Title: Star Decompositions of the Complete Split Graph

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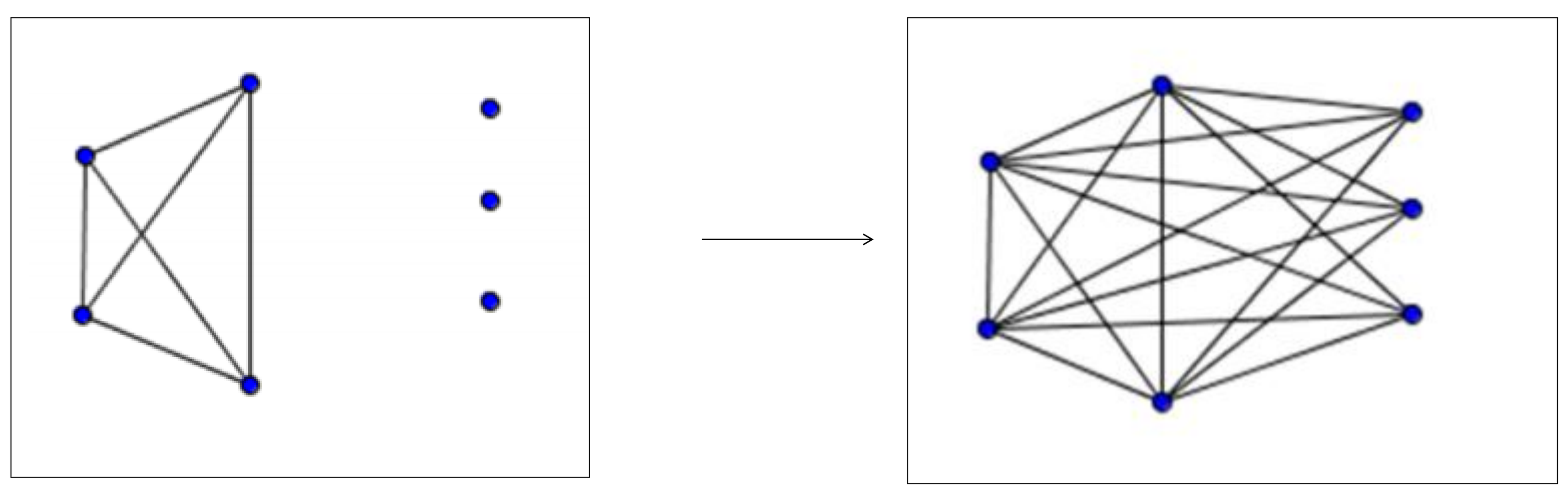
Introduction

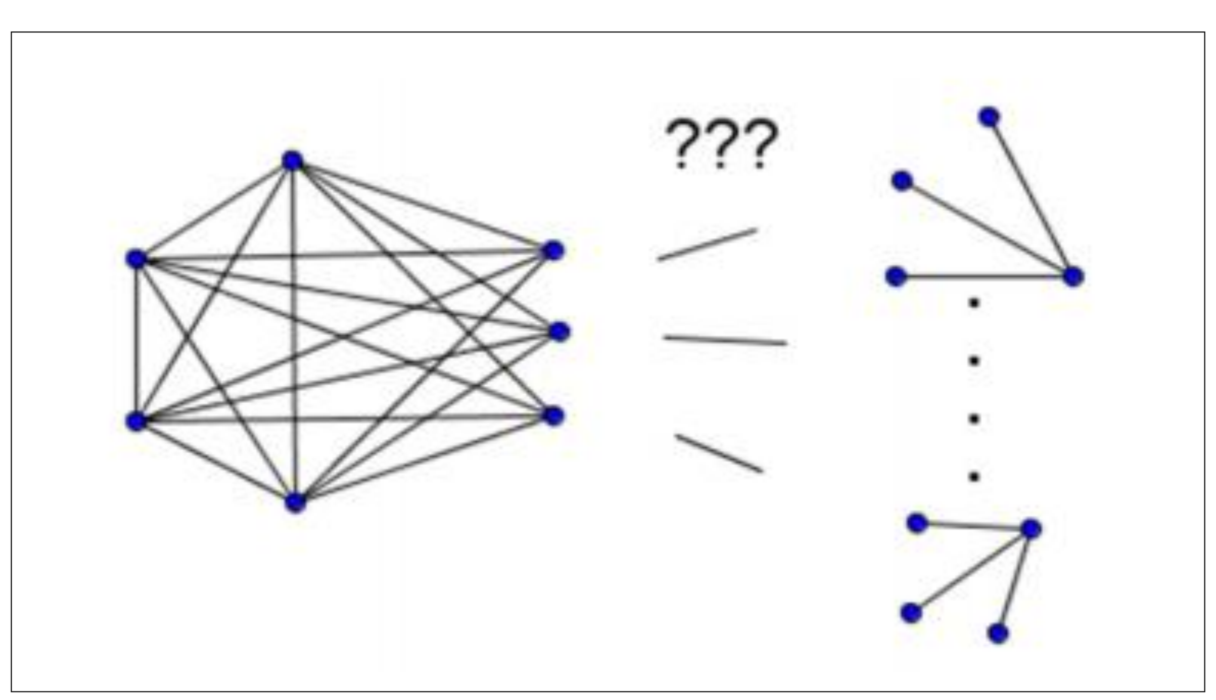
A graph is a discrete mathematical structure that consists of a set of vertices and a set of edges between pairs of vertices. A graph decomposition is a partitioning of the edges of a graph into disjoint sets in such a way that the induced subgraphs produced are isomorphic to each other. The graphs we focus on here are stars and complete split graphs (see below).



A complete split graph as the join of a complete graph and independent set

The Problem

Let G be the complete split graph with clique of order $n-m$ and independent set of order m. For what values of n, m, and t can we decompose G into edge disjoint copies of $K_{1,t}$?



Special Cases

- $m = n - 1$: decomposable if and only if $t \mid m$
- $t = 1$: trivial
- $t = 2$: decomposable if and only if total number of edges is even [3]

Necessary Conditions

- If G can be decomposed into t-stars, then $t \mid \binom{n-m}{2} + m(n-m)$

Casework and Results

- $n-m<t$: decomposable if and only if
  - $t \mid \frac{n+m-1}{2}$
- $n-m=t$: decomposable if and only if
  - t is odd and $m \ge \frac{t+1}{2}$
- $t<n-m<2t$: decomposable if
  - $t \mid \frac{n+m-1}{2}$ or
  - t is odd, $t \mid m$, and $n-m=t+1$
- $n-m \ge 2t$: decomposable if
  - $t \mid \frac{(n-m)(n-m-1)}{2}$ and $t \mid m(n-m)$,
  - $t \mid \frac{n+m-1}{2}$, or
  - $n-m$ is odd and $m \equiv -1 \ (mod\ t)$

Future work

Since we were unable to completely solve the problem for two of our cases, this is one place to begin.

We could also consider a more general problem by removing a subgraph H belonging to a different class of graphs.

Rather that limiting the size of stars to be a fixed value, we could consider decomposing a graph into stars of size t where t comes from some finite set of positive integers.

References

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[4] H.-C. Lee, Decomposition of the complete bipartite multigraph into cycles and stars, Discrete Mathematics 338 (2015), 1362-1369.

[5] S. Yamamoto, H. Ikeda, S. Shige-eda, K. Ushio and N. Hamada, On claw-decomposition of complete graphs and complete bigraphs, Hiroshima Math. J. 5 (1975), 3342.