions to solicit desired data. Although computer databases are commonly used to identify uncertainty, this form of acquisition is generally appropriate for programs that have been in existence for some period of time. Problems arise in obtaining information in the appropriate format or context. Many one of a kind programs do not have the historical base to make this acquisition method useful. Computer simulations are generally an excellent method of obtaining uncertainty data. Simulations are cheaper than scientific experiments in most instances. However, the uncertainty information obtained is only as good as the model that has been built.

Using expert opinion to obtain uncertainty presents its own set of challenges. This method of acquisition is most appropriate when there is no historical data available. The data may have never been collected in the past or is too expensive to obtain. Past data may no longer be relevant or the data is sparse and requires expert opinion to fill in the holes. Expert judgment is also used when the area being modeled is new. For many of these reasons, expert judgment data acquisition approach is utilized for this research study. The problem involves the conceptual design phase of a new launch vehicle. The limited data on the input parameters is fairly new and very little data has been collected in the past. To experimentally collect data would not be feasible at the conceptual design phase.

1.2 Propagation of Uncertainties

Typically, a complete system design combines the results of numerous simulation tools that have been used by various disciplines. Each simulation tool has its own individual bias and precision errors (uncertainties). Consequently, the accumulation of those errors across the system has the potential of being significant.

When probability distributions are available for input parameters and the associated uncertainties, a simulation technique can be used for the propagation of uncertainties across multiple designs. The principles associated with the accumulation or propagation of uncertainty are well documented and this research selects a suitable strategy from existing techniques. The research examines alternative uncertainty propagation methodologies which fall in the categories of either analytic or simulation solutions.

Analytic solutions are most often used in reliability engineering and are an alternative to simulations. Current research is being conducted in the areas of First Order Reliability Methods (FORM), Second Order Reliability Methods (SORM) and Fast Probability Integration (FPI). These methods are effective when used with metamodels or response surfaces and are discussed in greater detail in the literature review.

Monte Carlo methods are simulations and comprise that branch of experimental mathematics which is concerned with experiments on random numbers [Hammersley and Handscomb (1964)]. For a probabilistic problem the simplest Monte Carlo approach is to observe random numbers chosen in such a way that they directly simulate the physical random processes of the original problem, and to infer the desired solution from the behavior of these random numbers. Monte Carlo methods can employ a variety of sampling techniques. Random sampling is the most commonly used sampling technique. In fact, Monte Carlo methods with random sampling are often referred to as Crude Monte Carlo or simply Monte Carlo. Several authors use this more restrictive definition of Monte Carlo methods [Law and Kelton (1991)]. Other sampling techniques used with Monte Carlo methods include stratified sampling, importance sampling and Latin Hypercube Sampling. Latin Hypercube Sampling is one of the more recently developed sampling techniques. The research examines these various sampling strategies and chooses an appropriate technique for the conceptual design environment.

1.3 Uncertainty Management Strategy

Uncertainties propagated across two or more disciplines results in the identification of parameters that impact risk associated with a certain response variable. By systematically varying controllable parameters, it is possible to identify the optimum input parameters that minimize risk. This stage of the research identifies an uncertainty risk reduction technique. The research attempts to identify parameters that influence the mean and variance of a response variable. Having identified those parameters, an optimization strategy is outlined. This optimization facilitates the selection of parameters that minimize the objective function of a Pareto solution and consequently minimizes risk.

Program decisions are determined based on a variety of factors. Some of those factors are quantitative in nature. The integration of factor parameters into a decision is typically subjective. Strategies that provide effective means of decision-making in a conceptual design environment are explored in conjunction with the mitigation strategy.

1.4 Problem Domain

The research is conducted in the conceptual design environment of a new launch vehicle. Figure 1 illustrates the conceptual design process for the assent phase of a launch vehicle. This process is used in the Vehicle Analysis Branch (VAB) at NASA Langley Research Center and there are several other phases of a launch vehicle design. Each design process involves multiple disciplines and requires multiple iterations to achieve a converged solution. Although these disciplines are not integrated directly, results from one discipline are passed on to one or more disciplines. These analysis codes are deterministic in nature. Each of these codes is developed in FORTRAN except SMART. SMART, a geometry code, was developed in the C++ programming language. A risk analysis tool (RAT) has also been developed in C++ that interfaces directly with the weights and sizing program (CONSIZ). This program was based on the research conducted by Monroe (1997), and facilitates calculation of vehicle weight based on uncertainty input parameters solicited from a single expert. The expert elicitation strategy of RAT has been incorporated into this research. This study uses the elicitation strategy in a multiple expert environment.

In the conceptual design phase, VAB makes use of state-of-the-art response surface methodology for optimization. Response surfaces have already been developed for a deterministic weight optimization with pitching moment coefficient constraints on hypersonic, supersonic and subsonic conditions.

In this study, two disciplines are coupled together to create a multidisciplinary environment. For methodology development and application purposes, C++ programs are developed which provide an output distribution for the coupled system. The C++ programs are used to compare output distributions generated using Monte Carlo simulations with output generated using Latin Hypercube Sampling. The C++ programs are also used to compare output generated for a single expert with output generated using multiple experts. This research demonstrates a risk analysis concept that can be extended for use in a complex engineered system such as a launch vehicle. Quantification of the benefits from employing the research methodology for more than two disciplines is not part of this study, but is recommended to be explored as future research.

The two disciplines used in this research are aerodynamics and weight & sizing for a launch vehicle. As illustrated by Figure 1, these disciplines are only two of several other disciplines that interface or impact one another during complicated system analysis. As input into one or both disciplines changes, the output of both disciplines is changed in an iterative fashion. The use of two disciplines in this research serves as a test bed for a more complex structure. The two chosen disciplines directly or indirectly impact weight which is very important in the design process. Weight is critical to launch vehicle system design success and engineers strive to keep weight to a minimum. Current focus of launch vehicle design is on optimizing weight while using other system requirements (e.g., pitching moment coefficient) as constraints [Unal et al, 1998]. The emphasis on weight optimization verses other design parameter optimizations adds risk to vehicle performance which to date has yet to be explored. The deterministic designs of the past are based on perfection. Input parameters have to be exact in order for the launch vehicle to function as designed. Uncertainties in parameters can push the vehicle out of performance feasibility regions. There is a need to identify how well these requirements are actually being met when uncertainty is taken into consideration for launch vehicle design.

Engineering organizations involved in the design of complex systems (i.e., NASA) would be interested in this methodology. Successful implementation of this risk analysis methodology could conceivably impact the design process of every system having two or more disciplines as risk is inherent in every engineered system. The uncertainty identification and quantification element is applicable to the conceptual design of any system. The uncertainty propagation strategy is relevant to any process where a Monte Carlo simulation can be implemented. Finally, the optimization strategy can be used when tradeoffs between optimal mean and optimal variance are needed for system design requirements.

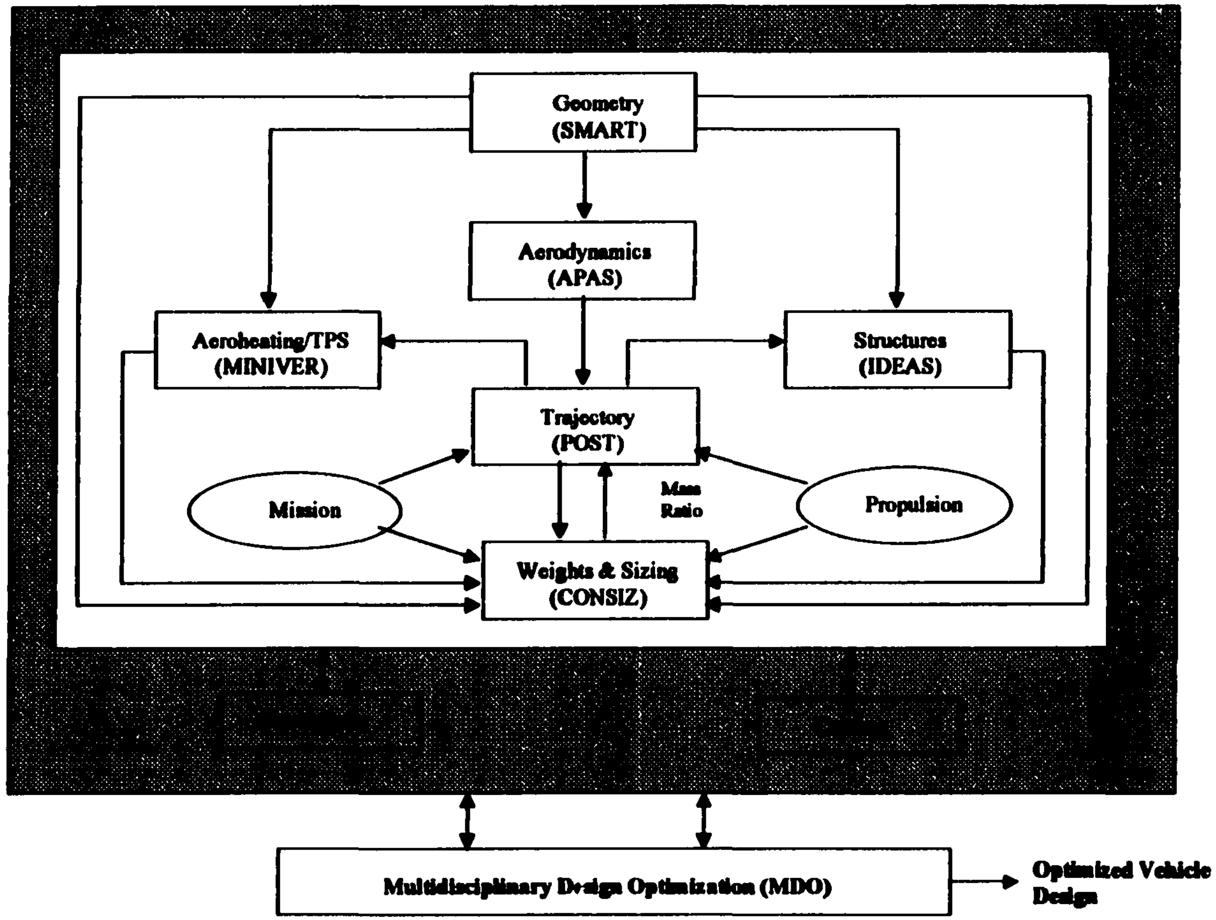

Figure 1 Launch Vehicle Conceptual Design Process [Rowell et al, 1996]

CHAPTER II
LITERATURE REVIEW

The purpose of this literature review is to survey earlier studies relating quantitative risk analysis and the concomitant elements to multidisciplinary design. Quantitative risk analysis is probabilistic in nature and falls into four categories: cost, schedule, technical parameter and reliability-based (or safety) risk analysis. Quantitative risk analysis entails the propagation of probabilistic input distributions within a risk analysis model or algorithm. For cost, schedule and technical parameter analysis, the mean and variance of the response variable are the measures of interest. Reliability-based risk analysis evaluates the probability of component failures (risk) within a mechanism or structural system. The probability of failures and the associated failure consequences are the parameters of interest. Technical parameter risk analysis is the focus of this research study. Reliability-based risk analysis literature is briefly reviewed as some of the computational procedures are analogous to methods used in technical parameter risk analysis. It was believed that the reliability methods could potentially be utilized in a technical parameter risk study.

This literature review will frame the current research topic within the context of the overall body of knowledge. Additionally, the review will act as a filter through the expanse of related literature and provide convergence to, and thus justification for, a specific integrated risk analysis strategy for the multidisciplinary design environment of this research. The eight sections of this literature review include 1) risk analysis applications, 2) uncertainty identification/quantification, 3) uncertainty propagation, 4) uncertainty management, 5) current risk analysis research, 6) available risk analysis software, 7) literature review summary and 8) contribution. Literature from the four categories of probabilistic risk analysis was examined in an effort to identify pertinent strategies that could be implemented in this study. Figure 2 identifies some of the noted authors within each category and relevant strategies are documented in subsequent paragraphs. Technical Parameter literature specifically focused on multidisciplinary design. Figure 3 identifies the strategies documented in the literature for each element.

2.1 Risk Analysis Applications

Cooper and Chapman (1987) stated that " ... risk analysis models manipulate probabilities and probability distributions, in order to assess the combined impact of risks.... The exact manner in which this is done depends on the purpose of the analysis." More succinctly, no single risk analysis model is suitable for every purpose. Some models are simplistic while others are required to be complicated due to the nature of the problem. Quantitative risk analysis has been applied in the fields of project management, and finances. Most recently, it has been applied to the field of multidisciplinary design optimization (MDO). MDO of large systems is characterized by interdisciplinary couplings, multiple objectives, large design variable space and a number of design constraints (Tappeta and Renaud, 1997). Reliability-based risk analysis is a growing field with a different approach than traditional technical quantitative risk analysis methods.

2.1.1 Quantitative Risk Analysis

Winston (1996) and Cooper and Chapman (1987) address risk modeling for project management and financial endeavors. Hertz and Thomas (1983) provide coverage of financial risk analysis techniques. There are other text written about risk with respect to these disciplines. Those text cited here are a representative sampling.

While there are several articles and texts written on project management, and financial risk analysis, there appear to be no specific text available on risk analysis pertaining to multidisciplinary design optimization. A few journal articles have been identified. The primary article cited here is by Du and Chen (1999). This article addresses the elements of uncertainty quantification, uncertainty propagation and uncertainty management. It addresses the decision analysis process within the uncertainty management element. Du and Chen (1999) use both the extreme condition approach and Monte Carlo simulation to propagate uncertainties. Finally, a robust design mitigation element was employed in the risk analysis strategy. Gu, Renaud and Batill (1998) also address the identification and propagation of uncertainties within their article.

Putko et al (2001) extend the research of Du and Chen (2000) by applying the three stages of risk analysis to Computational Fluid Dynamics (CFD). The authors use different propagation techniques from that of Du and Chen (2000). These techniques are discussed further in 2.3.2. The authors, here, apply risk analysis in a single discipline setting, but refer the readers to articles where the propagation technique has been used in Finite Element Analysis (FEA) as well. The methodology, therefore, has the potential of being used in a multidisciplinary design environment.

A review of the risk analysis literature revealed common threads between the applications. Each application had the elements of uncertainty quantification, uncertainty propagation and risk management within the respective strategies. Historical data was most typically used as the method of acquiring uncertainty data and data characteristics. Crude Monte Carlo simulation methods were the chosen techniques for propagating uncertainty. Finally, in financial applications, sensitivity analysis was used as the form of risk management. Here, efforts to understand the sensitivity of the solutions or responses to variations in input data were undertaken with no attempt to actually control the input parameters. In MDO applications, full factorial designs and Response Surface Methodology (RSM) were employed to manage risk.

2.1.2 Reliability-based Risk Analysis

Reliability-based risk applications utilize different techniques for quantifying and propagating uncertainty (probability of failure) than other quantitative risk analyses. Ayyub and McCuen (1997), Henley and Kumamoto (1981), Gnedendo etal (1999), Kumamoto and Henly (1996) and Lewis (1996) have written text that contain elements of risk analysis pertaining to reliability engineering. Haldar and Mahadevan (2000a and

2000b) have written two books on reliability and probabilistic methods in engineering design. Mahadevan and Han (1997) were funded by NASA to study multidisciplinary system reliability analysis and documented their work in a final report. Software programs, such as NESSUS, have been written to perform many of the computational procedures associated with reliability-based risk analysis.

Reliability-based risk analysis also makes use of Monte Carlo simulations. Random sampling, importance sampling and antithetic variates are all used as the sampling techniques associated with Monte Carlo estimators.

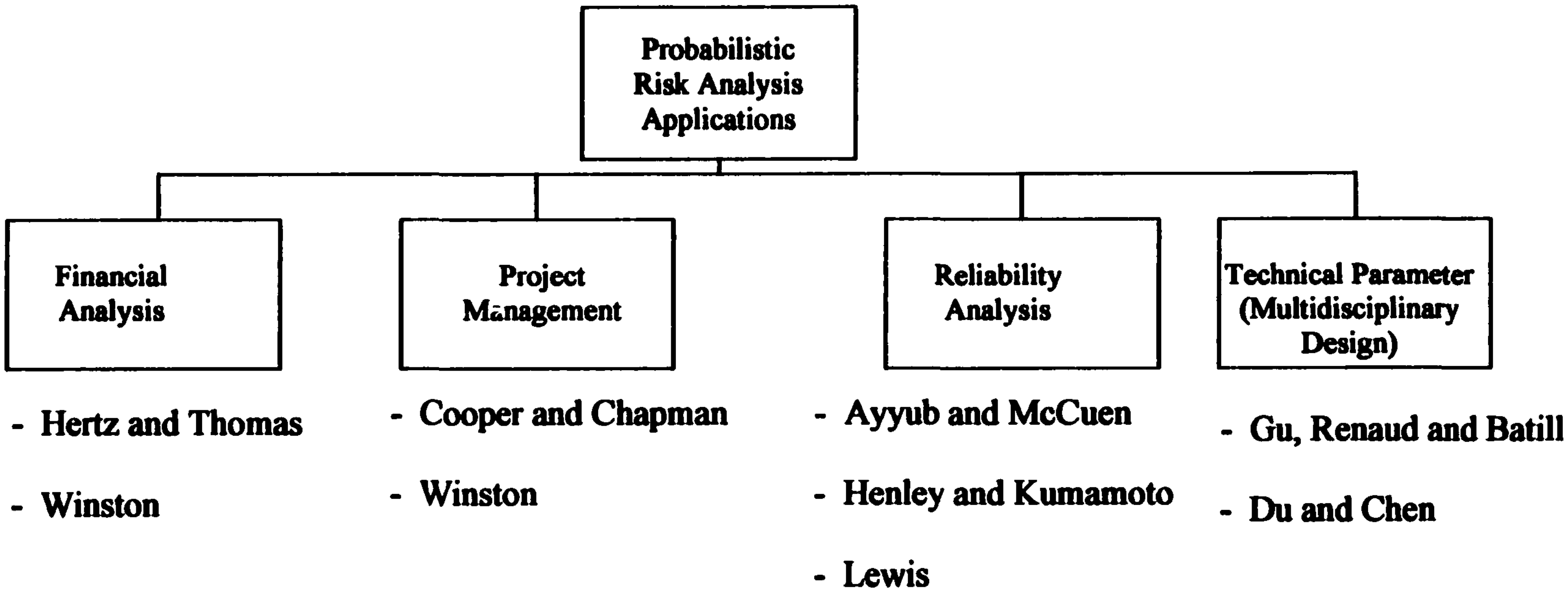

Figure 2 - Risk Analysis Categories

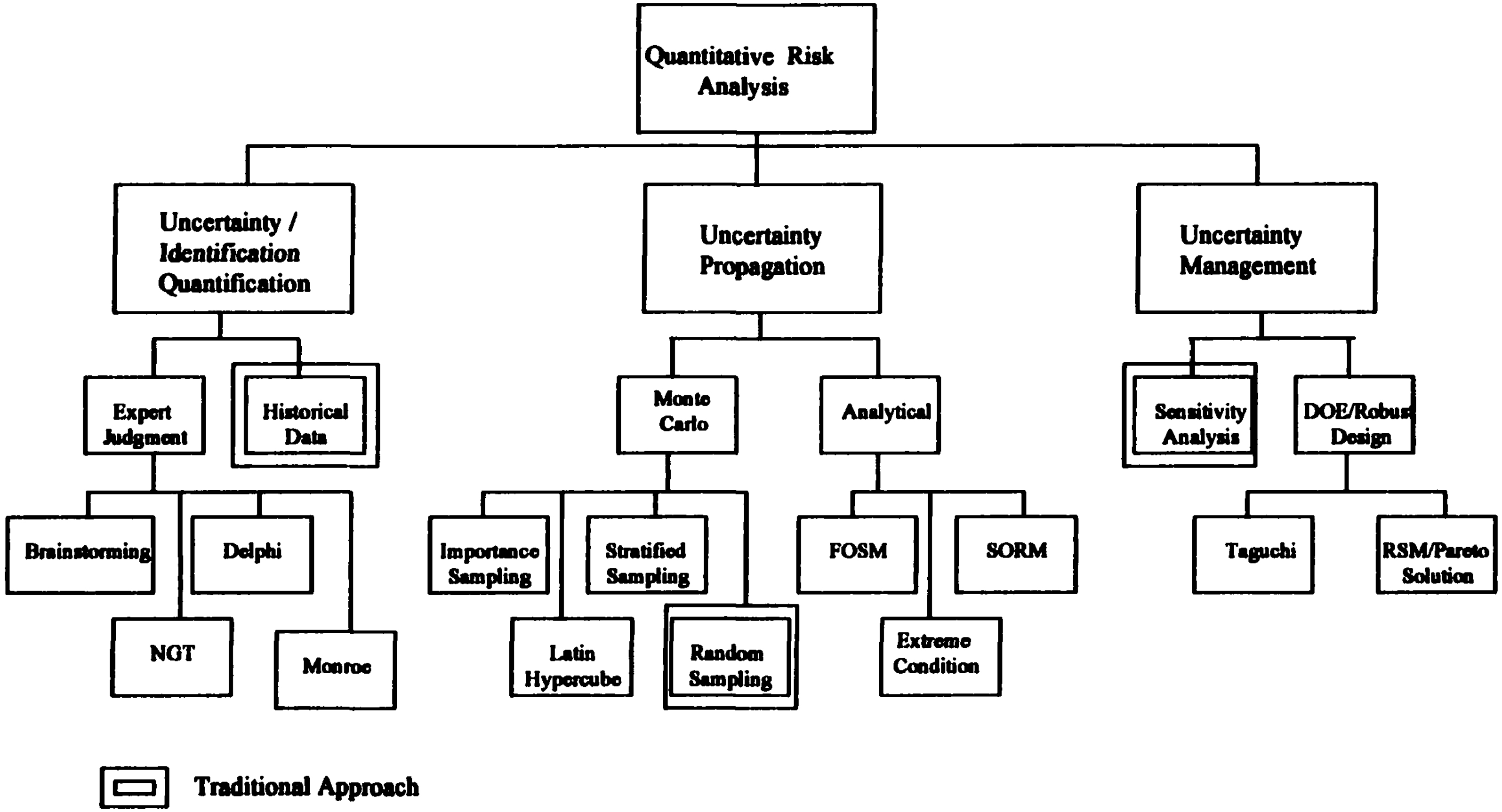

Figure 3 - Elemental Risk Analysis Map

2.2 Uncertainty Identification/Quantification

The first element of a risk analysis is the identification/quantification of uncertainties. This element also includes data gathering techniques.

2.2.1 Uncertainty Identification

Uncertainty is the inability to determine the true state of a system. It is caused by incomplete knowledge or stochastic variability [Haimes (1998)]. Uncertainties must be identified before they can be quantified. Ayyub (1994) outlines a variety of uncertainty types encountered in engineering design problems. Du and Chen (1999) further categorized Ayyub's uncertainty types into internal and external uncertainties. Gu et al. (1998) provide illustrations of the various categories of error (uncertainties) associated with simulation and modeling. Simulation tool uncertainties stem from model approximation error and algorithmic error associated with optimization techniques. Computational error also exists, but this type of uncertainty can be minimized and is typically neglected. A failure to account for uncertainties associated with simulation based tools and input data parameters can produce poor analysis results.

A significant source of uncertainty often ignored is how well the model used actually represents the real system's significant behavior. This uncertainty is introduced through model topology, parameters, data, optimization technique and human subjectivity [Haimes (1998)]. Model topology refers to the form, order and type of equations used to model a system. The decision to use polynomials, partial differential equations, linear or nonlinear equations is a source of uncertainties and error in the accuracy of a model. Once the topology has been selected, the choice of model parameters impacts the accuracy of the model to the real system. The parameter estimation process is discussed further in section 2.2.2 and affects the calculated values of the parameters as well as the model itself. Having enough representative data for model construction, calibration and validation is very important to risk analysis. A lack of data due to collecting, processing or analyzing techniques can cause substantial errors. Once the mathematical model has been constructed and parameters identified, selecting and applying a suitable optimization strategy introduces another source of uncertainties. Human subjectivity plays a huge role in the selection of major model characteristics. Human judgments are affected by the background, training and experience of the analyst. It is very difficult to measure the impact of human subjectivity on model errors.

2.2.2 Uncertainty Data Acquisition

This literature review focuses on the use of expert opinion or expert judgment in the gathering of uncertainty input parameters. "Expert judgment methods utilize recognized or identifiable experts(s) in a given domain to provide an informed judgment about some variable of interest…" (Monroe 1997). When there is no statistical information, input parameter distributions will be obtained using the expert judgment methodology. There are several expert judgment acquisition techniques such as the Delphi method [Dalkey (1969)], the Nominal Group Technique [Lock (1987)], brainstorming [Lock (1987)] and Monroe's approach (1997). Each of these techniques, except brainstorming, elicits expert opinion using questionnaires. Delphi is accomplished at a distance. It is a method of dialogue with feedback restrictions. Open discussion is not permitted. The feedback consists of summary statistics such as group means or quantiles. Each person then reassesses their distribution and the process is repeated until the different opinions converge toward a common distribution. This approach can be inexpensive compared to group techniques since the experts need not communicate directly and social pressure is reduced. Winkler (1986) points out that it is difficult to limit the feedback to summary statistics if a specific family of distributions is not already known.

Nominal Group Technique and brainstorming are accomplished in a group setting. The Nominal Group Technique (NGT) combines aspects of silent voting with limited discussion to help build consensus and arrive at a team decision. Using NGT, the first round of opinions is generated silently, and no discussion is held until all opinions have been presented [P. K. Kelly (1994)]. Each opinion is then discussed separately.

Next, each expert ranks the list of opinions silently. Then, the members call out their rankings. Rankings are then totaled and the opinion with the lowest ranking is taken as the consensus opinion. NGT ensures equal participation and minimizes controversy.

The objective of brainstorming sessions is to ensure that everyone has the same information on which to base their opinions. Pertinent information is gathered prior to the meeting and disseminated to group members. At the meeting discussions are held on the uncertainties of each variable. Discussions are held until consensus opinions have been reached. Brainstorming sessions can drag on when issues are controversial. Often, strong personalities dominate the discussion and good ideas or opinions can be missed.

The Monroe approach [Monroe (1997)] of soliciting expert opinion was specifically developed for risk analysis in a conceptual design environment. The technique presented by Monroe (1997) uses a set of questionnaires to qualify and quantify uncertainty associated with parameters as obtained from experts. This method elicits minimum, most likely and maximum values of an input parameter. Experts are asked for cues that help shape their opinion. Cues from each expert are then shared with other experts and each is asked to reexamine their first opinion and revise it if appropriate. The methodology was used in determining uncertainty associated with weight estimating relationships for a launch vehicle in the conceptual design phase. Monroe hypothesized the usefulness of the technique in other decision-making arenas analogous to the conceptual design phase of a launch vehicle.

In a conceptual design domain, solicitation is often required to be accomplished at a distance. The Delphi and Monroe methods would then be suitable. The Delphi method requires several iterations and can become time consuming to achieve convergence. The Monroe expert judgment methodology seeks to take advantage of the effectiveness of questionnaires while eliminating the repetitive steps of the Delphi method.

It should be noted that studies conducted by previous researchers indicate that using human judgment as a basis for making decisions can produce poor results.

Christensen-Szalanski and Beach (1984) point out that there have been a number of studies that support the use of human judgment, but these studies have not received as much attention as studies which do not support the practice. The authors' term for this phenomenon is citation bias. The fact that expert opinion is included in this research methodology is not an argument for or against the use of human judgment. That debate is beyond the scope of this study which assumes that there is a need to use expert opinion due to a lack of historical or statistical data. The use of expert judgment techniques in this manner coincides with Dalkey and Helmer (1962) recommended utilization. Studies by Ettenson and Shanteau (1987), Einhorn (2000) and Monroe (1997) highlight the conditions that are necessary to identify experts when judgments are the source of statistical data. In an effort to identify conditions which impact the validity of expert judgments, Beach (1975) found that experts do better when asked for an upper and lower bound around a midpoint rather than for probability distributions.

2.2.3 Uncertainty Quantification

Having elicited data from the experts concerning an uncertain variable, it is then necessary to fit probability distributions (risk profiles) to the information obtained. Fitting probability distributions to data assumes that sufficient information is available to perform the required analytical process and that an analytical technique (e.g., the extreme condition approach) is not being employed to quantify uncertainty. There are numerous articles and text that address probability modeling for both historical and expert judgment data. Examples include Vincent (1998), Law and Kelton (1991), Mendenhall and Sincich (1995) and Haldar and Mahadevan (2000a and 2000b). Vose (1996) states that as a rule for modeling expert judgment, non-parametric distributions are more flexible and reliable than parametric distributions. He also points out that there are exceptions to the rule. For example, a triangular distribution is the most commonly used distribution for modeling expert opinion. Other distributions appropriate for the task are the BetaPERT, the modified BetaPERT, the general, the cumulative and the discrete distributions. Beach (1975) and Monroe (1997) are proponents of eliciting expert judgments by requesting minimum, most likely and maximum values. Three distributions fit this requirement and are discussed below.

The triangular distribution is an approximate modeling tool used when end points and most likely value can be estimated. It has no theoretical basis, but derives its statistical properties from its geometry. The flexibility of the shape of this distribution coupled with its ease of use make this a popular distribution. Estimating end points, which are absolute extremes, is sometimes difficult. This is a drawback to the use of this tool. The ability of the triangular distribution to maintain skewness is a strength when considering its use as an input distribution. Figure 4 illustrates examples of some triangular distributions.

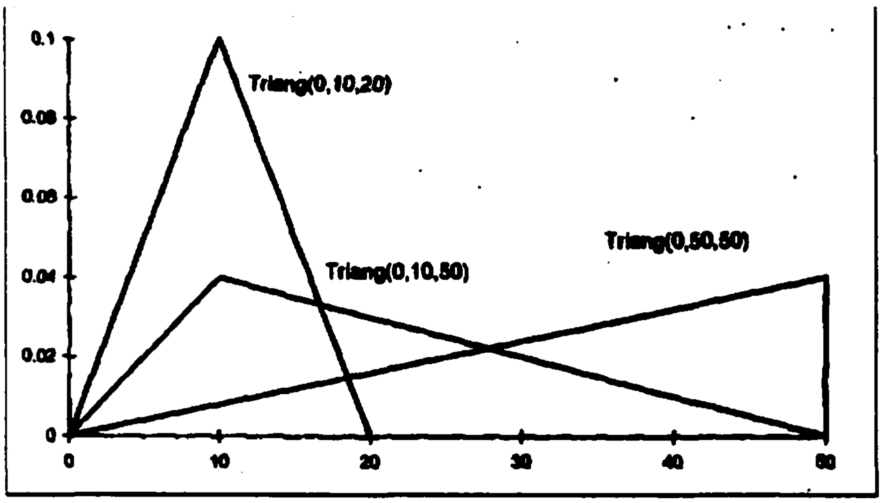

Figure 4 Triangular Distributions [Vose, 1996]

The BetaPERT is a four-parameter version of the Beta distribution. It rescales the beta distribution to model a variable that runs between two points. The formula used is provided by the probability density function. The BetaPERT has been used to model activity duration in PERT networks. It assumes that mean = (minimum + 4* most_likely + maximum)/6. The mean of the BetaPERT distribution is four times more sensitive to the most likely value than the end values. This is different from the triangular distribution mean which is equally sensitive to all three points. The standard deviation of a BetaPERT is also less sensitive to the estimates of the extreme. The BetaPERT can produce shapes with varying degrees of uncertainty (Figure 5).

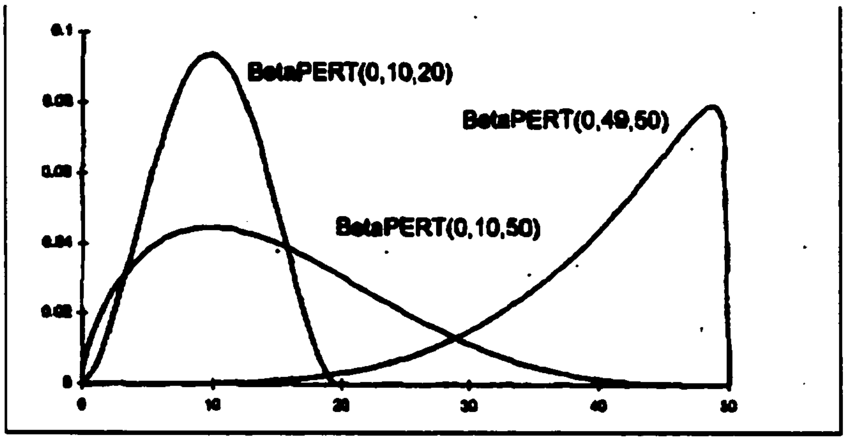

Figure 5 BetaPERT Distributions [Vose, 1996]

The modified BetaPERT (Figure 6) allows the user to vary the degree of peakness of the distribution. The modified BetaPERT has a mean = (minimum + γ*most likely + maximum)/(γ+2). In the standard BetaPERT, γ =4. As γ increases, the distribution becomes more peaked around the most likely value. Experts estimate the same three values of minimum, most likely and maximum. The values of γ are varied and the distribution plotted for each γ. The expert is then allowed to choose which distribution best fits his opinion. This distribution is used when experts have a good understanding of statistical distributions.

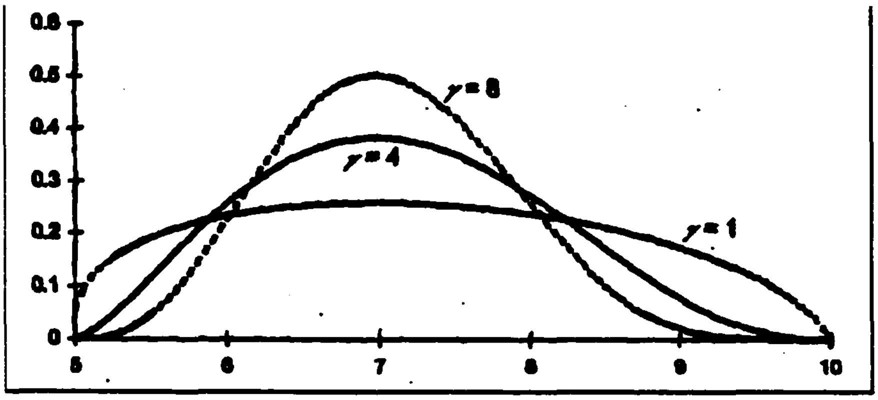

Figure 6 – Modified BetaPERT [Vose, 1996]

Of the three distributions just discussed, the triangular distribution is the most intuitive and easiest to use. Haimes (1998) states that the triangular distribution is an ideal approach for soliciting expert opinion when the expert is not comfortable with the assessment of probabilities.

Having quantified individual expert opinions, the next step in this methodology is to aggregate those opinions. Monroe (1997) did not implement a methodology for combining multiple expert judgments, although it was suggested that such an approach is needed. While the research performed by Monroe (1997) extracted opinions from several experts, analyses were conducted using each individual expert's opinion. There was no attempt to combine the expert judgments prior to conducting analysis. Vose (1996) provides recommendations on such methods that facilitate integration of multiple opinions into probability distributions.

There are mathematical, behavioral and mixed approaches to aggregating expert judgments. Mathematical approaches involve the statistical integration of a number of opinions into a single judgment. Behavioral approaches entail interaction of the entire group of experts until a consensus is achieved (Rowe 1992). With respect to mathematical aggregation, Rowe addresses the fact that composites formed by combining judgments have frequently been shown to outperform individual judgmental tasks requiring subjective input. Some researchers oppose the integration of expert judgments and infer that accurate models would eliminate the need for such integration. Other researchers believe that aggregation of opinions is simply a substitute for our inability to identify the most expert individual (Rowe 1992). Rowe, further, suggests that behavioral aggregation should be used when there is a variance of opinion in member expert groups. Additionally, there are two mixed approaches noted by Rowe (1992): Delphi and Nominal Group Technique. Monroe (1997) suggested a mixed approach that utilizes questionnaires to effectively ameliorate bias among experts.

Whether the aggregation approach is mathematical, behavioral or mixed, the integration technique must consider applying weighting factors to individual expert opinions. With respect to weighting factors, Genest and Zidek (1986) stated that preference-based opinion is not part of statistical science, but is treated as a group decision problem. This statement would lead one to explore decision theory and the concept of utility to a decision maker. In keeping with decision theory, the derivation, quantification and application of weighting factors should be determined by a process and it is that process that should be logical and repeatable. Of importance is whether the group must agree to the resulting aggregation opinion as an expression of consensus. This particular problem has not been treated in statistical literature [Genest and Zidek (1986)] and is not considered here.

Many researchers on the subject of aggregating expert opinion agree that modeling is the most appropriate method of combining opinions. A major concern in the expert resolution literature is whether probabilities should be combined via a multiplicative rule, weighted average or some other type of formula [Winkler (1986)]. Vose (1996) advocates the weighted average method while some researchers [Winkler (1986); Clemen and Winkler (1993)] advocate the Bayesian approach. The Bayesian approach is thought to be relatively straight forward, but difficulty exists in assessing the likelihood function [Clemen and Winkler (1993)]. Vose (1996) has used discrete distributions to aggregate opinions for a number of years with good results and it is relatively easy to implement.

In this research, the discrete distribution was chosen to aggregate multiple expert opinions as it provides for an accurate representation of the combined opinions given the relative importance of each expert judgment.

2.3 Uncertainty Propagation

Uncertainty propagation is a major element in the proposed research, consequently, it is essential to discuss relevant literature on the topic. Fortunately, the propagation of error is a classical problem and principles are well documented (Klir, 1994 and Evans, 1992). Although uncertainty can be modeled as either a linear or non-linear function, most analysis procedures assume linearity. This assumption significantly reduces the complexity of uncertainty modeling especially with respect to the propagation across multidisciplinary systems.

Ayyub and Chao (1997) present uncertainty modeling theory. Gu, Renaud and Batill (1998) elaborate on single discipline uncertainty modeling within their study. Box, Hunter and Hunter (1978) is an excellent text on statistics for experimenters and is still cited in many studies today when discussing uncertainty propagation. Gu, Renaud and Batill (1998), and Du and Chen (1999) develop theories for "worst case" uncertainty propagation across coupled systems. Balci (1998) explores the propagation of uncertainty modeling using the extreme condition approach in discrete event simulation.

2.3.1 Monte Carlo Methods

Monte Carlo simulation is one of the traditional tools for propagating uncertainties in risk analysis modeling. Monte Carlo algorithms are available for all of the distributions considered feasible for expert judgment data acquisition by Vose (1996). Emphasis on efficient Monte Carlo sampling dates back to the 1950s, and efficiency issues are just as important today as we strive to solve problems of larger scope and complexity (Gentle, 1998). By increasing the number of observations, the variance in computed results can be reduced (Hammersley and Handscomb, 1964). Consequently, to improve accuracy of Monte Carlo simulations, a large number of iterations are typically required. Techniques, such as importance sampling, stratified sampling, control variates, antithetic variates, and Latin Hypercube sampling have been used to reduce the number of iterations required to improve computational efficiency. These methods are known as variance reduction techniques and are addressed in Gentle (1998), Hammersley and Handscomb (1964), Law and Kelton (1991), Kleijnen (1998) and numerous other text. Other variance reduction techniques or sampling schemes are mentioned in some of the more recent articles and text. The techniques listed here are traditional in that they are covered in the older text and articles as well as the modern literature.

Hammersley and Handscomb (1964) demonstrate through calculative procedure that methods used as variance reduction techniques are more efficient than Monte Carlo experiments with random sampling. Here the terms experiment, methods and simulation are used interchangeably with regard to Monte Carlo techniques. Hammersley and Handscomb (1964) used a function for which there was an existing analytical solution to demonstrate the gains in efficiencies when employing stratified sampling, importance sampling, control variates or antithetic variates verses random sampling in Monte Carlo methods.

McKay, Beckman and Conover (1979) introduced Latin Hypercube sampling in a study where a comparison of the efficiency with that of random sampling and stratified sampling techniques was made. Latin Hypercube sampling was shown to reduce the sampling error significantly over the two comparative techniques. Beckman and McKay (1987) and Tang (1993) provided empirical evidence that Latin Hypercube sampling was more efficient than simple random sampling. Additionally, Stein (1987) showed that the variance of Monte Carlo estimators using Latin Hypercube sampling was smaller than the variance of Monte Carlo estimators using random sampling.

An objective of sampling is to reduce the variance of the estimators while preserving other good qualities, such as unbiasedness [Gentle (1998)]. When discussing Monte Carlo sampling procedures, we are discussing variance reduction in Monte Carlo applications. It should be noted that there are other Monte Carlo variance reduction methods that are not specifically sampling techniques. Analytic reduction, antithetic variates and common variates are examples of purely variance reduction techniques.

Random Sampling uses independent random numbers between 0 and 1 to generate variates from a specific input distribution. Using this technique, random numbers are equally likely to occur, but variates with higher probability of occurrence are more likely to be generated. With enough iterations, this sampling technique recreates the input distribution. When a small number of iterations are performed, variates tend to cluster around high probability outcomes and the input distribution is not recreated accurately enough. It takes many iterations when using random sampling for the mean to converge upon the true mean and stabilize. Other statistics used to assess convergence include skewness, percentiles and standard deviation.

In Stratified Sampling, the rule is to sample where values are likely to exhibit a lot of variability. In this sampling technique, distinct subregions (or strata) are formed. Within these strata, random sampling is conducted. As each region of the distribution function is sampled, convergence happens more rapidly with Stratified Sampling than it does with Random Sampling.

The objective of Importance Sampling is to concentrate the distribution of sample points in parts of the interval that correspond to large values or areas of more "importance." Importance Sampling also results in improved efficiencies over random sampling. Hammersley and Handscomb (1964) show that Importance Sampling and

Stratified Sampling result in about the same improvement in efficiencies over Random Sampling.

The principle behind Latin Hypercube Sampling is to sample equally along the entire distribution function. Latin Hypercube Sampling is a form of Stratified Sampling. The strata form equal probability regions. Contrary to stratified sampling, only one sample is taken from each strata. With Latin Hypercube Sampling, input distributions are more accurately reflected by the samples. It avoids the problem of clustering associated with the Random Sampling technique when insufficient iterations are conducted. McKay, Beckman and Conover (1979) show that Latin Hypercube Sampling is an improvement in efficiencies over random sampling and stratified sampling. Based on results by Hammersly and Handscomb (1964), one may infer that Latin Hypercube Sampling is also an improvement over Importance Sampling. Latin Hypercube Sampling forces simulation sampling to include low probability events which produces more accurate simulation outputs. Latin Hypercube Sampling provides for faster run times by requiring fewer iterations for convergence.

In a developing research-in-process, Du and Chen (1999) explore a statistical approach to propagating uncertainty which includes the use of a Monte Carlo simulation with random sampling techniques. Probability distributions were developed for input data and response surface equation errors. Then, a statistical analysis with Monte Carlo simulation was used for propagation of uncertainties across multiple designs. The sampling procedure used was random, but the use of Latin Hypercube Sampling to improve computational efficiencies was hypothesized. As the results of the literature review suggests, the use of Latin Hypercube Sampling, as well as other sampling techniques, provides improvements in computational efficiencies over random sampling.

2.3.2 Analytical Solutions

Analytical solutions are approximations and are not as accurate as simulations. Additionally, as the number of modeling input parameters increase, First Order and Second Order approximation methods can take as long as Monte Carlo simulations to converge upon an output variable.

2.3.2.1 FORM, SORM and FPI

Techniques such as First Order Reliability Method (FORM) (Ayyub and McCuen, 1997) and Second Order Reliability Method (SORM) (Ayyub and McCuen, 1997) are analytical approximation methods that have their roots in reliability engineering (See figure 7). FORM utilizes a reliability index (β) and the cumulative probability distribution function of the standard normal variate (Φ) to predict the probability of failure (Pf). Tables are used to obtain the cumulative probability distribution function of the standard normal variate. FORM uses a Taylor series expansion about the mean values of the basic random variables and truncates the series to the first order terms (Ayyub and McCuen, 1997). This yields a first order approximate mean and variance for inclusion into the reliability index equation. Using the second order mean (including the square term in the Taylor series expansion) improves the accuracy of the mean estimation.

Fast Probability Integration (FPI) is another analytical approximate solution that has been used in reliability-based risk applications. "Linearizing the failure function and approximating the non-normal variables by normal functions leads to very simple approximations" (Chen and Lind, 1983). It is this linearization and normalization that is called Fast Probability Integration (FPI). FPI gives good approximation to small probabilities in the 10-3 to the 10-7 range. This analysis technique approximates the tail portion of a function. Since probabilities in reliability analyses are typically small, they would fall within the tail section of the distribution. The errors have been shown to be within five percent of the Monte Carlo solution (Chen and Lind, 1983). Users should be cautioned of employing FPI outside of its intended range. The closer the probability is to the center of a non-normal distribution, the more the error increases. FPI methods have been developed by Rackwitz-Fiessler (R-F) and Chen-Lind (C-L). Wu (1986) examines a new FPI algorithm that was proposed by himself and Wirsching in an earlier 1985 effort. This new FPI is an extension of the R-F and C-L schemes. As a result of the inaccuracies outside of the tail portion of a non-normal distribution, FPI should be used with caution in risk analysis.

2.3.2.2 FOSM and SOSM

The First Order Second Moment (FOSM) and Second Order Second Moment (SOSM) methods have been used in Computational Fluid Dynamics (CFD) and Finite Element Analysis (FEA) to propagate uncertainty within risk analysis [Putko et al, 2001]. To use FO and SO analysis, CFD output solutions are approximated using Taylor series expansions. First-order Taylor series approximations are used for FOSM and second-order Taylor series approximations are used for SOSM. Expected values for the mean (first moment) and variance (second moment) of the output function are then obtained. The FOSM and SOSM methods are straight forward, but difficulty lies in computation of sensitivity derivatives (SDs) from the CFD codes. Putko et al (2001) use the approach suggested by Taylor et al (2001).

2.3.2.3 Extreme Condition Approach

The extreme condition approach for two coupled simulation tools is presented by Du and Chen (1999). The approach is appropriate across systems for which a range is known for each input variable. A range of error functions for the two simulation tools would also need to be available. The first simulation tool is both minimized and maximized over the range of the input variables plus the error. The output of this minimization and maximization effort is a range for the linking variable (Y). The second simulation tool is then minimized and maximized over the range of the linking variable (Y) plus the simulation tool error. The result is a range for the output variable (Z) of the coupled system.

2.4 Risk Management

Kumamoto and Henley (1996) refer to probabilistic risk assessment as "...more scientific, technical, formal, quantitative, and objective than the management phase, which involves value judgments and heuristics, and hence is more subjective, qualitative...." Using robust design techniques for risk management is a means of eliminating some of the subjectivity to the conduct of risk analysis.

In the past, mitigation strategies have focused on reducing the magnitude of response variable variations. More recently, techniques have been generated that reduce the impact of potential variations by manipulating controllable variables (Du and Chen, 1999). These mitigation strategies are based on the principles of robust design. Robust design uses mathematical formulations from statistical design of experiments to obtain information about design variables involved in making engineering decisions (Phadke, 1989).

2.4.1 Design of Experiments

There are several good texts on statistical design of experiments. Among the more noted authors are Box and Draper (1969), Box and Draper (1987), Box, Hunter and Hunter (1978), Hicks (1964) and John (1971). Methods cited in these publications include full factorial designs, fractional factorial designs and response surface techniques. Full factorial experiments require significant computational time when experiments involve large factor numbers accompanied with two or more factor levels (Law and Kelton, 1991). Fractional factorial designs are a variation on full factorial designs and require less computational effort. Unimportant factors are screened out in configuring the experiment and attention is then given to the remaining factors.

Design of experiments emphasizes the experiment, the design and the analysis. The experiment consists of a problem statement, identification of factors and response variables. The design focuses on the number of observations, the order of the observations, methods of randomization and mathematical model representations. Analysis entails the data collection methods, computation test statistics and the interpretation of results. The objective of design of experiments is to obtain more information for less cost than can be obtained by traditional experimental studies (Hicks, 1964).

2.4.1.1 Taguchi Methods

"The Taguchi Method uses orthogonal arrays (OA) from design of experiments theory to study parameter space with a small number of experiments" (Unal et al, 1993). Arrays are fractional factorial designs and illustrate that full factorial designs can be reduced while still maintaining statistical significance. The Taguchi method identifies controllable parameter settings that optimize the system response variable and reduce design sensitivity to variations in other uncontrollable parameters. Phadke (1989), Unal et al. (1993), as well as other authors, outline the process of performing Taguchi's method. Taguchi has been credited with making optimization user friendly for engineers who have little or no training in optimization methods (Chen et al, 1996). Box (1988) criticizes Taguchi, however, for the statistical methods being "sometimes unnecessarily inefficient and complicated." Shortcomings of the Taguchi method include the fact that it is not accurate for nonlinear design problems and that it involves a single performance measure. Chen et al. (1996) recommend that multiple performance measures be utilized as there are multiple objectives for design systems.

2.4.1.2 Response Surface Methodology (RSM)

Response Surface Methodology involves a dependent variable (the response variable) and several independent variables (control variables). By careful design and analysis of experiments, RSM seeks to relate a response or dependent variable to the levels of a number of controllable input variables that affect it (Box and Draper, 1987). The objective is to optimize the response variable through the use of an estimating algorithm. RSM is covered by Box and Draper (1987), Box, Hunter and Hunter (1978), Hicks (1964), and Law and Kelton (1991).

2.4.2 Sensitivity Analysis

Risk analysis models are constructed based on certain assumptions and premises. Since most systems are dynamic, assumptions for models may not be representative of changing conditions. Additionally, model output may be sensitive to certain parameters. Sensitivity analysis provides a methodological framework in order to evaluate the sensitivity of model output or constraints to changes in model parameters (Haimes, 1998).

2.4.3 Decision Analysis

Decision analysis is "a formalization of common sense for decision problems which are too complex for informal use of common sense" (Eppen et al, 1993). It entails assigning utilities to projected outcomes and optimizing the expected utilities. Raiffa (1968) provides an elementary discussion on the application of utility functions. Quantification of preferences is the precursor to developing utility functions. LaVille (1978) outlines the fundamentals of decision analysis which includes development of preferences and utility functions.

Decisions in a multidisciplinary design environment are, relatively, straight forward when the optimization problem has only one response or performance measure. Tradeoffs between multiple and often conflicting objectives is at the heart of risk decision-making (Haimes, 1998). When multiple performance measures are required, additional techniques to those used for single objective problems are required to make a decision or manage uncertainty.

Compromise programming is an interactive method appropriately used in a multiple linear objective problem. Compromise programming identifies solutions that are closest to the ideal solution as measured by some distance (Goicoechea, Hansen and Duckstein, 1982). The ideal solution is typically not attainable, but serves as an evaluation standard for nondominated or Pareto solutions.

The concept of multiple objective optimality is necessarily different from single objective optimization. A Pareto optimal solution falls in the category of multiple objective optimality. It is that solution that improves upon one objective function at the expense of another objective function (Haimes, 1998). Pareto solutions are also known as nondominated solutions. Chen et al. (1988) present a strategy by combining Response Surface Methodology with the compromise Decision Support Problem (DSP) for obtaining a multiobjective solution.

2.5 Current Risk Analysis Research

Figure 8 illustrates the research that is currently being conducted in the area of multidisciplinary design risk analysis. Many current researchers are using metamodels or response surfaces to simulate the modeling tools in the conceptual design phase. Analytical solutions are primarily being explored in an effort to speed up computations while propagating uncertainties. Analytic solutions are deemed appropriate in a conceptual design environment because of the need to obtain approximate, but adequate, information in this phase of design. More accurate and costly simulation solutions are typically performed in the detailed design phase.

Monte Carlo solutions can be implemented with both metamodels and the modeling tools. A goal of launch vehicle research is to perform risk analysis on existing modeling tools without the need for development of response surfaces. This research study and the work performed by Du and Chen appears to meet that need. The use of Monte Carlo simulation with Latin Hypercube Sampling can be used in the conceptual design phase of a complex system to propagate uncertainties. This technique is just as accurate as Monte Carlo simulation with random sampling but provides for faster convergence. Additionally, the technique could be used in the detailed design phase, thereby only requiring an update of data from the conceptual design phase. Savings in computational manpower should be realized.

Although work with modeling tools and Monte Carlo simulations is being conducted for single disciplines [Monroe (1997), Putko et al. (2001) and Smith and Maheadevan (2001)], no research has been identified with coupled multidisciplinary design tools using either Monte Carlo simulations or analytic methods to propagate uncertainties.

2.6 Available Risk Analysis Software

There are software programs developed to work with spreadsheets for simulating the simple risk analysis tasks associated with project management and financial applications. @Risk works with Microsoft Excel and Microsoft Project. Other software products include Monte Carlo, Opera, Predict!, Risk 7000, Risk+ and Crystal Ball.

The software used in this research and risk analysis application would have to support existing systems at NASA Langley Research Center used for launch vehicle conceptual design. Current analysis programs are written mostly in FORTRAN and some programs are in C++. Although commercial software does not interface with these existing systems, their use could serve to validate results of programs developed using C++ or FORTRAN.

2.7 Literature Review Summary

Figure 9 summarizes the results of the literature review. This risk analysis strategy will be further developed in Chapter III.

Uncertainties in multidisciplinary design can be identified as either internal or external uncertainties. These uncertainty types can be quantified using a variety of probability distributions following data collection using historical data or expert judgment. Expert judgment is suitable for this domain. In multidisciplinary design, there are a variety of methods by which the efficiency of Monte Carlo simulations can be improved. Utilizing any one of the variance reduction techniques cited in 2.3.2 in place of random sampling would facilitate an increase in computational efficiency. Latin Hypercube sampling appears to have the greatest opportunity for improved efficiency. Using robust design strategies such as Taguchi's orthogonal arrays or response surfaces to mitigate uncertainty are acceptable strategies used in multidisciplinary design. Response surfaces already exist for the specific problem under consideration and are therefore chosen to support this research for development and application. Finally, decision analysis for multiobjective criteria can be conducted by employing the compromise decision support problem and Pareto solutions as used by Du and Chen (1999).

This literature review has covered a broad range of topics. Each of the individual topics has a large volume of literature associated with it. This review was not intended to provide complete coverage of the individual risk analysis elements. The literature review is intended to identify techniques and procedures that are relevant to multidisciplinary design optimization risk analysis.

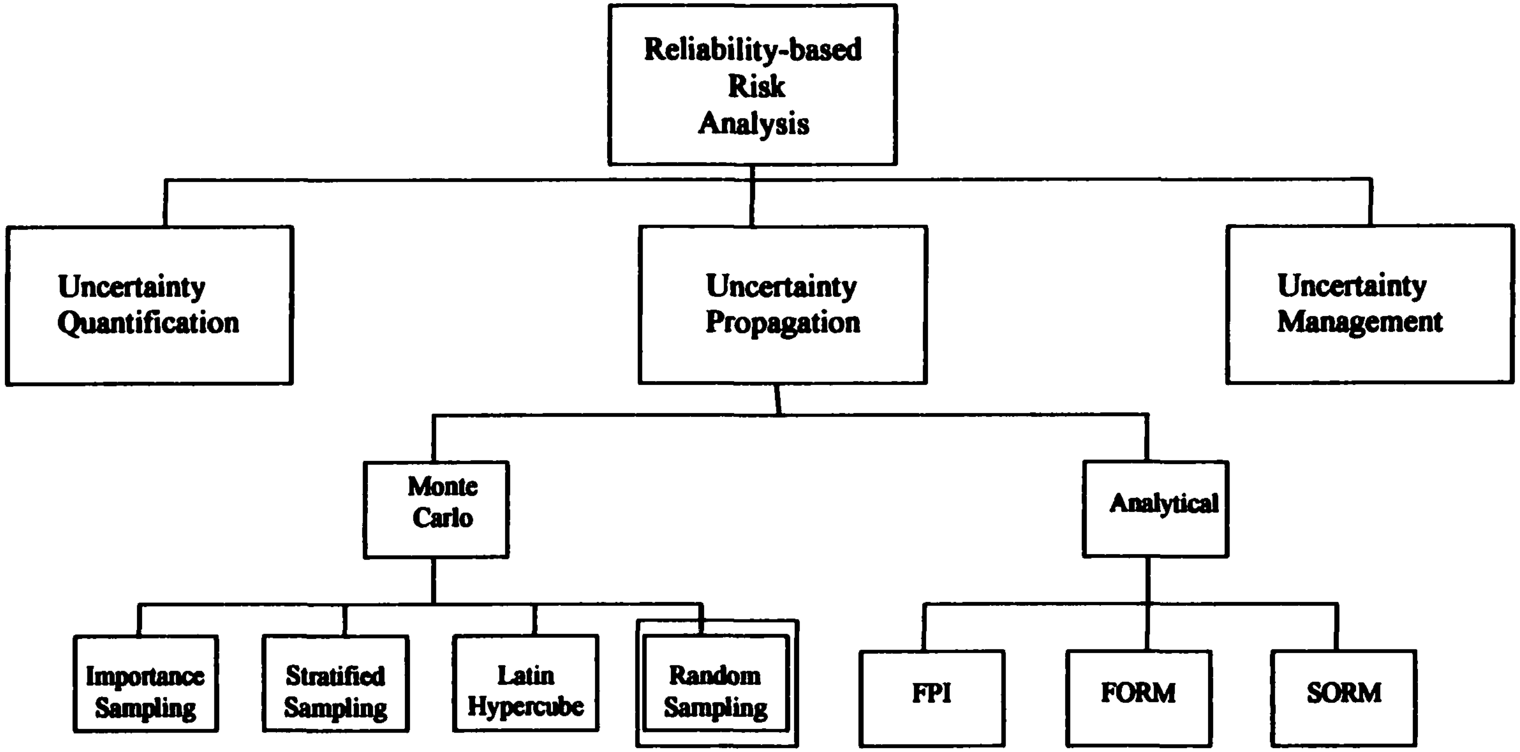

Figure 7 - Reliability-Based Risk Analysis

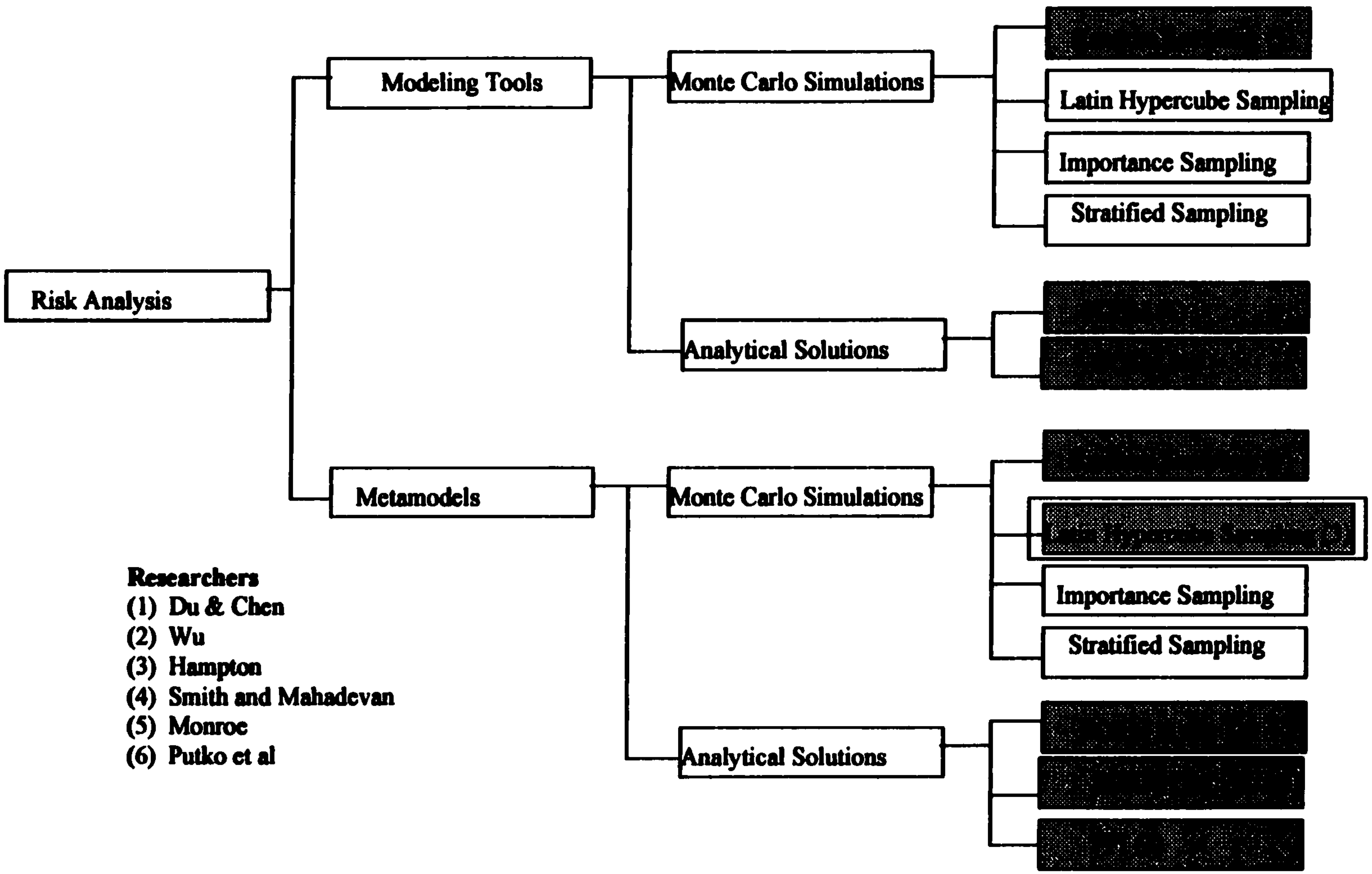

Figure 8 - Multidisciplinary Design Research

2.8 Contribution

The concept of using uncertainty analysis within a design environment is not new, but its extension to handle multiple disciplines within a complex and integrated engineering problem such as launch vehicle design has yet to be attempted. Stochastic optimization methods that use uncertainty information have been minimally developed; however, a general approach to create a multidisciplinary design capability which is based on uncertainty analysis currently does not exist. This research contributes to the literature of multidisciplinary design optimization (MDO) by promulgating a strategy for conducting uncertainty analysis in a multidisciplinary design environment.

The selection of the proposed modeling/optimization problem is an extension of the Du and Chen (1999) research in that it is applied to a real-world problem that is more complicated than the analytical model used in their study. This research study includes one variable input parameter and multiple input parameter distributions instead of one. The extension of the Du and Chen methodology to a real-world complex system was suggested by the authors themselves. This research also extends the work of Richard Monroe with expert judgment data collection techniques and incorporates those techniques into the methodology developed by Du and Chen (1999). The current research further extends the work of Unal, Lepsch and McMillin (1998) with respect to optimization of integrated response surfaces for a launch vehicle.

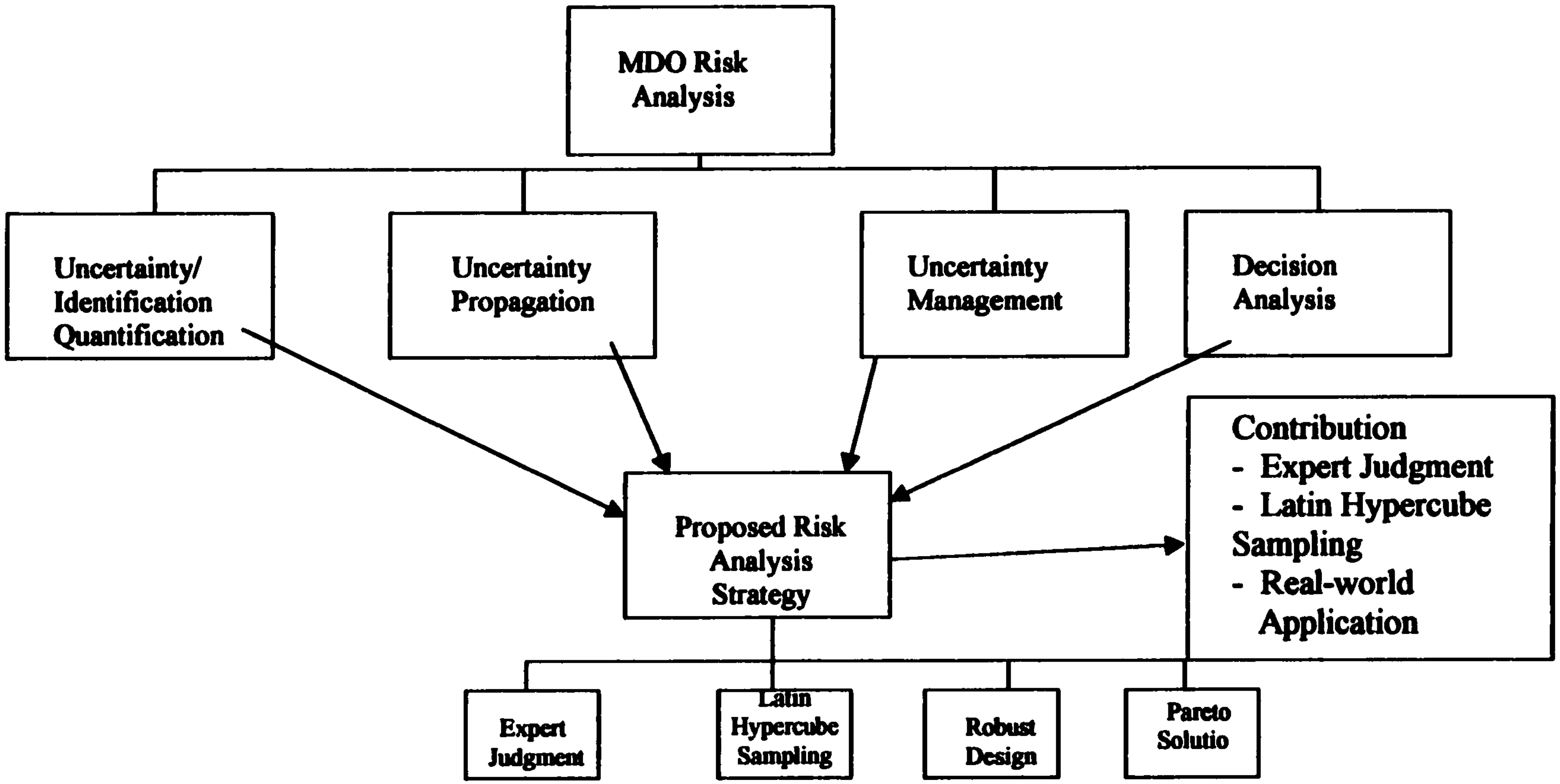

Figure 9 - Proposed Strategy Map

CHAPTER III
METHODOLOGY

Figure 10 illustrates the concept of uncertainty propagation and management within a multidisciplinary environment. The research will be divided into three stages or modules. The first two modules include the quantification and propagation of uncertainties. The final stage involves the management of uncertainties or response optimization.

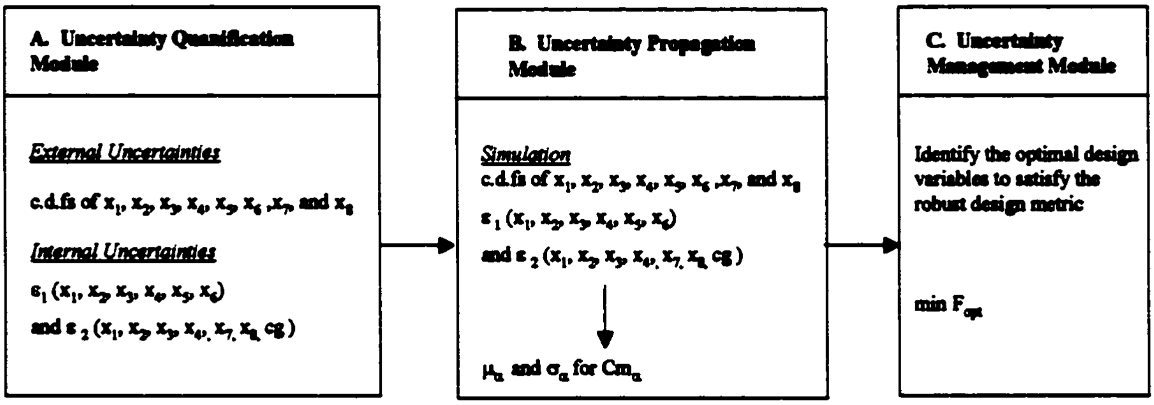

Figure 10 Integrated Uncertainty Mitigation Strategy

The practical application of this methodology is on a weights & sizing and aerodynamics optimization problem for a launch vehicle concept. The research uses the two response surface models that had been developed to approximate the disciplinary analysis codes used in the design process by Unal, Lepsch and McMillin (1998). The two simulation models selected were part of a configuration optimization study conducted on a single-stage-to-orbit launch vehicle and were second order equations. The first response surface equation was developed from 45 designed experiments using the Configuration & Sizing (CONSIZ) tool. The output of this tool is center of gravity with payload included (Cgin). The second response surface equation was developed from designed experiments of the Aerodynamic Preliminary Analysis System (APAS). The output of this tool is pitching moment coefficient (Cm) for specific Mach numbers. This research only examines Mach 0.3. Figure 11 illustrates the interaction of the two response surfaces with their associated uncertainties. The objective is to optimize the pitching moment coefficient (Cm) over the range of angles of attack (α) and other design variables. These variables include fineness ratio (FR), wing area ratio (WA), tip fin area ratio (TFA), body flap area ratio (BFA), ballast weight (BL), mass ratio (MR) and elevon deflection (DELEV). Modeling tool error for both CONSIZ (ε_1) and APAS (ε_2) are included in the solution of the problem.

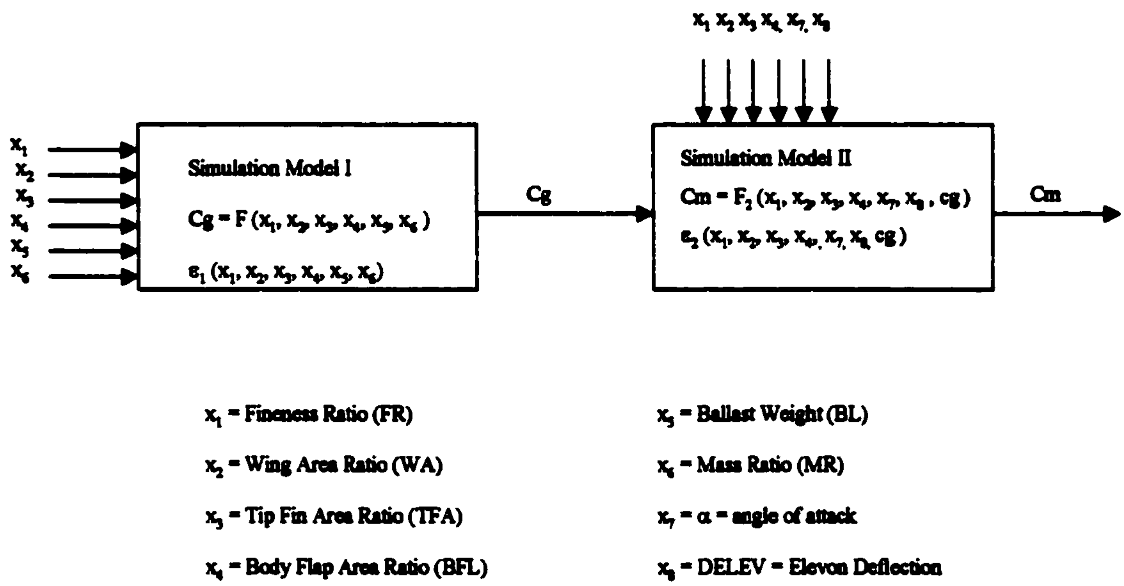

Figure 11 Integrated Simulation

3.1 Uncertainty Identification/Quantification

3.1.1 Uncertainty Identification

Uncertainty caused by variability takes three forms. Temporal, spatial and individual heterogeneous variability are the result of inherent fluctuations or differences in the quantity of concern [Haimes, 1998]. Temporal variability fluctuates with time. Spatial variability fluctuates according to geography and individual heterogeneous variability covers all other sources of fluctuation.

Uncertainty caused by a lack of knowledge also takes three forms. These forms are model, parameter and decision uncertainty. Model uncertainty is potentially the largest contributor of error if it is improperly treated. The use of surrogate variables or the exclusion of variables is potentially a source of modeling uncertainty. The impact of rare situations on models is a source of uncertainty. Modeling uncertainty can also be the result of the use of approximations, conflicting expert opinions or using an incorrect form for the basic model. Parameter uncertainties can be the result of random errors in direct measurements or systematic errors induced by the method of sampling. Parameter uncertainty also exists simply because of unpredictability.

Decision uncertainty arises when there is controversy over how to compare or weigh objectives. The first source of decision uncertainty is found in the selection of an index to determine risk. The second source of decision uncertainty is in the evaluation of the cost of risk. The final source of decision uncertainty is the quantification of value, the acceptable level of risk. Uncertainty specific to this research is discussed further in Chapter IV.

3.1.2 Uncertainty Quantification

In this study, external uncertainty quantification is accomplished using the expert judgment approach of Monroe (1997). The expert judgment technique used in this research replaces weight estimating parameters with weights & sizing parameters. The following steps have been derived from the steps suggested by Monroe (1997) in obtaining data from multiple experts:

i. Select the parameters for risk that will be evaluated for uncertainty.
ii. Rate the parameter for uncertainty using low, most likely and high values
iii. Document reason for uncertainty for each parameter rated
iv. Prompt expert for cues to further document the thinking process
v. Provide expert the opportunity to revise estimates

Monroe (1997) advocated a questionnaire approach to quantifying risk associated with internal uncertainties. This research will extend that principle to external uncertainties. An initial assessment of ranges for each design parameter will be requested of the experts. The assessments will include low, most likely and high values. Then, the experts will be requested to review the initial valuations of design parameter ranges and to consider revising them. Finally, the experts will be requested to describe any scenario that might change the valuations that they have applied to any of the design parameters. This last step will serve to de-bias the judgment.

This research, additionally, makes provision for more than one opinion on parameter distributions. This provision requires the aggregation of multiple expert opinions for the various design parameters. Aggregation is handled computationally versus having brainstorming sessions to arrive at consensus estimates. The method recommended by Vose (1996) is utilized as it avoids some potential pitfalls. Vose recommends that a discrete distribution be developed from the combination of the distributions from each expert opinion. The expert opinion could take any of the forms suggested in Chapter II for fitting distributions to data. Vose also recommends that weighting factors be applied to each individual expert opinion distribution based on level of confidence associated with each individual expert. For example, if three experts are used in the data collection process, one expert may be more senior than the remaining two experts. In such a case, weighting among expert opinions may be 50%, 25% and 25%. The most senior individual would have the 50% weighting associated with his or her opinion. Figure 12 illustrates the technique. Goodness-of-fit statistics are observed to verify the degree of conformity of the distribution curves with the discrete data points.

Voses's recommended method avoids three potential problems previously encountered in the literature. The first problem is choosing the most pessimistic estimate. Such caution should only be applied at the decision-making stage after reviewing the risk analysis results. The second incorrect method would be taking averages of the two distributions. This method ends up with a distribution that is too narrow. The third problem is the aggregated distribution provides a positive value over a range that all experts agree should be zero. If all experts agree on the values of input parameters at a specific location in the distribution, then the discrete distribution provides for the consensus value to be employed. Using different types of distributions (e.g., normal or beta) to represent the aggregated opinions can often result in portions of the distribution curve that all experts agree are incorrect.

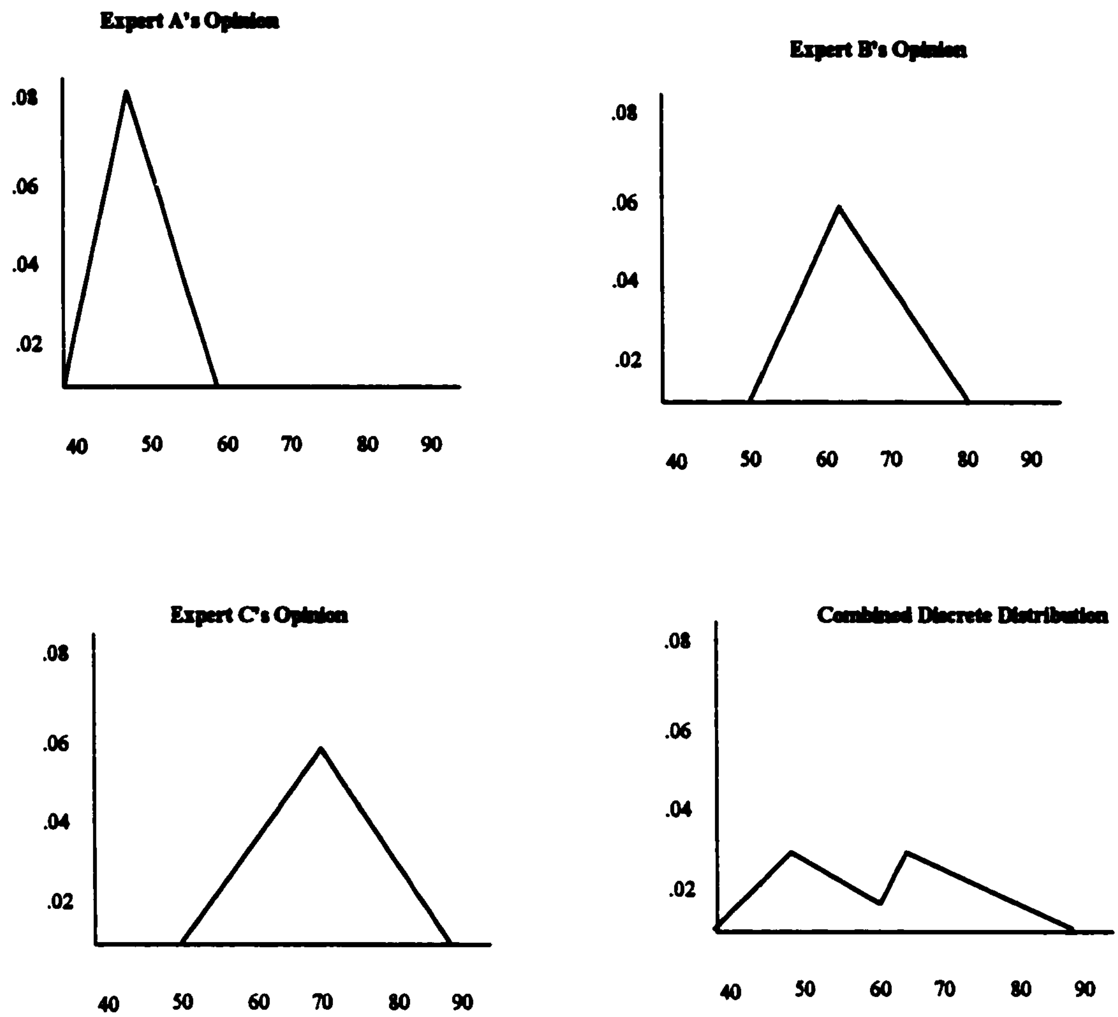

Figure 12 Expert Judgment Aggregation

Following quantification of the input parameter distributions, quantification of the error associated with the two response surfaces (internal error) is conducted. This quantification is accomplished by using samples from the input distributions. CONSIZ and APAS were executed for each of 45 and 180 design points respectively. The response surface equations were also executed using the same design points. The differences in the design points output of Cgin between the first response surface and CONSIZ is computed. An error function (ε_1) is developed for the first response surface equation under the assumption that the function is normally distributed. Next, the differences in the design points output of Cm between the second response surface and APAS is computed. An error function (ε_2) is developed for the second response surface equation, again, under the assumption that the function is normally distributed.

3.2 Uncertainty Propagation

Figure 13 was taken from Du and Chen (1999) and it provides an illustration of uncertainties being propagated between two disciplines or simulation tools. These uncertainties ($\varepsilon_1(x_1)$ and ε_2 (x_2, y)) impact the optimization of the system response variable, Z. Typically, a complete system design is a compilation of numerous simulation tools with their individual discipline bias and precision errors. Consequently, the system error accumulation has the potential of being significant. Figure 13 has been updated to suit the aerodynamic optimization problem and the changes reflected in Figure 11.

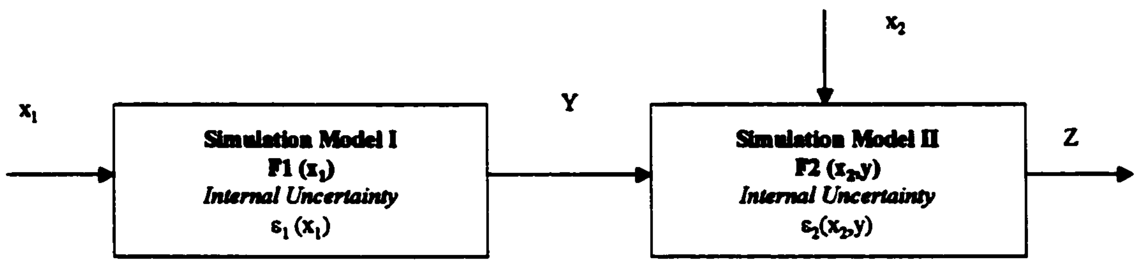

Figure 13 An illustrative simulation model chain [Du and Chen (1999)]

The propagation procedures for this launch vehicle analysis are shown in Figures 14 and 15. Figure 14 illustrates propagating uncertainties using LHS while Figure 15 illustrates the procedure using Random sampling. In each of the diagrams the following steps are applicable:

a) Sample from the eight external parameter distributions developed using the expert opinion elicitation strategy.
b) Sample from Simulation Model I (CONSIZ) error distribution.
c) Compute center of gravity using the first response surface equation and add the error computed in step (b).
d) Sample from Simulation Model II (APAS) error distribution.
e) Compute pitching moment coefficient using the second response surface and add the error computed in step (d). Return to step (a) for a specified number of iterations.
f) Obtain distribution for pitching moment coefficient for each of the various numbers of iterations.

This research compares two sampling techniques used when propagating uncertainties. Sampling is the process of drawing random values from an input distribution. With enough iterations, the sampled values for a probability distribution approximates the known input distribution. The specifics of the Latin Hypercube sampling and random sampling routines are outlined below.

3.2.1 Latin Hypercube Sampling

The technique used by Latin Hypercube sampling (LHS) is sampling without replacement. The number of stratifications of the cumulative distribution in LHS is equal to the number of iterations performed. A sample is taken from each stratification. Once a sample is taken from a stratification, this stratification is not sampled from again.

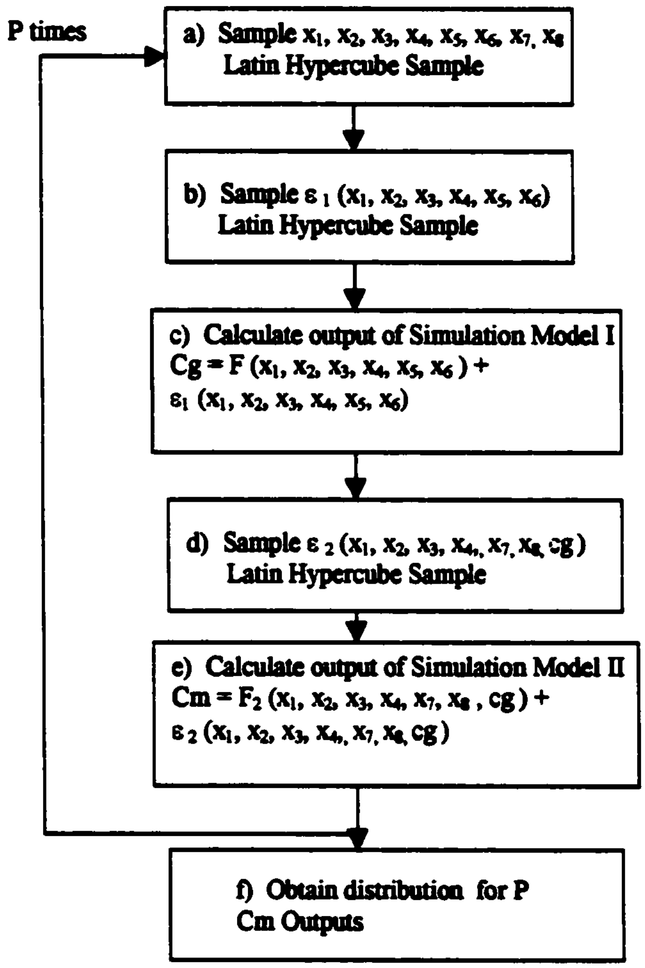

Figure 14– Uncertainty Propagation (Latin Hypercube Sampling)

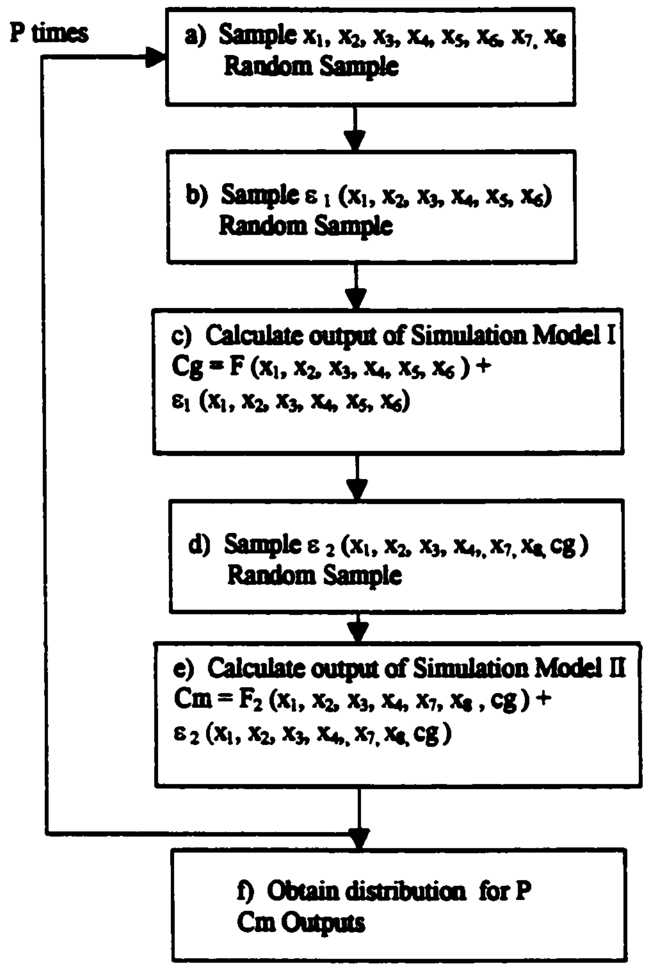

Figure 15– Uncertainty Propagation (Random Sampling)

When using the Latin Hypercube technique to sample from multiple variables, it is important to maintain independence between variables. The values sampled for one variable need to be independent of those sampled for another. This independence is maintained by randomly selecting the interval to draw a sample from for each variable. In a given iteration, variable #1 may be sampled from stratification #5, variable #2 may be sampled from stratification #7 and so on. This preserves randomness and independence and avoids unwanted correlation between variables [Palisade 2001].

Figure 16 illustrates the principle behind the technique. Here, the cumulative distribution curve is divided into five equal segments (stratifications). The sampling routine forces a design point to be selected from each stratification.

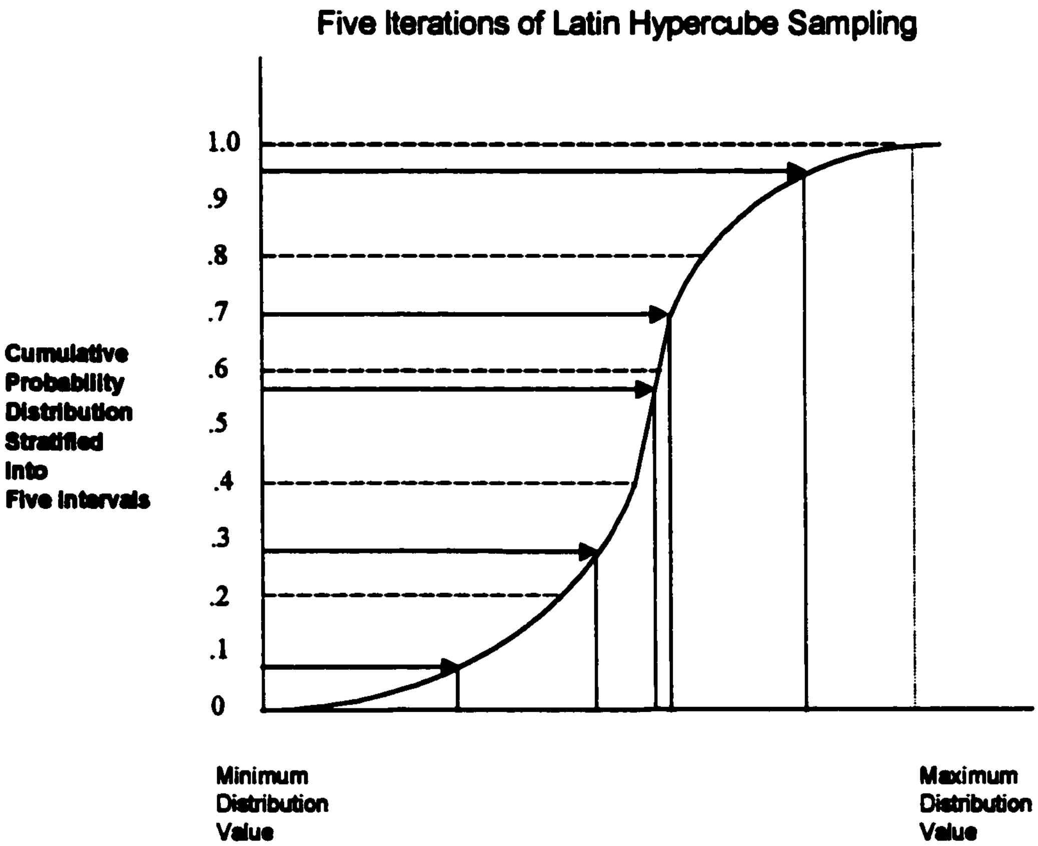

Figure 16– Latin Hypercube Sampling [Palisade 2001]

3.2.2 Random Sampling

Monte Carlo sampling refers to the traditional technique for using random numbers to sample from a probability distribution. Monte Carlo sampling techniques are entirely random and a sample can fall anywhere in the range of the input distribution. Samples occur must often in the areas of high probability. This results in what is known as clustering. With enough iterations, however, the input distribution can be represented accurately enough. Figure 17 illustrate the technique of Monte Carlo sampling (Random sampling) as five data points are taken from the cumulative distribution curve below.

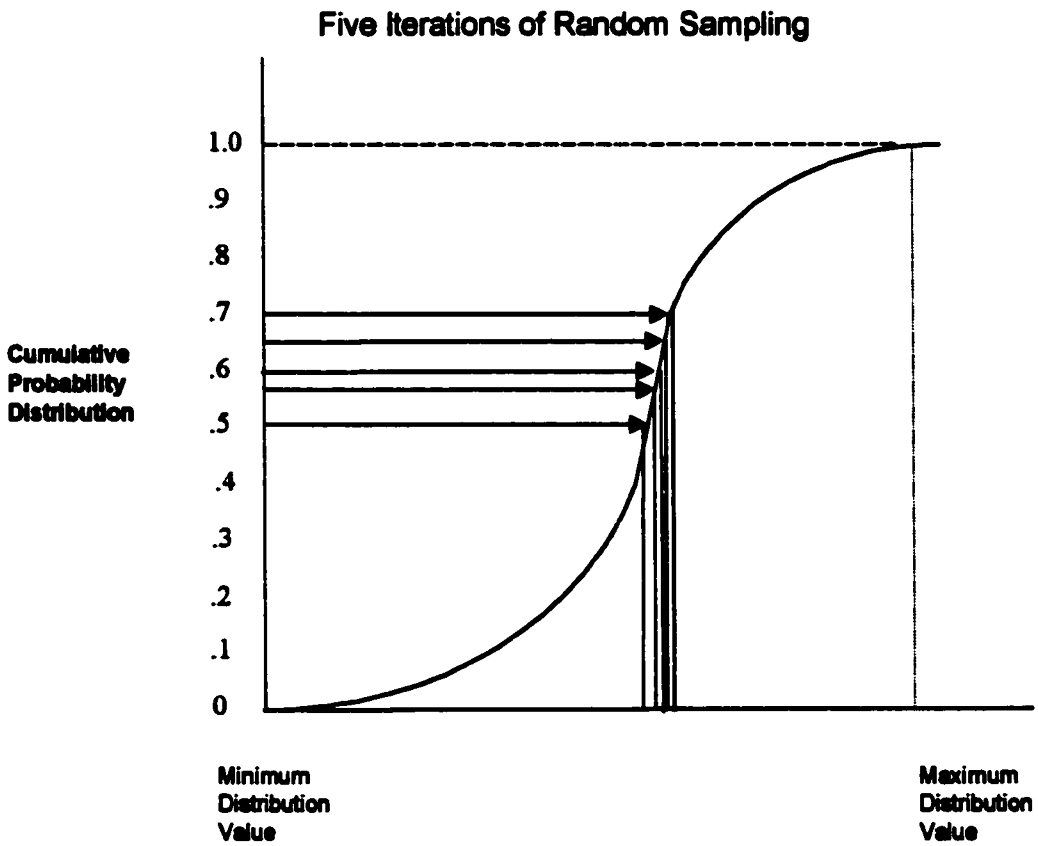

Figure 17 – Random Sampling [Palisade 2001]

3.3 Uncertainty Management

3.3.1 Response Surfaces

In order to facilitate rapid analysis capability and multidisciplinary integration of anaysis codes, approximation model building methods, called response surface methods (RSM), are utilized (Myers, 1971). Response surface methods have been used successfully in prior studies for approximation model building and multidisciplinary integration (Roux et al, 1996; Unal et al, 1998). The simplified response surface models and mathematical programming methods enable quick integration of disciplines and facilitate fast simulation studies.

A D-Optimal design matrix was constructed by Unal et al. (1998) to simulate configuration and sizing data and aerodynamics data for the launch vehicle. Aerodynamics were generated for Mach 0.3, Mach 2 and Mach 10. This research only makes use of the Mach 0.3 data. Center of gravity was obtained from CONSIZ and pitching moment coefficient was obtained from APAS. Regression Analysis was then used to determine the model coefficients for Cg and Cm in terms of six design parameters. These metamodels are used in steps d) and f) of Figure 18. The optimization process begins at step f). The optimization strategy is outlined in the paragraph below.

3.3.2 Pareto Optimal Solution

The Pareto solutions explored in this research study involve mean optimal solutions and variance optimal solutions. In any given problem, the solution that minimizes the target mean (mean optimal) and the solution that minimizes the variance (variance optimal) are two different solutions. The Pareto optimal solution is a compromise between the mean optimal and the variance optimal solution. An objective function is developed and used to identify the Pareto solutions.

The weighted sum method is used to model the multiple objectives of this optimization problem. The equation to be minimized is a modification of that used by Du and Chen (1999) and is provided below. Cmtarget here is zero. The weighting factors are chosen to place emphasis on either the closest solution to the targeted mean value or the smallest variance.

$$\text{Min } F(x_1, x_2, x_3, x_4, x_5, x_6, x_8) = w_1 \, [\mu_\alpha - Cm_{target}]^2 / [\mu_\alpha^* - Cm_{target}]^2 + w_2 \, \sigma_\alpha^2 / \Sigma\sigma_\alpha^{*2} \qquad (1)$$

$[\mu_\alpha^* - Cm_{target}]^2$ is the mean square error function for the ideal solution. It is used to normalize the mean square error of each design point. σ_α^{*2} is the variance for the ideal solution. It is used to normalize the variances for each design point. These ideal solutions are obtained from the iterative calculations of μ_α and σ_α. The design point that has the lowest mean square error yields $[\mu_\alpha^* - Cm_{target}]^2$. The design point that has the lowest variance yields σ_α^{*2}. The optimum solution for one variable input parameter is the design point that minimizes equation (1). Various combinations of W_1 and w_2 have been used here, and these values can vary as long as $w_1 + w_2 = 1$.

For two variable input parameters, the optimization equation becomes

$$\text{Min } F(x_1, x_2, x_3, x_4, x_5, x_6) = w_1 \, [\mu_\alpha - Cm_{target}]^2 / [\mu_\alpha^* - Cm_{target}]^2 + w_2 \, \sigma_\alpha^2 / \sigma_\alpha^{*2} + w_3 \, [\mu_d - Cm_{target}]^2 / [\mu_d^* - Cm_{target}]^2 + w_4 \, \sigma_d^2 / \sigma_d^{*2} \qquad (2)$$

With two variable parameters, $w1 + w2 + w3 + w4 = 1$. $[\mu_d - Cm_{target}]^2 / [\mu_d^* - Cm_{target}]^2$ and $\sigma_d^2 / \sigma_d^{*2}$ would be obtained by treating elevon deflection (d) as the sole variable input parameter and propagating the other seven design parameters much the same as when angle of attack was the sole variable input parameter.

Figure 18 is a diagram of the optimization procedure. One of the eight input parameters was chosen to be variable for this application. Angle of attack and elevon deflection appeared to be the parameters most variable during vehicle operation. Although it should be possible to optimize for both of these parameters, angle of attack was initially selected as the sole variable parameter. The following procedure is used:

(a) Design points are obtained by sampling from the seven fixed input parameters. Fixed parameters are set during the design of the launch vehicle. Variable parameters often change during vehicle operation within a specified range.

(b) Once the design point for the seven fixed parameters is selected, the variable parameter is changed by sampling from input parameter distributions.

(c) ε_1 is sampled from the error function of CONSIZ.

(d) Center of gravity including payload (Cgin) is then calculated for the design point. This includes the error computed in step (c).

(e) ε_2 is sampled from the error function of APAS.

(f) Pitching moment coefficient (Cm) is then calculated for the design point using Cgin. This includes ε_2 error function sampled in step (e).

(g) The mean (μ_α) and standard deviation (σ_α) can then be calculated from the distribution of Cmα.

(h) The variance and mean square error are calculated. This process of calculating Cmα is repeated as angle of attack is varied over its full range. These Cmα calculations are known as sensitivity derivatives. A new design point is then selected by sampling from the seven fixed parameters. The variable parameter (α) is then changed over its range and Cmα, μ_α and σ_α are calculated.

(i) Having determined the values of μ_α and σ_α at each design point, it is possible to identify the optimum design point over the range of the variable (α). The design point with the minimum variance and the design point with the minimum mean square error are identified.

(j) The minimum variance and minimum mean square error values are then used to normalize the objective function for each design point. The design point with the minimum objective function is selected.

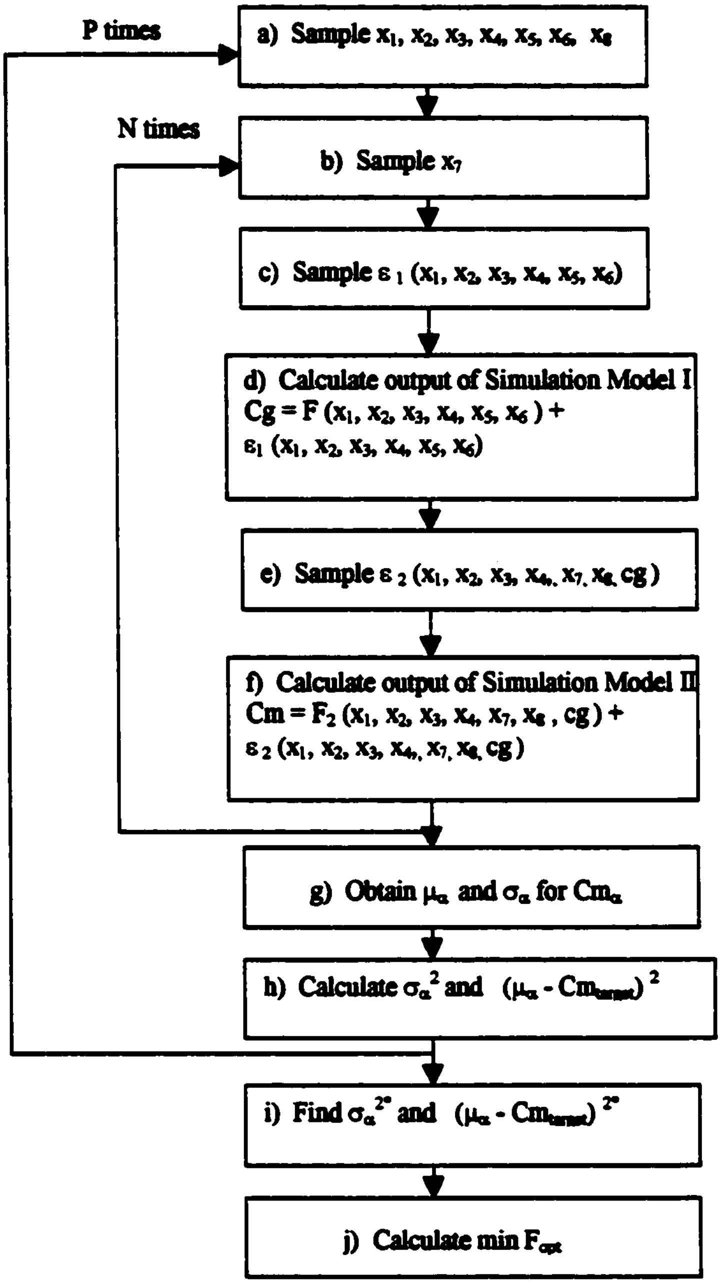

Figure 18 Uncertainty Propagation and Optimization Routine

3.4 Expected Results

The end result from this methodology will be a probabilistic confidence level for the critical subsystem performance characteristic estimate in S-curve form (or more appropriately in cumulative distribution function form) indicative of the risk.

While the use of multiple exert opinion is expected to provide a more realistic representation of input parameter distributions, this method totally changes the shape and breath of each input parameter distribution from the triangular distribution initially generated by a single expert. Extremes in each expert's opinion are muted by the weighting factors. Overlapping opinions, or portions thereof, would be reinforced by the weighting factors. The distribution associated with multiple expert opinions would tend to have a larger confidence interval as the standard deviation or measure of dispersion would increase. The distributions would span wider ranges.

Additionally, the Latin Hypercube Sampling routine is expected to produce a distribution that has a smaller variance than the random sampling routine for Monte Carlo simulations. This is an advantage of the methodology in that for large computer programs, run times are minimized and computer resources are used more efficiently.

A disadvantage to using Latin Hypercube sampling is that programming of the sampling routine is more complicated than programming of the random sampling routine. The disadvantage of increased initial programming effort should be offset by the savings that would be realized in computer run time especially for programs that are executed often.

CHAPTER IV
RESEARCH FINGINGS

This section promulgates the refinements made to the procedures outlined in Chapter III. It also documents the analysis results. This research does not examine the principles of response surface methodology (RSM) or design of experiments, but an elementary knowledge of both subjects is assumed. Text such as Box, Hunter and Hunter (1978) and Law and Kelton (1991) are excellent sources for additional information.

4.1 Methodology Refinement

This risk analysis study was accomplished using the C++ programming language in quantifying input parameter distributions, propagating uncertainties throughout the two disciplines and optimizing input parameter selection. C++ was chosen because NASA Langley makes use of this programming language in some of its existing programs for launch vehicle computation. FORTRAN is also used at NASA and C++ can be integrated with existing FORTRAN programs or legacy systems. These procedures specifically refer to the C++ program development. @Risk was used to model the same risk analysis procedures as a validation of the C++ programming. The use of @Risk also serves to demonstrate the adequacy of existing risk analysis tools for executing complex problems.

The proposed methodology makes use of random variates generated from probability distributions. In the case of external uncertainties, the probability distributions are triangular. Random variate generation for specific distributions is discussed extensively in Law and Kelton (1991) and Cheng (1998). The basic tool required in generating random variates is a statistically reliable U(0,1) random-number generator. With the identification of a suitable random-number generator, algorithms exist which utilize these random numbers to generate random variates.

4.1.1 External Uncertainty Identification

The initial stage of the methodology consists of the identification of internal and external uncertainties for the two coupled response surfaces. The external uncertainties are associated with the input parameters to both response surfaces and these were analyzed first. A questionnaire was developed in an effort to document expert opinion on the parameters sought. The questionnaire utilized is provided in Appendix A. The methodology uses a triangular distribution to simulate the input parameter uncertainties from each expert. Initially, only one set of opinions derived from a single expert was implemented. In subsequent analysis, multiple expert opinions were aggregated.

This research uses the Inverse Transform method of generating random variates. Employing $\beta = (b-a)/(c-a)$, triangular distributions random number variates are calculated using the following algorithms:

$$X = a + (c-a)\,(\beta u)^{1/2} \qquad \text{if } u < \beta \tag{3}$$

$$X = a + (c-a)\,[\,1-(\,(1-\beta)\,(1-u\,))^{1/2}] \qquad \text{if } u \geq \beta \tag{4}$$

Figure 19 illustrates the constants used in the distribution. The low value of the input parameter is "a." The most likely value for the parameter is "b," and the high value for the parameter is "c."

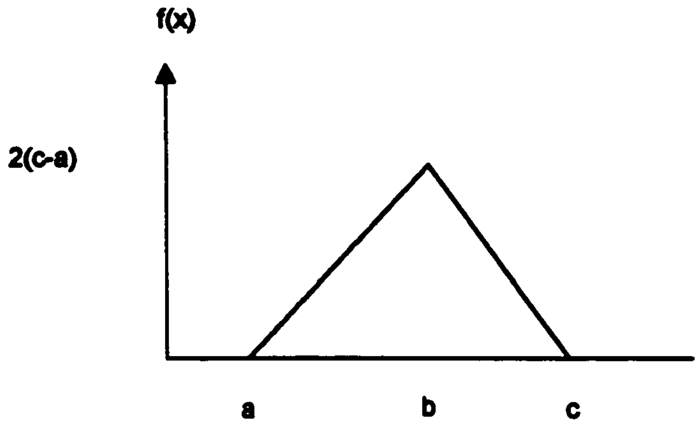

Figure 19 – Triangular Distribution Density Function

The constants associated with each input parameter for computation of the center of gravity (Cg) and pitching moment coefficient (Cm) are provided in Figure 20. These were obtained from the questionnaire of Appendix A.

parameter	a	b	c	beta
x1	4	5.5	7	0.5
x2	10	15	20	0.5
x3	0.5	1.75	3	0.5
x4	0	0.5	1	0.5
x5	0	0.02	0.04	0.5
x6	7.75	8	8.25	0.5
x7	5	12	15	0.7
x8	-14.68	-11.7004	-4.345	0.288302

Figure 20 – Input Parameter Triangular Distribution Factors

4.1.2 Internal Uncertainties Identification

The response surfaces were coded in an Excel spreadsheet. A printout of the spreadsheet format is provided in Appendix B. Forty-five data points were used to generate the first response surface (Cg), while 180 data points were used to generate the second response surface (Cm). Errors between the first response surface model (Cg) and actual output CONSIZ data points were calculated. Errors between the second response surface model (Cm) and actual output APAS data points were also calculated. Spreadsheets containing the error, error mean and error variance were developed in Excel, and printouts are provided in Appendix C. Histograms of the data are also provided in Appendix C. The Arena Input Analyzer was then used to fit the best distribution to the data based on mean square error. The Arena Input Analyzer fitted a lognormal distribution to the first response surface errors (Cg), and a normal distribution to the second response surface (Cm) errors. The Arena Input Analyzer was also used to fit a normal distribution to the first response surface error (Cg) data. Although the normal distribution was not the best fit to the data, it was an acceptable fit. Summary data from the Arena Input Analyzer are provided in Appendix D. The summary data provides the results of goodness-of-fit calculations for the fitted distributions. The Arena Input Analyzer executes the Kolmogorov-Smirnov goodness-of-fit test in addition to computing the mean square error. The results of that test indicate that the normal distribution is a good fit to the Cg error data. The K-S test indicates that the normal distribution is not a very good fit to the Cm error data, but it is the best fit out of the nine distributions attempted. These nine distributions include lognormal, normal, Erlang, gamma, Weibull, triangular, uniform, exponential and beta distributions.

Du and Chen (1999) recommend the use of normal distributions to represent internal uncertainties or model error for two response surfaces. Consistent with Du and Chen (1999), the normal distribution is selected as the distribution fit for Cm error data. A lognormal distribution was selected to simulate Cg error. This study uses the distribution that best fits the data (the lognormal distribution) instead of just using the assumed normal distribution for Cg error. As the data demonstrates, non-normal distributions may be more appropriate for modeling tool error.

The Box-Muller method [Cheng (1998)] is used to generate normal variates as there is no closed-form expression to accomplish the task. This technique returns pairs of independent normal variates and is accomplished using the following routine:

```
While (True) {

    Generate u1, u2, RN(0,1) variates.
    Let v1 = 2u1 – 1, v2 = 2u2 – 1, w = v1² + v2²
    If (w < 1) {
        Let y = [(-2 ln w) / w] ½
        Return X1 = μ + σv1y and X2 = μ + σv2y
        }
    }
```

There is no closed form expression for generating lognormal variates either. The procedure for generating such variates starts with generating normal variates and then takes the exponential of the normal variates. The mean and variance used to generate the normal variates are transformations of the lognormal mean and variance. If θ = the lognormal mean and τ = the lognormal standard deviation, then

$$\mu = \ln\left(\theta^2/(\theta^2 + \tau^2)^{1/2} \right) \tag{5}$$

and

$$\sigma^2 = \ln\left((\theta^2 + \tau^2) / \theta^2\right) \tag{6}$$

are the values placed in the Box-Muller method. This procedure generates pairs of variates just like the normal variates generation technique.

4.1.3 Uncertainty Propagation

There were five risk analysis programs developed in this research study. The first C++ risk analysis program utilized a single expert opinion and Monte Carlo simulation with Random sampling. This program, Risk_sm, was developed in stages. The first stage included uncertainty quantification. Each of the eight input parameter distributions and two error distributions were coded. The program wrote the variates to separate files so that the accuracy of the coding could be tested. Five hundred variates were generated for the triangular distributions and 1000 variates were generated for the normal and lognormal distributions. Triangular, normal and lognormal distributions were then fitted to the data as appropriate. The results are provided in Appendix E. Although none of the simulations had converged upon the mean, the summary results indicate that the quantification coding of uncertainties had been accomplished accurately.

The second stage of developing the first risk analysis program was to code the response surfaces and to transform the actual values of the input parameters into forms suitable for their respective response surfaces. This stage resulted in the computation of Cg and Cm distributions. Algorithms for the response surfaces are provided below. See Figure 11 for symbol definitions.

Cg = 0.7412279-0.014978*fr1+0.0124302*wa1+0.0098995*tfa1+0.001898*bfl1-0.010154*bl+0.0043786*mr+0.004716*fr1*fr1-0.001637*fr1*wa1-0.000729*wa1*wa1-0.002255*fr1*tfa1-0.001238*wa1*tfa1+0.0002298*tfa1*tfa1-0.000307*fr1*bfl1-0.000344*wa1*bfl1-0.000141*tfa1*bfl1-0.000054*bfl1*bfl1+0.0003703*fr1*bl-0.000252*wa1*bl-0.000246*tfa1*bl + 0.000102*bfl1*bl-0.000068*bl*bl + 0.0000505*fr1*mr + 0.0000505*fr1*mr + 0.0000231*wa1*mr-0.000081*tfa1*mr-0.000068*bfl1*mr-0.000019*bl*mr+0.0003477*mr*mr (7)

Cm = -0.032188-0.002886*fr2-0.008796*wa2-0.006746*alpha-0.053769*delev-0.00032*fr2*wa2-0.000768*fr2*alpha-0.000276*fr2*delev-0.000203*wa2*tfa2-0.001762*wa2*alpha-0.00284*wa2*delev-0.000346*tfa2*alpha-0.000858*tfa2*delev-0.000651*alpha*delev + 0.000509*fr2*fr2+0.001773*wa2*wa2+ 0.000748*tfa2*tfa2+

0.000319*bfa2*bfa2+0.023208*cg2-0.000171*cg2*fr2 + 0.000765*cg2*wa2 + 0.000288*cg2*tfa2 + 0.00468*cg2*alpha + 0.012162*cg2*delev-0.001566*tfa2 (8)

The next step in the risk analysis was to program errors into the two response surfaces to model the launch vehicle. A lognormal distribution was programmed for the Cg error and a normal distribution was programmed for the Cm error. The first risk analysis program was then complete and ready to be executed at various numbers of iterations for the Monte Carlo simulation routine.

Several iterations of the program were run in an effort to observe output parameter convergence. The risk analysis program was run for 10, 25, 50, 100, 200, 300, 400, 500, 1000, 1500 and 2000 iterations. The data was entered into the Arena Input Analyzer. The results are provided in Appendix F. The Arena Input Analyzer fitted an appropriate distribution to the Cm data points generated at the various iteration values. Graphs of the distribution functions are also provided to illustrate shape and skewness.

The quantification and propagation of uncertainties was also coded in @Risk. Printouts of the Excel spreadsheet used for Cg and Cm computation are provided in Appendix G. This spreadsheet has input distributions and output distributions coded the same as Risk_sm. @Risk was easy to use and similar to using Microsoft Excel.

The second C++ risk analysis program developed, Risk_mm, used Monte Carlo simulation with Random sampling, but incorporated opinions from multiple experts. The expert elicitation methodology resulted in the use of discrete distributions for five of the eight input parameters to the Cg and Cm response surfaces. Two experts' opinions were aggregated to obtain the discrete distribution functions. The experts agreed on the remaining three input parameter distributions, so triangular distributions were used for these three parameters. Diagrams which compare the triangular distributions for the two experts are provided in Appendix H. Additionally, diagrams of the aggregate discrete distribution functions are provided in this Appendix. This risk analysis program was run for the same number of iterations as the Risk_sm program. These Arena Input Analyzer results are provided in Appendix I.

Both Risk_sm and Risk_mm use Monte Carlo simulations with random sampling to propagate uncertainties. Two related programs were developed in C++ that used a Latin Hypercube Sampling (LHS) routine to propagate uncertainties. The first LHS program, Risk_sl, models a single expert opinion for the eight input parameters. The second LHS program, Risk_ml, models two expert opinions for the input parameters. Similar to Risk_mm, Risk_ml uses discrete distributions for five of the eight parameters. Risk_sl and Risk_ml Arena Input Analyzer results are provided in Appendix J and K respectively.

4.1.4 Uncertainty Management

The final stage in the development of this risk analysis methodology is the uncertainty management or optimization portion. A fifth program was developed using C++ that incorporated a Pareto optimization strategy. This program (Risk_pareto) is a modification of the Risk_ml program which utilized Latin Hypercube Sampling for the propagation routine and aggregated multiple expert opinions for the external uncertainty quantification. The management program identifies the design solution that optimizes the mean as well as the design solution that optimizes the variance. Weighting factors were assigned to both of these solutions in synthesizing the objective function to be minimized. Weighting factors of W1 = 1.0 and W2 =0.0 corresponds with the mean optimal solution. Weighting factors of W1 = 0.0 and W2 = 1.0 corresponds with the variance optimal solution. Four cases were run with varying factor weightings and the results are provided in Appendix O.

4.2 Analysis of Results

The analysis of this research is divided into three segments: 1) analysis of Monte Carlo simulations with random sampling verses Latin Hypercube Sampling, 2) analysis of single expert opinion results verses aggregated multiple expert opinions and 3) analysis of optimization routine.

4.2.1 Uncertainty Propagation

The Arena Input Analyzer results were plotted for each of the C++ Risk programs developed. The mean and standard deviation for both Cg and Cm were plotted in an effort to observe convergence as the number of iterations of the simulations were increased.

4.2.1.1 Single Expert Opinion (Random Sampling vs. Latin Hypercube Sampling)

Figure L-1 shows the plots of Cgmean and Cg standard deviation for a single expert's opinion input data as obtained using Random Sampling (CgM mean and CgM std dev) and Latin Hypercube Sampling (CgL mean and CgL std dev). It is obvious that the mean converges faster with Latin Hypercube Sampling. The standard deviation converges at approximately the same rate for Random Sampling and Latin Hypercube Sampling. The magnitude of difference between Cg mean at 10 iterations and Cg mean at 2000 iterations is small for Monte Carlo simulations with Random Sampling. Even this small change in the location of the mean coincides with approximately a 17% increase in the location of the Cm mean (See figure L-2).

Figure L-2 shows that Cm mean converges slightly faster using Latin Hypercube Sampling verses Monte Carlo simulations with Random Sampling. Cm standard deviation appears to converge slightly faster for Random Sampling than with Latin Hypercube Sampling. In general, Cm parameters converge faster than Cg parameters. It is possible for parameters that converge quickly that LHS does not result in any savings in efficiency. The larger a simulation takes to converge using Random Sampling, the greater is the opportunity for improvement in efficiency using Latin Hypercube Sampling.

4.2.1.2 Multiple Expert Opinion (Random Sampling vs. Latin Hypercube Sampling)

Figure L-3 plots Cgmean and Cg standard deviation for the case where expert opinions were aggregated into discrete distributions. Cg mean LHS converges faster than Cg mean Random Sampling. Again, Cg standard deviation appears to converge at the same rate.

Figure L-4 plots Cm mean and Cm standard deviation for the case of aggregated expert opinions. Cm mean converges slightly faster with LHS over Random Sampling. Cm standard deviation converges slightly faster with random sampling.

Distribution shape and skewness are also factors when considering convergence. It can be noted from the distribution plots that Cg and Cm converges upon shape and skewness within 100 iterations for both random sampling and Latin Hypercube Sampling. The exceptions to this fact are Cm single expert random sampling and Cm single expert Latin Hypercube sampling. The distributions converge upon shape and skewness at 200 and 300 iterations respectively.

4.2.2 Uncertainty Quantification

The results of the Monte Carlo simulation with Random Sampling were plotted for the risk analysis programs having a single expert's opinion as input and having multiple expert opinions as input. These plots are provided in Appendix M. Figure M-1 shows Cg mean and Cg standard deviation. Cg mean single expert (CgS mean) consistently resulted in an increase in the mean from that of the aggregated opinions. This is expected as aggregating opinions tends to mute the extremes of any one individual's judgment. Cg standard deviation was virtually identical between the two risk programs. The experts in these two risk programs agreed on the minimum and maximum for all eight input parameters. These experts only disagreed on the most likely value for five of the parameters. This explains the high level of agreement between the Cg standard deviation of the single expert and multiple expert distributions. Even with this large level of agreement, a noticeable difference in Cg mean was evident. Additionally, the difference in Cg mean corresponds to a 43% increase in Cm mean single expert. These differences were taken using the data corresponding with 2000 iterations.

Cm mean was noticeably different in the risk analyses as well. Cm mean single expert (CmS mean) was consistently lower when compared to Cm mean multiple experts (Cm A mean). The single expert's opinion appears to provide Cm results that are more favorable to the design while the aggregated expert opinion illustrates that Cm is probably less favorable to the design. The standard deviations for these analyses were almost identical. Theoretically, the aggregated opinions would result in larger standard deviations, but since the expert's opinions were largely in agreement, no significant difference in Cg standard deviation or Cm standard deviation was evident.

4.2.3 Uncertainty Management

Design performance is influenced by both the mean location and its variance. Dealing with the tradeoff between mean square error and variance is the essence of a Pareto optimal solution. Figure 21 illustrates the distribution functions for the Pareto solutions plotted as a function of weighting function. As shown, the mean optimal solution (W1 = 1.0) is closest to the targeted mean of 0.0. This solution has the largest variance. The variance optimal solution (W1 = 0.0) is farthest away from the targeted mean, but has the smallest variance.

The best solution is chosen based on the tolerances of the problem. The limits of this problem are at Cm values of –0.01 and +0.01, the mean optimal solution results in

88.32% of design conditions across the range of angle of attacks satisfying the problem limits. The Pareto optimal solution coinciding with W1 =0.9/0.95 results in 77.94% of conditions satisfying the design limits. In this case, the mean optimal solution is the best solution. As variance decreases, the distribution function moves farther away from the targeted mean and further outside of the problem constraints.

Cm limits (or tolerances) have been set by the Vehicle Analysis Branch (VAB) at NASA Langley Research Center based on good engineering practices and judgment. Limits can sometimes be relaxed and still maintain design integrity. Relaxation of limits may not be acceptable for this particular launch vehicle problem, but for limits between –0.015 and +0.015, the mean optimal solution results in 98.38% of design conditions satisfying the problem constraints. The Pareto optimal solution coinciding with W1=0.9/0.95 results in 99.22% of conditions satisfying the design limits. (Note: All percentages were derived assuming normally distributed output functions.) In this case, the Pareto optimal solution coinciding with W1=0.9/0.95 would be the best solution. The Pareto optimal solutions coinciding with W1 = 0.75, 0.5 and 0.0 are totally outside of the design limits and thus would not be considered feasible solutions.

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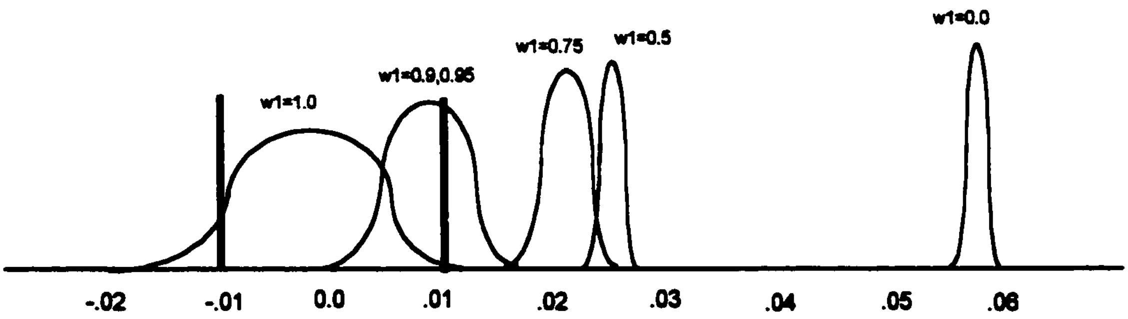

Figure 21 – Comparison of Cm Output Distributions as Function of W1

CHAPTER V

VERIFICATION AND VALIDATION OF RESULTS

Verification and validation of results were conducted by developing the same risk models in @Risk. This use of commercial software served to both verify the accuracy of the C++ programs developed and to validate the adequacy of @Risk to perform complex risk analysis problems. Verification of the uncertainty propagation results was accomplished first. @Risk was then used to verify the uncertainty quantification results.

The results of a weight optimization study (Unal et al, 1998) was used to compare the optimization results of this research. Weight is an important component in launch vehicle design. The results of a Cm optimization effort should be reviewed to identify any weight penalties that might be realized by modifying input parameters to suit Cm output.

Finally, the results of a weight optimization effort performed at Vanderbilt University on the same response surfaces is reviewed for relevance to this research.

5.1 Uncertainty Propagation

To verify the accuracy of the random sampling and Latin Hypercube sampling routines that were developed for the C++ risk programs, the same analysis was conducted in @Risk using the same number of iterations to plot results. These results are displayed in Appendix N. Figure N-1 compares Cg mean and Cg standard deviation using random sampling and using Latin Hypercube Sampling for a single expert's input parameters. Cg mean Latin Hypercube Sampling (CgL mean) converged faster than Cg mean random sampling (CgM mean). This figure is comparable to Figure L-1 for the C++ programs and the results are consistent. Cg standard deviation Latin Hypercube Sampling (CgL std dev), Figure N-1, converges slightly faster than Cg standard deviation random sampling

(CgM std dev). This was not consistent with Figure L-1 where the random sampling component converged at the same rate as the Latin Hypercube Sampling component. The researcher believes that the sequence of random numbers used in the C++ programs were uniquely suited to the quicker convergence of that risk analysis program (Risk_sm).

Figures N-2 through N-4 validate the findings of L2 through L-4. Cg mean and Cm mean converges faster using Latin Hypercube Sampling verses Random Sampling. In general, Cm converges faster than Cg. Greater efficiencies were noted in using Latin Hypercube Sampling for Cg parameters than for Cm parameters.

5.2 Uncertainty Quantification

Figures N-5 and N-6 compare the results of Cg and Cm computation using a single expert's opinion on input parameters versus using multiple expert opinions on input parameters. Monte Carlo simulation with random sampling was used as the propagation technique. Cg mean single expert was consistently higher than Cg mean aggregated expert opinions except at 50 and 100 iterations. Cg standard deviation single expert showed very little difference from that of Cg standard deviation aggregated expert opinions. Figure N-5 basically validates the results of Figure M-1 for the C++ programs.

Figure N-6 illustrates that Cm mean single expert and Cm mean aggregated expert opinion are virtually the same at all iterations. Cm standard deviation single expert is lower than Cm standard deviation aggregated expert opinion through 300 iterations. Above 300 iterations, the values are the same. Figure N-6 confirms that the similarity between single expert and aggregated expert opinions. The small difference in Cm mean would be expected as the largest difference between Cg mean single expert and Cg mean aggregated expert opinion is 0.002. This difference in the C++ programming was 0.004. Just this small increase in Cg mean location greatly impacted Cm mean difference between single expert and aggregated expert opinions in the C++ programs. C++ programs do not have a command that allows the programmer to set the random number seed. The absence of this feature results in the same random number stream being utilized in each of the programs. Under other circumstances this is not a desirable feature. In this instance, the use of the same random number stream is a variance reduction technique that facilitates better comparison of individual program results. The Cm mean single expert and Cm mean aggregated expert opinion comparison should be more accurate using the C++ programs.

5.3 Uncertainty Management

@Risk was used to optimize the solutions to the response surface equations. The results were compared to the C++ solutions and are provided in Table 1. @Risk does not have the capability to calculate a Pareto optimal solution when the Pareto solution is a compromise between the mean and variance optimal solutions. @Risk does have the capability to calculate both the mean optimal and variance optimal solutions.

The @Risk Optimizer can provide several simulations of the same model as it varies the random number stream. The C++ random number generator uses the same random number stream in the risk programs developed. The @Risk Optimizer identifies the best solution out of all of the simulations run, while the C++ program only has one simulation from which to choose a solution.

The C++ solutions satisfy the Cm tolerances of –0.01 to +0.01better than the @Risk solutions. The C++ mean optimal solution is 88.32% while the @Risk mean optimal solution is only 78.81%. If tolerances were increased to –0.015 to +0.015, then the C++ mean optimal solution increases its conformance rate to 98.38% and the @Risk mean optimal solution only increases to 90.66% conformance. The w1=0.9/0.95 Pareto solution provides the best conformance to these expanded limits with 99.22% compliance.

	@ Risk		C++				
	Mean Optimal	Variance Optimal	Mean Optimal	Variance Optimal	Pareto W1=.9,.95	Pareto W1=.75	Pareto W1=.5
FR	6.7575	4.833	6.5	5	5.75	5	4.25
WA	19.6342	10.622	15	10	12	13	13
TFA	1.75	2.405	1.9492	1.689469	1.249278	2.211422	1.995444
BFL	0.3245	0.335	0.6	0	0.2	0.41	0.2
BL	0.01854	0.0055	0.024844	0.022683	0.015466	0.001899	0.018767
MR	8.1541	7.833	8	8	7.85	8.03	7.85
Delev	-9.4857	-12.076	-6.63731	-14.06686	-5.256107	-7.661563	-8.270349
Mean	0.002397	0.047888	-0.00329	0.057642	0.007656	0.021212	0.025155
Variance	0.00009092	0.00000144	0.00003	0.000000166	0.000009207	0.000002061	0.000000664
Std Dev	0.0095351	0.0012012	0.005477	0.00040743	0.0030343	0.00143562	0.00081486
%Satisfied (.01)	78.81%	0.00%	88.32%	0.00%	77.94%	0.00%	0.00%
%Satisfied (.015)	90.66%	0.00%	98.38%	0.00%	99.22%	0.00%	0.00%
Empty Weight	276,840	248,772	249,628	243,990	211,790	269,627	310,848

Note: Mean, variance and std dev used here are for the pitching moment coefficient

Table 1 – Optimized Solutions

5.4 Weight Optimization Study Comparison

As the empty weight response surface was provided with the Cg and Cm response surfaces, empty weight was computed for each of the optimized C++ and @Risk solutions. The Pareto optimal solution with W1 = 0.9/0.95 provides the best solution to weight, but only has a 77.94% conformance within the –0.01 to +0.01 Cm tolerance. For constraints between –0.015 and 0.015, the w1 = 0.9/0.95 solution provides the optimal conformance for Cm requirements as well as weight.

The weight optimization effort performed by Unal et al. (1998) did not consider uncertainties. The optimized predicted weight for that study was 249,360 pounds. The w1=0.9/0.95 Pareto solution weight is projected to be less than the results of that optimization effort. It should be remembered that this research optimizes for Cm at Mach number of 0.3 only. The weight optimization effort of Unal et al. (1998) included Mach 2 and Mach 10 Cm requirements as constraints. Each of the Cm values were constrained to –0.005 and 0.005. It should also be noted that neither the mean optimal, variance optimal, nor Pareto optimal solutions provide 99% compliance to the constraints without relaxation of the original boundaries. This is a disturbing fact and has serious implications for the design requirements of the launch vehicle. The optimized parameters for this weight optimization effort were:

Fineness ratio	6.9
Wing area ratio	18.76
Tip Fin area ratio	1.99
Body Flap area ratio	0.0
Ballast weight	0.014
Mass ratio	8.0

The @Risk optimized parameters show a slight weight reduction for the variance optimal solution (0.23%). A large weight penalty is realized, however, for the mean optimal solution (10.7%). Conversely, the C++ mean optimal solution provides the best Cm optimization compliance results with virtually no weight penalty (0.11%).

5.5 Vanderbilt University Weight Optimization Results

Smith and Mahadevan (2001) performed a weight optimization analysis on the same response surfaces used in the deterministic optimization study of paragraph 5.4. This analysis included uncertainties and used the First Order Reliability Method (FORM) to propagate uncertainties. The objective of the study was to minimize mean weight such that pitching moment coefficient has a 99% probability of falling between –0.01 and 0.01. These limits were expanded after a solution could not be found between –0.005 and 0.005. Even with the expansion in boundaries, it was necessary to vary the input variable ranges to arrive at a solution. The optimized empty weight was 196,660 pounds. This predicted weight was found for the following parameters.

Fineness ratio	6.2796
Wing area ratio	16.1524

Tip Fin area ratio	0.5
Body Flap area ratio	0.0
Ballast weight	0.0
Mass ratio	7.75

The empty weight is much lower than the value predicted in the deterministic analysis, as well as, the Cm optimization effort of this research.

Smith and Mahadevan's (2001) optimization effort validates the difficulty in using deterministic design constraints in a probabilistic design environment.

CHAPTER VI

DISCUSSIONS AND CONCLUSIONS

6.1 Discussions

It should be noted that CONSIZ is itself a model for the actual system. The error calculated in this study is the error between two models. For purposes of this research, CONSIZ data is assumed to be an accurate representation of the system. The use of CONSIZ data as the real world system simply serves as a means of demonstrating the risk analysis methodology.

Several observations concerning the data are also note worthy. Cg(CONSIZ) is only given to three decimal places. The errors on the first response surface (Cg) would be slightly different if the values for Cg had not been truncated. There was not a lot of error observed between Cg(RSM) and Cg(CONSIZ). Greater error exists between Cm(RSM) and Cm(APAS). It was also noted from manipulating the Excel spreadsheet of the response surfaces that Cm is very sensitive to small changes in Cg. Consequently, although Cg error is small, the error is still significant. Further, the mean error for Cm is larger than that for Cg. The error standard deviation is greater as well. This increase in error is expected due to the cumulative nature of errors coupled in a system. Since Cm is smaller than Cg, this cumulative error represents a greater percentage of the response and is thus more significant as a matter of relative importance.

Internal uncertainties are less prevalent than external uncertainties in this particular risk analysis problem because the response surfaces were so well developed and produced little error. In a different problem where the metamodels are less accurate representations of the real system, internal uncertainties could have a severe impact upon output parameters and thus upon the stability of the system design.

It was surprising that the standard deviation statistic, and thus variance, converged at approximately the same rate for Latin Hypercube sampling and random sampling. It is believed that the Monte Carlo simulations with random sampling converged so quickly for this statistic in the problem examined that LHS could not improve upon the efficiency of the computation.

6.2 Summary

An objective of this research effort was to synthesize a methodology for conducting risk analysis in the conceptual design phase of a system such as a launch vehicle. A second objective was to demonstrate that methodology on a real world application. The methodology developed herein was primarily a compilation and extension of the research of authors such as Du and Chen (1999), Monroe (1997) and Vose (1996). Other authors such as Putko et al (2001), Haldar and Mahadevan (2000a and 2000b), and Hammersley and Handscomb (1964) provided alternative strategies for conducting one or more of the three stages of a risk analysis. The research uses expert judgment to elicit external input parameters from multiple experts. It, then, aggregates these individual distributions into a single distribution using a discrete distribution and weighted average approach.

Uncertainties were propagated through a coupled configuration & sizing and aerodynamics launch vehicle problem using a Monte Carlo simulation with Latin Hypercube sampling and Random sampling. Following propagation of uncertainties, a robust design technique was used to optimize input parameters over the range of a single variable input parameter. Latin Hypercube Sampling results were compared to Random sampling results. The research demonstrates that Latin Hypercube sampling converges upon distribution statistics faster than Random Sampling, particularly the mean.

The research also demonstrates that the use of multiple expert opinions verses a single experts' opinion impacts the final design enough to be important. Finally, the research shows that tradeoffs between optimal variance and optimal mean solutions can result in designs that are more robust and provides greater stability when considering the inevitable variability present in developing models for a complex system.

6.3 Conclusions

The use of multiple experts in determining the input parameters for a risk analysis is supported by the expected increase in the accuracy of the aggregated distribution. Aggregated opinions allow one to account for uncertainty among the experts. Using discrete distributions to combine multiple expert opinions is easily implemented in both C++ programming and @Risk.

Latin Hypercube Sampling results in faster convergence of distributions than using Monte Carlo Simulations with Random Sampling. The magnitude of the improvement in efficiencies increases as distributions take longer to converge using Random Sampling. Little improvement in efficiencies is expected for fast converging analysis. Distributions that take thousands of iterations for convergence will have greater efficiencies than distributions that only require a few hundred using Monte Carlo simulations. Latin Hypercube Sampling is recommended as a replacement for Monte Carlo Simulations with Random Sampling and should also be considered for replacement of analytic uncertainty propagation methods in the conceptual design phase.

@Risk software provides quick solutions for response surface models. When the need exists to compute optimum solutions to problems involving response surfaces, @Risk is a satisfactory product and is recommended to be used. When engineering problems involve existing software systems which do not interface with @Risk, then developing risk optimization routines using the methodology outlined in this research study is recommended.

Managing uncertainty requires that designs be optimized to satisfy ranges of conditions and ranges of input variables. This differs from deterministic solutions that only focus on a specific set of input variables and single set of design conditions. Designs optimized for uncertainty can focus on mean optimal solutions, variance optimal solutions or Pareto optimal solutions. Pareto optimal solutions provide opportunities for improvement in design robustness over both mean optimal and variance optimal solutions.

When risk analysis is examined and compared to deterministic designs, many existing deterministic design requirements will result in designs that have significant risk of failure. Using deterministic analysis, many systems are designed with less than 99% compliance on constraints, yet this is unknown to the designer. Risk analysis provides visibility into the true design feasibility region. The use of the risk analysis methodology developed in this research will allow designers to make reliable decisions under uncertain conditions representative of complex systems.

6.4 Limitations

A limitation of the research is associated with the expert opinion aggregation strategy. In order to use this methodology with a simulation tool such as CONSIZ, this aggregation strategy would have to be computerized. In this study, expert opinions were combined manually.

Another limitation of the research involved using C++ in a laptop environment. The usable memory is limited and therefore the number of iterations allowed in the optimization effort is limited as well.

The use of expert judgment elicitation techniques should be limited to environments where little is known about the parameters of interest. Where sufficient data exists, historical data uncertainty acquisition strategies is preferable to using expert judgment.

This research methodology is not limited to use on response surfaces, but can be used with design analysis tools with no change in implementation strategy. Although this methodology has been demonstrated on a problem that involves only two disciplines, it is not anticipated that greater numbers of disciplines will increase the complexity of the implementation strategy. Obviously the matrices involved in C++ program development will increase in size and dimension. The C++ language is, however, less straightforward as matrix dimensions increase.

6.5 Future Extensions

The ultimate extension of this research is that the methodology will be applied in a simulation tool environment versus the response surface environment for the management of uncertainties. Additionally, the research could be applied to a problem having more than two disciplines and more than two experts to provide opinions. This research might also be extended by including the Mach 2 and Mach 10 aerodynamic constraints into the risk analysis methodology. To consider Mach 2 and Mach 10, the methodology would generate three distribution functions for Cm. The optimal solution would then be the one that provides the greatest percentage of satisfaction for the three cases. An objective function could be used that would be maximized. Weighting on which Mach number was most important would have to be considered.

The research might be extended to perform FORM and SORM analysis on the Cm optimization problem. The results could then be compared to simulation results with both Latin Hypercube sampling and random sampling. FORM results for the weight optimization study [Smith and Mahadevan, 2001] did not include tool error, but a method for incorporating this type of uncertainty could probably be devised. It would also be interesting to apply the optimization approach outlined in this research to the weight optimization problem studied by Vanderbilt [Smith and Mahadevan, 2001] and compare the results of risk analysis using Monte Carlo simulation methods to propagate uncertainties with those obtained using FORM to propagate uncertainties.

Additionally, this research could be extended by fitting distribution functions, other than the triangular distribution, to external input parameters and comparing the results of the risk analysis.

A further area of interest is in developing a process for determining the weighting factors for aggregating multiple expert opinions used in the uncertainty quantification phase.

Designers need a risk analysis tool that can compute the percentage of compliance anticipated when specific design parameters are chosen. The development of such a tool for complex systems using strategies contained herein would augment this research.

When using risk analysis methodologies, it is important to determine when it is worth the added effort to include the uncertainties into an analysis. Consequently, extending this study to incorporate levels of fidelity into risk analysis research is also recommended.

Finally, the risk analysis methodology developed in this research should be applied in other applications or problem domains in order to determine consistency of results between related environments.

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APPENDIX A
Input Parameter Questionnaire

INPUT PARAMETER QUESTIONNAIRE

Part I.

1. Provide an estimate of ranges for the fineness ratio (FR). The range of values should include low, most likely and high values.

X1 low	4
X1 most likely	5.5
X1 high	7

2. Provide an estimate of ranges for the wing area ratio (WA). The range of values should include low, most likely and high values.

X2 low	10
X2 most likely	15
X2 high	20

3. Provide an estimate of ranges for the tip fin area ratio (TFA). The range of values should include low, most likely and high values.

X3 low	0.5
X3 most likely	1.75
X3 high	3.0

4. Provide an estimate of ranges for the body flap area ratio (BFL). The range of values should include low, most likely and high values.

X4 low	0
X4 most likely	0.5
X4 high	1.0

5. Provide an estimate of ranges for the ballast weight (BL). The range of values should include low, most likely and high values.

X5 low	0
X5 most likely	.02
X5 high	.04

6. Provide an estimate of ranges for the mass ratio (MR). The range of values should include low, most likely and high values.

X6 low	7.75
X6 most likely	8.0
X6 high	8.25

7. **Provide an estimate of ranges for the angle of attack (alpha). The range of values should include low, most likely and high values.**

X7 low	5
X7 most likely	12
X7 high	15

8. **Provide an estimate of ranges for the elevon deflection (DELEV). The range of values should include low, most likely and high values.**

X8 low	-14.68
X8 most likely	-11.7004
X8 high	-4.345

Part II.

9. **Revisit the values provided for fineness ratio. Revise these values if you deem it appropriate.**

X1 low	same
X1 most likely	same
X1 high	same

10. **Describe any scenario that might change the valuations that you applied to fineness ratio.**

11. **Revisit the values provided for wing area ratio. Revise these values if you deem it appropriate.**

X2 low	same
X2 most likely	same
X2 high	same

12. **Describe any scenario that might change the valuations that you applied to wing area ratio.**

13. **Revisit the values provided for tip fin area ratio. Revise these values if you deem it appropriate.**

X3 low	**same**
X3 most likely	**same**
X3 high	**same**

14. Describe any scenario that might change the valuations that you applied to tip fin area ratio.

15. Revisit the values provided for body flap area ratio. Revise these values if you deem it appropriate.

X4 low	**same**
X4 most likely	**same**
X4 high	**same**

16. Describe any scenario that might change the valuations that you applied to body flap area ratio.

17. Revisit the values provided for ballast weight. Revise these values if you deem it appropriate.

X5 low	**same**
X5 most likely	**same**
X5 high	**same**

18. Describe any scenario that might change the valuations that you applied to ballast weight.

19. Revisit the values provided for mass ratio. Revise these values if you deem it appropriate.

X6 low	**same**
X6 most likely	**same**
X6 high	**same**

20. Describe any scenario that might change the valuations that you applied to mass ratio.

21. Revisit the values provided for angle of attack. Revise these values if you deem it appropriate.

X7 low	**same**
X7 most likely	**same**
X7 high	**same**

22. Describe any scenario that might change the valuations that you applied to angle of attack.

23. Revisit the values provided for elevon deflection. Revise these values if you deem it appropriate.

X8 low	**same**
X8 most likely	**same**
X8 high	**same**

24. Describe any scenario that might change the valuations that you applied to elevon deflection.

APPENDIX B
Cg and Cm Response Surface Spreadsheet

Empty Wt.	Coded	Actual
FR - X1	0.926939	6.890409
WA - X2	0.751727	18.75864
TFA - X3	0.195766	1.994707
BFL - X4	-1	0
BL - X5	-0.27995	0.014401
MR - X6	0	8
Empty wt	249791.6	
Empty wt	246863.6	

Cm, M 0.3	Coded	Actual	Coded	Actual	Coded	Actual
fr	2.204261	6.890409	2.204261		2.204261	
wa	1.787607	18.75864	1.787607		1.787607	
tfa	0.465531	1.994707	0.465531		0.465531	
bfa	-2.378	0	-2.378		-2.378	
			aoa limit		aoa limit	
alpha	0.9512	12	-2.378	5	2.378	15
delev	-0.92746	-11.7004	-0.34442	-4.34503	-1.16374	-14.6814
cg	0.366636	0.741998	0.366636		0.366636	
Predicted	0.005		0.005		0.005	

Cl, M 0.3	Coded	Actual
fr	2.204261	6.890409
wa	1.787607	18.75864
tfa	0.465531	1.994707
bfa	-2.378	0
alpha	0.9512	12
delev	-0.92746	-11.7004
Predicted	0.279903	

Cd, M 0.3	Coded	Actual
fr	2.204261	6.890409
wa	1.787607	18.75864
tfa	0.465531	1.994707
bfa	-2.378	0
alpha	0.9512	12
delev	-0.92746	-11.7004
Predicted	0.036455	

Landed W	Coded	Actual
FR - X1	0.926939	6.890409
WA - X2	0.751727	18.75864
TFA - X3	0.195766	1.994707
BFL - X4	-1	0
BL - X5	-0.27995	0.014401
MR - X6	0	8
Landed W	302059.2	

Cm, M 2	Coded	Actual	Coded	Actual	Coded	Actual
fr	2.204261	6.890409	2.204261		2.204261	
wa	1.787607	18.75864	1.787607		1.787607	
tfa	0.465531	1.994707	0.465531		0.465531	
bfa	-2.378	0	-2.378		-2.378	
			aoa limit		aoa limit	
alpha	0	10	-2.378	5	2.378	15
delev	-0.31707	-4	-0.76145	-9.60818	0.039633	0.5
cg	0.366636	0.741998	0.366636		0.366636	
bflap		-5		-10		0
Predicted	-0.00018		-0.00365		0.004737	
With body	0.004339		0.005		0.004737	

P/L in.cg	Coded
FR - X1	0.926939
WA - X2	0.751727
TFA - X3	0.195766
BFL - X4	-1
BL - X5	-0.27995
MR - X6	0
Predicted	0.741998

Cm, M 10	Coded
fr	2.204261
wa	1.787607
tfa	0.465531
bfa	-2.378
alpha	-0.79267
delev	0.792667
cg	0.526261
bflap	
Predicted	0.01365
With body	-0.005

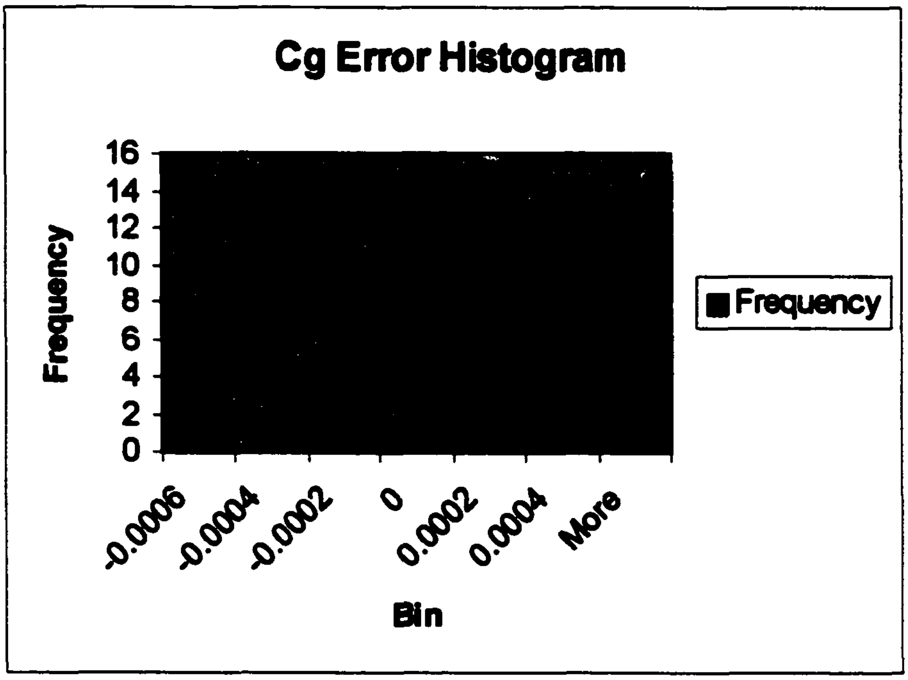
Cl, M 10	Coded
fr	2.204261
wa	1.787607
tfa	0.465531
bfa	-2.378
alpha	-0.79267
delev	0.792667
bflap	
Predicted	0.902525
With body	0.932615

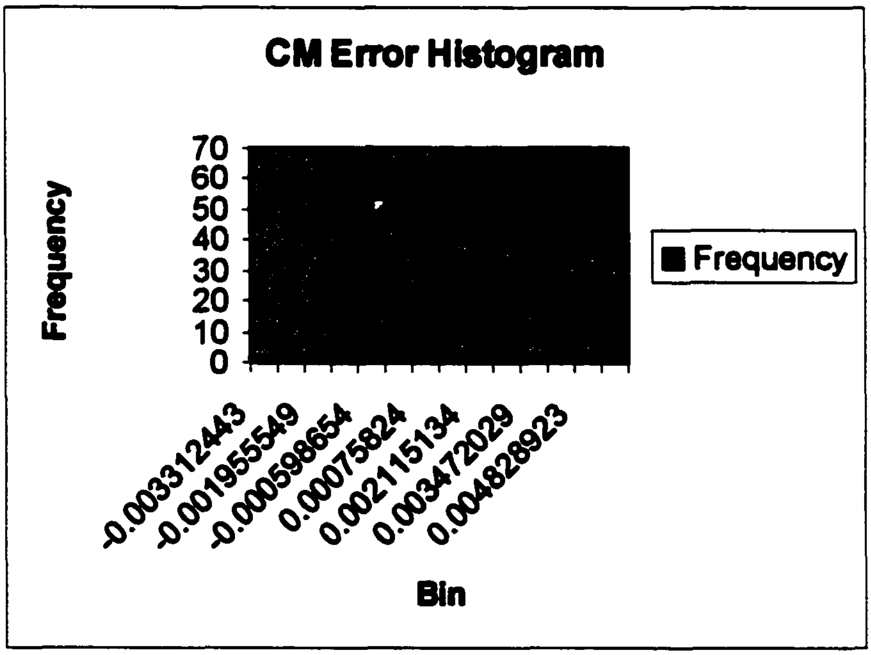
Cd, M 10	Coded
fr	2.204261
wa	1.787607
tfa	0.465531
bfa	-2.378
alpha	-0.79267
delev	0.792667
bflap	
Predicted	0.69089
With body	0.744778

APPENDIX C
Cg and Cm Response Surface Error

	x1	x2	x3	x4	x5	x6	cgplin (consize)	cgplin (RSM)	error
1	-1	-1	-1	-1	-1	1	0.745	0.745	0
2	-1	-1	-1	-1	1	-1	0.716	0.716	0
3	-1	-1	-1	1	-1	0	0.745	0.7455	-0.0005
4	-1	-1	-1	1	1	1	0.73	0.7302	-0.0002
5	-1	-1	0	1	-1	-1	0.755	0.7549	1E-04
6	-1	-1	1	-1	-1	1	0.773	0.7724	0.0006
7	-1	-1	1	-1	1	1	0.751	0.7512	-0.0002
8	-1	-1	1	1	0	1	0.767	0.7666	0.0004
9	-1	-1	1	1	1	-1	0.748	0.7479	1E-04
10	-1	0	-1	-1	1	1	0.741	0.7409	1E-04
11	-1	0	0	1	-1	1	0.778	0.778	0
12	-1	0	1	-1	-1	-1	0.778	0.7781	-1E-04
13	-1	0	1	0	0	0	0.773	0.7733	-0.0003
14	-1	1	-1	-1	-1	1	0.777	0.7769	1E-04
15	-1	1	-1	0	1	-1	0.749	0.749	0
16	-1	1	-1	1	-1	-1	0.772	0.7719	1E-04
17	-1	1	0	-1	0	-1	0.768	0.7683	-0.0003
18	0	-1	1	0	-1	-1	0.745	0.7456	-0.0006
19	0	0	-1	0	0	-1	0.728	0.7274	0.0006
20	0	0	0	1	1	0	0.733	0.733	0
21	0	1	-1	1	1	1	0.741	0.7408	0.0002
22	0	1	0	-1	-1	0	0.762	0.7618	0.0002
23	0	1	1	-1	0	1	0.765	0.7651	-1E-04
24	0	1	1	0	1	-1	0.747	0.7471	-1E-04
25	0	1	1	1	-1	1	0.778	0.7783	-0.0003
26	1	-1	-1	-1	-1	-1	0.714	0.7137	0.0003
27	1	-1	-1	-1	1	1	0.704	0.7041	-1E-04
28	1	-1	-1	0	1	0	0.702	0.7014	0.0006
29	1	-1	-1	1	-1	-1	0.718	0.7178	0.0002
30	1	-1	-1	1	-1	1	0.727	0.7268	0.0002
31	1	-1	-1	1	1	-1	0.699	0.6995	-0.0005
32	1	-1	0	-1	0	-1	0.713	0.7133	-0.0003
33	1	-1	1	-1	-1	1	0.741	0.7413	-0.0003
34	1	-1	1	-1	1	-1	0.713	0.7127	0.0003
35	1	-1	1	1	-1	0	0.74	0.7399	1E-04
36	1	-1	1	1	1	1	0.725	0.7252	-0.0002
37	1	0	-1	-1	-1	1	0.736	0.7363	-0.0003
38	1	1	-1	-1	1	-1	0.719	0.7192	-0.0002
39	1	1	-1	0	-1	-1	0.74	0.7403	-0.0003
40	1	1	-1	1	0	0	0.736	0.7362	-0.0002
41	1	1	0	0	-1	1	0.756	0.7559	1E-04
42	1	1	1	-1	-1	-1	0.753	0.7527	0.0003
43	1	1	1	-1	1	1	0.741	0.7409	1E-04
44	1	1	1	1	0	-1	0.745	0.7447	0.0003
45	1	1	1	1	1	1	0.743	0.7432	-0.0002
							mean =		-6.66667E-06
							standdard dev =		0.000286356

x1	x2	x3	x4	x7	x8	cmAPAS 66% c.g.	cm RSM 66% c.g.	error 66% c.g.	cmAPAS 72% c.g.	cm RSM 72% c.g.	error 72% c.g.	cmAPAS 75% c.g.	cm RSM 75% c.g.	error 75% c.g.	cmAPAS 78% c.g.	cm RSM 78% c.g.	error 78% c.g.
-1	-1	-1	-1	-1	-1	0.029696	0.031416	-0.00152	0.03672	0.0369	-0.00018	0.040132	0.03964	0.000492	0.043544	0.042384	0.00116
-1	-1	-1	-1	-1	1	-0.09084	-0.0912	0.000355	-0.06168	-0.06139	-0.00029	-0.04709	-0.04648	-0.00061	-0.03251	-0.03158	-0.00093
-1	-1	-1	1	1	-1	0.015226	0.015618	-0.00039	0.03061	0.030462	0.000148	0.038302	0.037884	0.000418	0.045994	0.045306	0.000688
-1	-1	-1	1	1	1	-0.10865	-0.1096	0.000948	-0.07006	-0.07043	0.000367	-0.05077	-0.05085	7.6E-05	-0.03148	-0.03126	-0.00021
-1	-1	1	-1	1	-1	0.013344	0.01334	3.75E-06	0.028998	0.02876	0.000238	0.036825	0.03647	0.000355	0.044652	0.04418	0.000472
-1	-1	1	-1	1	1	-0.1144	-0.11531	0.000904	-0.07496	-0.07556	0.000603	-0.05524	-0.05569	0.000453	-0.03552	-0.03582	0.000303
-1	-1	1	1	-1	-1	0.029431	0.030522	-0.00109	0.036302	0.036582	-0.00028	0.039738	0.039612	0.000126	0.043174	0.042642	0.000532
-1	-1	1	1	-1	1	-0.09471	-0.09552	0.000811	-0.06496	-0.06514	0.000178	-0.05009	-0.04995	-0.00014	-0.03521	-0.03475	-0.00046
-1	1	-1	-1	1	-1	0.000642	-0.0003	0.000944	0.016535	0.016072	0.000463	0.024481	0.024259	0.000222	0.032427	0.032446	-1.9E-05
-1	1	-1	-1	1	1	-0.13668	-0.13688	0.000198	-0.09524	-0.09618	0.000942	-0.07452	-0.07583	0.001314	-0.0538	-0.05548	0.001686
-1	1	-1	1	-1	-1	0.022737	0.022544	0.000193	0.029476	0.029558	-8.2E-05	0.032846	0.033065	-0.00022	0.036216	0.036572	-0.00036
-1	1	-1	1	-1	1	-0.11115	-0.11143	0.000281	-0.07981	-0.08009	0.000278	-0.06414	-0.06442	0.000277	-0.04848	-0.04875	0.000276
-1	1	1	-1	-1	-1	0.021836	0.020838	0.000998	0.028699	0.028428	0.000271	0.032131	0.032223	-9.2E-05	0.035563	0.036018	-0.00046
-1	1	1	-1	-1	1	-0.11654	-0.11657	2.61E-05	-0.08436	-0.08465	0.000289	-0.06827	-0.0687	0.000421	-0.05219	-0.05274	0.000553
-1	1	1	1	1	-1	-0.00184	-0.00339	0.001547	0.014424	0.013558	0.000866	0.022558	0.022033	0.000525	0.030692	0.030508	0.000184
-1	1	1	1	1	1	-0.14382	-0.1434	-0.00042	-0.10126	-0.10213	0.000861	-0.07999	-0.08149	0.001502	-0.05871	-0.06085	0.002143
1	-1	-1	-1	1	-1	0.009966	0.009844	0.000122	0.024883	0.024346	0.000537	0.032341	0.031597	0.000744	0.039799	0.038848	0.000951
1	-1	-1	-1	1	1	-0.1152	-0.11648	0.001275	-0.07702	-0.07765	0.000632	-0.05793	-0.05824	0.00031	-0.03884	-0.03882	-1.2E-05
1	-1	-1	1	-1	-1	0.027345	0.028714	-0.00137	0.033894	0.033856	3.82E-05	0.037169	0.036427	0.000742	0.040444	0.038998	0.001446
1	-1	-1	1	-1	1	-0.09451	-0.095	0.00049	-0.06557	-0.06554	-3.7E-05	-0.0511	-0.0508	-0.0003	-0.03663	-0.03607	-0.00056
1	-1	1	-1	-1	-1	0.026927	0.02782	-0.00089	0.033524	0.033538	-1.4E-05	0.036822	0.036397	0.000425	0.04012	0.039256	0.000864
1	-1	1	-1	-1	1	-0.0983	-0.09933	0.00103	-0.06877	-0.06929	0.000516	-0.05401	-0.05427	0.000259	-0.03924	-0.03924	1.98E-06
1	-1	1	1	1	-1	0.008155	0.007566	0.000589	0.023342	0.022644	0.000698	0.030936	0.030183	0.000753	0.03853	0.037722	0.000808
1	-1	1	1	1	1	-0.12082	-0.12219	0.001371	-0.08177	-0.08278	0.00101	-0.06225	-0.06308	0.000829	-0.04273	-0.04338	0.000648
1	1	-1	-1	-1	-1	0.018936	0.018562	0.000374	0.025452	0.025234	0.000218	0.02871	0.02857	0.00014	0.031968	0.031906	6.21E-05
1	1	-1	-1	-1	1	-0.11602	-0.11651	0.000498	-0.08465	-0.08552	0.000865	-0.06927	-0.07002	0.000749	-0.05369	-0.05452	0.000833
1	1	-1	1	1	-1	-0.00611	-0.00736	0.001243	0.009404	0.008676	0.000728	0.017162	0.016692	0.00047	0.02492	0.024708	0.000212
1	1	-1	1	1	1	-0.14472	-0.14504	0.000314	-0.10358	-0.10468	0.001098	-0.08301	-0.0845	0.00149	-0.06244	-0.06432	0.001882
1	1	1	-1	1	-1	-0.00853	-0.01045	0.001921	0.007367	0.006162	0.001205	0.015313	0.014466	0.000847	0.023259	0.02277	0.000489
1	1	1	-1	1	1	-0.15172	-0.15156	-0.00016	-0.10946	-0.11063	0.001162	-0.08834	-0.09016	0.001824	-0.06721	-0.06969	0.002486
1	1	1	1	-1	-1	0.018083	0.016856	0.001227	0.024722	0.024104	0.000618	0.028042	0.027728	0.000314	0.031362	0.031352	9.64E-06
1	1	1	1	-1	1	-0.12131	-0.12165	0.000341	-0.0893	-0.09008	0.000778	-0.0733	-0.07429	0.000997	-0.05729	-0.05851	0.001216
-2.378	0	0	0	0	0	-0.04614	-0.04606	-7.8E-05	-0.02264	-0.02245	-0.00019	-0.01089	-0.01639	0.005507	0.000864	0.001168	-0.0003
2.378	0	0	0	0	0	-0.06099	-0.05897	-0.00202	-0.03855	-0.03617	-0.00237	-0.02732	-0.02477	-0.00255	-0.0161	-0.01337	-0.00273
0	-2.378	0	0	0	0	-0.0217	-0.02263	0.000937	-0.00123	-0.00125	1.59E-05	0.009005	0.009449	-0.00044	0.019236	0.020144	-0.00091
0	2.378	0	0	0	0	-0.07006	-0.06811	-0.00195	-0.04566	-0.04308	-0.00258	-0.03347	-0.03057	-0.0029	-0.02127	-0.01805	-0.00322
0	0	-2.378	0	0	0	-0.04637	-0.04676	0.000389	-0.02478	-0.02423	-0.00055	-0.01399	-0.01297	-0.00102	-0.0032	-0.00171	-0.00149
0	0	2.378	0	0	0	-0.05699	-0.05557	-0.00141	-0.0337	-0.03168	-0.00202	-0.02206	-0.01974	-0.00232	-0.01041	-0.00779	-0.00262
0	0	0	-2.378	0	0	-0.05462	-0.05359	-0.00103	-0.03167	-0.03038	-0.00129	-0.02019	-0.01878	-0.00141	-0.00872	-0.00718	-0.00154
0	0	0	2.378	0	0	-0.05462	-0.05359	-0.00103	-0.03167	-0.03038	-0.00129	-0.02019	-0.01878	-0.00141	-0.00872	-0.00718	-0.00154
0	0	0	0	-2.378	0	-0.02789	-0.02822	0.000332	-0.01555	-0.01615	0.000593	-0.00938	-0.01011	0.000723	-0.00321	-0.00407	0.000853
0	0	0	0	2.378	0	-0.08588	-0.08257	-0.00331	-0.05126	-0.04823	-0.00303	-0.03394	-0.03106	-0.00288	-0.01663	-0.01389	-0.00274
0	0	0	0	0	-2.378	0.100418	0.101388	-0.00097	0.095254	0.095675	-0.00042	0.092672	0.092818	-0.00015	0.09009	0.089961	0.000129
0	0	0	0	0	2.378	-0.21497	-0.21218	-0.00279	-0.16197	-0.16005	-0.00192	-0.13547	-0.13399	-0.00148	-0.10897	-0.10792	-0.00105
0	0	0	0	0	0	-0.05462	-0.0554	0.000776	-0.03167	-0.03219	0.000519	-0.02019	-0.02058	0.00039	-0.00872	-0.00898	0.000261


Cg Error Histogram
Frequency
16
14
12
10
8
6
4
2
0
-0.0006
-0.0004
-0.0002
0
0.0002
0.0004
More
Bin
Frequency


CM Error Histogram
Frequency
70
60
50
40
30
20
10
0
-0.003312443
-0.001955549
-0.000598654
0.000758824
0.002115134
0.003472029
0.004828923
Bin
Frequency

APPENDIX D
Cg and Cm Error Distribution Fit

Cg Error Distribution Summary

Distribution: Normal
Expression: NORM(-6.67e-006, 0.000283)
Square Error: 0.004258

Chi Square Test

Number of intervals = 2
Degrees of freedom = -1
Test Statistic = 0.579
Corresponding p-value < 0.005

Kolmogorov-Smirnov Test

Test Statistic = 0.122
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 45
Min Data Value = -0.0006
Max Data Value = 0.0006
Sample Mean = -6.67e-006
Sample Std Dev = 0.000286

Histogram Summary

Histogram Range = -0.01 to 0.01
Number of Intervals = 6

==

Fit All Summary
Data File: G:\cgdata.txt

Function	Sq Error
Lognormal	0.00326
Normal	0.00426
Erlang	0.00882
Gamma	0.00947
Weibull	0.0164
Triangular	0.167
Uniform	0.34
Exponential	0.414
Beta	-1.#J

Cm Error Distribution Summary

Distribution:	**Normal**
Expression:	**NORM(3.04e-005, 0.0012)**
Square Error:	**0.018730**

Chi Square Test

Number of intervals	**= 3**
Degrees of freedom	**= 0**
Test Statistic	**= 5.27**
Corresponding p-value	**< 0.005**

Kolmogorov-Smirnov Test

Test Statistic = 0.145	
Corresponding p-value	**< 0.01**

Data Summary

Number of Data Points	**= 180**
Min Data Value	**= -0.0033**
Max Data Value	**= 0.00551**
Sample Mean	**= 3.04e-005**
Sample Std Dev	**= 0.0012**

Histogram Summary

Histogram Range	**= -0.01 to 0.01**
Number of Intervals	**= 13**

Fit All Summary
Data File: G:\Cm_error.txt

Function	Sq Error
Normal	0.0187
Beta	0.0207
Lognormal	0.0292
Weibull	0.0315
Erlang	0.0923
Gamma	0.0926
Triangular	0.213
Uniform	0.31
Exponential	0.348

APPENDIX E
C++ Input Parameter Coding Validation

Fineness Ratio (FR) Distribution Summary

Distribution: Triangular
Expression: TRIA(4, 5.45, 7)
Square Error: 0.001674

Chi Square Test
Number of intervals = 18
Degrees of freedom = 16
Test Statistic = 17.3
Corresponding p-value = 0.376

Kolmogorov-Smirnov Test
Test Statistic = 0.0238
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = 4
Max Data Value = 7
Sample Mean = 5.48
Sample Std Dev = 0.604

Histogram Summary

Histogram Range = 4 to 7
Number of Intervals = 22

Wing Area (WA) Distribution Summary

Distribution: Triangular
Expression: TRIA(10, 14.3, 20)
Square Error: 0.001640

Chi Square Test
Number of intervals = 18
Degrees of freedom = 16
Test Statistic = 24.1
Corresponding p-value = 0.0904

Kolmogorov-Smirnov Test
Test Statistic = 0.0542
Corresponding p-value = 0.103

Data Summary

Number of Data Points = 500
Min Data Value = 10
Max Data Value = 19.7
Sample Mean = 15
Sample Std Dev = 2.15

Histogram Summary

Histogram Range = 10 to 20
Number of Intervals = 22

Tip Fin Area (TFA)Ratio Distribution Summary

Distribution: Triangular
Expression: TRIA(0.25, 1.81, 3)
Square Error: 0.002988

Chi Square Test
Number of intervals = 18
Degrees of freedom = 16
Test Statistic = 39
Corresponding p-value < 0.005

Kolmogorov-Smirnov Test
Test Statistic = 0.0861
Corresponding p-value < 0.01

Data Summary

Number of Data Points	= 500
Min Data Value	= 0.5
Max Data Value	= 3
Sample Mean	= 1.78
Sample Std Dev	= 0.505

Histogram Summary

Histogram Range	= 0.25 to 3
Number of Intervals	= 22

Body Flap Area (BFL) Ratio Distribution Summary

Distribution: Triangular
Expression: TRIA(0, 0.571, 1)
Square Error: 0.001369

Chi Square Test
Number of intervals = 18
Degrees of freedom = 16
Test Statistic = 11.6
Corresponding p-value > 0.75

Kolmogorov-Smirnov Test
Test Statistic = 0.0323
Corresponding p-value > 0.15

Data Summary

Number of Data Points	= 500
Min Data Value	= 0.0224
Max Data Value	= 1
Sample Mean	= 0.524
Sample Std Dev	= 0.201

Histogram Summary

Histogram Range	= 0 to 1
Number of Intervals	= 22

Ballast Weight (BL) Distribution Summary

Distribution: Triangular
Expression: TRIA(-0.001, 0.0187, 0.05)
Square Error: 0.003536

Chi Square Test
- Number of intervals = 19
- Degrees of freedom = 17
- Test Statistic = 68.9
- Corresponding p-value < 0.005

Kolmogorov-Smirnov Test
- Test Statistic = 0.116
- Corresponding p-value < 0.01

Data Summary

Number of Data Points = 500
Min Data Value = 0
Max Data Value = 0.04
Sample Mean = 0.02
Sample Std Dev = 0.00865

Histogram Summary

Histogram Range = -0.001 to 0.05
Number of Intervals = 22

Mass Ratio (MR) Distribution Summary

Distribution: Triangular
Expression: TRIA(7.7, 7.99, 8.29)
Square Error: 0.002611

Chi Square Test
Number of intervals = 18
Degrees of freedom = 16
Test Statistic = 42
Corresponding p-value < 0.005

Kolmogorov-Smirnov Test
Test Statistic = 0.053
Corresponding p-value = 0.119

Data Summary

Number of Data Points = 500
Min Data Value = 7.75
Max Data Value = 8.23
Sample Mean = 7.99
Sample Std Dev = 0.103

Histogram Summary

Histogram Range = 7.7 to 8.29
Number of Intervals = 22

Angle of Attack (alpha) Distribution Summary

Distribution: Triangular
Expression: TRIA(5, 11.9, 15)
Square Error: 0.001459

Chi Square Test
Number of intervals = 19
Degrees of freedom = 17
Test Statistic = 12.9
Corresponding p-value = 0.742

Kolmogorov-Smirnov Test
Test Statistic = 0.0326
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = 5.46
Max Data Value = 14.8
Sample Mean = 10.6
Sample Std Dev = 1.99

Histogram Summary

Histogram Range = 5 to 15
Number of Intervals = 22

Elevon Deflection (DELEV) Distribution Summary

Distribution: Triangular
Expression: TRIA(-15, -11.3, -4)
Square Error: 0.002720

Chi Square Test
Number of intervals = 19
Degrees of freedom = 17
Test Statistic = 27.7
Corresponding p-value = 0.0492

Kolmogorov-Smirnov Test
Test Statistic = 0.0572
Corresponding p-value = 0.0764

Data Summary

Number of Data Points = 500
Min Data Value = -14.3
Max Data Value = -4.73
Sample Mean = -10.1
Sample Std Dev = 2.1

Histogram Summary

Histogram Range = -15 to -4
Number of Intervals = 22

Cg Error Distribution Summary

Distribution: Lognormal
Expression: -0.01 + LOGN(0.00999, 2.82e-005)
Square Error: 0.000007

Chi Square Test
Number of intervals = 1
Degrees of freedom = -2
Test Statistic = 0.015
Corresponding p-value < 0.005

Kolmogorov-Smirnov Test
Test Statistic = 0.0671
Corresponding p-value < 0.01

Data Summary

Number of Data Points = 1000
Min Data Value = -0.000103
Max Data Value = 7.3e-005
Sample Mean = -1.4e-005
Sample Std Dev = 2.82e-005

Histogram Summary

Histogram Range = -0.01 to 0.01
Number of Intervals = 31

Cm Error Distribution Summary

Distribution: Normal
Expression: NORM(5.57e-005, 0.00124)
Square Error: 0.000586

Chi Square Test
Number of intervals = 10
Degrees of freedom = 7
Test Statistic = 9.54
Corresponding p-value = 0.224

Kolmogorov-Smirnov Test
Test Statistic = 0.0329
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = -0.00349
Max Data Value = 0.00428
Sample Mean = 5.57e-005
Sample Std Dev = 0.00124

Histogram Summary

Histogram Range = -0.01 to 0.01
Number of Intervals = 31

APPENDIX F
Cg and Cm Distributions (Single Expert, Random Sampling)

Cg Distribution Summary (10 Iterations)

Distribution: Beta
Expression: 0.71 + 0.05 * BETA(1.61, 1.13)
Square Error: 0.063322

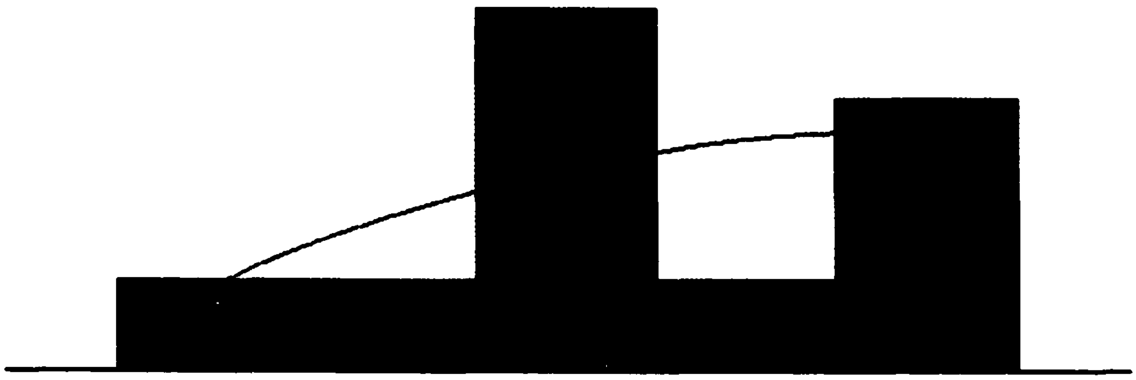
Kolmogorov-Smirnov Test
Test Statistic = 0.155
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = 0.717
Max Data Value = 0.755
Sample Mean = 0.739
Sample Std Dev = 0.0127

Histogram Summary

Histogram Range = 0.71 to 0.76
Number of Intervals = 5



Cg Distribution Summary (25 Iterations)

Distribution: Normal
Expression: NORM(0.744, 0.0106)
Square Error: 0.005002

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 0.304
Corresponding p-value < 0.005

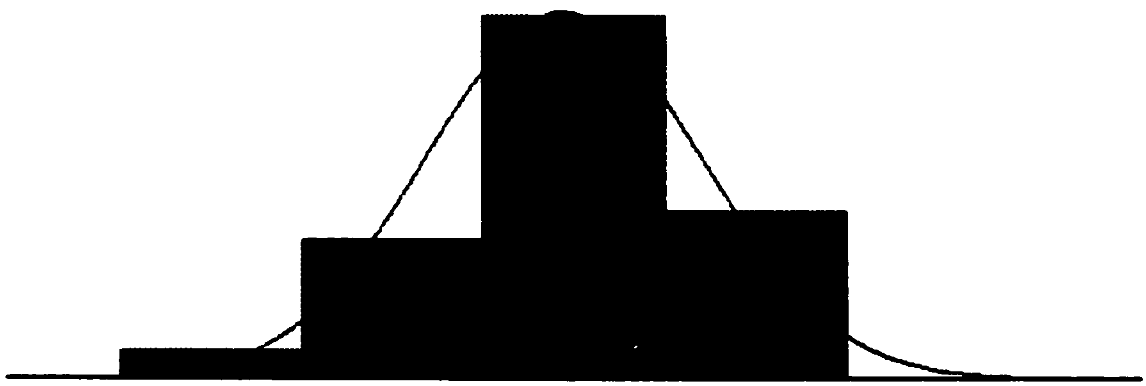
Kolmogorov-Smirnov Test
Test Statistic = 0.0987
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = 0.717
Max Data Value = 0.766
Sample Mean = 0.744
Sample Std Dev = 0.0108

Histogram Summary

Histogram Range = 0.71 to 0.78
Number of Intervals = 5



Cg Distribution Summary (50 Iterations)

Distribution: Normal
Expression: NORM(0.744, 0.0105)
Square Error: 0.005654

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 0.451
Corresponding p-value < 0.005

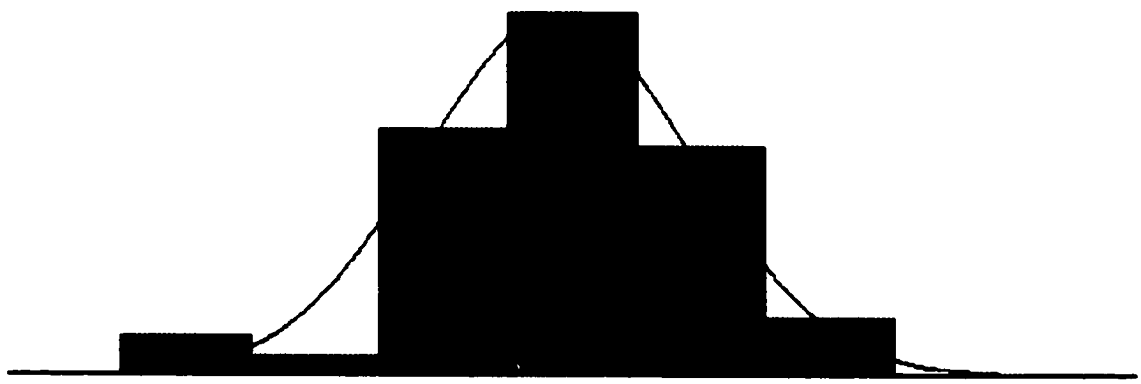
Kolmogorov-Smirnov Test
Test Statistic = 0.0682
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = 0.717
Max Data Value = 0.767
Sample Mean = 0.744
Sample Std Dev = 0.0106

Histogram Summary

Histogram Range = 0.71 to 0.78
Number of Intervals = 7



Cg Distribution Summary (100 Iterations)

Distribution: Beta
Expression: 0.71 + 0.07 * BETA(6.38, 6.85)
Square Error: 0.001006

Chi Square Test
Number of intervals = 4
Degrees of freedom = 1
Test Statistic = 0.29
Corresponding p-value = 0.617

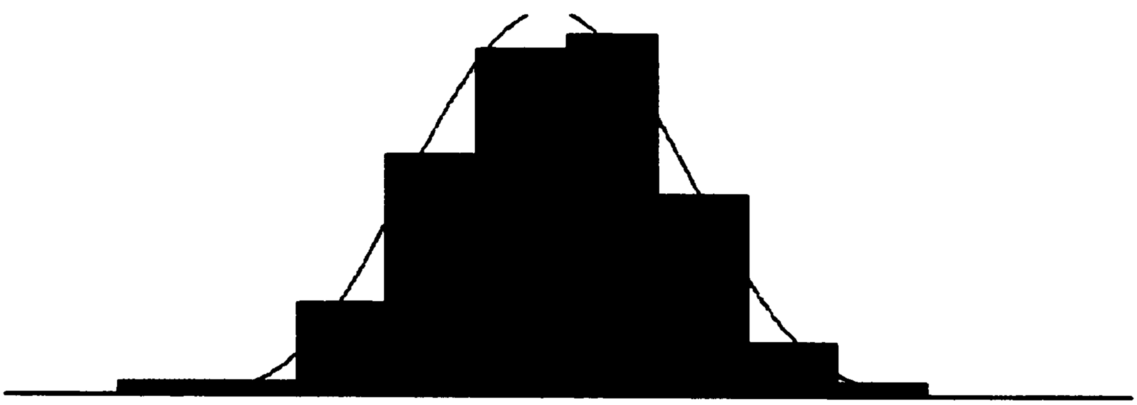
Kolmogorov-Smirnov Test
Test Statistic = 0.0429
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = 0.717
Max Data Value = 0.767
Sample Mean = 0.744
Sample Std Dev = 0.00994

Histogram Summary

Histogram Range = 0.71 to 0.78
Number of Intervals = 10



Cg Distribution Summary (200 Iterations)

Distribution: Weibull
Expression: 0.71 + WEIB(0.0357, 3.8)
Square Error: 0.005117

Chi Square Test
Number of intervals = 7
Degrees of freedom = 4
Test Statistic = 6.68
Corresponding p-value = 0.169

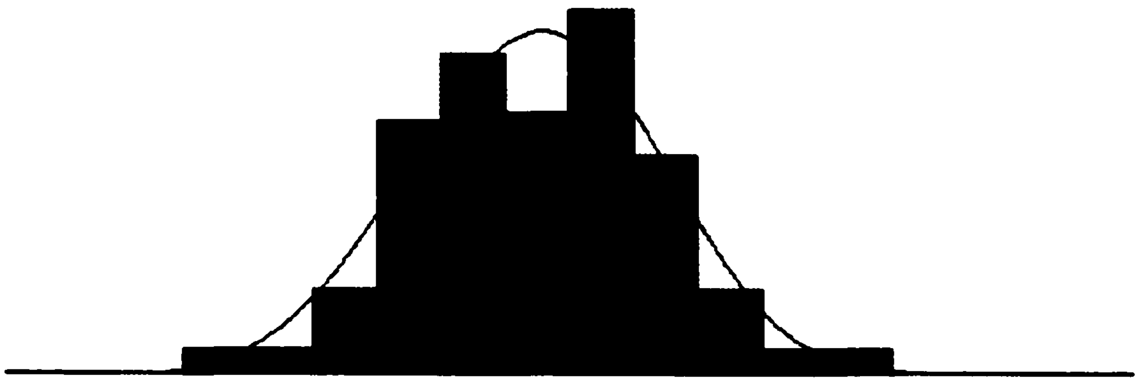
Kolmogorov-Smirnov Test
Test Statistic = 0.0433
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = 0.715
Max Data Value = 0.767
Sample Mean = 0.742
Sample Std Dev = 0.00942

Histogram Summary

Histogram Range = 0.71 to 0.78
Number of Intervals = 14



Cg Distribution Summary (300 Iterations)

Distribution: Beta
Expression: 0.71 + 0.07 * BETA(5.55, 6.25)
Square Error: 0.002486

Chi Square Test
Number of intervals = 10
Degrees of freedom = 7
Test Statistic = 5.75
Corresponding p-value = 0.571

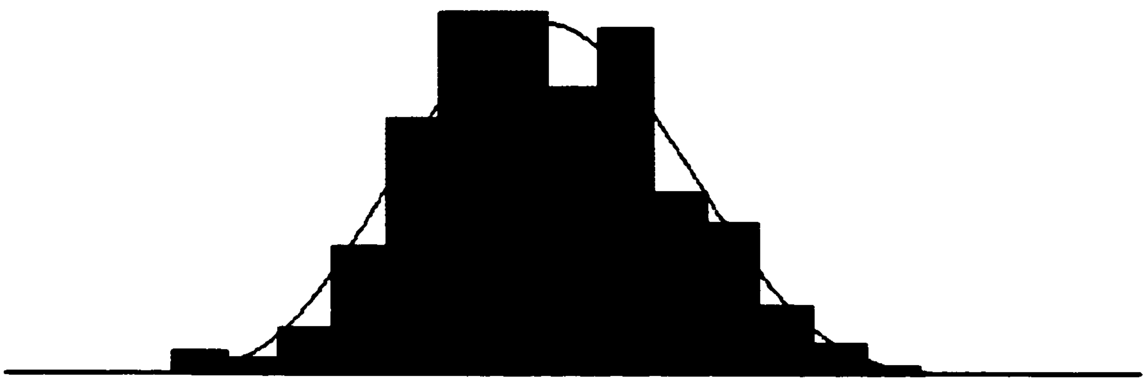
Kolmogorov-Smirnov Test
Test Statistic = 0.0501
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 300
Min Data Value = 0.715
Max Data Value = 0.769
Sample Mean = 0.743
Sample Std Dev = 0.00977

Histogram Summary

Histogram Range = 0.71 to 0.78
Number of Intervals = 17



Cg Distribution Summary (400 Iterations)

Distribution: Normal
Expression: NORM(0.742, 0.0102)
Square Error: 0.001702

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 6.32
Corresponding p-value = 0.612

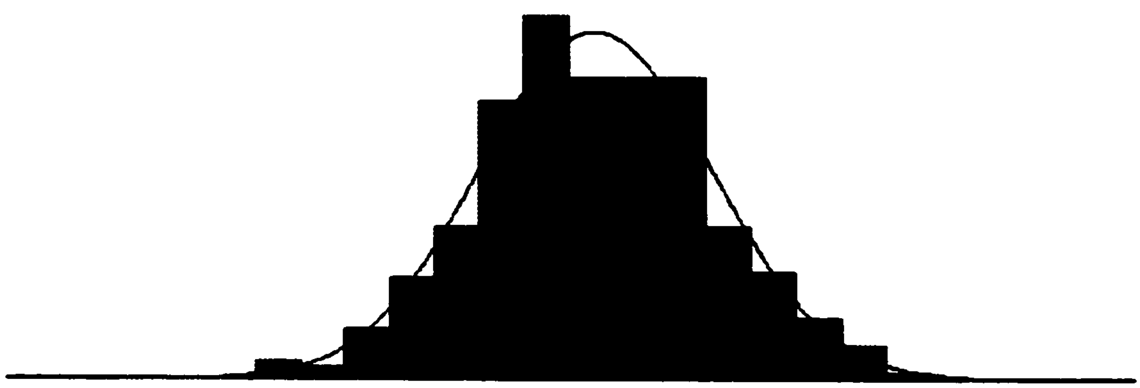
Kolmogorov-Smirnov Test
Test Statistic = 0.0375
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = 0.713
Max Data Value = 0.769
Sample Mean = 0.742
Sample Std Dev = 0.0102

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 20



Cg Distribution Summary (500 iterations)

Distribution: Beta
Expression: 0.7 + 0.09 * BETA(8.4, 9.5)
Square Error: 0.002937

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 11.3
Corresponding p-value = 0.197

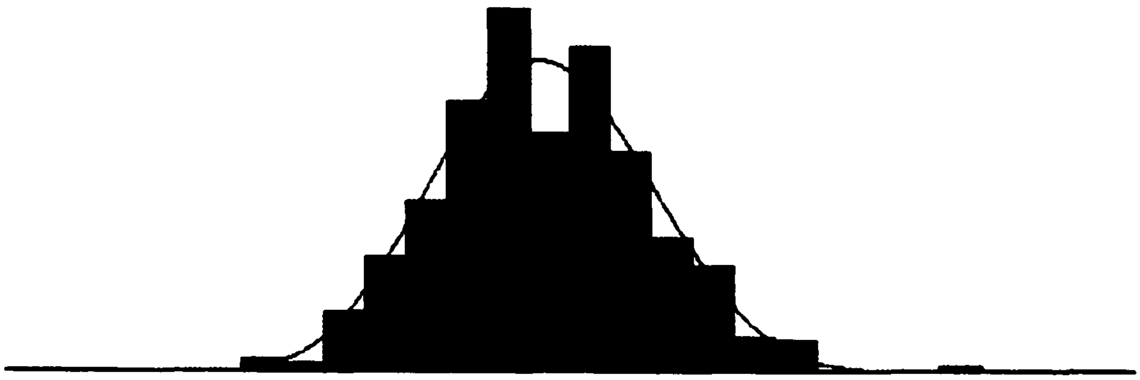
Kolmogorov-Smirnov Test
Test Statistic = 0.0371
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = 0.713
Max Data Value = 0.782
Sample Mean = 0.742
Sample Std Dev = 0.0103

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 22



Cg Distribution Summary (1000 iterations)

Distribution: Beta
Expression: 0.7 + 0.09 * BETA(8.59, 9.64)
Square Error: 0.000596

Chi Square Test
Number of intervals = 17
Degrees of freedom = 14
Test Statistic = 9.31
Corresponding p-value > 0.75

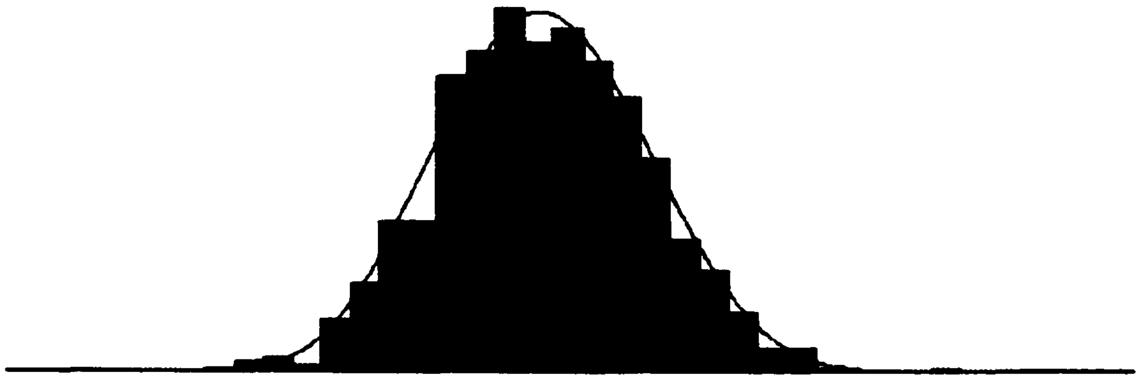
Kolmogorov-Smirnov Test
Test Statistic = 0.0234
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = 0.71
Max Data Value = 0.782
Sample Mean = 0.742
Sample Std Dev = 0.0102

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 31



Cg Distribution Summary (1500 iterations)

Distribution:	Beta
Expression:	0.7 + 0.09 * BETA(8.65, 9.82)
Square Error:	0.000835

Chi Square Test

Number of intervals	= 22
Degrees of freedom	= 19
Test Statistic	= 20.2
Corresponding p-value	= 0.393

Kolmogorov-Smirnov Test

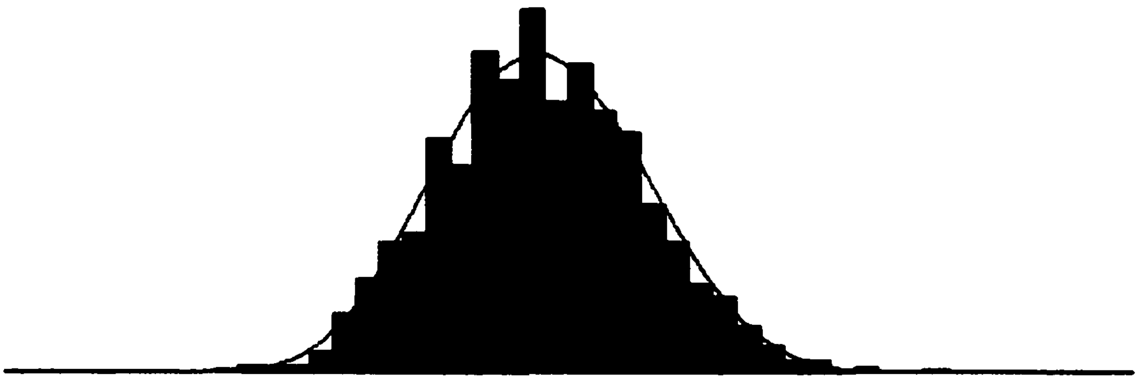
Test Statistic = 0.0182
Corresponding p-value > 0.15

Data Summary

Number of Data Points	= 1500
Min Data Value	= 0.71
Max Data Value	= 0.782
Sample Mean	= 0.742
Sample Std Dev	= 0.0102

Histogram Summary

Histogram Range	= 0.7 to 0.79
Number of Intervals	= 38



Cg Distribution Summary (2000 iterations)

Distribution: Beta
Expression: 0.7 + 0.09 * BETA(8.85, 10.1)
Square Error: 0.000677

Chi Square Test
Number of intervals = 23
Degrees of freedom = 20
Test Statistic = 29
Corresponding p-value = 0.0905

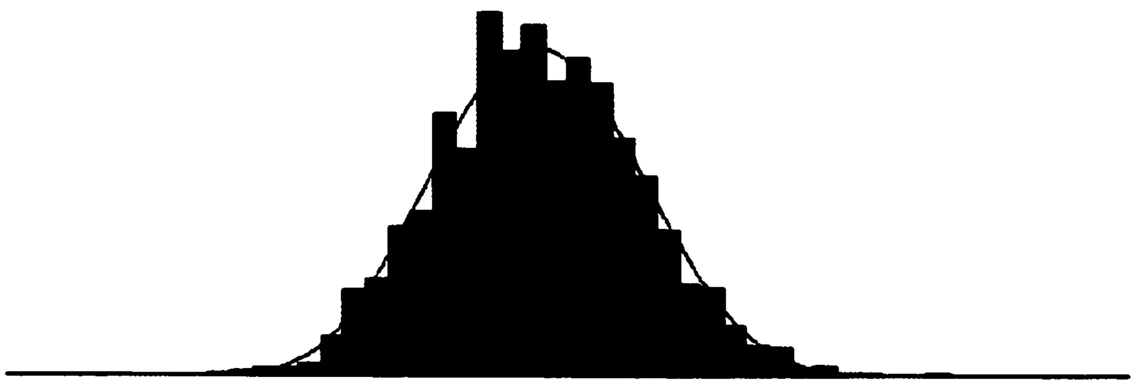
Kolmogorov-Smirnov Test
Test Statistic = 0.0215
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = 0.71
Max Data Value = 0.782
Sample Mean = 0.742
Sample Std Dev = 0.0101

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 40



Cm Distribution Summary (10 Iterations)

Distribution: Beta
Expression: -0.02 + 0.06 * BETA(1.82, 1.33)
Square Error: 0.011000

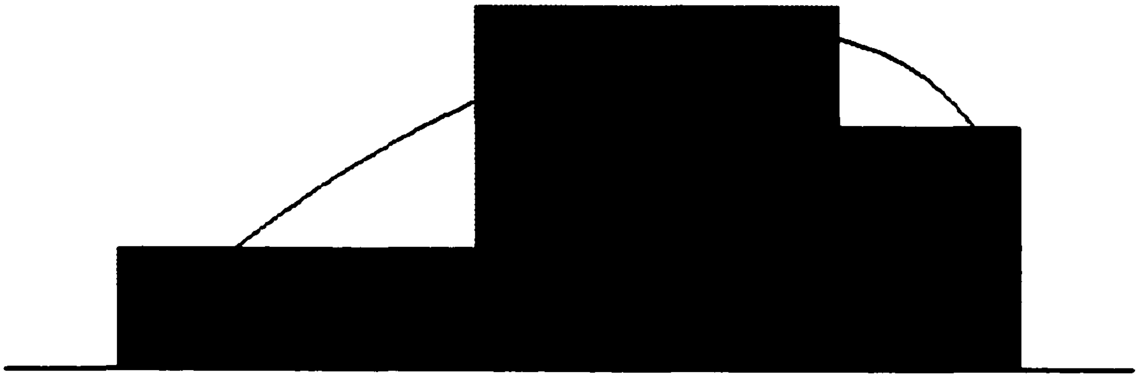
Kolmogorov-Smirnov Test
Test Statistic = 0.145
Corresponding p-value > 0.15

Data Summary

Number of Data Points	= 10
Min Data Value	= -0.0141
Max Data Value	= 0.0327
Sample Mean	= 0.0147
Sample Std Dev	= 0.0146

Histogram Summary

Histogram Range	= -0.02 to 0.04
Number of Intervals	= 5



Cm Distribution Summary (25 Iterations)

Distribution: Beta
Expression: -0.02 + 0.06 * BETA(2.1, 1.58)
Square Error: 0.006224

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 0.742
Corresponding p-value < 0.005

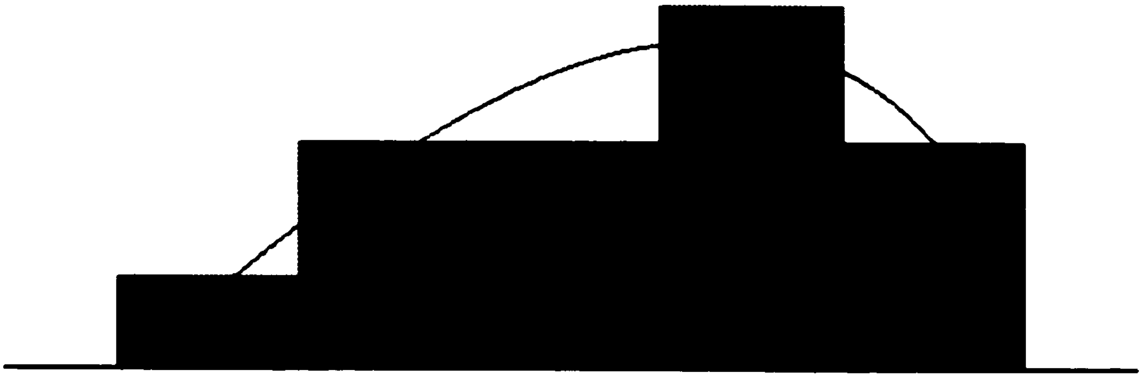
Kolmogorov-Smirnov Test
Test Statistic = 0.105
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = -0.0141
Max Data Value = 0.0327
Sample Mean = 0.0143
Sample Std Dev = 0.0137

Histogram Summary

Histogram Range = -0.02 to 0.04
Number of Intervals = 5



Cm Distribution Summary (50 Iterations)

Distribution: Triangular
Expression: TRIA(-0.02, 0.0242, 0.05)
Square Error: 0.007382

Chi Square Test
Number of intervals = 5
Degrees of freedom = 3
Test Statistic = 1.87
Corresponding p-value = 0.607

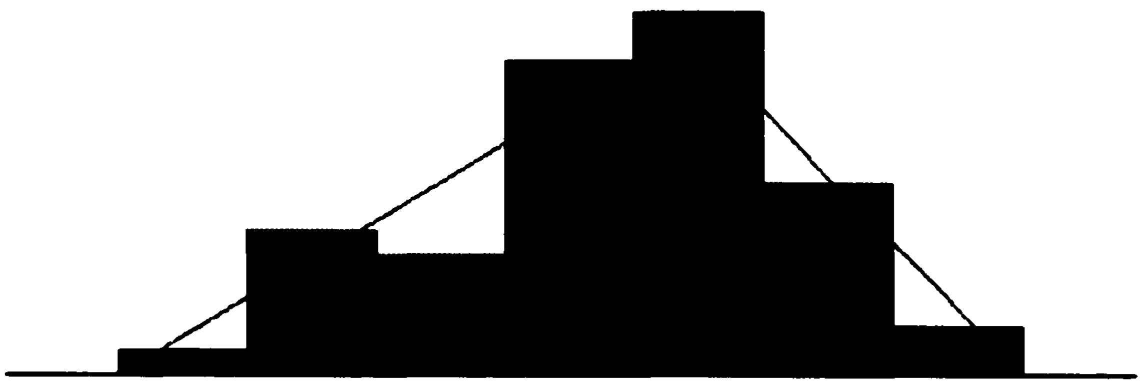
Kolmogorov-Smirnov Test
Test Statistic = 0.085
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = -0.0141
Max Data Value = 0.0426
Sample Mean = 0.0181
Sample Std Dev = 0.0135

Histogram Summary

Histogram Range = -0.02 to 0.05
Number of Intervals = 7



Cm Distribution Summary (100 Iterations)

Distribution: Triangular
Expression: TRIA(-0.02, 0.0225, 0.05)
Square Error: 0.014538

Chi Square Test
Number of intervals = 7
Degrees of freedom = 5
Test Statistic = 10.3
Corresponding p-value = 0.0709

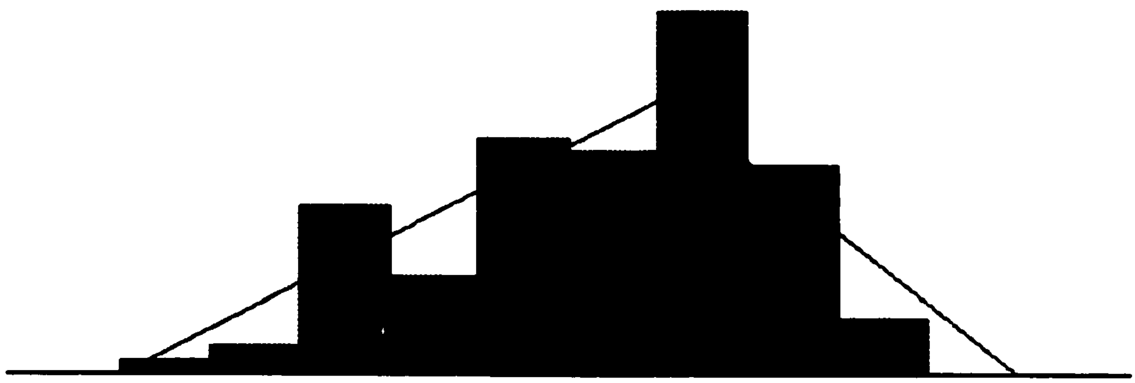
Kolmogorov-Smirnov Test
Test Statistic = 0.0733
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = -0.0141
Max Data Value = 0.0426
Sample Mean = 0.0175
Sample Std Dev = 0.013

Histogram Summary

Histogram Range = -0.02 to 0.05
Number of Intervals = 10



Cm Distribution Summary (200 Iterations)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0526, 4.6)
Square Error: 0.004635

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 7.16
Corresponding p-value = 0.219

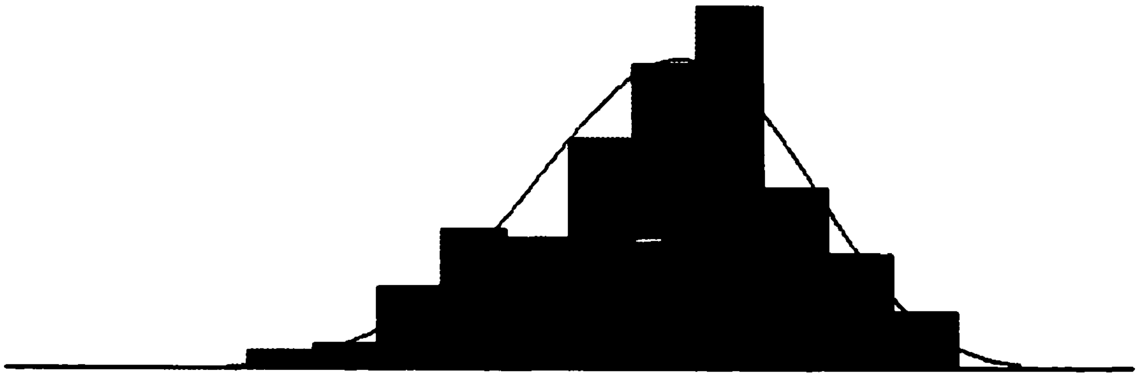
Kolmogorov-Smirnov Test
Test Statistic = 0.0555
Corresponding p-value > 0.15

Data Summary

Number of Data Points	= 200
Min Data Value	= -0.0144
Max Data Value	= 0.0437
Sample Mean	= 0.018
Sample Std Dev	= 0.0122

Histogram Summary

Histogram Range	= -0.03 to 0.05
Number of Intervals	= 14



Cm Distribution Summary (300 Iterations)

Distribution: Weibull
Expression: -0.03 + WEIB(0.053, 4.61)
Square Error: 0.004852

Chi Square Test
Number of intervals = 10
Degrees of freedom = 7
Test Statistic = 11.6
Corresponding p-value = 0.119

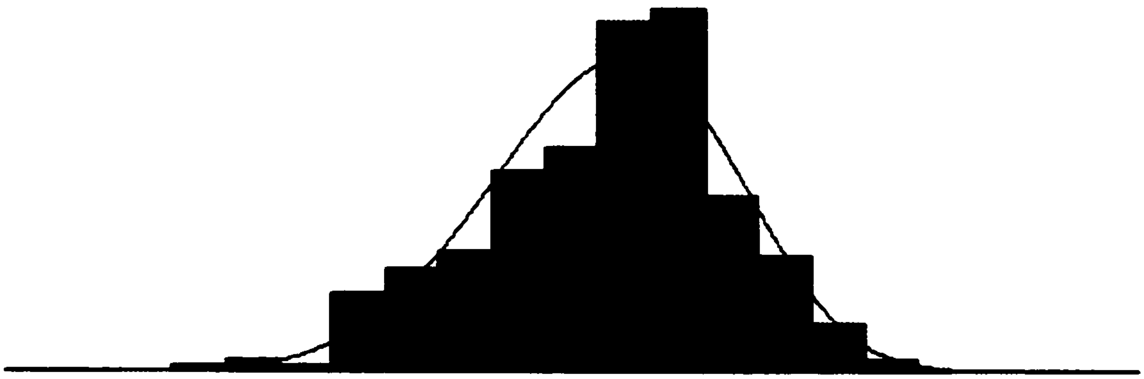
Kolmogorov-Smirnov Test
Test Statistic = 0.0573
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 300
Min Data Value = -0.0214
Max Data Value = 0.0474
Sample Mean = 0.0184
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.03 to 0.06
Number of Intervals = 17



Cm Distribution Summary (400 Iterations)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0528, 4.56)
Square Error: 0.002321

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 9.54
Corresponding p-value = 0.309

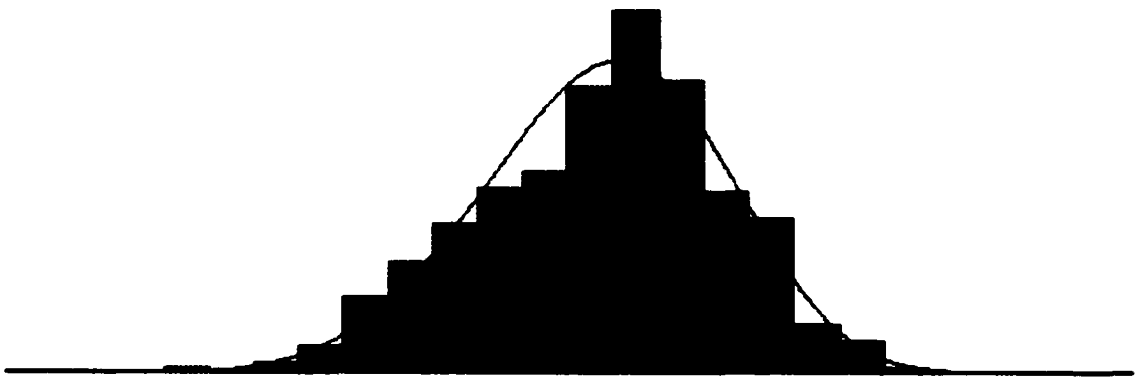
Kolmogorov-Smirnov Test
Test Statistic = 0.0401
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = -0.0214
Max Data Value = 0.0474
Sample Mean = 0.0182
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.03 to 0.06
Number of Intervals = 20



Cm Distribution Summary (500 iterations)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0527, 4.64)
Square Error: 0.001370

Chi Square Test
Number of intervals = 13
Degrees of freedom = 10
Test Statistic = 7.93
Corresponding p-value = 0.636

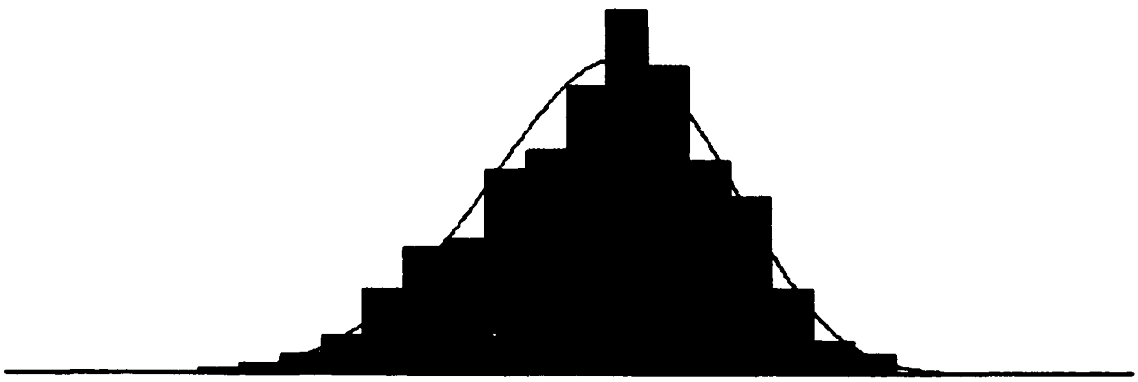
Kolmogorov-Smirnov Test
Test Statistic = 0.0365
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = -0.0214
Max Data Value = 0.0474
Sample Mean = 0.0182
Sample Std Dev = 0.0121

Histogram Summary

Histogram Range = -0.03 to 0.06
Number of Intervals = 22



Cm Distribution Summary (1000 iterations)

Distribution: Weibull
Expression: -0.04 + WEIB(0.063, 5.62)
Square Error: 0.000641

Chi Square Test
Number of intervals = 18
Degrees of freedom = 15
Test Statistic = 17.8
Corresponding p-value = 0.277

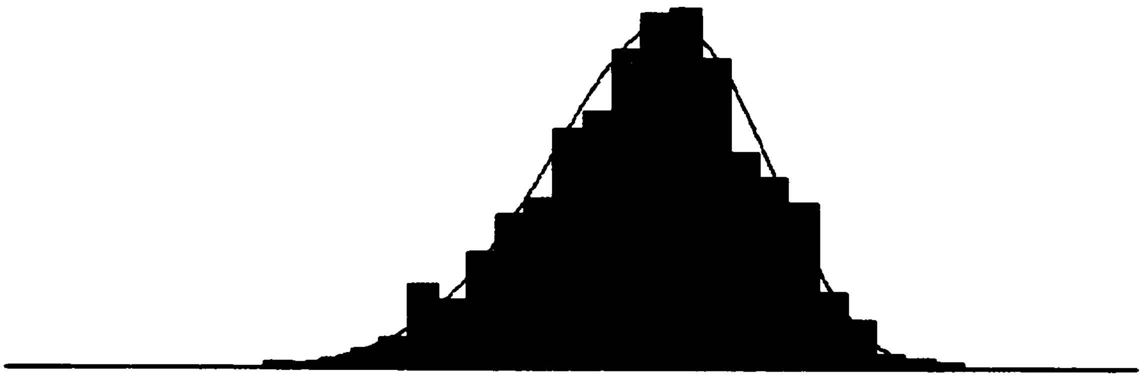
Kolmogorov-Smirnov Test
Test Statistic = 0.0228
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = -0.0234
Max Data Value = 0.0507
Sample Mean = 0.0184
Sample Std Dev = 0.0124

Histogram Summary

Histogram Range = -0.04 to 0.06
Number of Intervals = 31



Cm Distribution Summary (1500 iterations)

Distribution: Weibull
Expression: -0.04 + WEIB(0.063, 5.62)
Square Error: 0.000492

Chi Square Test
Number of intervals = 22
Degrees of freedom = 19
Test Statistic = 20.6
Corresponding p-value = 0.371

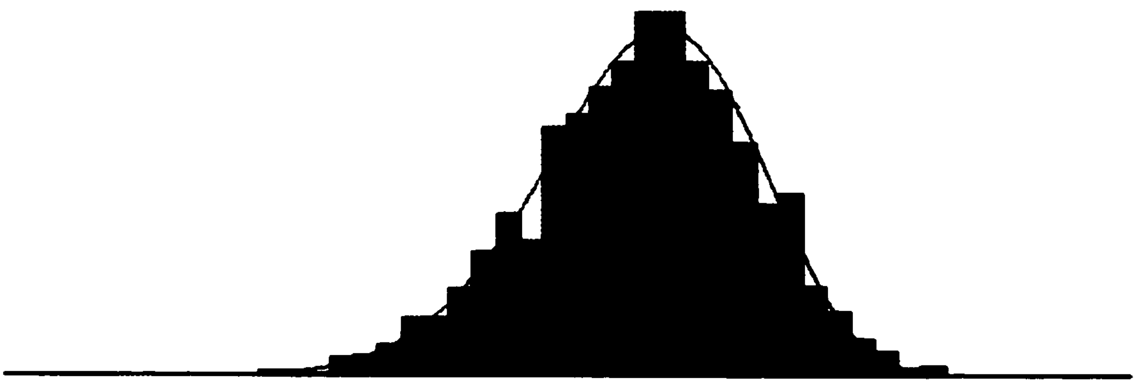
Kolmogorov-Smirnov Test
Test Statistic = 0.0166
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = -0.0236
Max Data Value = 0.0521
Sample Mean = 0.0183
Sample Std Dev = 0.0123

Histogram Summary

Histogram Range = -0.04 to 0.06
Number of Intervals = 38



Cm Distribution Summary (2000 iterations)

Distribution: Weibull
Expression: -0.04 + WEIB(0.063, 5.62)
Square Error: 0.000390

Chi Square Test
Number of intervals = 24
Degrees of freedom = 21
Test Statistic = 27.9
Corresponding p-value = 0.154

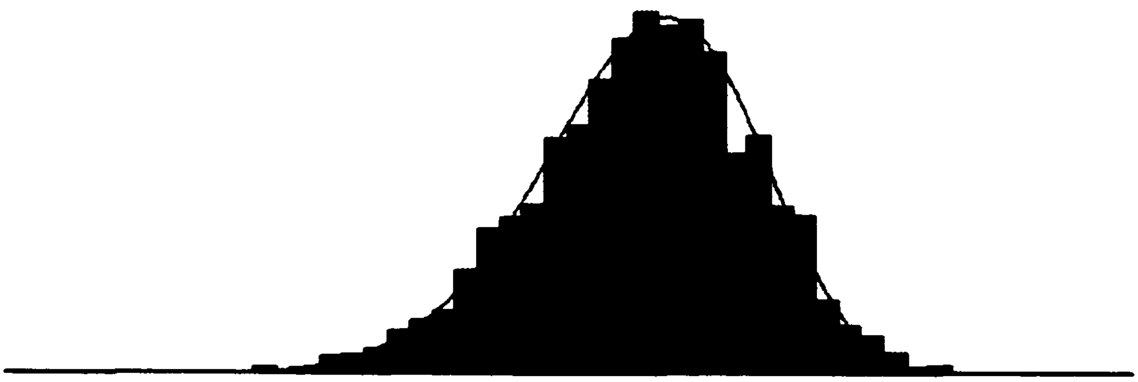
Kolmogorov-Smirnov Test
Test Statistic = 0.0146
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = -0.0236
Max Data Value = 0.0521
Sample Mean = 0.0183
Sample Std Dev = 0.0124

Histogram Summary

Histogram Range = -0.04 to 0.06
Number of Intervals = 40



APPENDIX G
@Risk Cg and Cm Distributions Spreadsheet

	Actual	Code 1	Code2
FR	5.5	0	0
WA	15	0	0
TFA	1.75	0	0
BFL	0.5	0	0
BL	0.02	-1.73472E-16	
MR	8	0	
alpha	10.66666667		0.316667
delev	-10.2418		-0.81081
erro1	-1E-05		
error2	0.000032		
cg		0.7412179	0.353632
cm			0.014715

APPENDIX H
Multiple Expert Opinion Aggregation Data

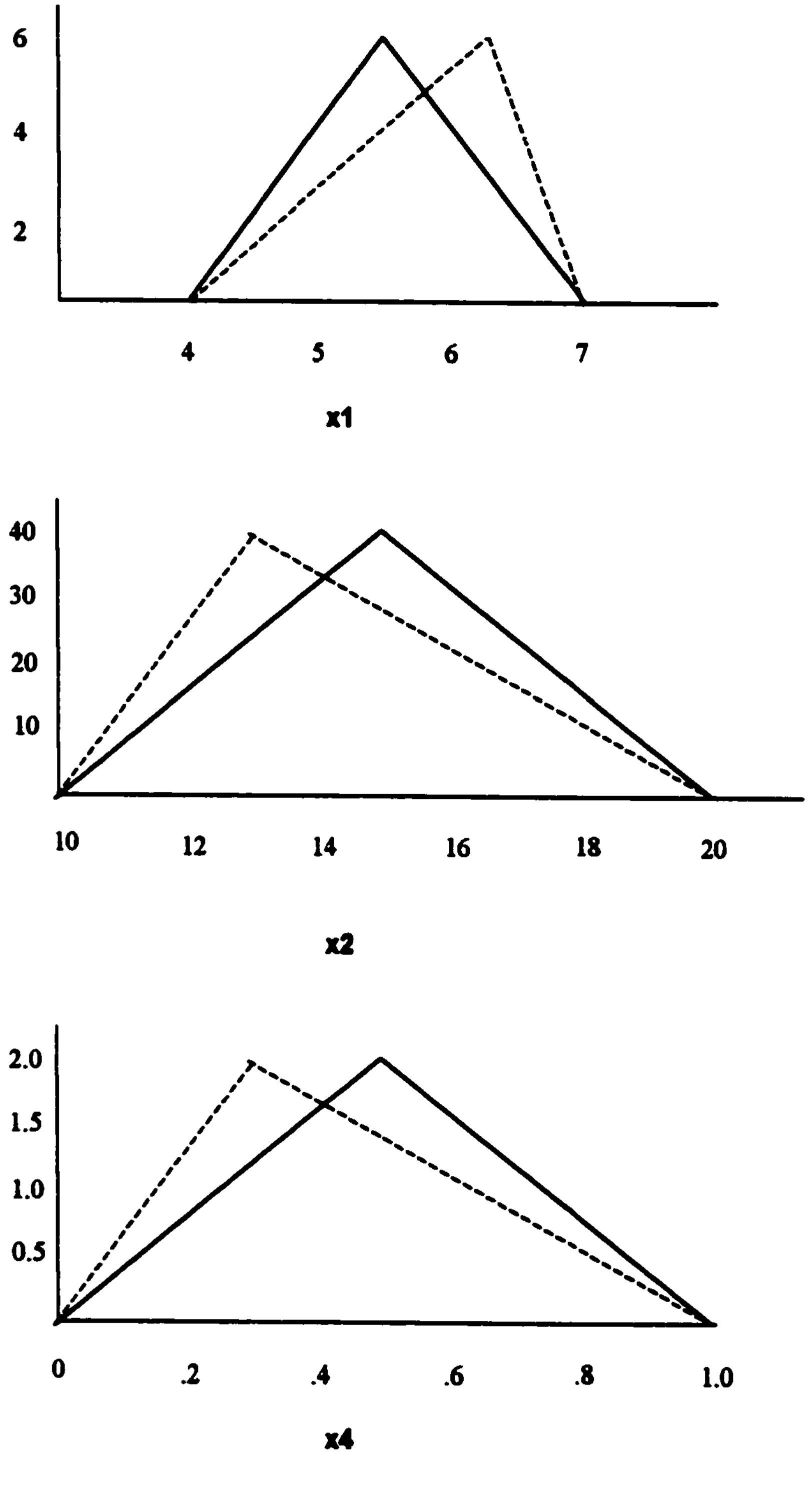
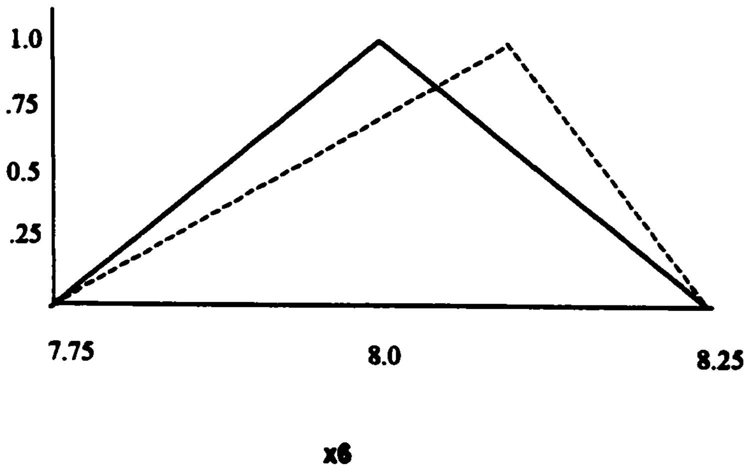


Figure H1 – Expert Opinion Comparison



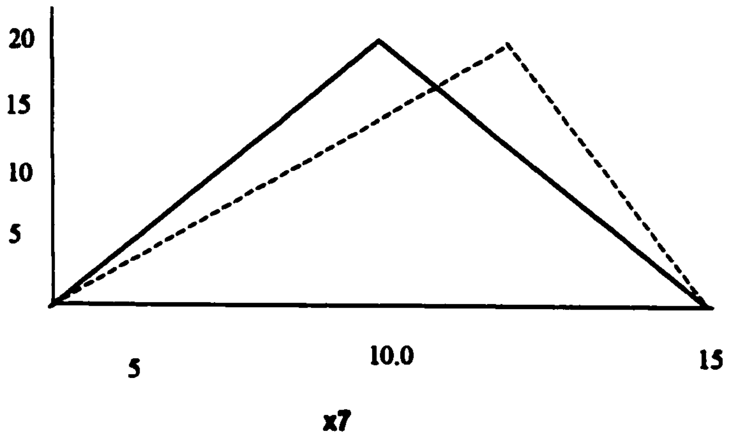
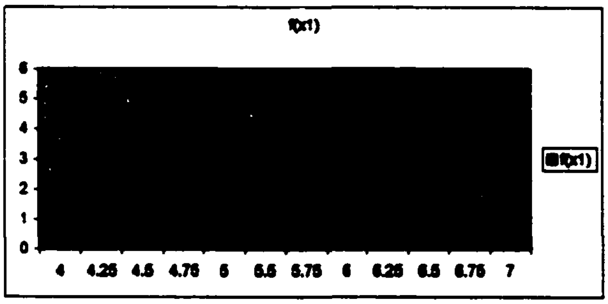
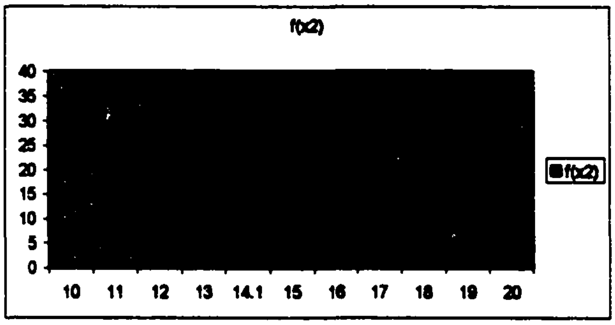


Figure H1(Continued) – Expert Opinion Comparison

x1	f(x1)	F(x)	U
4	0	0	0
4.25	1	1	0.029851
4.5	1.9	2.9	0.086567
4.75	2.6	5.5	0.164179
5	3.25	8.75	0.261194
5.5	5	13.75	0.410448
5.75	5	18.75	0.559701
6	5	23.75	0.708955
6.25	4.5	28.25	0.843284
6.5	3	31.25	0.932836
6.75	2.25	33.5	0.986418
7	0	33.5	1



x2	f(x2)	F(x)	U
10	0	0	0
11	11.5	11.5	0.055961
12	22	33.5	0.163017
13	32.5	66	0.321168
14.1	34	100	0.486618
15	35	135	0.656934
16	28	163	0.793187
17	21	184	0.895377
18	14.5	198.5	0.965937
19	7	205.5	0.982968
20	0	205.5	1



x4	f(x4)	F(x)	U
0	0	0	0
0.1	0.5	0.5	0.049628
0.2	1	1.5	0.148883
0.3	1.625	3.125	0.310174
0.41	1.65	4.775	0.473945
0.5	1.75	6.525	0.647643
0.6	1.4	7.925	0.7866
0.7	1.025	8.95	0.888337
0.8	0.675	9.625	0.955335
0.9	0.45	10.075	0.977667
1	0	10.075	1

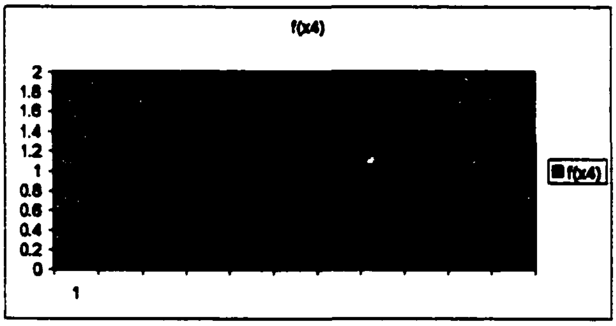
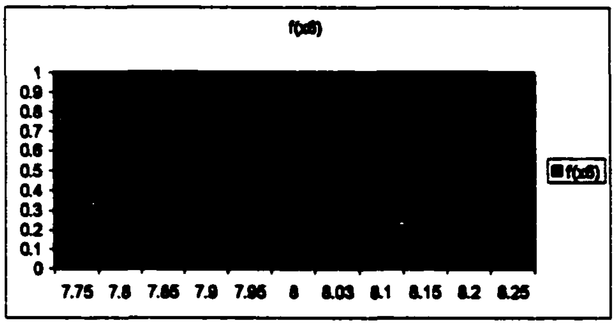


Figure H2 – Aggregated Discrete Distributions

x8	f(x8)	F(x)	U
7.75	0	0	0
7.8	0.125	0.125	0.025
7.85	0.325	0.45	0.09
7.9	0.5	0.95	0.19
7.95	0.675	1.625	0.325
8	0.875	2.5	0.5
8.03	0.85	3.35	0.67
8.1	0.8125	4.1625	0.8325
8.15	0.5375	4.7	0.94
8.2	0.3	5	0.97
8.25	0	5	1



x7	f(x7)	F(x)	U
5	0	0	0
6	3	3	0.030691
7	6.25	9.25	0.094629
8	10	19.25	0.196931
9	13	32.25	0.329923
10	17	49.25	0.503836
10.8	16.5	65.75	0.672634
12	16	81.75	0.836317
13	11	92.75	0.948849
14	5	97.75	0.974425
15	0	97.75	1

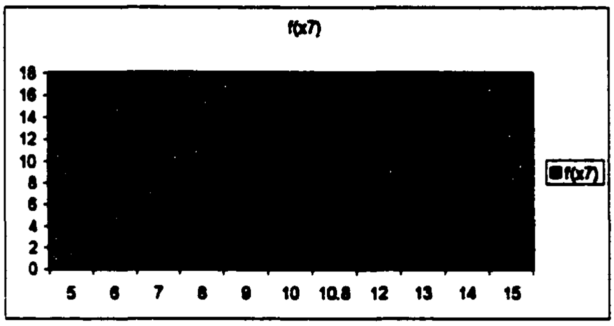


Figure H2(Continued) – Aggregated Discrete Distributions

APPENDIX I
Cg and Cm Distributions (Multiple Experts, Random Sampling)

Cg Distribution Summary (10 Iterations – Discrete Monte Carlo)

Distribution: Lognormal
Expression: 0.7 + LOGN(0.0365, 0.0148)
Square Error: 0.080960

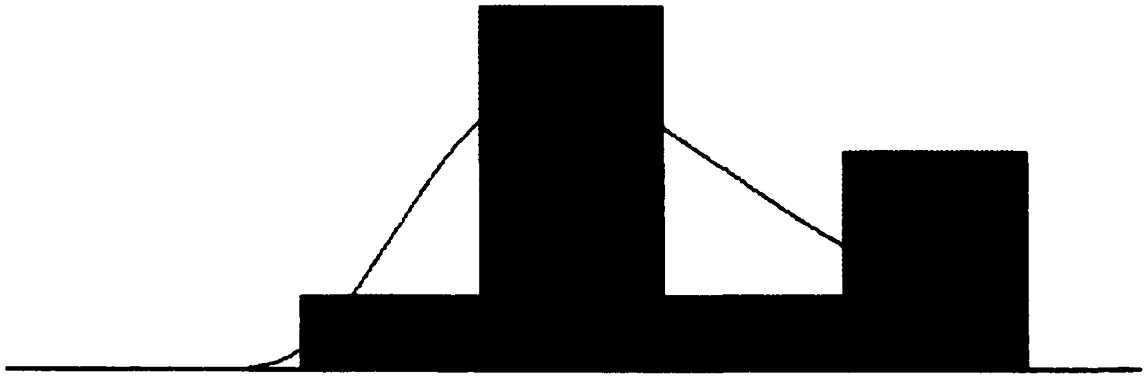
Kolmogorov-Smirnov Test
Test Statistic = 0.149
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = 0.714
Max Data Value = 0.753
Sample Mean = 0.736
Sample Std Dev = 0.0129

Histogram Summary

Histogram Range = 0.7 to 0.76
Number of Intervals = 5



Cg Distribution Summary (25 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: 0.7 + WEIB(0.0439, 4.33)
Square Error: 0.003925

Chi Square Test
Number of intervals = 2
Degrees of freedom = -1
Test Statistic = 0.0256
Corresponding p-value < 0.005

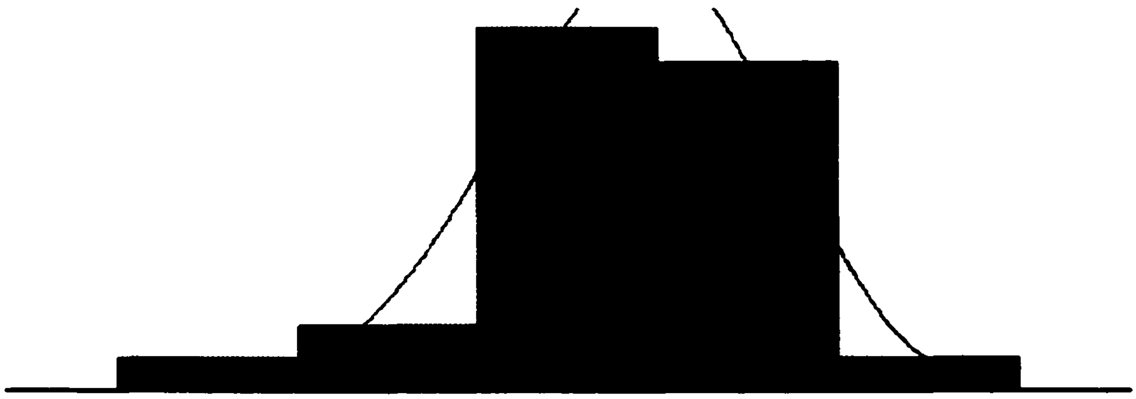
Kolmogorov-Smirnov Test
Test Statistic = 0.099
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = 0.714
Max Data Value = 0.761
Sample Mean = 0.74
Sample Std Dev = 0.0108

Histogram Summary

Histogram Range = 0.7 to 0.77
Number of Intervals = 5



Cg Distribution Summary (50 Iterations – Discrete Monte Carlo)

Distribution: Erlang
Expression: 0.7 + ERLA(0.0031, 13)
Square Error: 0.005862

Chi Square Test
Number of intervals = 4
Degrees of freedom = 1
Test Statistic = 1.8
Corresponding p-value = 0.198

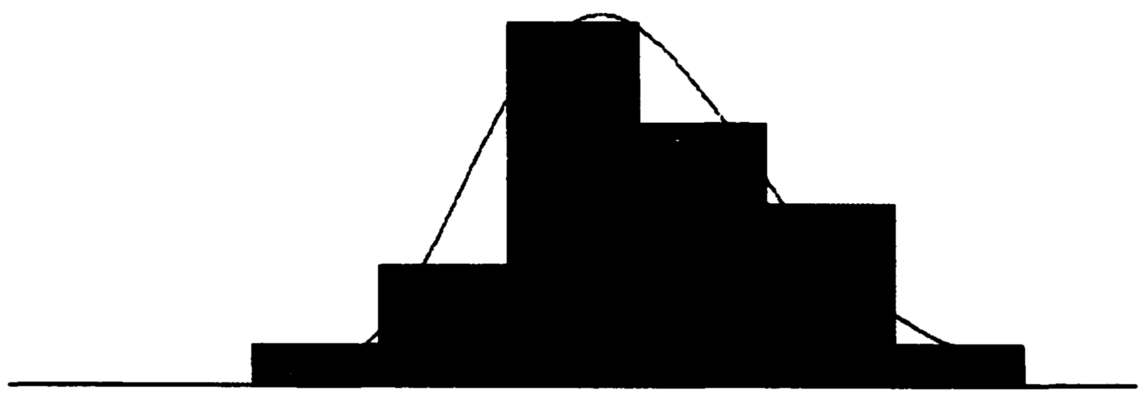
Kolmogorov-Smirnov Test
Test Statistic = 0.0701
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = 0.714
Max Data Value = 0.762
Sample Mean = 0.74
Sample Std Dev = 0.0109

Histogram Summary

Histogram Range = 0.7 to 0.77
Number of Intervals = 7



Cg Distribution Summary (100 Iterations – Discrete Monte Carlo)

Distribution: Gamma
Expression: 0.7 + GAMM(0.00276, 14.3)
Square Error: 0.003873

Chi Square Test
Number of intervals = 5
Degrees of freedom = 2
Test Statistic = 1.23
Corresponding p-value = 0.547

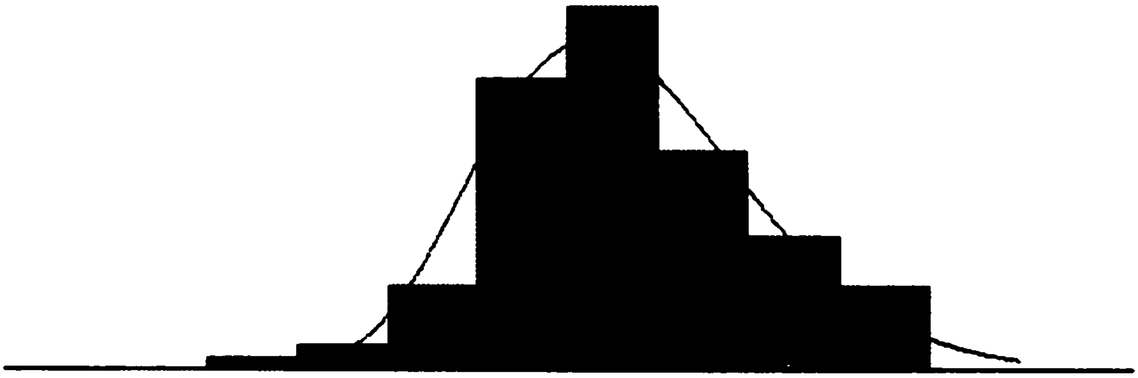
Kolmogorov-Smirnov Test
Test Statistic = 0.064
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = 0.714
Max Data Value = 0.763
Sample Mean = 0.74
Sample Std Dev = 0.0101

Histogram Summary

Histogram Range = 0.7 to 0.77
Number of Intervals = 10



Cg Distribution Summary (200 Iterations – Discrete Monte Carlo)

Distribution: Lognormal
Expression: 0.7 + LOGN(0.0385, 0.0102)
Square Error: 0.000825

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 1.33
Corresponding p-value > 0.75

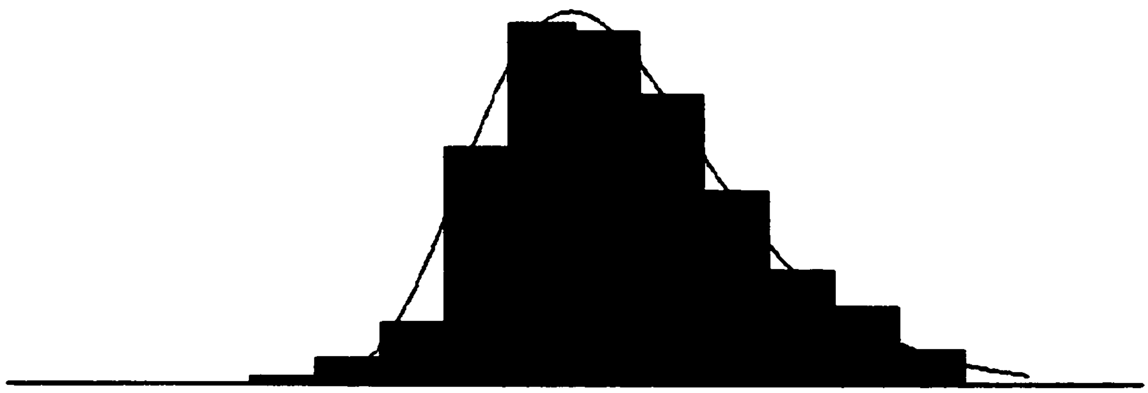
Kolmogorov-Smirnov Test
Test Statistic = 0.0442
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = 0.714
Max Data Value = 0.763
Sample Mean = 0.738
Sample Std Dev = 0.00964

Histogram Summary

Histogram Range = 0.7 to 0.77
Number of Intervals = 14



Cg Distribution Summary (300 Iterations – Discrete Monte Carlo)

Distribution: Erlang
Expression: 0.7 + ERLA(0.00262, 15)
Square Error: 0.002702

Chi Square Test
Number of intervals = 10
Degrees of freedom = 7
Test Statistic = 7.46
Corresponding p-value = 0.397

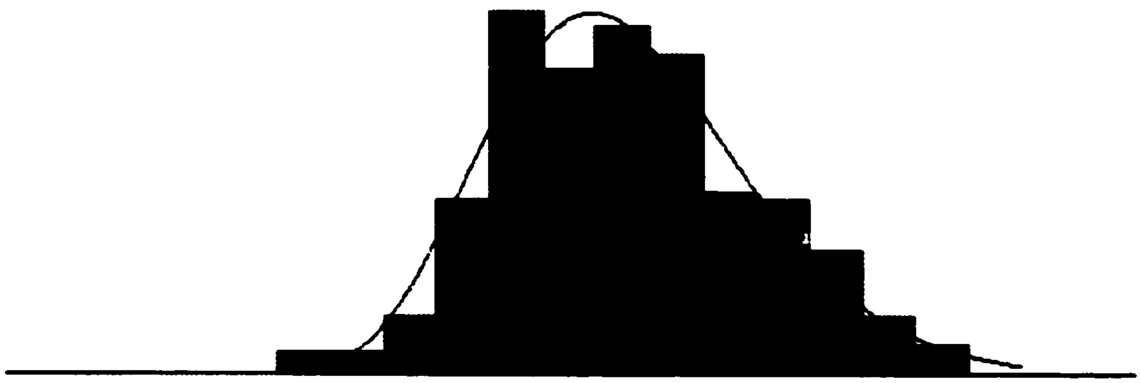
Kolmogorov-Smirnov Test
Test Statistic = 0.0288
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 300
Min Data Value = 0.714
Max Data Value = 0.764
Sample Mean = 0.739
Sample Std Dev = 0.00993

Histogram Summary

Histogram Range = 0.7 to 0.77
Number of Intervals = 17



Cg Distribution Summary (400 Iterations – Discrete Monte Carlo)

Distribution: Erlang
Expression: 0.7 + ERLA(0.00277, 14)
Square Error: 0.001799

Chi Square Test
Number of intervals = 12
Degrees of freedom = 9
Test Statistic = 7.4
Corresponding p-value = 0.597

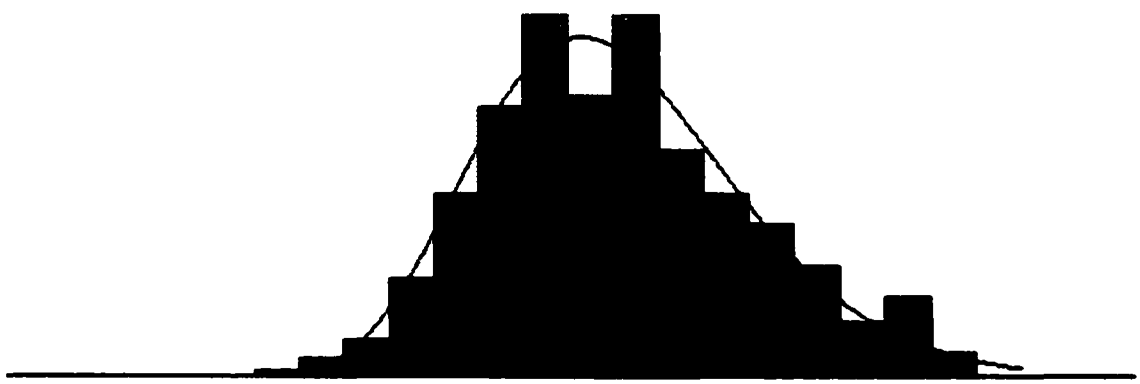
Kolmogorov-Smirnov Test
Test Statistic = 0.0212
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = 0.714
Max Data Value = 0.764
Sample Mean = 0.739
Sample Std Dev = 0.0102

Histogram Summary

Histogram Range = 0.7 to 0.77
Number of Intervals = 20



Cg Distribution Summary (500 Iterations – Discrete Monte Carlo)

Distribution: Gamma
Expression: 0.7 + GAMM(0.00283, 13.7)
Square Error: 0.001065

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 4.18
Corresponding p-value > 0.75

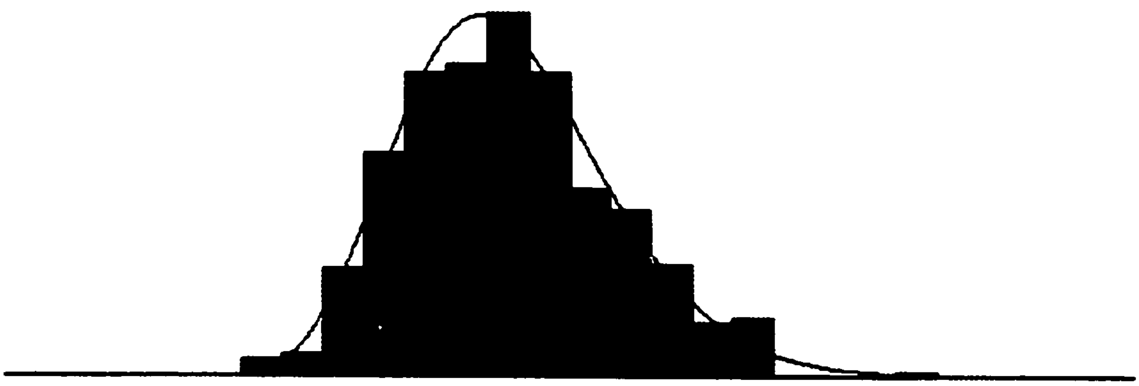
Kolmogorov-Smirnov Test
Test Statistic = 0.026
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = 0.714
Max Data Value = 0.779
Sample Mean = 0.739
Sample Std Dev = 0.0104

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 22



Cg Distribution Summary (1000 Iterations – Discrete Monte Carlo)

Distribution: Erlang
Expression: 0.7 + ERLA(0.00279, 14)
Square Error: 0.000967

Chi Square Test
Number of intervals = 17
Degrees of freedom = 14
Test Statistic = 26
Corresponding p-value = 0.0262

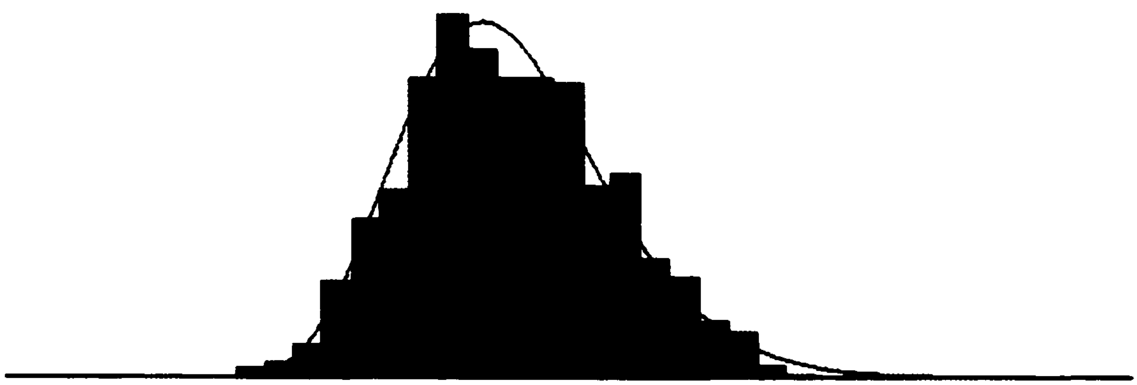
Kolmogorov-Smirnov Test
Test Statistic = 0.0257
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = 0.714
Max Data Value = 0.779
Sample Mean = 0.739
Sample Std Dev = 0.0103

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 31



Cg Distribution Summary (1500 Iterations – Discrete Monte Carlo)

Distribution: Erlang
Expression: 0.7 + ERLA(0.00277, 14)
Square Error: 0.000522

Chi Square Test
Number of intervals = 22
Degrees of freedom = 19
Test Statistic = 31.8
Corresponding p-value = 0.0352

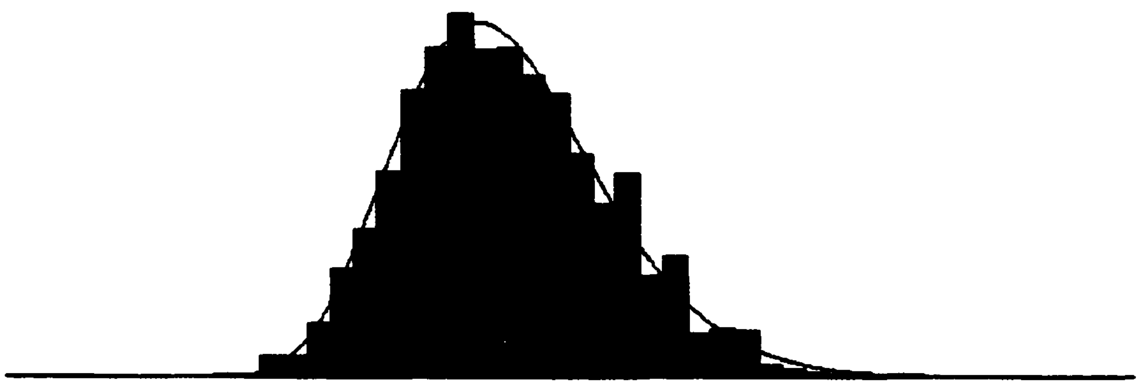
Kolmogorov-Smirnov Test
Test Statistic = 0.015
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = 0.711
Max Data Value = 0.779
Sample Mean = 0.739
Sample Std Dev = 0.0102

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 38



Cg Distribution Summary (2000 Iterations – Discrete Monte Carlo)

Distribution: Erlang
Expression: 0.7 + ERLA(0.00277, 14)
Square Error: 0.000439

Chi Square Test
Number of intervals = 24
Degrees of freedom = 21
Test Statistic = 33.6
Corresponding p-value = 0.0421

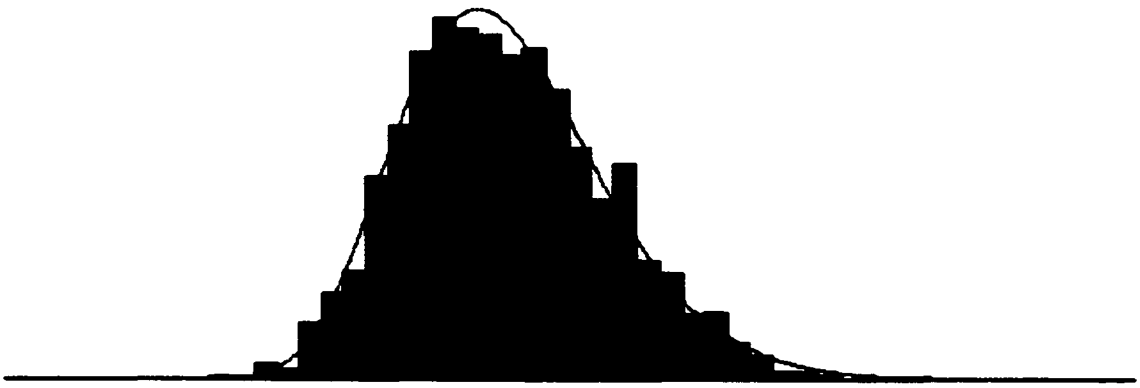
Kolmogorov-Smirnov Test
Test Statistic = 0.0146
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = 0.711
Max Data Value = 0.779
Sample Mean = 0.739
Sample Std Dev = 0.0102

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 40



Cm Distribution Summary (10 Iterations – Discrete Monte Carlo)

Distribution: Triangular
Expression: TRIA(-0.02, 0.0346, 0.05)
Square Error: 0.010670

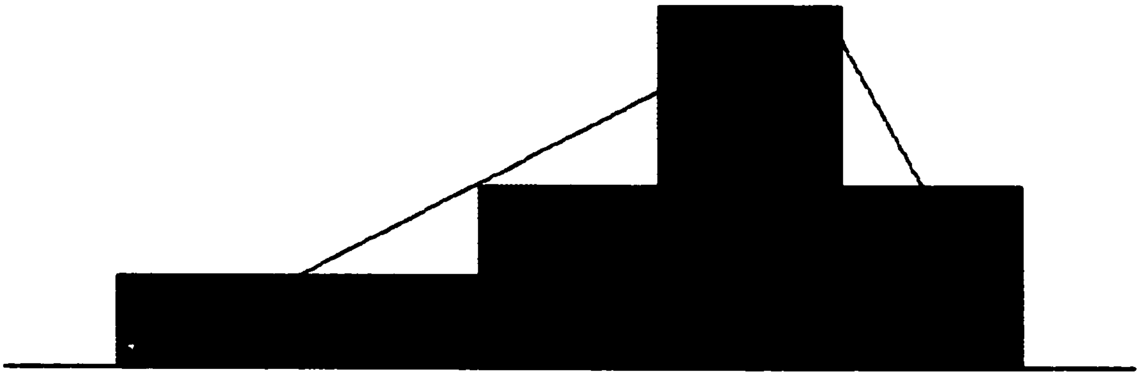
Kolmogorov-Smirnov Test
Test Statistic = 0.121
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = -0.00687
Max Data Value = 0.041
Sample Mean = 0.0215
Sample Std Dev = 0.0147

Histogram Summary

Histogram Range = -0.02 to 0.05
Number of Intervals = 5



Cm Distribution Summary (25 Iterations – Discrete Monte Carlo)

Distribution: Triangular
Expression: TRIA(-0.02, 0.032, 0.05)
Square Error: 0.008093

Chi Square Test
Number of intervals = 3
Degrees of freedom = 1
Test Statistic = 0.724
Corresponding p-value = 0.423

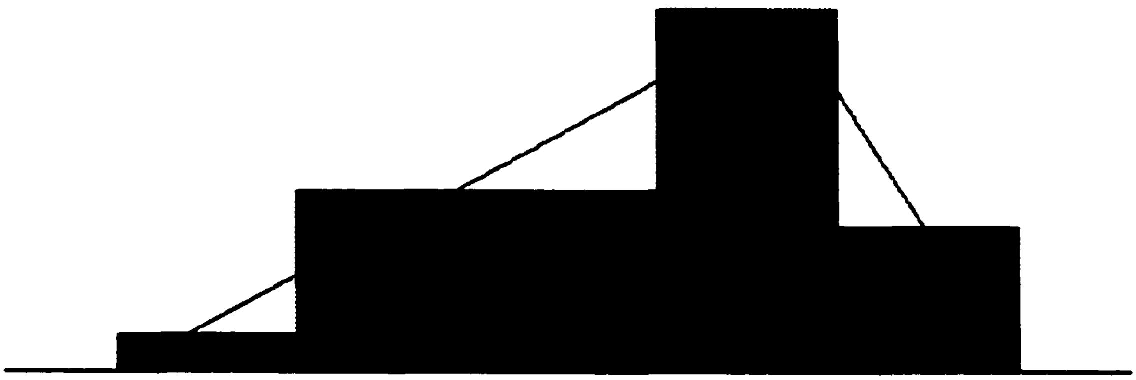
Kolmogorov-Smirnov Test
Test Statistic = 0.0944
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = -0.00687
Max Data Value = 0.041
Sample Mean = 0.0207
Sample Std Dev = 0.0135

Histogram Summary

Histogram Range = -0.02 to 0.05
Number of Intervals = 5



Cm Distribution Summary (50 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.02 + WEIB(0.049, 4.05)
Square Error: 0.016433

Chi Square Test
Number of intervals = 4
Degrees of freedom = 1
Test Statistic = 1.49
Corresponding p-value = 0.232

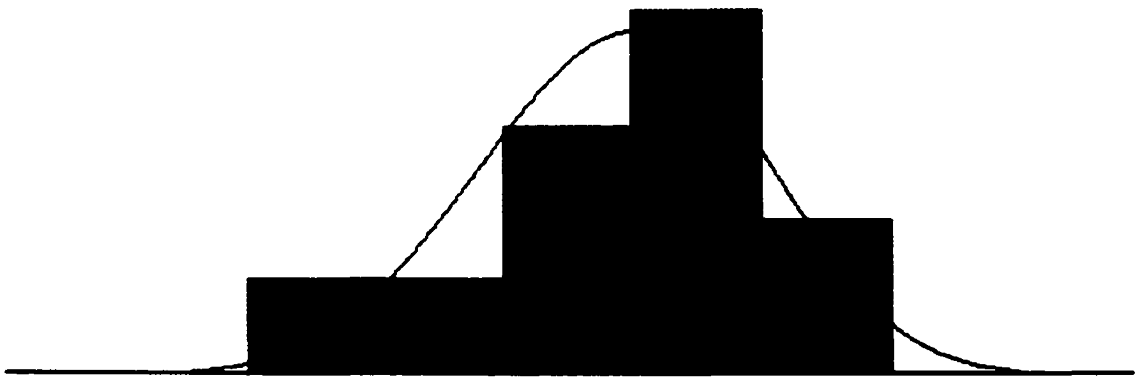
Kolmogorov-Smirnov Test
Test Statistic = 0.0996
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = -0.00687
Max Data Value = 0.0478
Sample Mean = 0.0244
Sample Std Dev = 0.013

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 7



Cm Distribution Summary (100 Iterations – Discrete Monte Carlo)

Distribution: Beta
Expression: -0.02 + 0.08 * BETA(4.73, 3.85)
Square Error: 0.002259

Chi Square Test
Number of intervals = 5
Degrees of freedom = 2
Test Statistic = 0.822
Corresponding p-value = 0.674

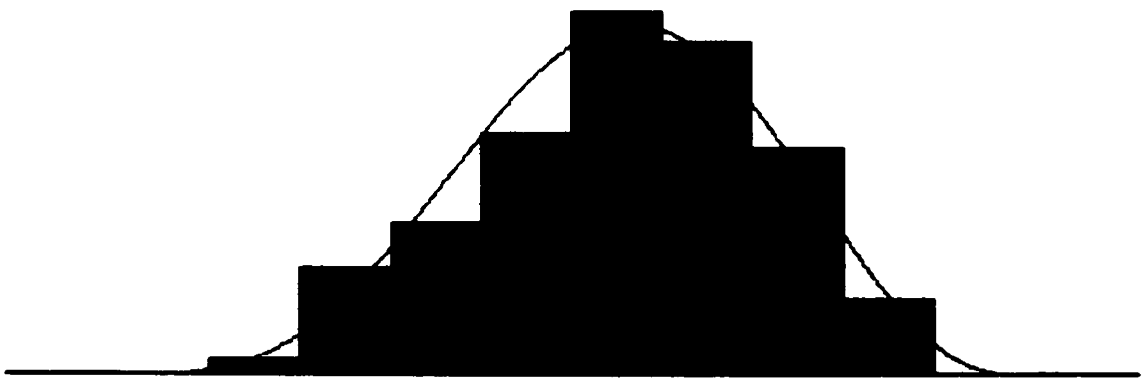
Kolmogorov-Smirnov Test
Test Statistic = 0.0683
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = -0.00687
Max Data Value = 0.049
Sample Mean = 0.0241
Sample Std Dev = 0.0129

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 10



Cm Distribution Summary (200 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0494, 4.25)
Square Error: 0.004625

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 8.3
Corresponding p-value = 0.154

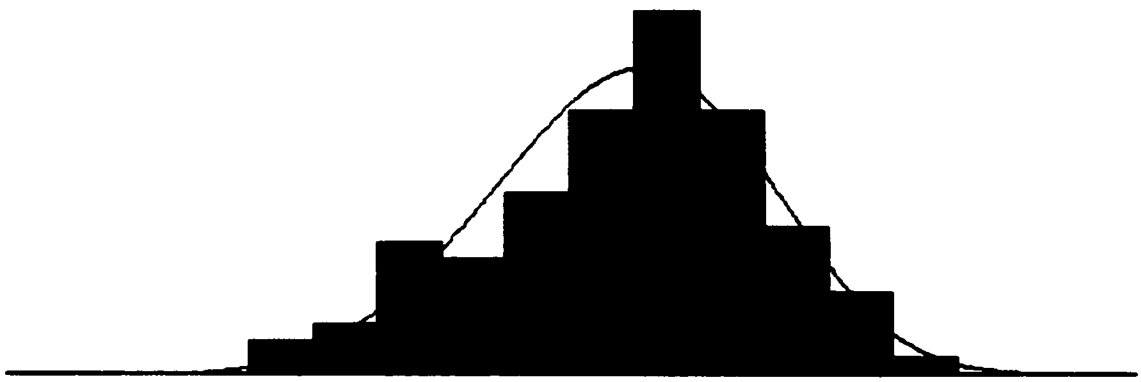
Kolmogorov-Smirnov Test
Test Statistic = 0.0547
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = -0.00738
Max Data Value = 0.049
Sample Mean = 0.0249
Sample Std Dev = 0.0124

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 14



Cm Distribution Summary (300 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.03 + WEIB(0.06, 5.24)
Square Error: 0.001081

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 2.37
Corresponding p-value > 0.75

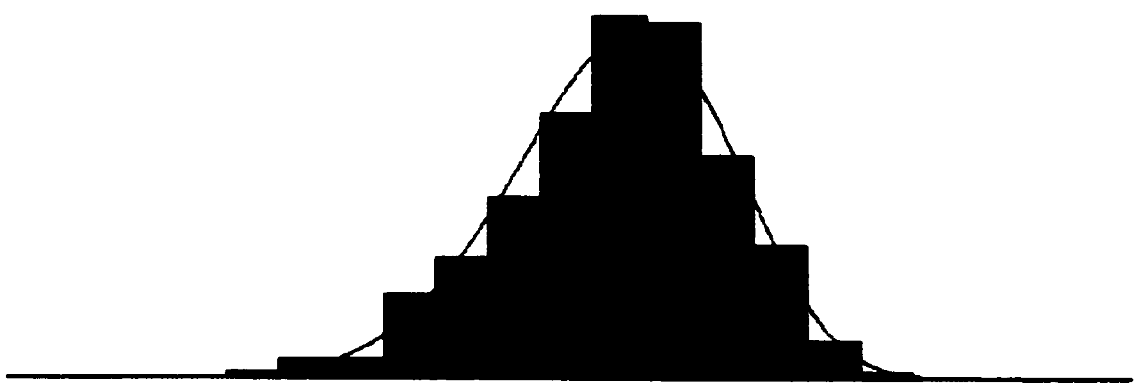
Kolmogorov-Smirnov Test
Test Statistic = 0.0405
Corresponding p-value > 0.15

Data Summary

Number of Data Points	= 300
Min Data Value	= -0.0134
Max Data Value	= 0.0553
Sample Mean	= 0.0252
Sample Std Dev	= 0.0123

Histogram Summary

Histogram Range	= -0.03 to 0.07
Number of Intervals	= 17



Cm Distribution Summary (400 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0598, 5.27)
Square Error: 0.001182

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 5.34
Corresponding p-value = 0.721

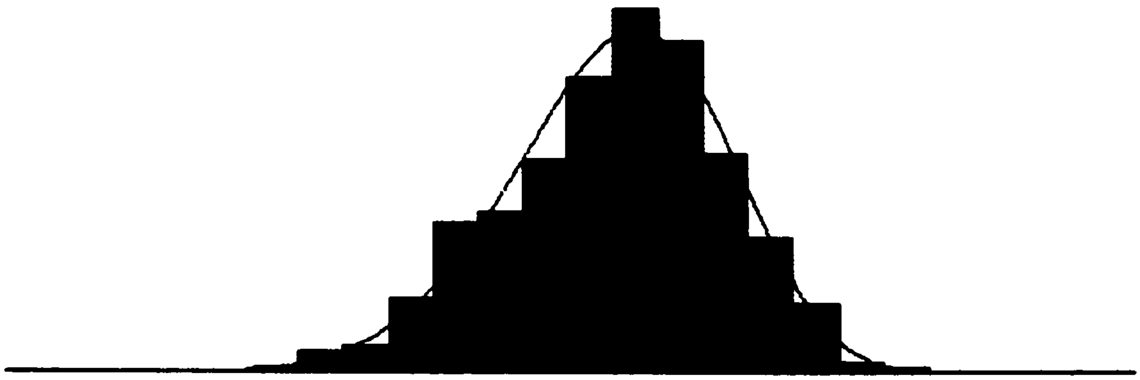
Kolmogorov-Smirnov Test
Test Statistic = 0.0314
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = -0.0134
Max Data Value = 0.0553
Sample Mean = 0.0251
Sample Std Dev = 0.0121

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 20



Cm Distribution Summary (500 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0599, 5.39)
Square Error: 0.001335

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 9.04
Corresponding p-value = 0.353

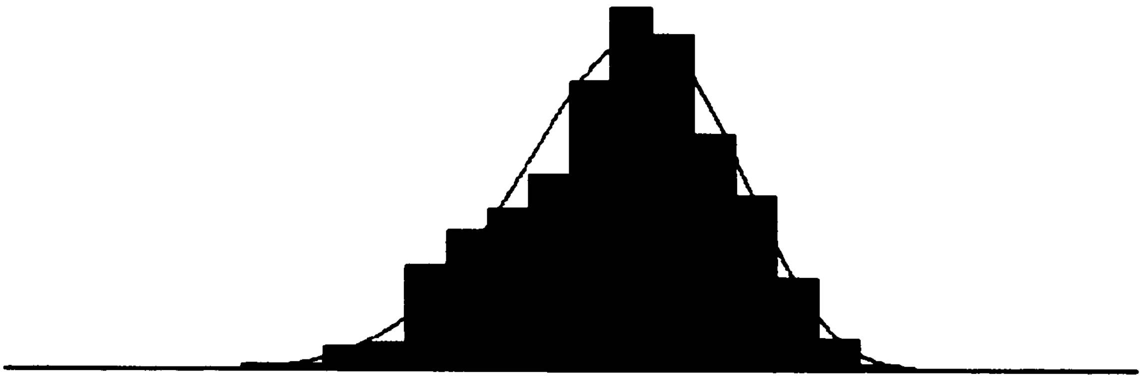
Kolmogorov-Smirnov Test
Test Statistic = 0.0304
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = -0.0134
Max Data Value = 0.0553
Sample Mean = 0.0252
Sample Std Dev = 0.0119

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 22



Cm Distribution Summary (1000 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0599, 5.39)
Square Error: 0.001214

Chi Square Test
Number of intervals = 17
Degrees of freedom = 14
Test Statistic = 18.3
Corresponding p-value = 0.206

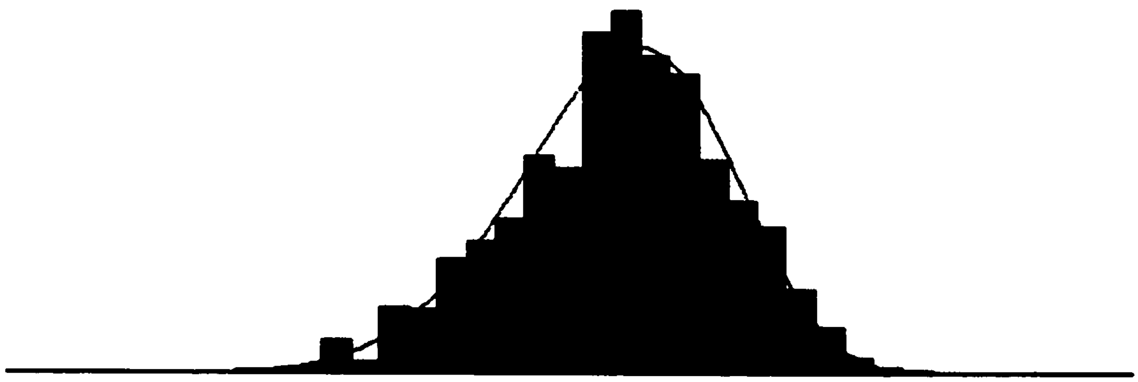
Kolmogorov-Smirnov Test
Test Statistic = 0.0298
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = -0.0142
Max Data Value = 0.0584
Sample Mean = 0.0255
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 31



Cm Distribution Summary (1500 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0599, 5.39)
Square Error: 0.000990

Chi Square Test
Number of intervals = 22
Degrees of freedom = 19
Test Statistic = 36.3
Corresponding p-value = 0.00975

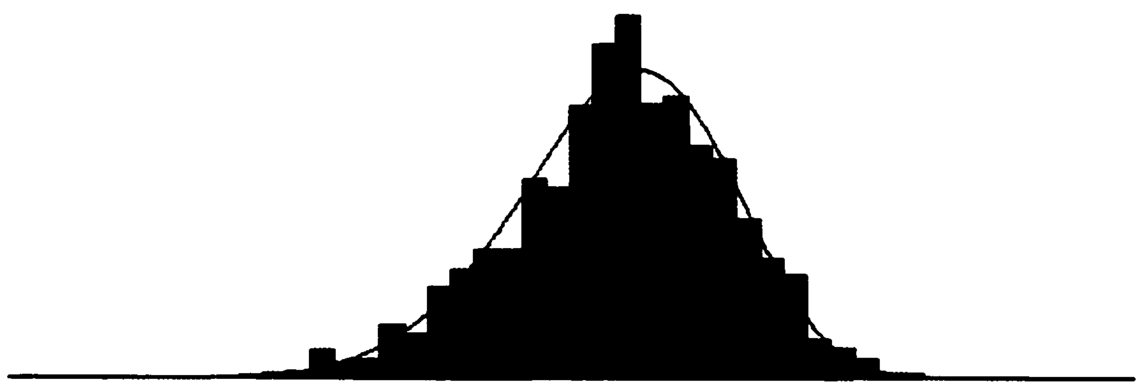
Kolmogorov-Smirnov Test
Test Statistic = 0.0214
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = -0.0142
Max Data Value = 0.0586
Sample Mean = 0.0254
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 38



Cm Distribution Summary (2000 Iterations – Discrete Monte Carlo)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0599, 5.39)
Square Error: 0.000917

Chi Square Test
Number of intervals = 24
Degrees of freedom = 21
Test Statistic = 36.2
Corresponding p-value = 0.0219

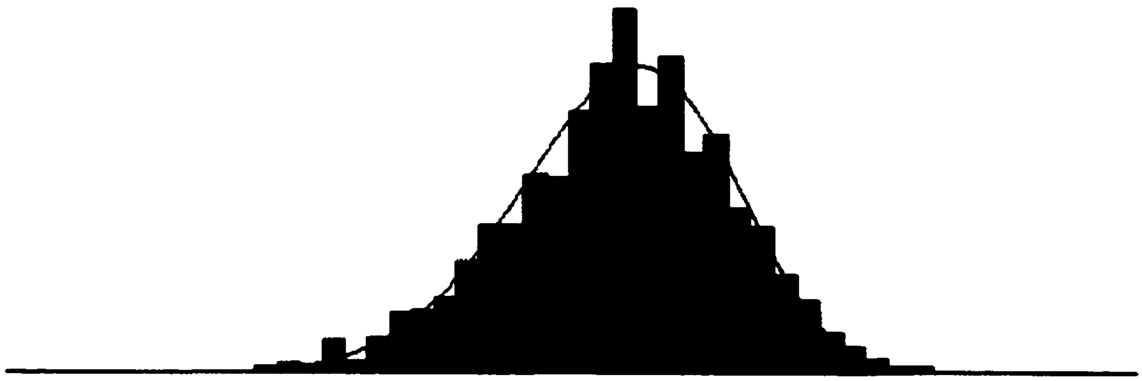
Kolmogorov-Smirnov Test
Test Statistic = 0.0163
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = -0.0142
Max Data Value = 0.0586
Sample Mean = 0.0254
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 40



APPENDIX J
Cg and Cm Distributions (Single Expert, Latin Hypercube Sampling)

Cg Distribution Summary (10 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.742, 0.0104)
Square Error: 0.022486

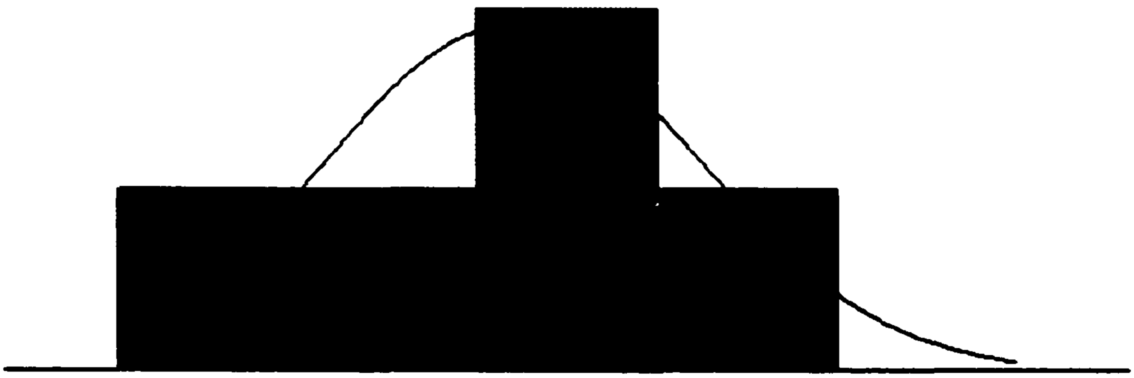
Kolmogorov-Smirnov Test
Test Statistic = 0.151
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = 0.725
Max Data Value = 0.758
Sample Mean = 0.742
Sample Std Dev = 0.011

Histogram Summary

Histogram Range = 0.72 to 0.77
Number of Intervals = 5



Cg Distribution Summary (25 Iterations – Latin Hypercube)

Distribution: Lognormal
Expression: 0.72 + LOGN(0.022, 0.011)
Square Error: 0.003368

Chi Square Test
Number of intervals = 2
Degrees of freedom = -1
Test Statistic = 0.054
Corresponding p-value < 0.005

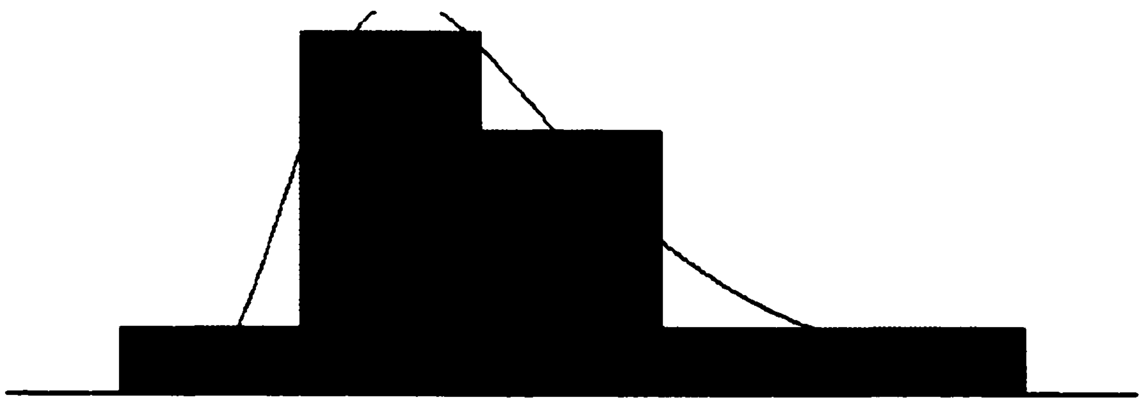
Kolmogorov-Smirnov Test
Test Statistic = 0.0981
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = 0.728
Max Data Value = 0.765
Sample Mean = 0.742
Sample Std Dev = 0.0102

Histogram Summary

Histogram Range = 0.72 to 0.77
Number of Intervals = 5



Cg Distribution Summary (50 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.742, 0.00987)
Square Error: 0.004057

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 0.262
Corresponding p-value < 0.005

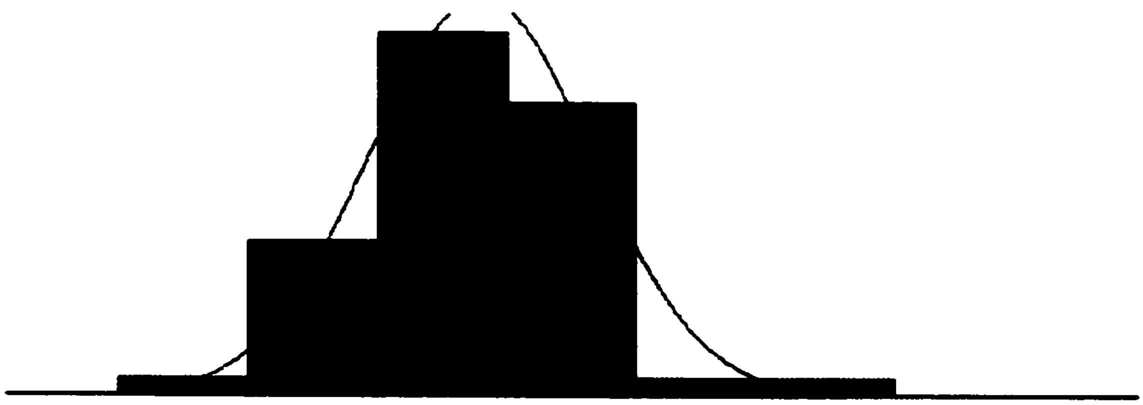
Kolmogorov-Smirnov Test
Test Statistic = 0.0749
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = 0.718
Max Data Value = 0.775
Sample Mean = 0.742
Sample Std Dev = 0.00997

Histogram Summary

Histogram Range = 0.71 to 0.79
Number of Intervals = 7



Cg Distribution Summary (100 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.742, 0.0109)
Square Error: 0.007487

Chi Square Test
Number of intervals = 4
Degrees of freedom = 1
Test Statistic = 2.16
Corresponding p-value = 0.16

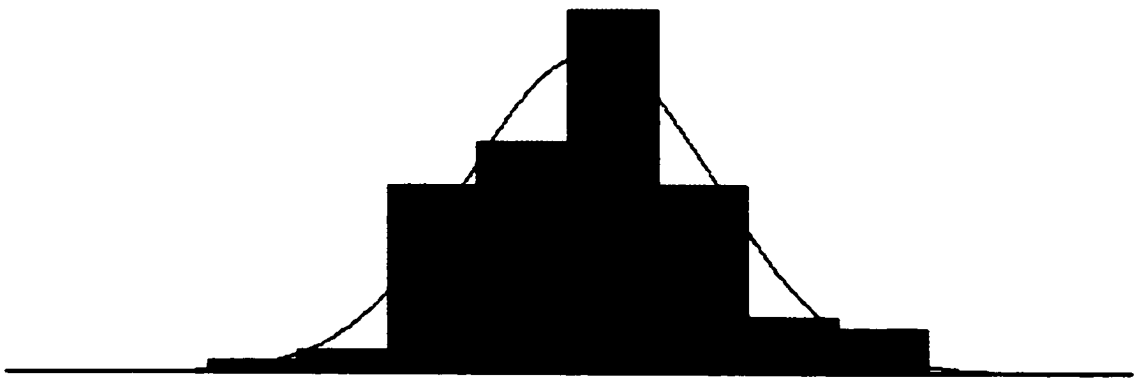
Kolmogorov-Smirnov Test
Test Statistic = 0.0551
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = 0.709
Max Data Value = 0.767
Sample Mean = 0.742
Sample Std Dev = 0.011

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 10



Cg Distribution Summary (200 Iterations – Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.08 * BETA(6.84, 6.21)
Square Error: 0.005346

Chi Square Test
Number of intervals = 7
Degrees of freedom = 4
Test Statistic = 6.4
Corresponding p-value = 0.186

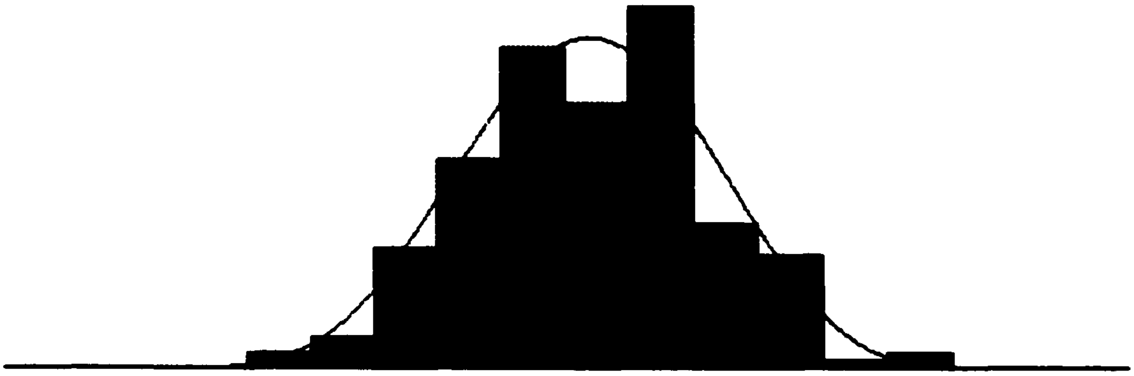
Kolmogorov-Smirnov Test
Test Statistic = 0.0408
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = 0.714
Max Data Value = 0.772
Sample Mean = 0.742
Sample Std Dev = 0.0107

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 14



Cg Distribution Summary (300 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.742, 0.0101)
Square Error: 0.002972

Chi Square Test
Number of intervals = 9
Degrees of freedom = 6
Test Statistic = 7.06
Corresponding p-value = 0.329

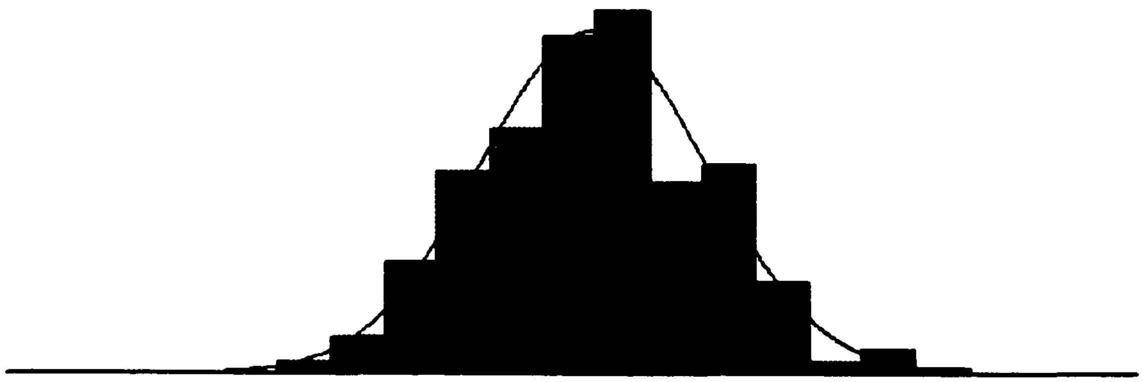
Kolmogorov-Smirnov Test
Test Statistic = 0.0229
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 300
Min Data Value = 0.715
Max Data Value = 0.772
Sample Mean = 0.742
Sample Std Dev = 0.0101

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 17



Cg Distribution Summary (400 Iterations – Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.08 * BETA(7.18, 6.7)
Square Error: 0.000758

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 4.22
Corresponding p-value > 0.75

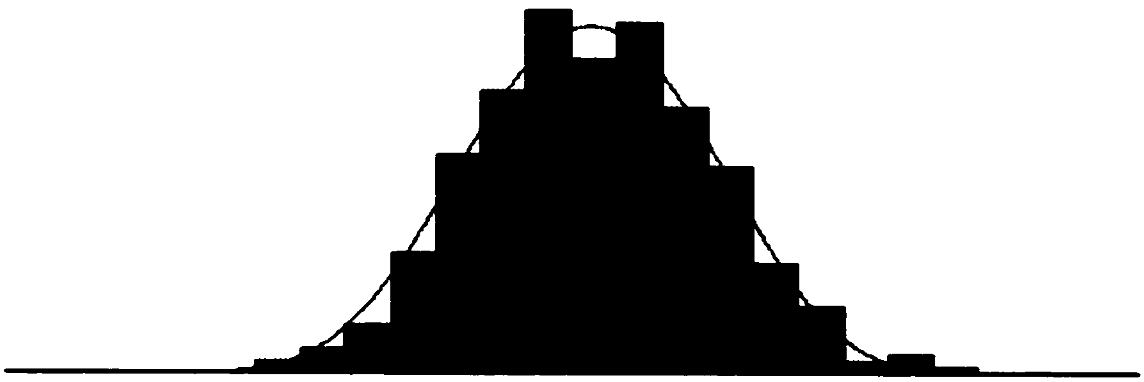
Kolmogorov-Smirnov Test
Test Statistic = 0.0314
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = 0.714
Max Data Value = 0.773
Sample Mean = 0.742
Sample Std Dev = 0.0103

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 20



Cg Distribution Summary (500 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.742, 0.0104)
Square Error: 0.002027

Chi Square Test
Number of intervals = 13
Degrees of freedom = 10
Test Statistic = 12.2
Corresponding p-value = 0.278

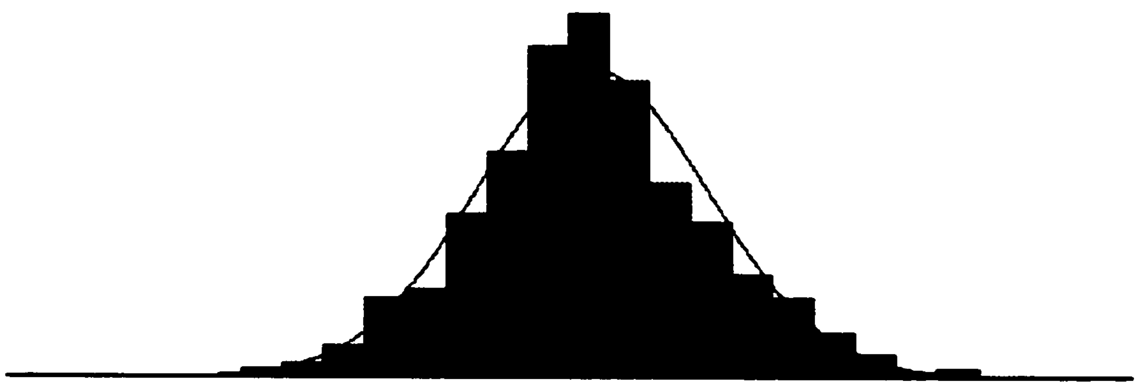
Kolmogorov-Smirnov Test
Test Statistic = 0.043
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = 0.712
Max Data Value = 0.773
Sample Mean = 0.742
Sample Std Dev = 0.0104

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 22



Cg Distribution Summary (1000 Iterations – Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.09 * BETA(8.43, 9.64)
Square Error: 0.001182

Chi Square Test
Number of intervals = 17
Degrees of freedom = 14
Test Statistic = 24.9
Corresponding p-value = 0.038

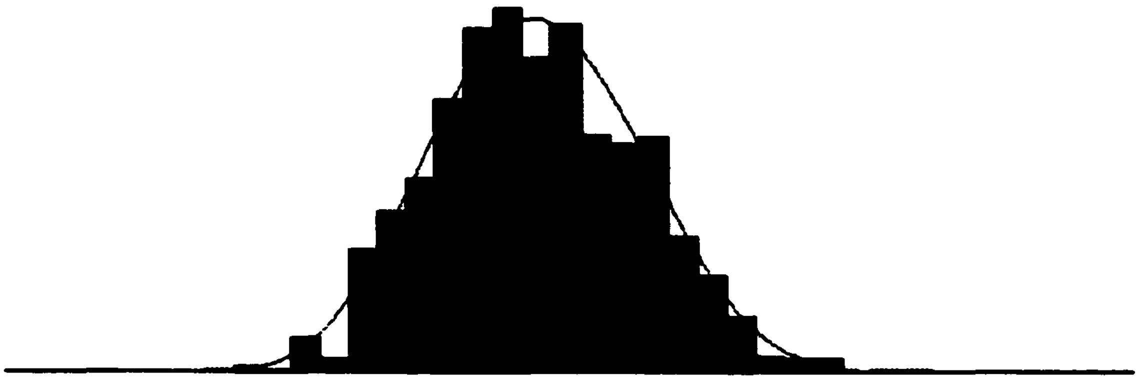
Kolmogorov-Smirnov Test
Test Statistic = 0.027
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = 0.711
Max Data Value = 0.779
Sample Mean = 0.742
Sample Std Dev = 0.0103

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 31



Cg Distribution Summary (1500 Iterations – Latin Hypercube)

Distribution: Erlang
Expression: 0.7 + ERLA(0.00247, 17)
Square Error: 0.000367

Chi Square Test
Number of intervals = 22
Degrees of freedom = 19
Test Statistic = 29.4
Corresponding p-value = 0.0625

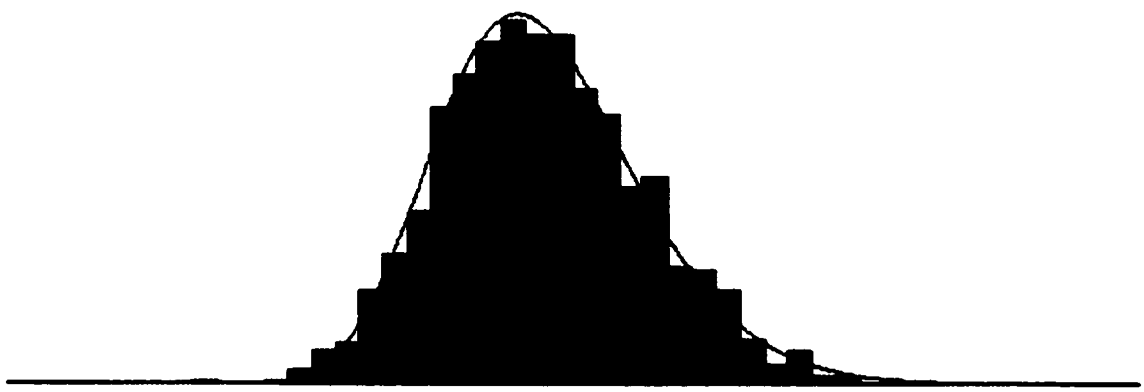
Kolmogorov-Smirnov Test
Test Statistic = 0.0162
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = 0.708
Max Data Value = 0.777
Sample Mean = 0.742
Sample Std Dev = 0.0101

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 38



Cg Distribution Summary (2000 Iterations – Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.09 * BETA(7.84, 8.97)
Square Error: 0.000354

Chi Square Test
Number of intervals = 24
Degrees of freedom = 21
Test Statistic = 15.2
Corresponding p-value > 0.75

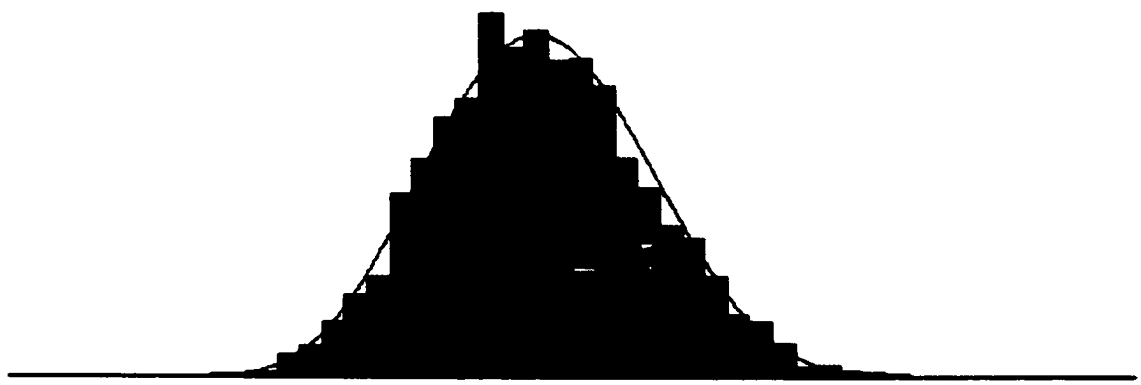
Kolmogorov-Smirnov Test
Test Statistic = 0.0213
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = 0.71
Max Data Value = 0.777
Sample Mean = 0.742
Sample Std Dev = 0.0106

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 40



Cm Distribution Summary (10 Iterations – Latin Hypercube)

Distribution: Beta
Expression: 0.01 + 0.03 * BETA(1.6, 1.59)
Square Error: 0.014427

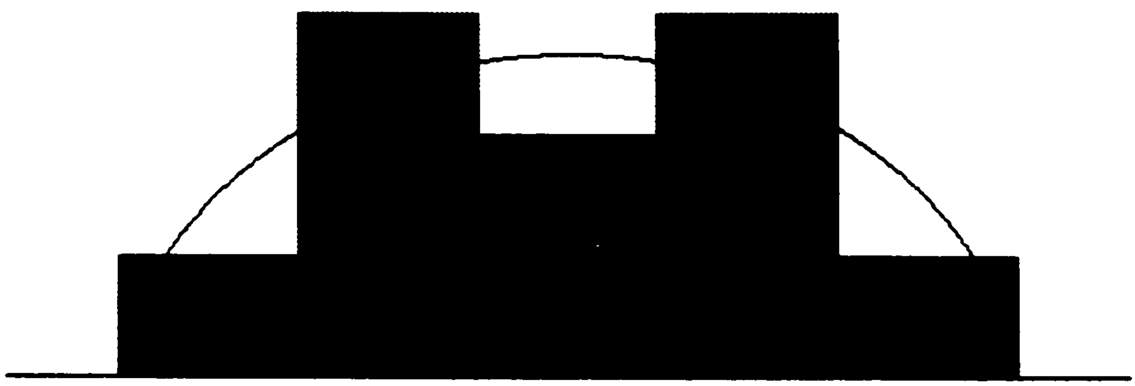
Kolmogorov-Smirnov Test
Test Statistic = 0.131
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = 0.0142
Max Data Value = 0.0346
Sample Mean = 0.025
Sample Std Dev = 0.00733

Histogram Summary

Histogram Range = 0.01 to 0.04
Number of Intervals = 5



Cm Distribution Summary (25 Iterations – Latin Hypercube)

Distribution: Beta
Expression: -0.03 + 0.08 * BETA(3.15, 1.88)
Square Error: 0.009663

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 0.194
Corresponding p-value < 0.005

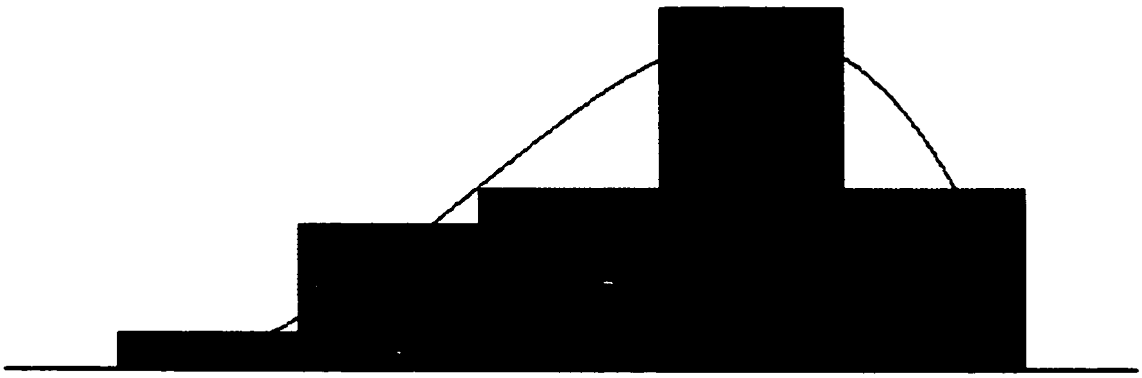
Kolmogorov-Smirnov Test
Test Statistic = 0.13
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = -0.0153
Max Data Value = 0.044
Sample Mean = 0.0203
Sample Std Dev = 0.017

Histogram Summary

Histogram Range = -0.03 to 0.05
Number of Intervals = 5



Cm Distribution Summary (50 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0448, 3.27)
Square Error: 0.004883

Chi Square Test
Number of intervals = 4
Degrees of freedom = 1
Test Statistic = 1.26
Corresponding p-value = 0.269

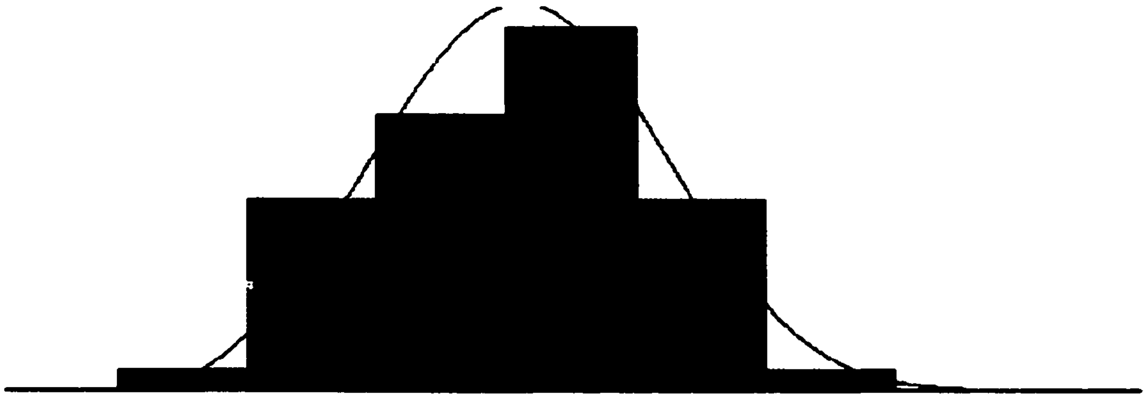
Kolmogorov-Smirnov Test
Test Statistic = 0.0902
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = -0.00743
Max Data Value = 0.0547
Sample Mean = 0.0201
Sample Std Dev = 0.0135

Histogram Summary

Histogram Range = -0.02 to 0.07
Number of Intervals = 7



Cm Distribution Summary (100 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.0191, 0.0127)
Square Error: 0.007921

Chi Square Test
Number of intervals = 5
Degrees of freedom = 2
Test Statistic = 2.15
Corresponding p-value = 0.362

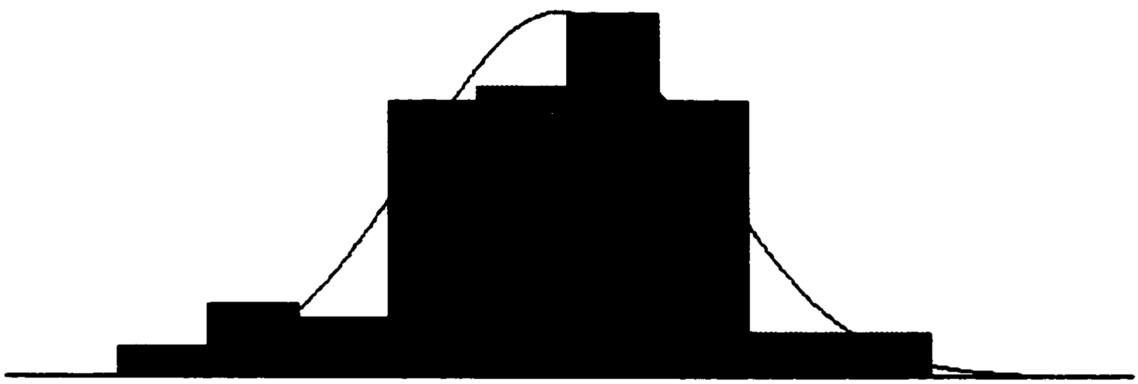
Kolmogorov-Smirnov Test
Test Statistic = 0.063
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = -0.0127
Max Data Value = 0.0499
Sample Mean = 0.0191
Sample Std Dev = 0.0127

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 10



Cm Distribution Summary (200 Iterations – Latin Hypercube)

Distribution: Normal
Expression: NORM(0.0182, 0.0126)
Square Error: 0.002321

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 3.61
Corresponding p-value = 0.611

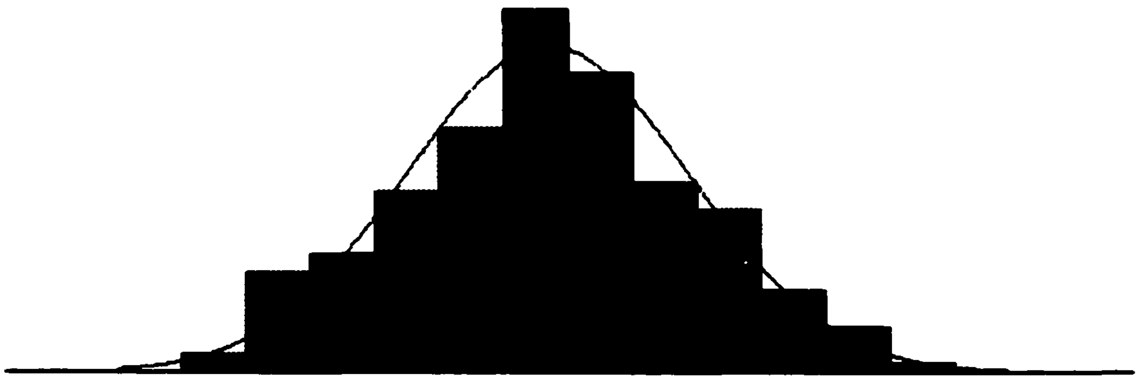
Kolmogorov-Smirnov Test
Test Statistic = 0.0404
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = -0.012
Max Data Value = 0.0504
Sample Mean = 0.0182
Sample Std Dev = 0.0127

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 14



Cm Distribution Summary (300 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.04 + WEIB(0.0632, 5.23)
Square Error: 0.002137

Chi Square Test
Number of intervals = 9
Degrees of freedom = 6
Test Statistic = 5.57
Corresponding p-value = 0.478

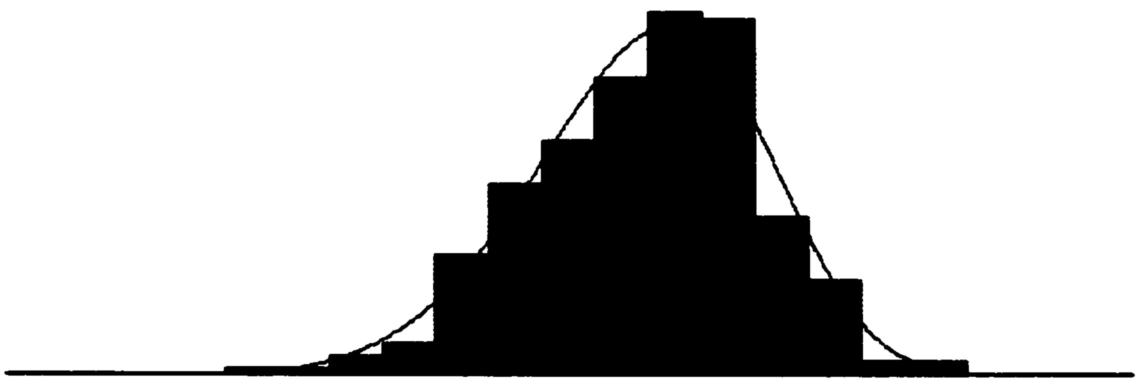
Kolmogorov-Smirnov Test
Test Statistic = 0.0319
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 300
Min Data Value = -0.0226
Max Data Value = 0.0519
Sample Mean = 0.0182
Sample Std Dev = 0.0127

Histogram Summary

Histogram Range = -0.04 to 0.06
Number of Intervals = 17



Cm Distribution Summary (400 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0529, 4.53)
Square Error: 0.002480

Chi Square Test
Number of intervals = 11
Degrees of freedom = 8
Test Statistic = 12.6
Corresponding p-value = 0.134

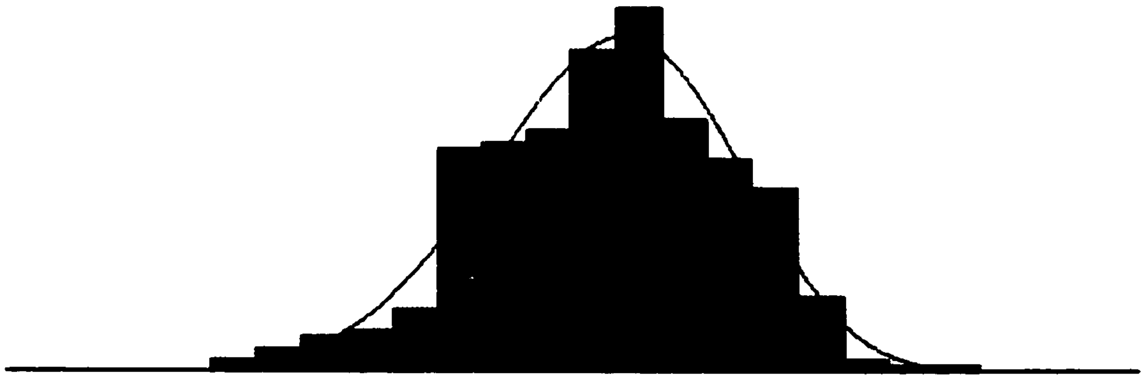
Kolmogorov-Smirnov Test
Test Statistic = 0.0327
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = -0.0206
Max Data Value = 0.0513
Sample Mean = 0.0183
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.03 to 0.06
Number of Intervals = 20



Cm Distribution Summary (500 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.04 + WEIB(0.0633, 5.44)
Square Error: 0.001421

Chi Square Test
Number of intervals = 12
Degrees of freedom = 9
Test Statistic = 7.31
Corresponding p-value = 0.605

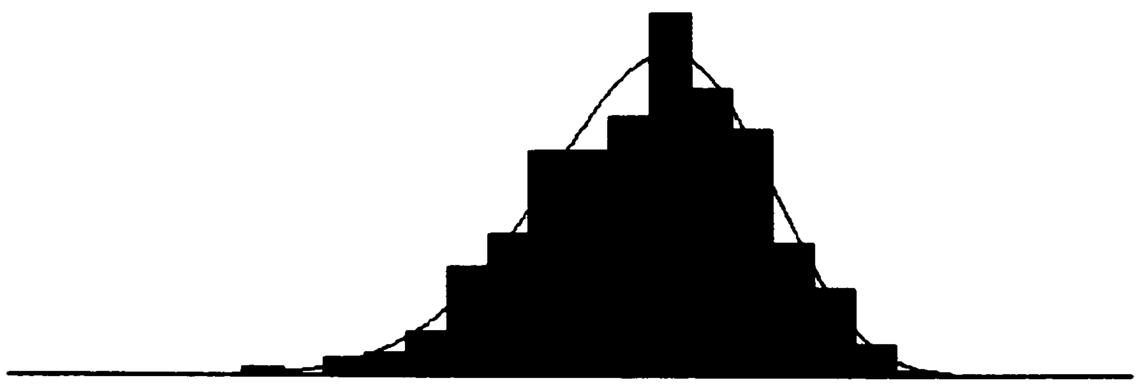
Kolmogorov-Smirnov Test
Test Statistic = 0.0249
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = -0.0256
Max Data Value = 0.0465
Sample Mean = 0.0184
Sample Std Dev = 0.0125

Histogram Summary

Histogram Range = -0.04 to 0.06
Number of Intervals = 22



Cm Distribution Summary (1000 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0531, 4.86)
Square Error: 0.000689

Chi Square Test
Number of intervals = 17
Degrees of freedom = 14
Test Statistic = 20.8
Corresponding p-value = 0.109

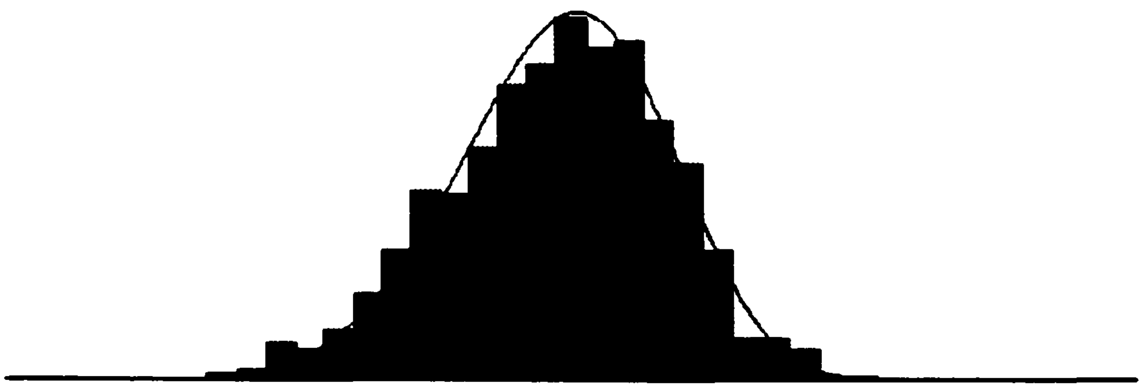
Kolmogorov-Smirnov Test
Test Statistic = 0.0378
Corresponding p-value = 0.114

Data Summary

Number of Data Points = 1000
Min Data Value = -0.0175
Max Data Value = 0.0537
Sample Mean = 0.0183
Sample Std Dev = 0.0121

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 31



Cm Distribution Summary (1500 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0528, 4.36)
Square Error: 0.000768

Chi Square Test
Number of intervals = 26
Degrees of freedom = 23
Test Statistic = 33.7
Corresponding p-value = 0.0738

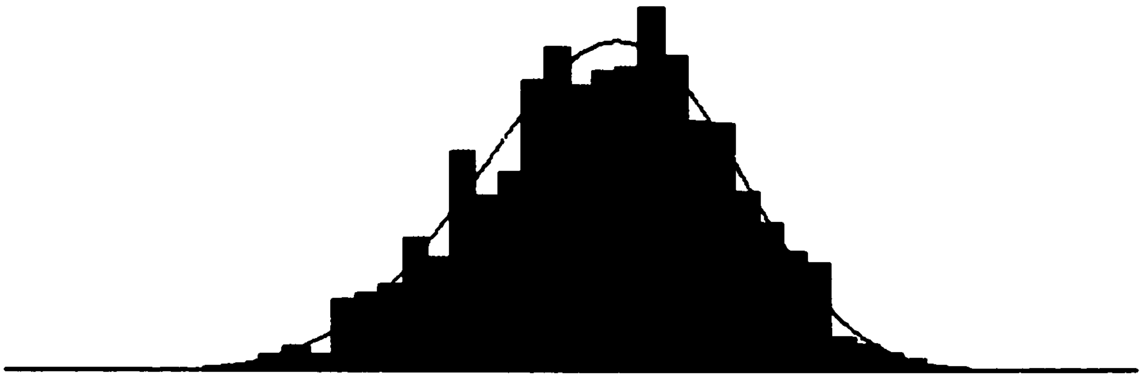
Kolmogorov-Smirnov Test
Test Statistic = 0.0211
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = -0.0165
Max Data Value = 0.0515
Sample Mean = 0.0183
Sample Std Dev = 0.0125

Histogram Summary

Histogram Range = -0.03 to 0.06
Number of Intervals = 38



Cm Distribution Summary (2000 Iterations – Latin Hypercube)

Distribution: Weibull
Expression: -0.04 + WEIB(0.0624, 5.49)
Square Error: 0.000485

Chi Square Test
Number of intervals = 22
Degrees of freedom = 19
Test Statistic = 22.7
Corresponding p-value = 0.251

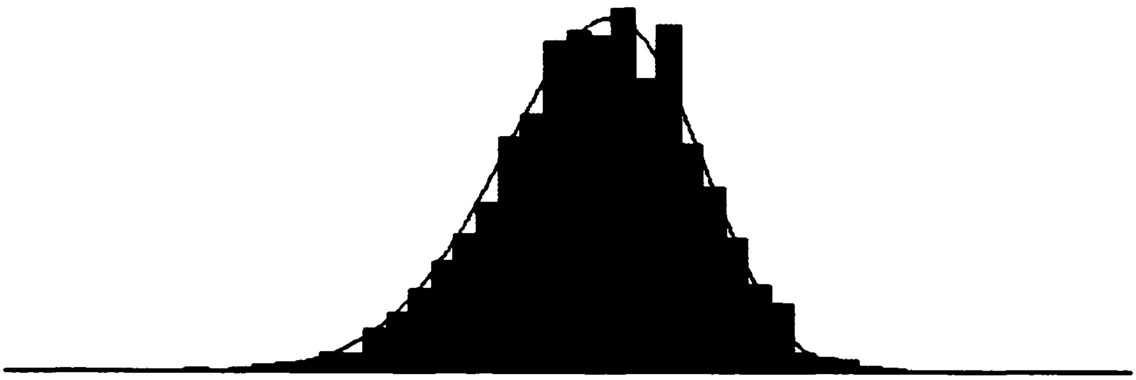
Kolmogorov-Smirnov Test
Test Statistic = 0.0269
Corresponding p-value = 0.111

Data Summary

Number of Data Points = 2000
Min Data Value = -0.0299
Max Data Value = 0.0542
Sample Mean = 0.0182
Sample Std Dev = 0.0123

Histogram Summary

Histogram Range = -0.04 to 0.07
Number of Intervals = 40



APPENDIX K
Cg and Cm Distributions (Multiple Experts, Latin Hypercube Sampling)

Cg Distribution Summary (10 Iterations – Discrete Latin Hypercube)

Distribution: Normal
Expression: NORM(0.739, 0.0119)
Square Error: 0.036328

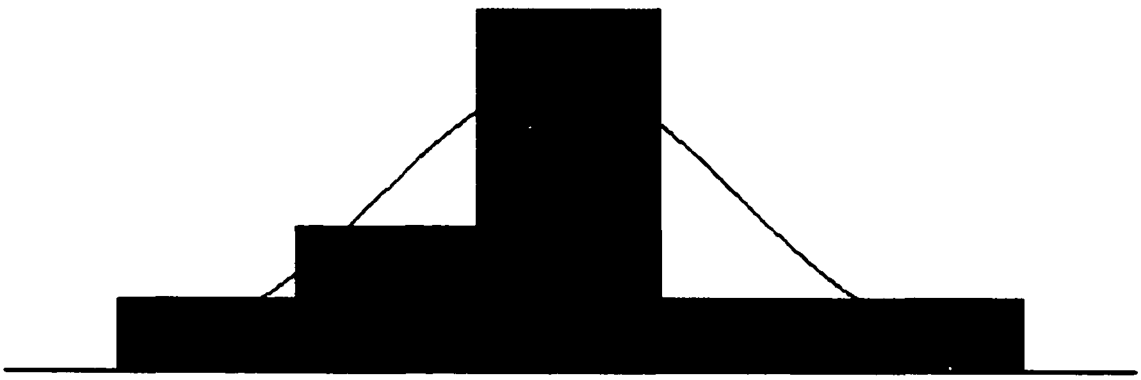
Kolmogorov-Smirnov Test
Test Statistic = 0.177
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = 0.718
Max Data Value = 0.759
Sample Mean = 0.739
Sample Std Dev = 0.0126

Histogram Summary

Histogram Range = 0.71 to 0.77
Number of Intervals = 5



Cg Distribution Summary (25 Iterations – Discrete Latin Hypercube)

Distribution: Lognormal
Expression: 0.71 + LOGN(0.029, 0.0112)
Square Error: 0.001411

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 0.167
Corresponding p-value < 0.005

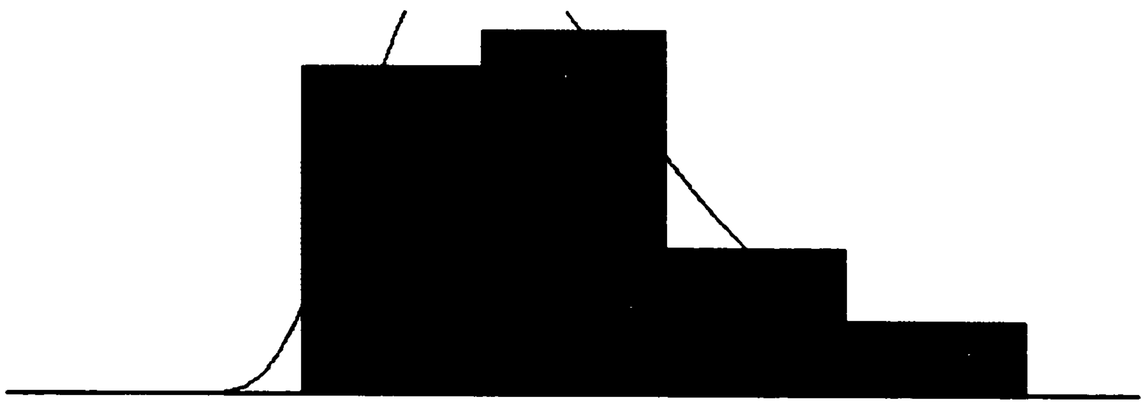
Kolmogorov-Smirnov Test
Test Statistic = 0.0658
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = 0.722
Max Data Value = 0.764
Sample Mean = 0.739
Sample Std Dev = 0.0112

Histogram Summary

Histogram Range = 0.71 to 0.77
Number of Intervals = 5



Cg Distribution Summary (50 Iterations – Discrete Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.08 * BETA(6.42, 6.83)
Square Error: 0.013684

Chi Square Test
Number of intervals = 3
Degrees of freedom = 0
Test Statistic = 1.22
Corresponding p-value < 0.005

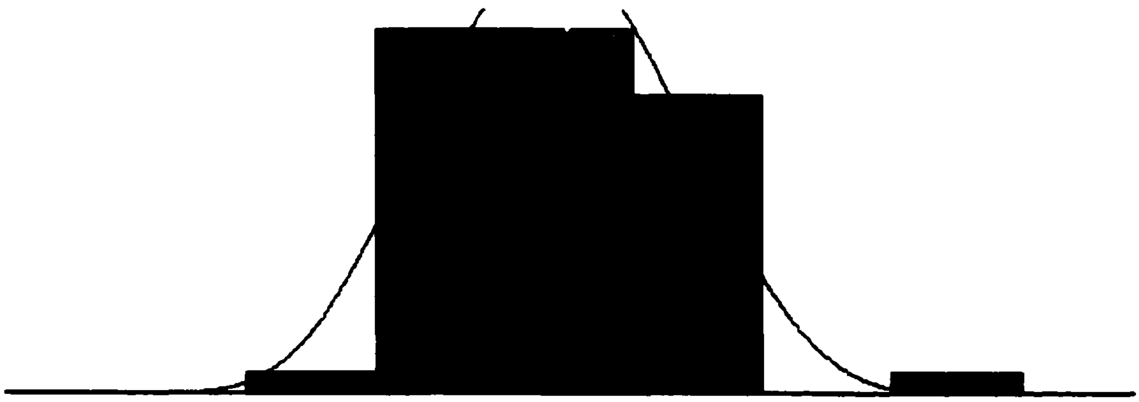
Kolmogorov-Smirnov Test
Test Statistic = 0.0657
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = 0.713
Max Data Value = 0.771
Sample Mean = 0.739
Sample Std Dev = 0.0106

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 7



Cg Distribution Summary (100 Iterations – Discrete Latin Hypercube)

Distribution: Normal
Expression: NORM(0.739, 0.0119)
Square Error: 0.011008

Chi Square Test
Number of intervals = 5
Degrees of freedom = 2
Test Statistic = 6.23
Corresponding p-value = 0.0456

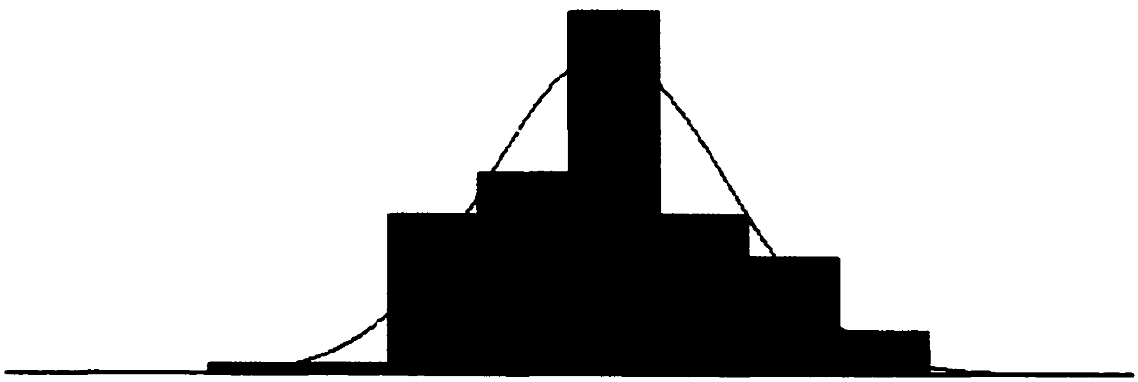
Kolmogorov-Smirnov Test
Test Statistic = 0.059
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = 0.705
Max Data Value = 0.769
Sample Mean = 0.739
Sample Std Dev = 0.012

Histogram Summary

Histogram Range = 0.69 to 0.78
Number of Intervals = 10



Cg Distribution Summary (200 Iterations – Discrete Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.08 * BETA(5.14, 5.47)
Square Error: 0.006677

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 10.4
Corresponding p-value = 0.069

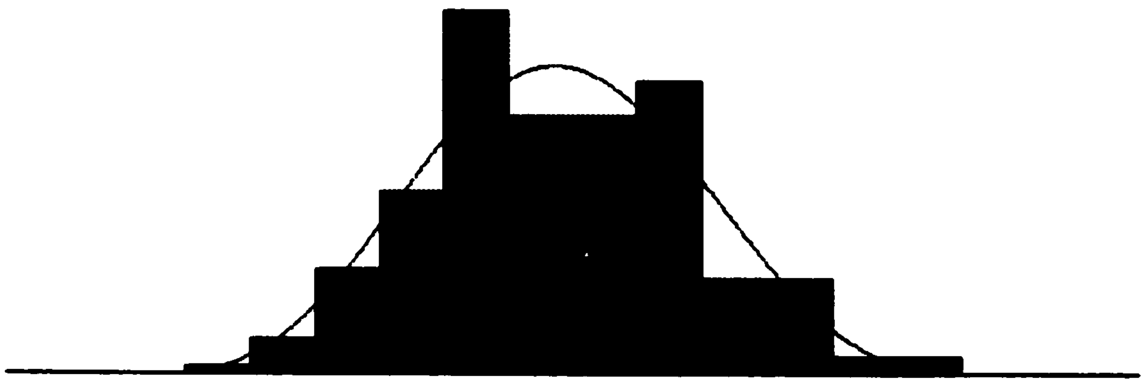
Kolmogorov-Smirnov Test
Test Statistic = 0.0545
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = 0.71
Max Data Value = 0.771
Sample Mean = 0.739
Sample Std Dev = 0.0117

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 14



Cg Distribution Summary (300 Iterations – Discrete Latin Hypercube)

Distribution:	Normal
Expression:	NORM(0.739, 0.0111)
Square Error:	0.000970

Chi Square Test

Number of intervals	= 10
Degrees of freedom	= 7
Test Statistic	= 2.64
Corresponding p-value	> 0.75

Kolmogorov-Smirnov Test

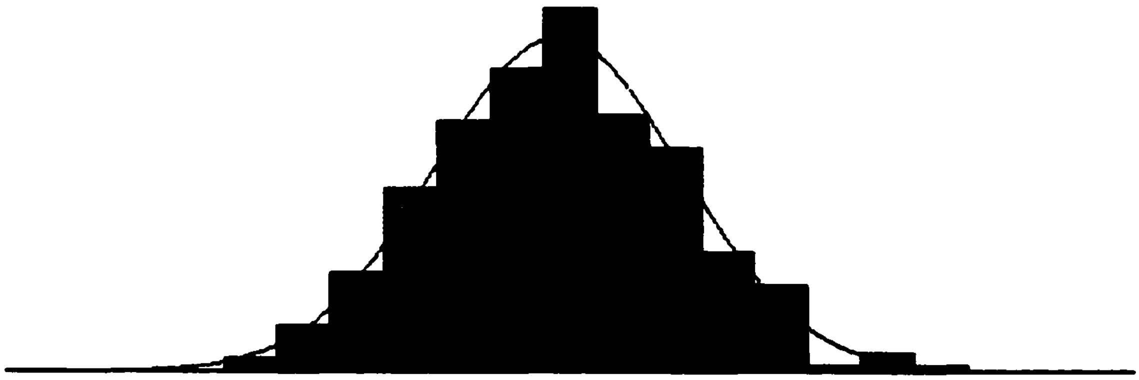
Test Statistic	= 0.0296
Corresponding p-value	> 0.15

Data Summary

Number of Data Points	= 300
Min Data Value	= 0.711
Max Data Value	= 0.771
Sample Mean	= 0.739
Sample Std Dev	= 0.0111

Histogram Summary

Histogram Range	= 0.7 to 0.78
Number of Intervals	= 17



Cg Distribution Summary (400 Iterations – Discrete Latin Hypercube)

Distribution: Beta
Expression: 0.7 + 0.08 * BETA(5.66, 6.01)
Square Error: 0.001593

Chi Square Test
Number of intervals = 12
Degrees of freedom = 9
Test Statistic = 7.67
Corresponding p-value = 0.569

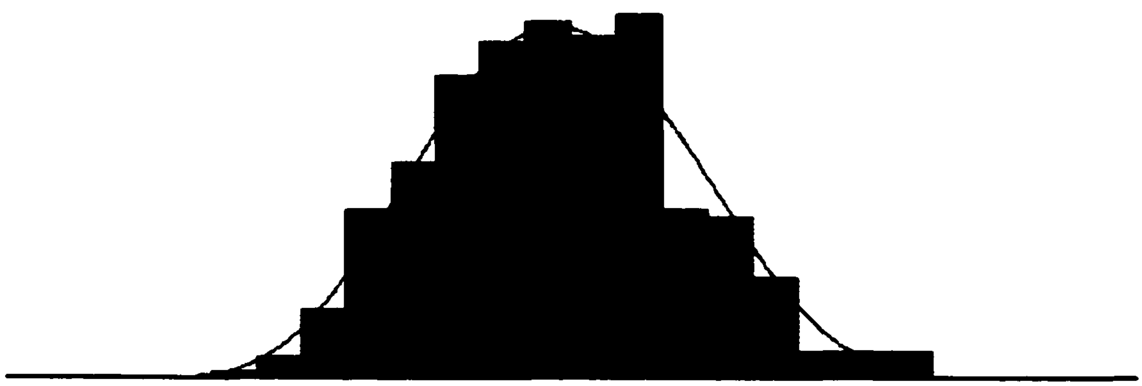
Kolmogorov-Smirnov Test
Test Statistic = 0.0426
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = 0.71
Max Data Value = 0.771
Sample Mean = 0.739
Sample Std Dev = 0.0112

Histogram Summary

Histogram Range = 0.7 to 0.78
Number of Intervals = 20



Cg Distribution Summary (500 Iterations – Discrete Latin Hypercube)

Distribution: Normal
Expression: NORM(0.739, 0.0114)
Square Error: 0.002775

Chi Square Test
Number of intervals = 13
Degrees of freedom = 10
Test Statistic = 13.3
Corresponding p-value = 0.218

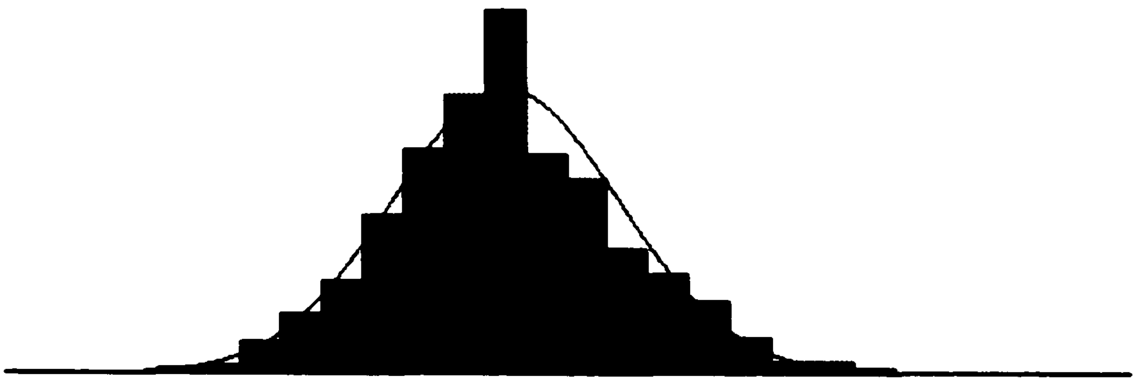
Kolmogorov-Smirnov Test
Test Statistic = 0.0531
Corresponding p-value = 0.117

Data Summary

Number of Data Points = 500
Min Data Value = 0.707
Max Data Value = 0.774
Sample Mean = 0.739
Sample Std Dev = 0.0114

Histogram Summary

Histogram Range = 0.7 to 0.79
Number of Intervals = 22



Cg Distribution Summary (1000 Iterations – Discrete Latin Hypercube)

Distribution: Erlang
Expression: 0.69 + ERLA(0.00271, 18)
Square Error: 0.000525

Chi Square Test
Number of intervals = 16
Degrees of freedom = 13
Test Statistic = 18.9
Corresponding p-value = 0.136

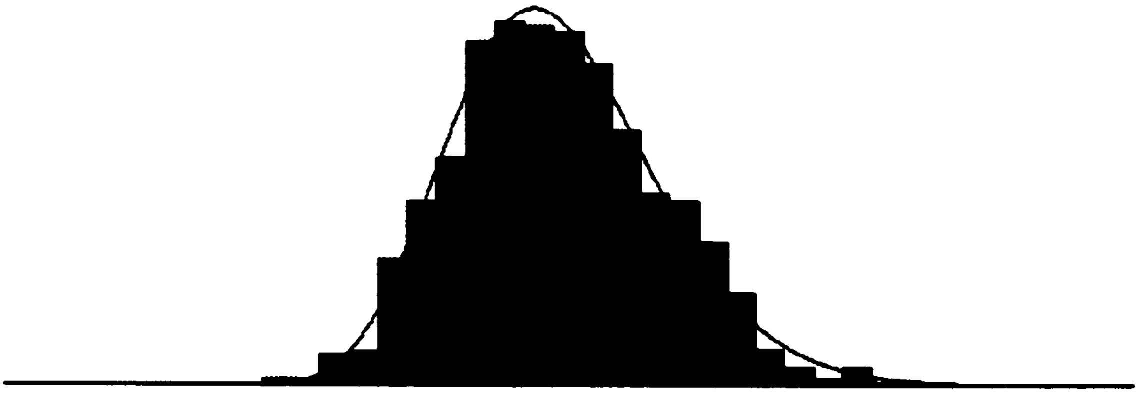
Kolmogorov-Smirnov Test
Test Statistic = 0.0166
Corresponding p-value > 0.15

Data Summary

Number of Data Points	= 1000
Min Data Value	= 0.706
Max Data Value	= 0.778
Sample Mean	= 0.739
Sample Std Dev	= 0.0113

Histogram Summary

Histogram Range	= 0.69 to 0.79
Number of Intervals	= 31



Cg Distribution Summary (1500 Iterations – Discrete Latin Hypercube)

Distribution: Erlang
Expression: 0.69 + ERLA(0.00257, 19)
Square Error: 0.000403

Chi Square Test
Number of intervals = 23
Degrees of freedom = 20
Test Statistic = 11.6
Corresponding p-value > 0.75

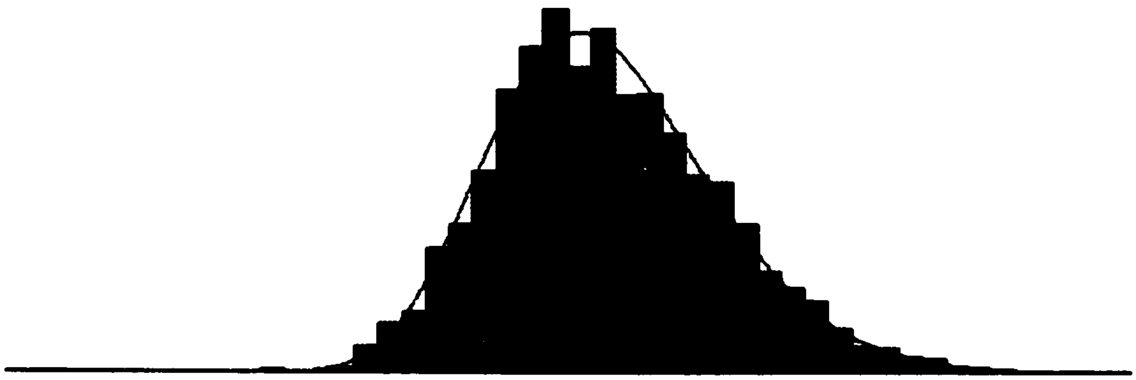
Kolmogorov-Smirnov Test
Test Statistic = 0.0118
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = 0.706
Max Data Value = 0.773
Sample Mean = 0.739
Sample Std Dev = 0.0111

Histogram Summary

Histogram Range = 0.69 to 0.78
Number of Intervals = 38



Cg Distribution Summary (2000 Iterations – Discrete Latin Hypercube)

Distribution: Erlang
Expression: 0.69 + ERLA(0.00287, 17)
Square Error: 0.000459

Chi Square Test
Number of intervals = 24
Degrees of freedom = 21
Test Statistic = 27.4
Corresponding p-value = 0.172

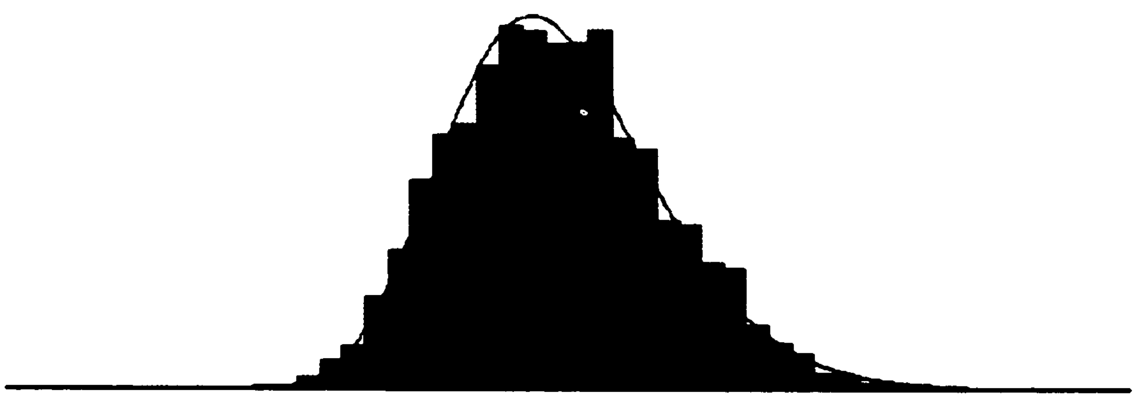
Kolmogorov-Smirnov Test
Test Statistic = 0.0188
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = 0.706
Max Data Value = 0.776
Sample Mean = 0.739
Sample Std Dev = 0.0117

Histogram Summary

Histogram Range = 0.69 to 0.79
Number of Intervals = 40



Cm Distribution Summary (10 Iterations – Discrete Latin Hypercube)

Distribution: Gamma
Expression: 0.01 + GAMM(0.00226, 8.96)
Square Error: 0.041730

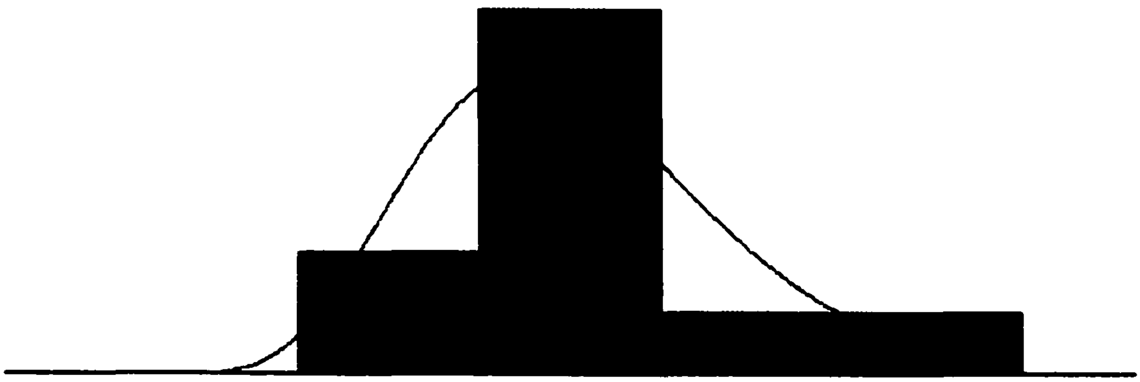
Kolmogorov-Smirnov Test
Test Statistic = 0.205
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 10
Min Data Value = 0.0191
Max Data Value = 0.042
Sample Mean = 0.0302
Sample Std Dev = 0.00667

Histogram Summary

Histogram Range = 0.01 to 0.05
Number of Intervals = 5



Cm Distribution Summary (25 Iterations – Discrete Latin Hypercube)

Distribution: Triangular
Expression: TRIA(-0.02, 0.0393, 0.06)
Square Error: 0.001594

Chi Square Test
Number of intervals = 3
Degrees of freedom = 1
Test Statistic = 0.151
Corresponding p-value = 0.715

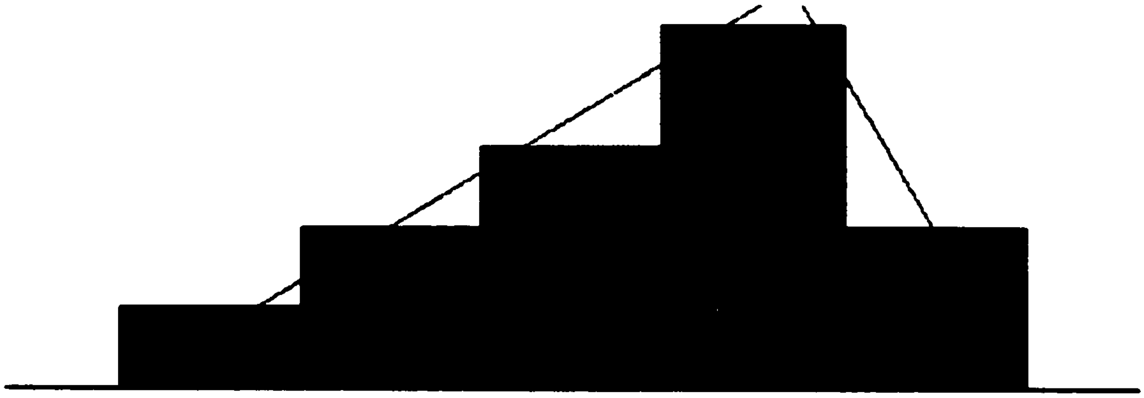
Kolmogorov-Smirnov Test
Test Statistic = 0.0812
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 25
Min Data Value = -0.00704
Max Data Value = 0.0484
Sample Mean = 0.0264
Sample Std Dev = 0.0166

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 5



Cm Distribution Summary (50 Iterations – Discrete Latin Hypercube)

Distribution: Beta
Expression: BETA(2.02, 3.21369)
Square Error: 0.001710

Chi Square Test
Number of intervals = 5
Degrees of freedom = 2
Test Statistic = 0.63
Corresponding p-value = 0.733

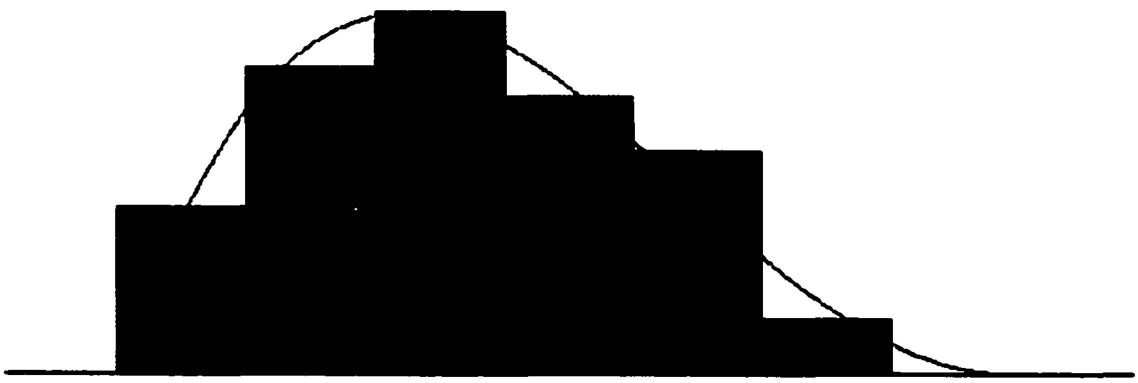
Kolmogorov-Smirnov Test
Test Statistic = 0.0454
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 50
Min Data Value = 0.000726
Max Data Value = 0.0572
Sample Mean = 0.027
Sample Std Dev = 0.0136

Histogram Summary

Histogram Range = 0 to 0.07
Number of Intervals = 7



Cm Distribution Summary (100 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0506, 4.3)
Square Error: 0.011926

Chi Square Test
Number of intervals = 5
Degrees of freedom = 2
Test Statistic = 3.47
Corresponding p-value = 0.193

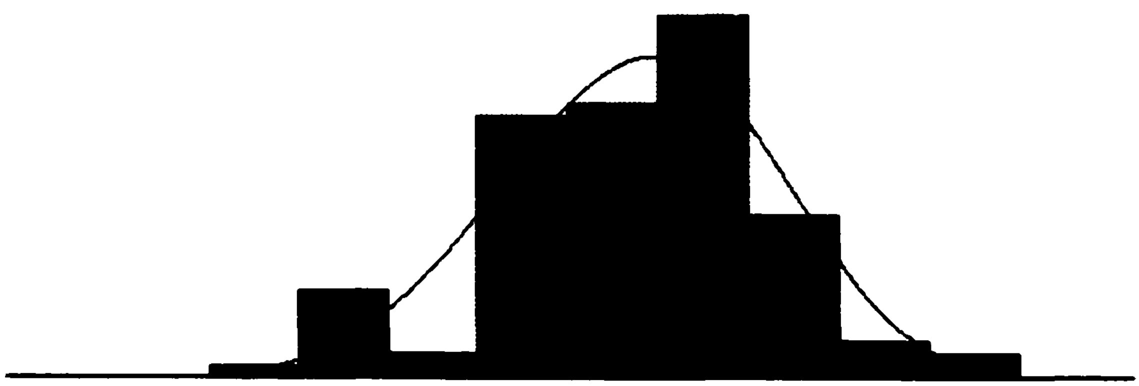
Kolmogorov-Smirnov Test
Test Statistic = 0.0662
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 100
Min Data Value = -0.00468
Max Data Value = 0.0524
Sample Mean = 0.026
Sample Std Dev = 0.0124

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 10



Cg Distribution Summary (200 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0499, 4.16)
Square Error: 0.005053

Chi Square Test
Number of intervals = 8
Degrees of freedom = 5
Test Statistic = 9.72
Corresponding p-value = 0.0869

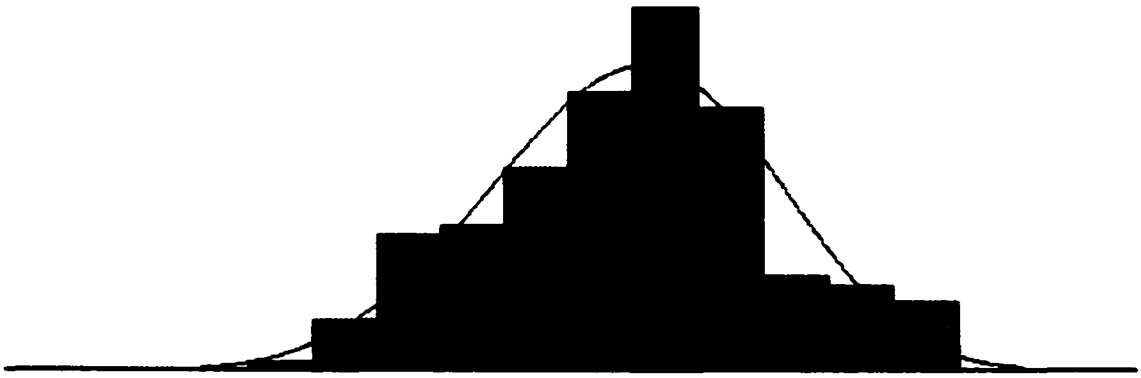
Kolmogorov-Smirnov Test
Test Statistic = 0.046
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 200
Min Data Value = -0.00513
Max Data Value = 0.0529
Sample Mean = 0.0253
Sample Std Dev = 0.0123

Histogram Summary

Histogram Range = -0.02 to 0.06
Number of Intervals = 14



Cm Distribution Summary (300 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0602, 5.03)
Square Error: 0.004455

Chi Square Test
Number of intervals = 9
Degrees of freedom = 6
Test Statistic = 10.3
Corresponding p-value = 0.118

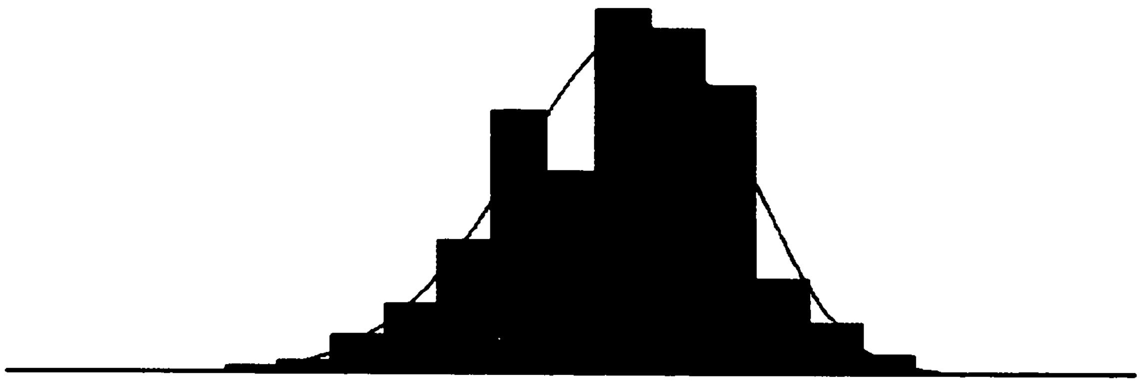
Kolmogorov-Smirnov Test
Test Statistic = 0.0346
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 300
Min Data Value = -0.0161
Max Data Value = 0.0577
Sample Mean = 0.0253
Sample Std Dev = 0.0127

Histogram Summary

Histogram Range = -0.03 to 0.07
Number of Intervals = 17



Cm Distribution Summary (400 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0498, 4.24)
Square Error: 0.001300

Chi Square Test
Number of intervals = 12
Degrees of freedom = 9
Test Statistic = 10.5
Corresponding p-value = 0.325

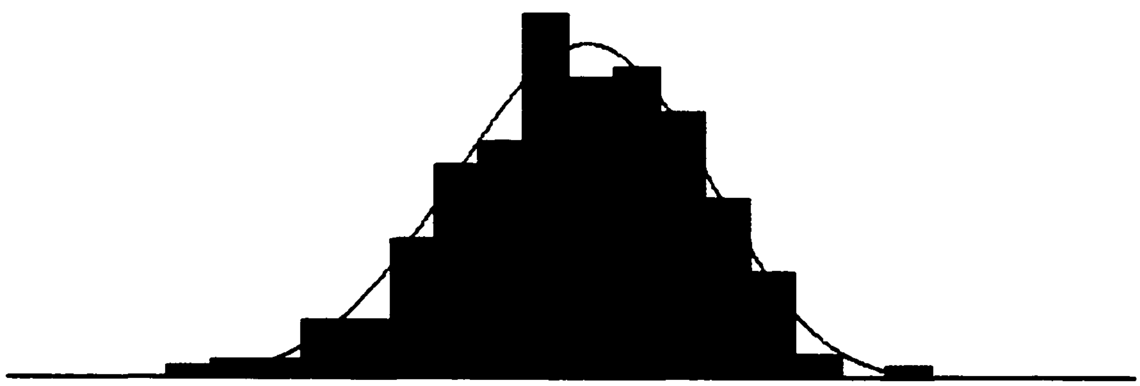
Kolmogorov-Smirnov Test
Test Statistic = 0.0285
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 400
Min Data Value = -0.0117
Max Data Value = 0.0576
Sample Mean = 0.0253
Sample Std Dev = 0.0122

Histogram Summary

Histogram Range = -0.02 to 0.07
Number of Intervals = 20



Cm Distribution Summary (500 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.03 + WEIB(0.0603, 5.18)
Square Error: 0.001562

Chi Square Test
Number of intervals = 14
Degrees of freedom = 11
Test Statistic = 9.75
Corresponding p-value = 0.553

Kolmogorov-Smirnov Test
Test Statistic = 0.0243
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 500
Min Data Value = -0.0216
Max Data Value = 0.0526
Sample Mean = 0.0254
Sample Std Dev = 0.0123

Histogram Summary

Histogram Range = -0.03 to 0.06
Number of Intervals = 22

Cm Distribution Summary (1000 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0502, 4.57)
Square Error: 0.000596

Chi Square Test
Number of intervals = 19
Degrees of freedom = 16
Test Statistic = 24.6
Corresponding p-value = 0.081

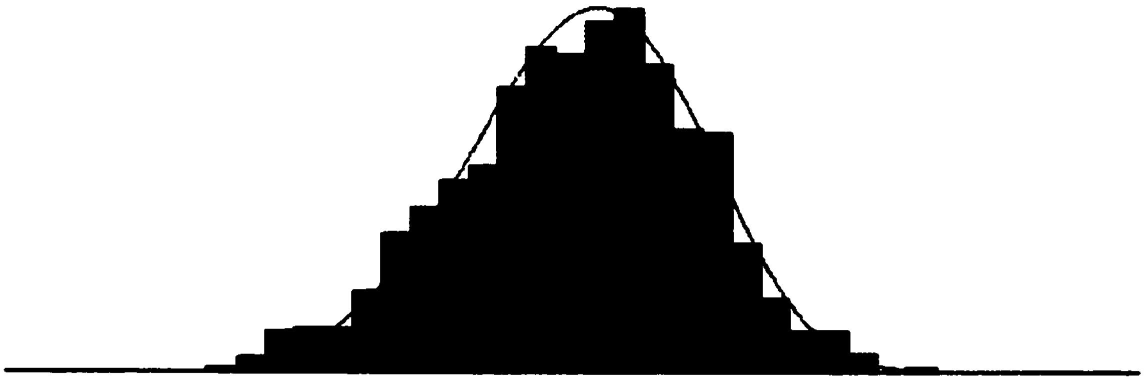
Kolmogorov-Smirnov Test
Test Statistic = 0.0335
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1000
Min Data Value = -0.0111
Max Data Value = 0.0599
Sample Mean = 0.0254
Sample Std Dev = 0.012

Histogram Summary

Histogram Range = -0.02 to 0.07
Number of Intervals = 31



Cm Distribution Summary (1500 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.02 + WEIB(0.0497, 4.05)
Square Error: 0.000277

Chi Square Test
Number of intervals = 25
Degrees of freedom = 22
Test Statistic = 9.7
Corresponding p-value > 0.75

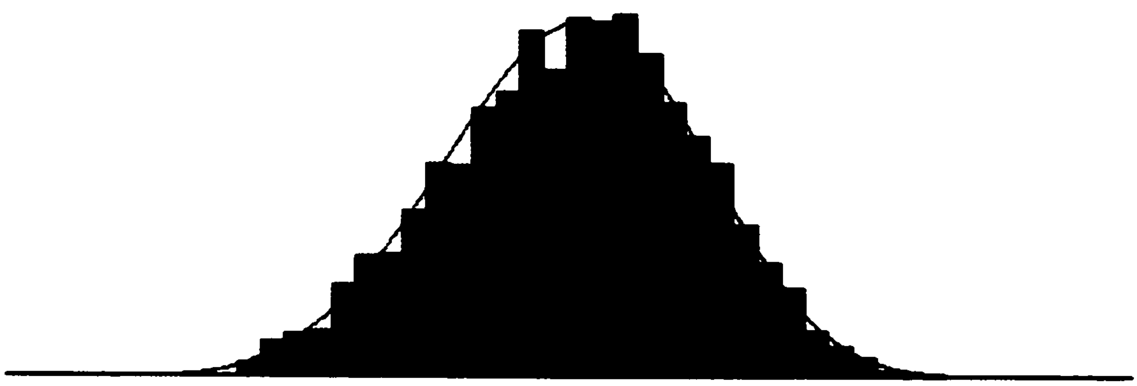
Kolmogorov-Smirnov Test
Test Statistic = 0.0276
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 1500
Min Data Value = -0.0111
Max Data Value = 0.0584
Sample Mean = 0.0254
Sample Std Dev = 0.0124

Histogram Summary

Histogram Range = -0.02 to 0.07
Number of Intervals = 38



Cm Distribution Summary (2000 Iterations – Discrete Latin Hypercube)

Distribution: Weibull
Expression: -0.04 + WEIB(0.0698, 6.08)
Square Error: 0.000283

Chi Square Test
Number of intervals = 23
Degrees of freedom = 20
Test Statistic = 26.3
Corresponding p-value = 0.17

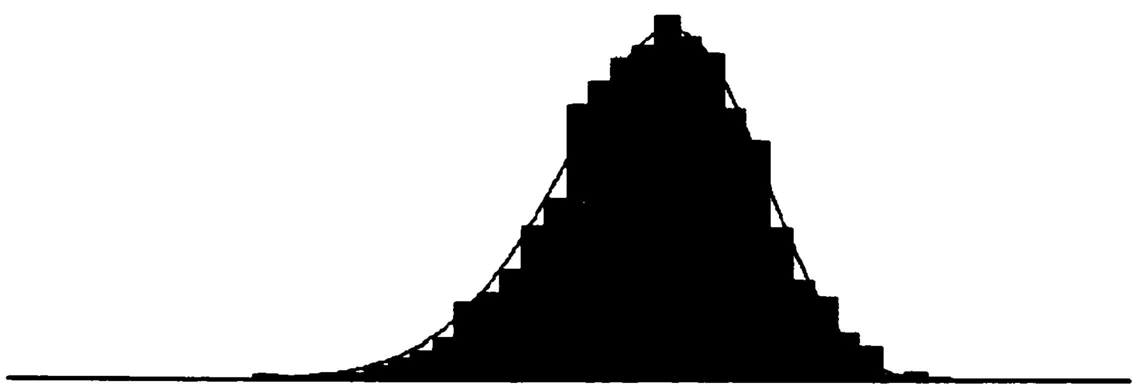
Kolmogorov-Smirnov Test
Test Statistic = 0.0251
Corresponding p-value > 0.15

Data Summary

Number of Data Points = 2000
Min Data Value = -0.0219
Max Data Value = 0.0614
Sample Mean = 0.0253
Sample Std Dev = 0.0122

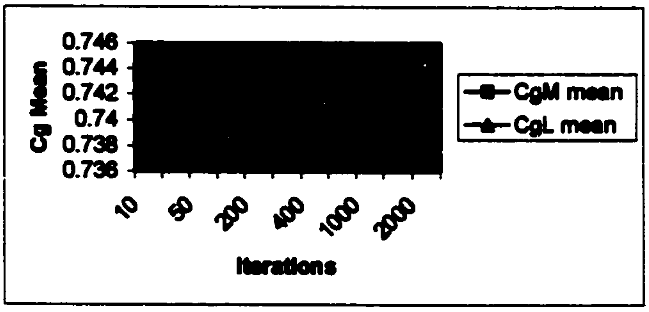
Histogram Summary

Histogram Range = -0.04 to 0.07
Number of Intervals = 40



APPENDIX L
Cg and Cm Statistics Comparison (Random Sampling vs Latin Hypercube Sampling)

Iterations	CgM mean	CgL mean
10	0.739	0.742
25	0.744	0.742
50	0.744	0.742
100	0.744	0.742
200	0.742	0.742
300	0.743	0.742
400	0.742	0.742
500	0.742	0.742
1000	0.742	0.742
1500	0.742	0.742
2000	0.742	0.742



Iterations	CgM Std Dev	CgL Std Dev
10	0.0127	0.011
25	0.0108	0.0102
50	0.0108	0.00997
100	0.00994	0.011
200	0.00942	0.0107
300	0.00977	0.0101
400	0.0102	0.0103
500	0.0103	0.0104
1000	0.0102	0.0103
1500	0.0102	0.0101
2000	0.0101	0.0108

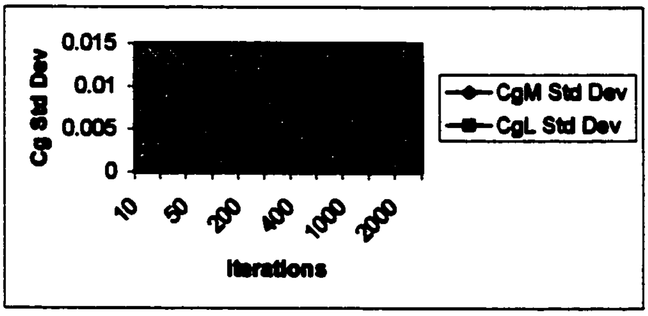
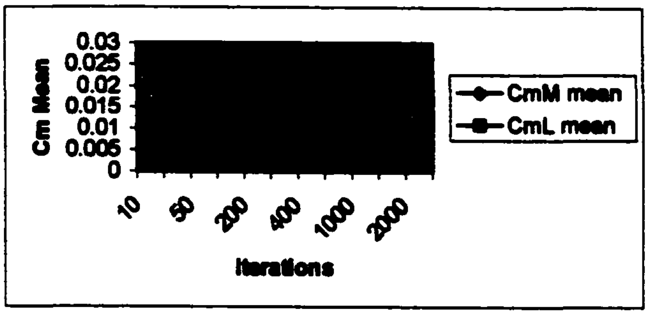


Figure L-1 Single Expert Cg Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CmM mean	CmL mean
10	0.0147	0.025
25	0.0143	0.0203
50	0.0181	0.0201
100	0.0175	0.0191
200	0.018	0.0182
300	0.0184	0.0182
400	0.0182	0.0183
500	0.0182	0.0184
1000	0.0184	0.0183
1500	0.0183	0.0183
2000	0.0183	0.0182



Iterations	CmM Std Dev	CmL Std Dev
10	0.0146	0.00733
25	0.0137	0.017
50	0.0135	0.0135
100	0.013	0.0127
200	0.0122	0.0127
300	0.0122	0.0127
400	0.0122	0.0122
500	0.0121	0.0125
1000	0.0124	0.0121
1500	0.0123	0.0125
2000	0.0124	0.0123

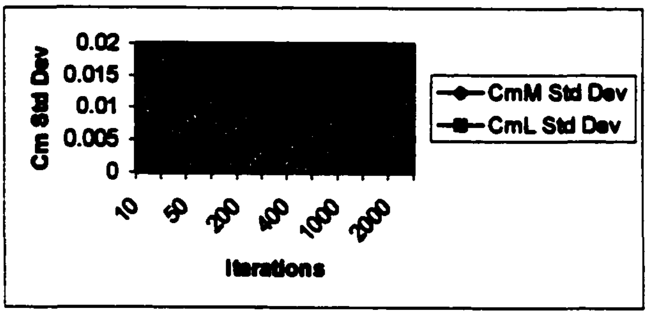
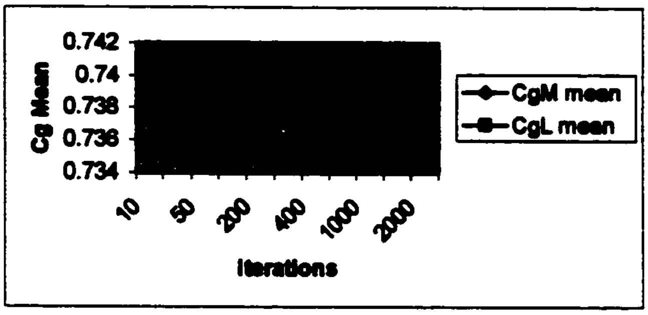


Figure L-2 Single Expert Cm Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CgM mean	CgL mean
10	0.738	0.739
25	0.74	0.739
50	0.74	0.739
100	0.74	0.739
200	0.738	0.739
300	0.739	0.739
400	0.739	0.739
500	0.739	0.739
1000	0.739	0.739
1500	0.739	0.739
2000	0.739	0.739



Iteations	CgM Std Dev	CgL Std Dev
10	0.0129	0.0126
25	0.0108	0.0112
50	0.0109	0.0106
100	0.0101	0.012
200	0.00964	0.0117
300	0.00993	0.0111
400	0.0102	0.0112
500	0.0104	0.0114
1000	0.0103	0.0113
1500	0.0102	0.0111
2000	0.0102	0.0117

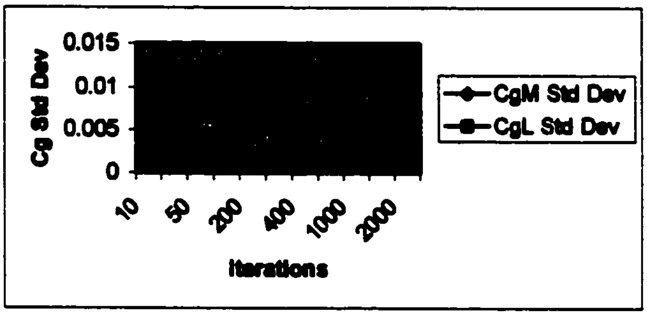
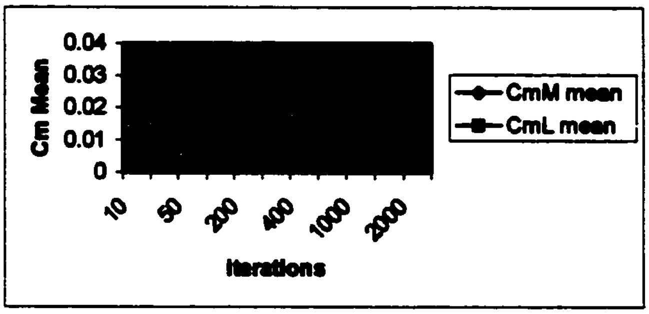


Figure L-3 Multiple Expert Cg Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CmM mean	CmL mean
10	0.0215	0.0302
25	0.0207	0.0284
50	0.0244	0.027
100	0.0241	0.026
200	0.0249	0.0253
300	0.0252	0.0253
400	0.0251	0.0253
500	0.0252	0.0254
1000	0.0255	0.0254
1500	0.0254	0.0254
2000	0.0254	0.0253



Iterations	CmM Std Dev	CmL Std Dev
10	0.0147	0.00867
25	0.0135	0.0166
50	0.013	0.0136
100	0.0129	0.0124
200	0.0124	0.0123
300	0.0123	0.0127
400	0.0121	0.0122
500	0.0119	0.0123
1000	0.0122	0.012
1500	0.0122	0.0124
2000	0.0122	0.0122

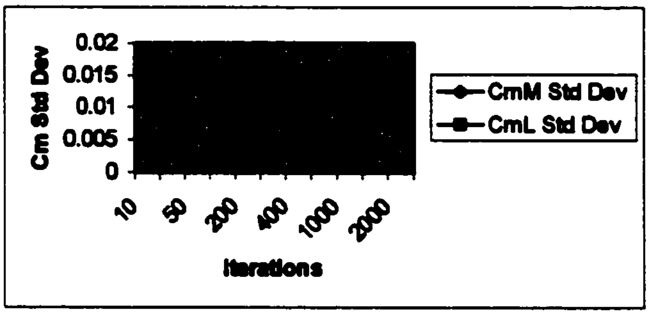
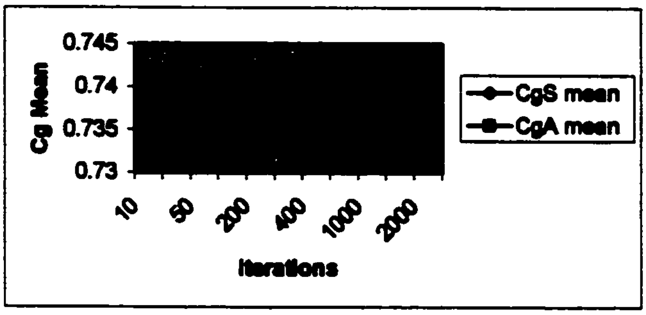


Figure L-4 Multiple Expert Cm Comparison (Random Sampling vs. Latin Hypercube Sampling)

APPENDIX M
Cg and Cm Statistics Comparison (Multiple Expert vs Single Expert)

Iterations	CgS mean	CgA mean
10	0.739	0.736
25	0.744	0.74
50	0.744	0.74
100	0.744	0.74
200	0.742	0.738
300	0.743	0.739
400	0.742	0.739
500	0.742	0.739
1000	0.742	0.739
1500	0.742	0.739
2000	0.742	0.739



Iterations	CgS Std Dev	CgA Std Dev
10	0.0127	0.0129
25	0.0108	0.0108
50	0.0106	0.0109
100	0.00994	0.0101
200	0.00942	0.00984
300	0.00977	0.00993
400	0.0102	0.0102
500	0.0103	0.0104
1000	0.0102	0.0103
1500	0.0102	0.0102
2000	0.0101	0.0102

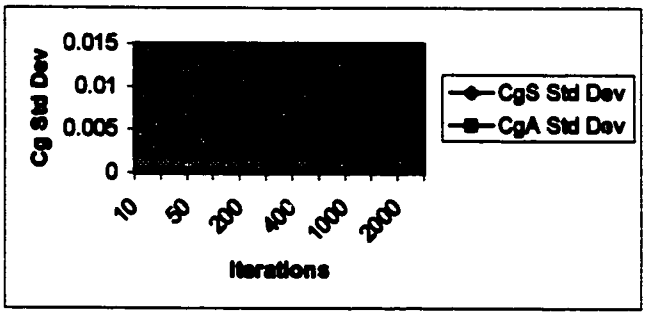
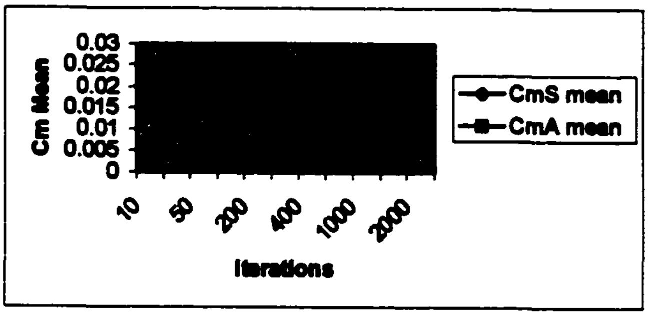


Figure M-1 Cg Comparison (Single Expert vs. Aggregated Expert Opinion)

Iterations	CmS mean	CmA mean
10	0.0147	0.0215
25	0.0143	0.0207
50	0.0181	0.0244
100	0.0175	0.0241
200	0.018	0.0249
300	0.0184	0.0252
400	0.0182	0.0251
500	0.0182	0.0252
1000	0.0184	0.0255
1500	0.0183	0.0254
2000	0.0183	0.0254



Itеations	CmS Std Dev	CmA Std Dev
10	0.0146	0.0147
25	0.0137	0.0135
50	0.0135	0.013
100	0.013	0.0129
200	0.0122	0.0124
300	0.0122	0.0123
400	0.0122	0.0121
500	0.0121	0.0119
1000	0.0124	0.0122
1500	0.0123	0.0122
2000	0.0124	0.0122

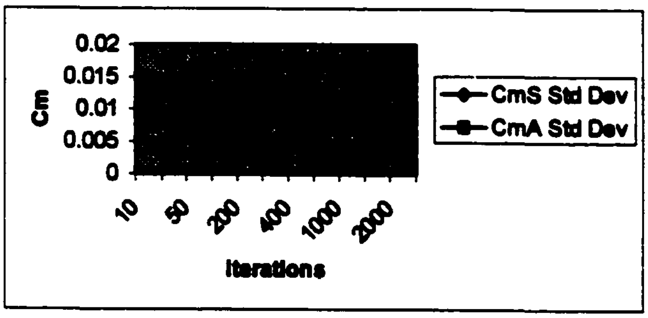
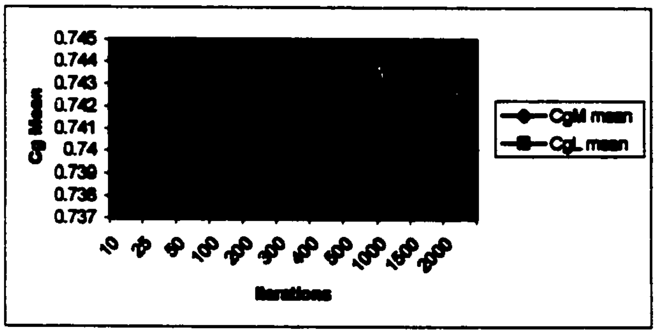


Figure M-2 Cm Comparison (Single Expert vs. Aggregated Expert Opinion)

APPENDIX N
@Risk Cg and Cm Statistics Comparisons

Iterations	CgM mean	CgL mean
10	0.7441	0.7417
25	0.7436	0.7417
50	0.7399	0.7417
100	0.7399	0.7417
200	0.7428	0.7417
300	0.7417	0.7417
400	0.742	0.7417
500	0.7415	0.7417
1000	0.7417	0.7417
1500	0.7419	0.7417
2000	0.7422	0.7417



Iterations	CgM Std Dev	CgL Std Dev
10	0.01106	0.0086
25	0.00909	0.0113
50	0.01056	0.0174
100	0.0139	0.0103
200	0.00932	0.0106
300	0.00966	0.00959
400	0.00963	0.00983
500	0.0102	0.0097
1000	0.01023	0.0101
1500	0.0101	0.01
2000	0.01	0.01032

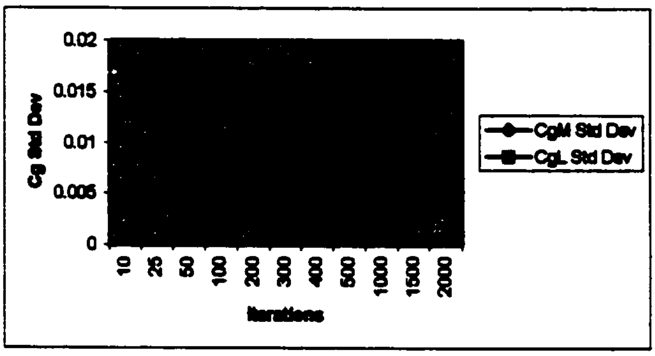
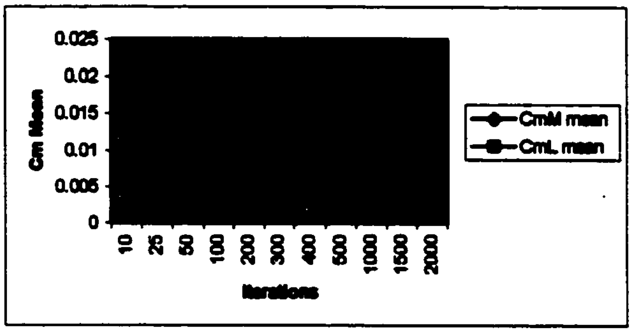


Figure N-1 Single Expert Cg Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CmM mean	CmL mean
10	0.01777	0.01776
25	0.0176	0.01782
50	0.0185	0.01795
100	0.0205	0.0181
200	0.0162	0.01831
300	0.0175	0.01817
400	0.01847	0.01816
500	0.0178	0.0182
1000	0.01845	0.01808
1500	0.0176	0.01814
2000	0.01796	0.018



Iteations	CmM Std Dev	CmL Std Dev
10	0.0088	0.01382
25	0.01297	0.01061
50	0.0121	0.0128
100	0.0116	0.01403
200	0.01142	0.0116
300	0.01165	0.0122
400	0.01219	0.01227
500	0.01229	0.01217
1000	0.0124	0.01248
1500	0.01228	0.01231
2000	0.0125	0.0125

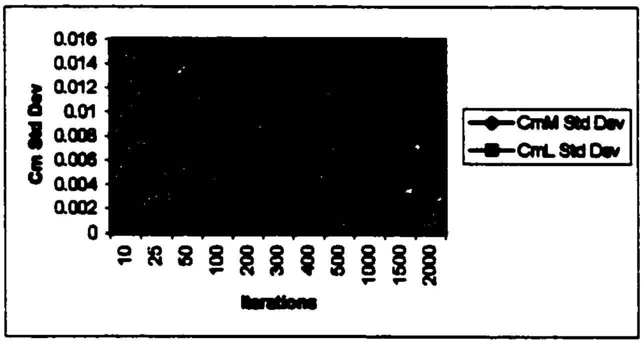
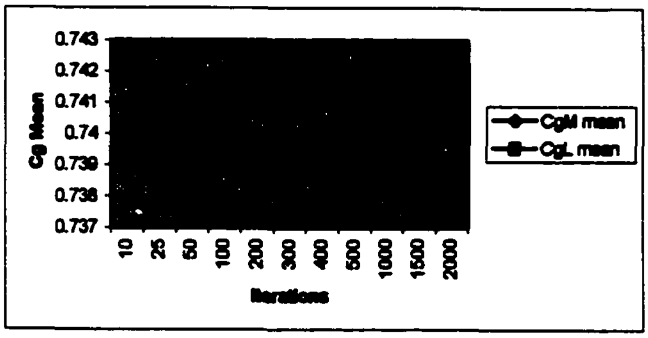


Figure N-2 Single Expert Cm Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CgM mean	CgL mean
10	0.7422	0.741
25	0.742	0.7401
50	0.741	0.7398
100	0.7401	0.7397
200	0.739	0.7398
300	0.74	0.7398
400	0.74	0.7398
500	0.74	0.7398
1000	0.7395	0.7398
1500	0.74	0.7397
2000	0.739	0.7398



Iterations	CgM Std Dev	CgL Std Dev
10	0.00956	0.01347
25	0.00953	0.0099
50	0.0138	0.0093
100	0.0102	0.0101
200	0.00975	0.0104
300	0.0105	0.0105
400	0.0105	0.0102
500	0.0126	0.01013
1000	0.0108	0.0101
1500	0.0109	0.0102
2000	0.0129	0.0103

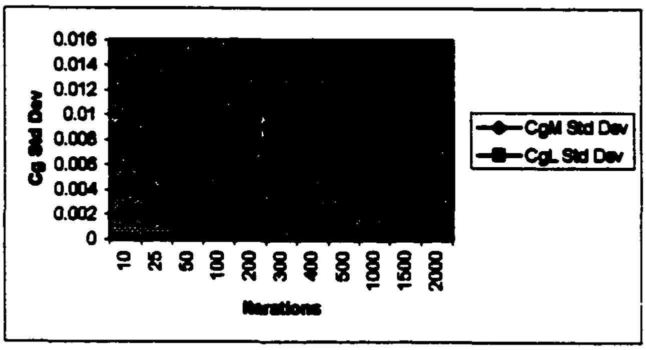
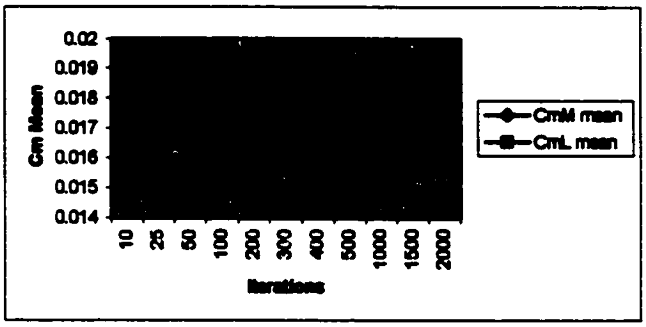


Figure N-3 Multiple Expert Cg Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CmM mean	CmL mean
10	0.01647	0.018996
25	0.017386	0.0195
50	0.0183	0.01923
100	0.0195	0.01866
200	0.0165	0.01867
300	0.0182	0.01871
400	0.0192	0.01879
500	0.0183	0.01866
1000	0.0184	0.0186
1500	0.0189	0.0187
2000	0.0191	0.0187



Iteations	CmM Std Dev	CmL Std Dev
10	0.01285	0.00627
25	0.01277	0.0125
50	0.01342	0.01046
100	0.01416	0.01339
200	0.0125	0.0116
300	0.0127	0.01246
400	0.01187	0.01217
500	0.0124	0.01205
1000	0.01271	0.01234
1500	0.0125	0.01231
2000	0.0123	0.01246

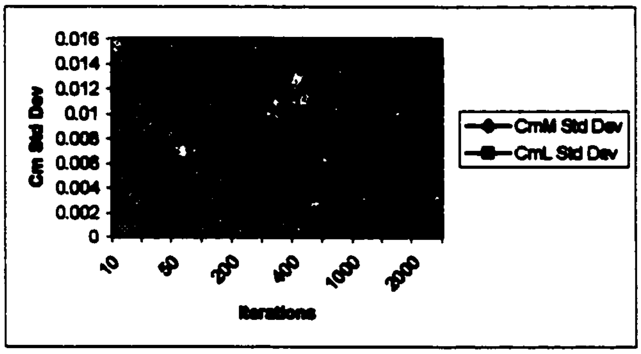
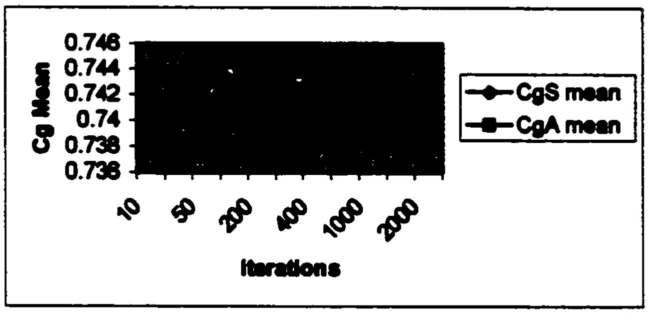


Figure N-4 Multiple Expert Cm Comparison (Random Sampling vs. Latin Hypercube Sampling)

Iterations	CgS mean	CgA mean
10	0.7441	0.7422
25	0.7436	0.742
50	0.7399	0.741
100	0.7399	0.7401
200	0.7428	0.739
300	0.7417	0.74
400	0.742	0.74
500	0.7415	0.74
1000	0.7417	0.7395
1500	0.7419	0.74
2000	0.7422	0.739



Iteations	CgS Std Dev	CgA Std Dev
10	0.01106	0.00956
25	0.00909	0.00953
50	0.01056	0.0138
100	0.0139	0.0102
200	0.00932	0.00975
300	0.00966	0.0105
400	0.00963	0.0105
500	0.0102	0.0126
1000	0.01023	0.0108
1500	0.0101	0.0109
2000	0.01	0.0129

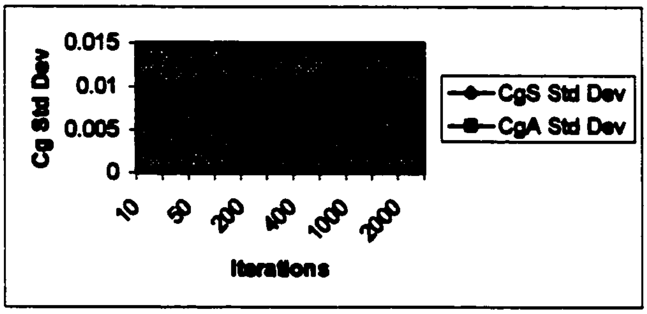
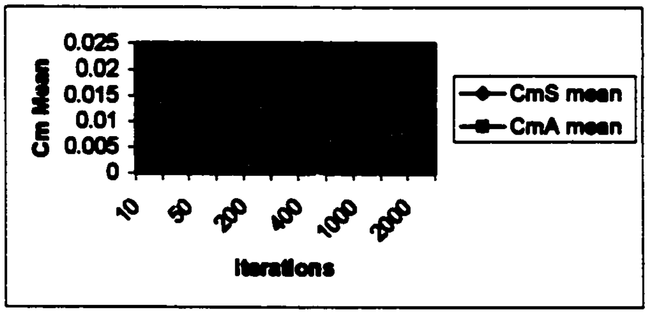


Figure N-6 Cg Comparison (Single Expert vs. Aggregated Expert Opinion)

Iterations	CmS mean	CmA mean
10	0.01777	0.01647
25	0.0176	0.017396
50	0.0195	0.0183
100	0.0205	0.0195
200	0.0162	0.0165
300	0.0175	0.0182
400	0.01847	0.0192
500	0.0178	0.0183
1000	0.01845	0.0184
1500	0.0176	0.0189
2000	0.01796	0.0191



Iteations	CmS Std Dev	CmA Std Dev
10	0.0088	0.01285
25	0.01297	0.01277
50	0.0121	0.01342
100	0.0116	0.01416
200	0.01142	0.0125
300	0.01165	0.0127
400	0.01219	0.01187
500	0.01229	0.0124
1000	0.0124	0.01271
1500	0.01228	0.0125
2000	0.0125	0.0123

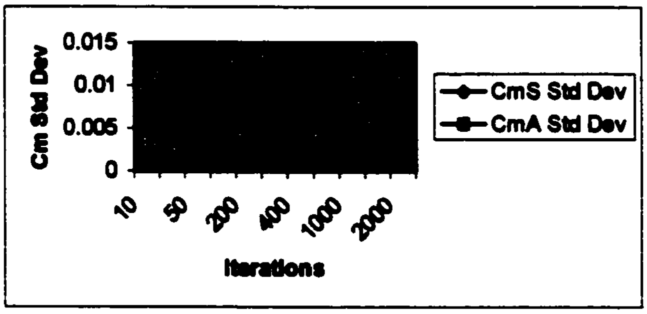


Figure N-6 Cm Comparison (Single Expert vs. Aggregated Expert Opinion)

APPENDIX O
Pareto Optimal Solutions

PITCHING MOMENT COEFFICIENT RISK ANALYSIS OPTIMIZATION
(Weighting Factors: W1 = 0.5, W2 = 0.5)

The mean square error of the mean optimal solution is: 0.002008

The mean of the mean optimal solution is: -0.003290
The variance of the mean optimal solution is: 0.000030

Fineness Ratio (x1) is: 6.500000
Wing Area Ratio (x2) is: 15.000000
Tip Fin Area Ratio (x3) is: 1.949199
Body Flap Area Ratio (x4) is: 0.600000
Ballast Weight (x5) is: 0.024844
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -6.637315

The variance of the variance optimal solution is: 0.000000166

The mean of the variance optimal solution is: 0.057642

Fineness Ratio (x1) is: 5.000000
Wing Area Ratio (x2) is: 10.000000
Tip Fin Area Ratio (x3) is: 1.689469
Body Flap Area Ratio (x4) is: 0.000000
Ballast Weight (x5) is: 0.022683
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -14.066860

The objective function of the Pareto optimum solution is: 9.892864

The variance of the pareto optimal solution is: 0.000000664

The mean of the pareto optimal solution is: 0.025155

Fineness Ratio (x1) is: 4.250000
Wing Area Ratio (x2) is: 13.000000
Tip Fin Area Ratio (x3) is: 1.995444
Body Flap Area Ratio (x4) is: 0.200000
Ballast Weight (x5) is: 0.018767
Mass Ratio (x6) is: 7.850000
Elevon Deflection (x8) is: -8.270349

PITCHING MOMENT COEFFICIENT RISK ANALYSIS OPTIMIZATION
(Weighting Factors: W1 = 0.75, W2 = 0.25)

The mean square error of the mean optimal solution is: 0.002008

The mean of the mean optimal solution is: -0.003290
The variance of the mean optimal solution is: 0.000030

Fineness Ratio (x1) is: 6.500000
Wing Area Ratio (x2) is: 15.000000
Tip Fin Area Ratio (x3) is: 1.949199
Body Flap Area Ratio (x4) is: 0.600000
Ballast Weight (x5) is: 0.024844
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -6.637315

The variance of the variance optimal solution is: 0.000000166

The mean of the variance optimal solution is: 0.057642

Fineness Ratio (x1) is: 5.000000
Wing Area Ratio (x2) is: 10.000000
Tip Fin Area Ratio (x3) is: 1.689469
Body Flap Area Ratio (x4) is: 0.000000
Ballast Weight (x5) is: 0.022683
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -14.066860

The objective function of the Pareto optimum solution is: 11.552882

The variance of the pareto optimal solution is: 0.000002061

The mean of the pareto optimal solution is: 0.021212

Fineness Ratio (x1) is: 5.000000
Wing Area Ratio (x2) is: 13.000000
Tip Fin Area Ratio (x3) is: 2.211422
Body Flap Area Ratio (x4) is: 0.410000
Ballast Weight (x5) is: 0.001899
Mass Ratio (x6) is: 8.030000
Elevon Deflection (x8) is: -7.661563

PITCHING MOMENT COEFFICIENT RISK ANALYSIS OPTIMIZATION
(Weighting Factors: W1 = 0.9, W2 = 0.1)

The mean square error of the mean optimal solution is: 0.002008

The mean of the mean optimal solution is: -0.003290
The variance of the mean optimal solution is: 0.000030

Fineness Ratio (x1) is: 6.500000
Wing Area Ratio (x2) is: 15.000000
Tip Fin Area Ratio (x3) is: 1.949199
Body Flap Area Ratio (x4) is: 0.600000
Ballast Weight (x5) is: 0.024844
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -6.637315

The variance of the variance optimal solution is: 0.000000166

The mean of the variance optimal solution is: 0.057642

Fineness Ratio (x1) is: 5.000000
Wing Area Ratio (x2) is: 10.000000
Tip Fin Area Ratio (x3) is: 1.689469
Body Flap Area Ratio (x4) is: 0.000000
Ballast Weight (x5) is: 0.022683
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -14.066860

The objective function of the Pareto optimum solution is: 7.073742

The variance of the pareto optimal solution is: 0.000009207

The mean of the pareto optimal solution is: 0.007656

Fineness Ratio (x1) is: 5.750000
Wing Area Ratio (x2) is: 12.000000
Tip Fin Area Ratio (x3) is: 1.249278
Body Flap Area Ratio (x4) is: 0.200000
Ballast Weight (x5) is: 0.015466
Mass Ratio (x6) is: 7.850000
Elevon Deflection (x8) is: -5.256107

PITCHING MOMENT COEFFICIENT RISK ANALYSIS OPTIMIZATION
(Weighting Factors: W1 = 0.95, W2 = 0.05)

The mean square error of the mean optimal solution is: 0.002008

The mean of the mean optimal solution is: -0.003290
The variance of the mean optimal solution is: 0.000030

Fineness Ratio (x1) is: 6.500000
Wing Area Ratio (x2) is: 15.000000
Tip Fin Area Ratio (x3) is: 1.949199
Body Flap Area Ratio (x4) is: 0.600000
Ballast Weight (x5) is: 0.024844
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -6.637315

The variance of the variance optimal solution is: 0.000000166

The mean of the variance optimal solution is: 0.057642

Fineness Ratio (x1) is: 5.000000
Wing Area Ratio (x2) is: 10.000000
Tip Fin Area Ratio (x3) is: 1.689469
Body Flap Area Ratio (x4) is: 0.000000
Ballast Weight (x5) is: 0.022683
Mass Ratio (x6) is: 8.000000
Elevon Deflection (x8) is: -14.066860

The objective function of the Pareto optimum solution is: 4.379223

The variance of the pareto optimal solution is: 0.000009207

The mean of the pareto optimal solution is: 0.007656

Fineness Ratio (x1) is: 5.750000
Wing Area Ratio (x2) is: 12.000000
Tip Fin Area Ratio (x3) is: 1.249278
Body Flap Area Ratio (x4) is: 0.200000
Ballast Weight (x5) is: 0.015466
Mass Ratio (x6) is: 7.850000
Elevon Deflection (x8) is: -5.256107

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Hampton, K. R. 1999. *Military Craft Acquisitions: A Systems Test and Evaluation Approach.* ASEM National Conference Proceedings. October 21-23.

Hampton, K., R. Wilson, D. Vann. 1988. *Analysis of Side-By-Side Roll Tests of 120TWR833 and 85WR102.* NAVSEACOMBATSYSENGSTA Report # 60-209.