Andrews University
Digital Commons @ Andrews University

Faculty Publications

Mathematics

4-2013

Measuring a Circle: A Math Lesson for Grades 5-10

Robert C. Moore
Andrews University, moorer@andrews.edu

MEASURING A CIRCLE

A Math Lesson for Grades 5-10

The *Principles and Standards for School Mathematics*[1] of the National Council of Teachers of Mathematics, the Common Core State Standards for Mathematical Practice,[2] and the NAD Curriculum Guides for mathematics[3] are consistent in calling for instructional activities that allow students to experience mathematical processes, like exploring, solving, justifying, measuring, estimating, representing, applying, and explaining. An important objective of engaging students in these processes is the development of conceptual understanding along with factual knowledge and procedural skills.

This article is designed to promote teaching methods that engage students in active learning and result in deep conceptual understanding by offering a sample lesson to help students (grades 5-10, ages 10-15) answer questions about and gain a deeper understanding of how to measure the circumference and area of a circle.

Lawrence and Hennessey's book, *Sizing Up Measurement: Activities for Grades 6–8 Classrooms,*[4] has activities similar to those in this article, as well as a rich collection of other measuring activities. For measurement activities for grades K-5, see Bachman[5] and Confer.[6] For other activities, lesson plans, and resources, I recommend the *Illuminations* Website of the National Council of Teachers of Mathematics.[7]

Getting Started

Give your students a circle cut from heavy paper. Ask:

"Do you know how to measure its circumference and area? Do you know more than one way to do it?"

"Do you know the difference between the two formulas, $2\pi r$ and πr^2?"

"Do you know the correct units to use for circumference and area?"

"What is the definition of π, and why is this symbol used in the formulas for both circumference and area?"

Circumference

According to Dunham,[8] the ancient Greeks knew that the ratio of the circumference C of a circle to its diameter d is constant. Today we call this ratio π, that is, $\pi = C/d$, or $C = \pi d$. Because the diameter is twice the radius r, we also have the formula $C = 2\pi r$.

Activity 1: Circumference (Estimated time: 10 minutes)

This activity will help students understand the important concept of circumference and its relationship with π.

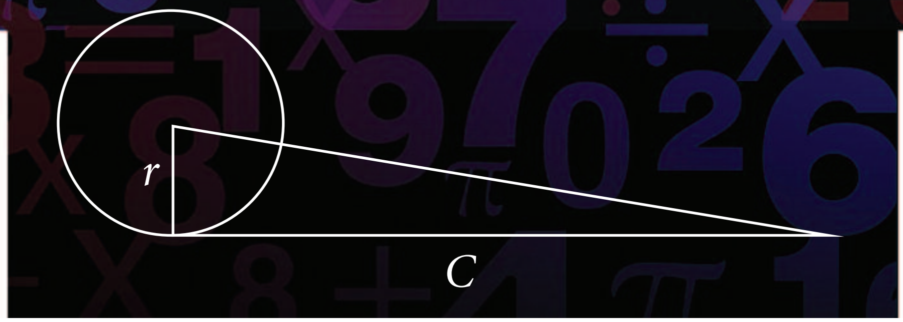
Give the students several circles of different diameters; for example, plastic lids from storage containers. With each circle, ask them to follow these steps:

1. Wrap a narrow strip of paper tape around the circle and tear it off to show the circumference, C.

2. Lay another strip of tape along a diameter and tear it off to show the length of the diameter, d.

3. Use the shorter piece of tape, d, to measure the longer one, C.

BY ROBERT C. MOORE



4. Measure the circumference and diameter with a ruler, then use a calculator to divide the circumference by the diameter. *[Make sure students use the correct units.]*

5. Write a conclusion about the investigation.

The students should find, of course, that the circumference is always a bit longer than three diameters, that is, π is slightly larger than 3. Students should also realize that circumference is the same as length and thus should be measured using units like centimeters or inches.

Another way for students to arrive at an estimate of π is to use a flexible measuring tape to measure the circumference and diameter and then divide the two quantities.

Area

Dunham[9] describes how the ancient Greek mathematician Archimedes proved the number π that is used in the formula for circumference is also used in the formula for area. In his proof, he considered a right triangle whose base was the circumference *C* of the circle and whose height was the radius *r* of the circle, as shown in the figure above. Then, using an ingenious argument that is too complex to repeat here, he showed that the area of the circle can be neither greater than nor less than the area of the triangle. He concluded, therefore, that the area of the circle was identical to the area of the triangle.

Using the familiar formula for the area of a triangle, $A = ½ \times base \times height$, and the formula for the circumference of a circle, $C = 2\pi r$, we find the formula for the area of a circle as follows: *Area of circle = Area of right triangle* $= ½ \times base \times height = ½ \times C \times r = ½ \times (2\pi r) \times r = \pi r^2$.

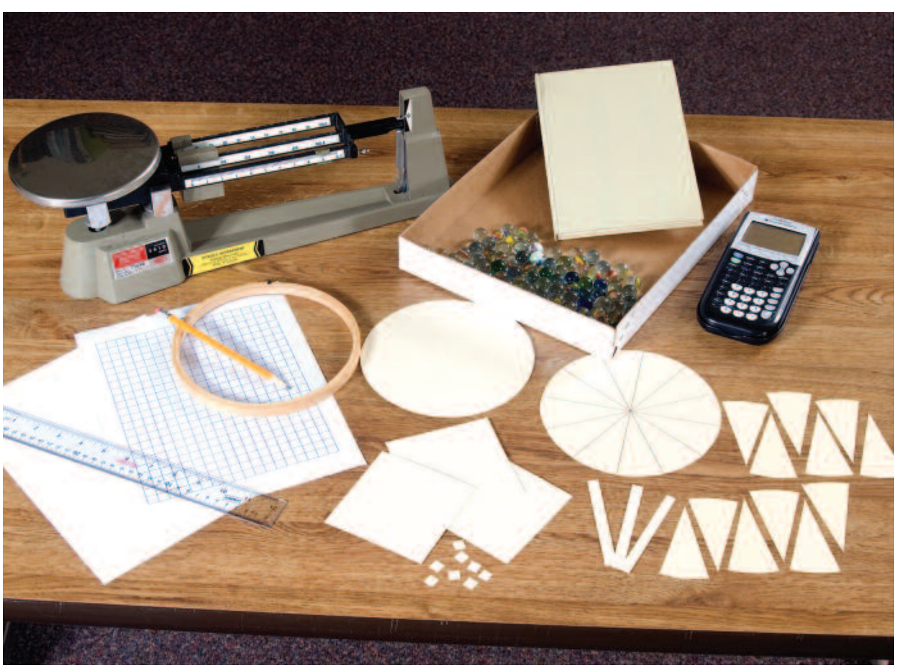
When we simply give students this area formula, $A = \pi r^2$, and ask them to calculate the areas of circles drawn in textbooks, how much do they really understand about circular area? I have found that they understand very little, which is why I developed the following laboratory activity.

Activity 2: Circular Area (Estimated time: 60 minutes or longer)

The activity sheet on page 33 shows a sequence of five methods for measuring the area of a circle. The goal of the activity is for the students to develop a deeper understanding of the concept of circular area, as well as area in general.

Each group will need a circle cut from heavy paper, like card stock or a manila file folder. To draw the circles, use an embroidery hoop about 15 cm in diameter. (The hoop will also be used for Method 5.) Lay the hoop on the heavy paper and draw around the *inside* of the hoop. Then cut out the circles. (If you choose to omit Method 5, you do not need an embroidery hoop. Instead, you can use a compass to draw the circles.)

It will be helpful for you to mark the center of each circle in advance. One way to locate the center is to use the embroidery hoop to draw a circle on plain paper or tracing paper. Carefully fold the circle in half so that it coincides with itself, and then fold again in a perpendicular direction. The two fold lines should meet at the center. Use a ruler to check. Now, with a sharp pencil or other object, punch a small hole through the



Supplies for this math lesson.

center. Use this as a template to mark the centers of the heavy paper circles.

Activity 2: Basic Materials Needed for Each Group

Paper circle

Metric ruler (centimeters)

Additional materials will be needed for each method, as listed below along with comments.

Method 1: Covering with squares

Square centimeter grid paper (the grid should be larger than the circle)

Comments: This first method highlights the concept of area—covering with squares. Emphasize the area unit, square centimeters (cm^2).

Activity 1.

Activity 2, Method 5.

Method 2: Using a formula

Calculator

Method 3: Weighing

Balance or scale for weighing a paper circle

Rectangular pieces of paper cut from the same paper that was used to make the circles

You need enough of these shapes to balance a paper circle:

- 10 cm × 10 cm squares (probably 2 or 3)
- 1 cm × 10 cm rectangles (probably 4 or 5)
- 1 cm × 1 cm squares (probably 9)

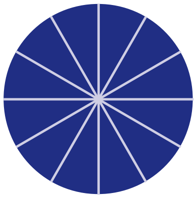
Comments: Do this method with the whole class. Otherwise, you will need several scales or will have to develop a plan whereby the students circulate from station to station. For best results, cut the strips as accurately as possible, and use a sensitive balance.

Method 4: Cutting and rearranging the pieces

Scissors, preferably one pair for each student

Protractor or other means of dividing the circle into 12 sectors of the same size

Comments: (a) You can save much lab time by marking six diameters on the circles in advance, or by providing each group with a copy of the diagram below. They can lay it over the center of their circle, mark 12 (or just six) points on their circle, and then draw the diameters.



(b) Notice that after the circle is cut, the pieces can be rearranged to form something that looks like a parallelogram. The area of a parallelogram is given by $A = base \times height$, and the base of the "parallelogram" above is half the circumference, ½ × 2π*r*, while the height of the "parallelogram" is *r*. Thus the area is approximately ½ × 2π*r* × *r* = π*r*². Nice!

Method 5: Transforming circular area to rectangular area

Enough marbles about 1.5 cm in diameter to fill the embroidery hoop one layer deep

Small cardboard box (i.e., shoe box)

Stiff piece of cardboard or book to push the marbles into one end of the box

Comment: As with Method 3, I suggest you do this activity with the whole class. Otherwise, it requires many marbles, boxes, and embroidery hoops or a circulation plan. ✐

The author would like to thank Richard Wright for helpful comments on an initial draft of this article and Marcy Moore for the privilege of doing these activities with her students.

Robert C. Moore, Ed.D. *(Mathematics Education), is Professor of Mathematics at Andrews University in Berrien Springs, Michigan, where he teaches mathematics to preservice elementary and secondary teachers. He particularly enjoys geometry and has enjoyed giving talks and workshops on geometry and measurement to teachers. In 2001, he served as the Coordinator of an issue of the* Journal *on Mathematics.*

NOTES AND REFERENCES

1. National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics* (Reston, Va.: NCTM, 2000).

2. Common Core State Standards Initiative, "Mathematics » Mathematics » Standards for Mathematical Practice": http://www.corestandards.org/Math/Practice. Accessed February 8, 2013.

3. The NAD Curriculum Guide for Mathematics, Grades K-12, is available at http://www.nadeducation.org/client_data/files/725_mathematicsk12.pdf. The process standards begin on page 29. Accessed February 8, 2013.

4. Ann Lawrence and Charlie Hennessey, *Sizing Up Measurement: Activities for Grades 6-8 Classrooms* (Sausalito, Calif.: Math Solutions Publications, 2007).

5. Vicki Bachman, *Sizing Up Measurement: Activities for Grades K-2 Classrooms* (Sausalito, Calif.: Math Solutions Publications, 2007).

6. Chris Confer, *Sizing Up Measurement: Activities for Grades 3-5 Classrooms* (Sausalito, Calif.: Math Solutions Publications, 2007).

7. *Illuminations: Resources for Teaching Math:* http://illuminations.nctm.org. Accessed February 8, 2013. This Website has many activities, lessons, links to online resources, and mathematics standards espoused by the National Council of Teachers of Mathematics. The materials are categorized by grade level and mathematics content. Many of the lessons have built-in applets that students can use for exploration and discovery.

8. William Dunham, *Journey Through Genius: The Great Theorems of Mathematics* (New York: Wiley, 1990), p. 89.

9. Ibid., pp. 89-99.

Five Ways to Measure the Area of a Circle

Student Activity Sheet

Each group will have a paper circle and a metric ruler. Your job is to use five methods to find the area of the circle. Be sure to give the area in square centimeters (cm^2).

Method 1: Covering With Squares

Lay your circle on a square centimeter grid, trace around the circle, and count the squares that cover the circle. You will have to estimate the parts of the squares that lie along the edge and are partly inside and partly outside.

area = ____________

Method 2: Using a Formula

Measure the diameter of the circle to the nearest tenth of a centimeter. Then calculate the area using the formula $A = \pi r^2$. (Use the π button on a calculator, or use 3.14 for π.)

diameter = ____________ radius = ____________ area = ____________

Method 3: Weighing

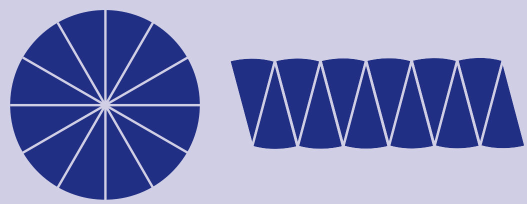
Weigh a circle, then place rectangular pieces of paper on the scale until you get the same weight as the circle. Calculate the total area of the rectangles.

area = ____________

Method 4: Cutting and Rearranging the Pieces

Locate the center of the circle. Use a protractor, pencil, and scissors to cut the circle into 12 equal pie-shaped pieces. Arrange the pieces as shown below. Notice that the pieces form a figure that is approximately a parallelogram. Measure the base and height of the "parallelogram" and then calculate its area (*A* = *base* × *height*).

base = __________
height = __________
area = __________



Method 5: Transforming a Circular Area to a Rectangular Area

Fill the embroidery hoop one layer deep with marbles. Make them fit tightly. Transfer the marbles to the box, then push them tightly to one end of the box to form a rectangular shape. Measure the length and width of the rectangle. Calculate the area of the rectangle.

area = ____________

Conclusion: Write a short report. Discuss questions such as the following:

- How close are the five areas to one another?
- Which method was your favorite?
- What did you learn?