



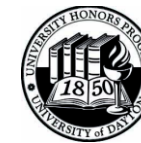
Generalized Multi-Latin Squares

Lydia Kindelin

Department: Mathematics

Advisor: Dr. Atif Abueida

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ABSTRACT

The research explores properties of generalized multi-latin squares and proposes ways to construct them. A (n, t, m, p, q) -generalized multi-latin square is an array consisting of n rows and n columns, where each cell is filled with m symbols from a collection consisting of t different symbols, any symbol appears in each row and in each column p times, and any pair of different symbols occur together q times. Understanding trivial examples, the properties, and the mathematical relationships behind the problem reveals multiple examples and a systematic way to build generalized multi-latin squares.

RELATED DESIGNS

BIBD is a pair (V, B) where V is a collection of v symbols and B is a collection of b k -subsets of V (blocks) such that each element of V is contained in exactly r blocks and any 2-subset of V is contained in exactly λ blocks. **Parallel classes** partition the set of blocks so that a symbol appears once in each class. **RBIBD** is a $BIBD(v, k, \lambda)$ whose blocks can be partitioned into parallel classes

CONSTRUCTION METHOD

Let c be the number of parallel classes and ℓ be the number of block in each parallel class from the $RBIBD(v, k, \lambda)$.

1. Build $c, \ell \times \ell$ square permuting the blocks each row
2. Arrange the $\ell \times \ell$ squares into a $c \times c$ square
3. To extend the generalized multi-latin square, expand the $c \times c$ square by a factor of c to use every parallel class the same number of times.
4. Let $N \times N$ be the size of the square filled with $\ell \times \ell$ squares. Therefore, $N^2 = c \cdot s$ for some natural number s .

The result is a $(N \cdot \ell, v, k, N, s \cdot \ell)$ -generalized multi-latin square.

GENERALIZED MULTI-LATIN SQUARES

RBIBD(9,3,1)

(12, 9, 3, 4, 12)-generalized multi-latin square

Symbols: $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Block size: $k = 3$

Number of block in each class:

$$\ell = v/k = 9/3 = 3$$

Number of blocks:

$$\binom{v}{2} / \binom{k}{2} = \binom{9}{2} / \binom{3}{2} = 12$$

Number of Parallel classes:

$$c = \left[\binom{9}{2} / \binom{3}{2} \right] / 3 = 4$$

1,2,3	4,5,6	7,8,9	1,4,7	2,5,8	3,6,9	1,5,9	2,6,7	3,4,8	1,6,8	2,4,9	3,5,7
4,5,6	7,8,9	1,2,3	2,5,8	3,6,9	1,4,7	2,6,7	3,4,8	1,5,9	2,4,9	3,5,7	1,6,8
7,8,9	1,2,3	4,5,6	3,6,9	1,4,7	2,5,8	3,4,8	1,5,9	2,6,7	3,5,7	1,6,8	2,4,9
1,4,7	2,5,8	3,6,9	1,5,9	2,6,7	3,4,8	1,6,8	2,4,9	3,5,7	1,2,3	4,5,6	7,8,9
2,5,8	3,6,9	1,4,7	2,6,7	3,4,8	1,5,9	2,4,9	3,5,7	1,6,8	4,5,6	7,8,9	1,2,3
3,6,9	1,4,7	2,5,8	3,4,8	1,5,9	2,6,7	3,5,7	1,6,8	2,4,9	7,8,9	1,2,3	4,5,6
1,5,9	2,6,7	3,4,8	1,6,8	2,4,9	3,5,7	1,2,3	4,5,6	7,8,9	1,4,7	2,5,8	3,6,9
2,6,7	3,4,8	1,5,9	2,4,9	3,5,7	1,6,8	4,5,6	7,8,9	1,2,3	2,5,8	3,6,9	1,4,7
3,4,8	1,5,9	2,6,7	3,5,7	1,6,8	2,4,9	7,8,9	1,2,3	4,5,6	3,6,9	1,4,7	2,5,8
1,6,8	2,4,9	3,5,7	1,2,3	4,5,6	7,8,9	1,4,7	2,5,8	3,6,9	1,5,9	2,6,7	3,4,8
2,4,9	3,5,7	1,6,8	4,5,6	7,8,9	1,2,3	2,5,8	3,6,9	1,4,7	2,6,7	3,4,8	1,5,9
3,5,7	1,6,8	2,4,9	7,8,9	1,2,3	4,5,6	3,6,9	1,4,7	2,5,8	3,4,8	1,5,9	2,6,7

RBIBD(51,3,1)

(425, 51, 3, 25, 425)-generalized multi-latin square

(595, 51, 3, 35, 833)-generalized multi-latin square

FUTURE RESEARCH

& APPLICATIONS

- ❖ To classify and characterize the existence of generalized multi-latin squares for any given parameters.
- ❖ Experiment variable design

REFERENCES

- Lindner, C. C., and C. A. Rodger. Design Theory. Boca Raton: Chapman and HallCRC, 2009. Print.
- L.D. Andersen, Latin squares and their generalizations, Ph.D. Thesis, Reading, 1979
- N. Cavenagh, C. Hamalainen, J. G. Lefevre, and D. S. Stones, Multi-latin squares, Discrete Mathematics, 311 (2011), 1164-1171.
- Wallis, W. D., and W. D. Wallis. Introduction to Combinatorial Designs. Boca Raton: Chapman and HallCRC, 2007. Print.

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