Generalized Multi-Latin Squares

Lydia Kindelin
Department: Mathematics
Advisor: Dr. Atif Abueida
April 2013

ABSTRACT

The research explores properties of generalized multi-latin squares and proposes ways to construct them. A *(n, t, m, p, q)-generalized multi-latin square* is an array consisting of n rows and n columns, where each cell is filled with m symbols from a collection consisting of t different symbols, any symbol appears in each row and in each column p times, and any pair of different symbols occur together q times. Understanding trivial examples, the properties, and the mathematical relationships behind the problem reveals multiple examples and a systematic way to build generalized multi-latin squares.

RELATED DESIGNS

BIBD is a pair (V, B) where V is a collection of v symbols and B is a collection of b k-subsets of V (blocks) such that each element of V is contained in exactly r blocks and any 2-subset of V is contained in exactly λ blocks.

Parallel classes partition the set of blocks so that a symbol appears once in each class.

RBIBD is a BIBD(v, k, λ) whose blocks can be partitioned into parallel classes

CONSTRUCTION METHOD

Let c be the number of parallel classes and ℓ be the number of block in each parallel class from the *RBIBD(v,k,λ).*

1. Build c, $\ell \times \ell$ square permuting the blocks each row
2. Arrange the $\ell \times \ell$ squares into a $c \times c$ square
3. To extend the generalized multi-latin square, expand the $c \times c$ square by a factor of c to use every parallel class the same number of times.
4. Let $N \times N$ be the size of the square filled with $\ell \times \ell$ squares. Therefore, $N^2 = c \cdot s$ for some natural number s.

The result is a $(N \cdot \ell, v, k, N, s \cdot \ell)$-generalized multi-latin square.

GENERALIZED MULTI-LATIN SQUARES

RBIBD(9,3,1)

Symbols: V ={1,2,3,4,5,6,7,8,9}

Block size: k = 3

Number of block in each class:

$\ell = v/k = 9/3 = 3$

Number of blocks:

$\binom{v}{2} / \binom{k}{2} = \binom{9}{2} / \binom{3}{2} = 12$

Number of Parallel classes:

$c = [\binom{9}{2} / \binom{3}{2}] / 3 = 4$

(12, 9, 3, 4, 12)-generalized multi-latin square

1,2,3	4,5,6	7,8,9	1,4,7	2,5,8	3,6,9	1,5,9	2,6,7	3,4,8	1,6,8	2,4,9	3,5,7
4,5,6	7,8,9	1,2,3	2,5,8	3,6,9	1,4,7	2,6,7	3,4,8	1,5,9	2,4,9	3,5,7	1,6,8
7,8,9	1,2,3	4,5,6	3,6,9	1,4,7	2,5,8	3,4,8	1,5,9	2,6,7	3,5,7	1,6,8	2,4,9
1,4,7	2,5,8	3,6,9	1,5,9	2,6,7	3,4,8	1,6,8	2,4,9	3,5,7	1,2,3	4,5,6	7,8,9
2,5,8	3,6,9	1,4,7	2,6,7	3,4,8	1,5,9	2,4,9	3,5,7	1,6,8	4,5,6	7,8,9	1,2,3
3,6,9	1,4,7	2,5,8	3,4,8	1,5,9	2,6,7	3,5,7	1,6,8	2,4,9	7,8,9	1,2,3	4,5,6
1,5,9	2,6,7	3,4,8	1,6,8	2,4,9	3,5,7	1,2,3	4,5,6	7,8,9	1,4,7	2,5,8	3,6,9
2,6,7	3,4,8	1,5,9	2,4,9	3,5,7	1,6,8	4,5,6	7,8,9	1,2,3	2,5,8	3,6,9	1,4,7
3,4,8	1,5,9	2,6,7	3,5,7	1,6,8	2,4,9	7,8,9	1,2,3	4,5,6	3,6,9	1,4,7	2,5,8
1,6,8	2,4,9	3,5,7	1,2,3	4,5,6	7,8,9	1,4,7	2,5,8	3,6,9	1,5,9	2,6,7	3,4,8
2,4,9	3,5,7	1,6,8	4,5,6	7,8,9	1,2,3	2,5,8	3,6,9	1,4,7	2,6,7	3,4,8	1,5,9
3,5,7	1,6,8	2,4,9	7,8,9	1,2,3	4,5,6	3,6,9	1,4,7	2,5,8	3,4,8	1,5,9	2,6,7

RBIBD(51,3,1) →

(425, 51, 3, 25, 425)-generalized multi-latin square

(595, 51, 3, 35, 833)-generalized multi-latin square

FUTURE RESEARCH & APPLICATIONS

- ❖To classify and characterize the existence of generalized multi-latin squares for any given parameters.
- ❖Experiment variable design

REFERENCES

- Lindner, C. C., and C. A. Rodger.Design Theory. Boca Raton: Chapman and HallCRC, 2009. Print.
- L.D. Andersen, Latin squares and their generalizations, Ph.D. Thesis, Reading, 1979
- N. Cavenagh, C. Hamalainen, J. G. Lefevre, and D. S. Stones, Multi-latin squares, Discrete Mathematics, 311 (2011), 1164-1171.
- Wallis,W. D., and W. D.Wallis. Introduction to Combinatorial Designs. Boca Raton: Chapman and HallCRC, 2007. Print.

ACKNOWLEDGEMENT

•Dr. Atif A. Abueida
•University of Dayton Honors Department
•University of Dayton Department of Mathematics
•Family and Friends