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James P. Clemens
Miami University - Oxford

Perry R. Rice
Miami University - Oxford

Pranaw Kumar Rungta
Miami University - Oxford

Robert J. Brecha
University of Dayton, rbrecha1@udayton.edu

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Two-level atom in an optical parametric oscillator: Spectra of transmitted and fluorescent fields in the weak-driving-field limit

J. P. Clemens,¹ P. R. Rice,¹ P. K. Rungta,¹ and R. J. Brecha²

¹*Department of Physics, Miami University, Oxford, Ohio 45056*

²*Department of Physics, University of Dayton, Dayton, Ohio 45469*

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We consider the interaction of a two-level atom inside an optical parametric oscillator. In the weak-driving-field limit, we essentially have an atom-cavity system driven by the occasional pair of correlated photons, or weakly squeezed light. We find that we may have holes, or dips, in the spectrum of the fluorescent and transmitted light. This occurs even in the strong-coupling limit when we find holes in the vacuum-Rabi doublet. Also, spectra with a subnatural linewidth may occur. These effects disappear for larger driving fields, unlike the spectral narrowing obtained in resonance fluorescence in a squeezed vacuum; here it is important that the squeezing parameter N tends to zero so that the system interacts with only one correlated pair of photons at a time. We show that a previous explanation for spectral narrowing and spectral holes for incoherent scattering is not applicable in the present case, and propose an alternative explanation. We attribute these anomalous effects to quantum interference in the two-photon scattering of the system.

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I. INTRODUCTION

In recent years, squeezed light sources have become available in the laboratory and attention has turned to their interaction with optical systems. In particular, much attention has been directed at modifying the radiative properties of an atom via interaction with squeezed light. Examples include the seminal work of Gardiner [1], who showed that the decay rate of the atomic polarization quadratures was phase-dependent. Carmichael, Lane, and Walls [2] (hereafter referred to as CLW) considered resonance fluorescence when the atom is immersed in squeezed vacuum. They predicted that for weak driving fields, independent of the relative phase between the driving field and the squeezed vacuum, the incoherent spectrum would narrow as the amount of squeezing was increased. In the limit of strong squeezing, the incoherent spectrum is a very narrow peak on top of a very broad background, essentially a δ function. For stronger driving fields, the central peak of the Mollow spectrum could be broadened or narrowed, depending on the relative phase between the strong driving field and the squeezed vacuum. The photon number distribution $P(n)$ has been calculated by Jagatap and Lawande [3], showing phase-sensitive behavior for strong fields.

It was realized early on that experiments would probably require some sort of cavity system, as it is impractical to squeeze all of the vacuum modes that interact with an atom. Several theoretical calculations having to do with squeezing only some of the vacuum modes have been presented. Savage [4] has calculated that for large Jaynes-Cummings coupling g and strong excitation, the width of the Rabi sidebands could be narrowed, but not below the natural linewidth. In a cavity of moderate Q , Courty and Reynaud [5] found that one of the Rabi sidebands could be suppressed for the proper detuning, essentially turning off spontaneous emission from one of the dressed states. Kennedy [6] has found similar behavior in the many-atom case. Rice and Pedrotti [7] have

considered an extension of the work of CLW, again for an atom in a cavity of moderate Q . They found that it was possible to squeeze away the cavity enhancement part of the linewidth, but that to obtain measurably subnatural linewidths, the fraction of 4π sr for the cavity mode subtends must be significant. This system has also been considered by Cirac [8], who investigated both the fluorescent spectrum and the steady-state population inversion, as discussed by Savage and Lindberg [9]. For very strong driving fields and finite-bandwidth squeezed light centered on the Rabi sidebands, Parkins [10] and Cirac and Sanchez-Soto [11] have found narrowing of one of the Rabi sidebands. Parkins *et al.* [12] have calculated the fluorescent spectrum of a strongly coupled atom-cavity system, where the driving field is tuned to resonance with the one-photon dressed-state resonance and have predicted narrowing. One notable example of a calculation in which the atom interacts with only one mode that is squeezed is the work of Vyas and Singh [13], who considered resonance fluorescence in the weak-field limit when the coherent driving field was replaced by the squeezed output of an optical parametric oscillator.

It was then suggested by Jin and Xiao [14,15] that the atom could be placed inside the source of the squeezing. They considered phase/intensity bistability in the case of a two-level atom inside an optical parametric oscillator (OPO). Further, they considered the spectrum of squeezing and incoherent spectra for that system. It was decided that it would be fruitful to examine this system in the weak driving field limit. Agarwal [16] had previously considered the two-level atom in an OPO, with a strong driving field incident directly on the atoms, from the side of the cavity. He considered the strong driving limit when the external field dressed the atoms, and found modifications of the Mollow triplet in that case. Our work is limited to the weak-field limit, and is closely related to that of Smyth and Swain [17], who have found anomalous spectra in optical systems driven by squeezed light. They have considered the cases of broadband and narrowband squeezing and both cavity and free-space

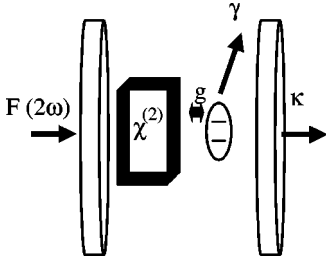


FIG. 1. A schematic of the physical system under consideration. We have a single two-level atom in a resonant cavity. $F(2\omega)$ is a classical driving field at twice the resonant frequency. The nonlinear crystal has a second-order susceptibility $\chi^{(2)}$. g is the atom-field coupling, κ is the field decay rate through the right-hand mirror, and γ is the spontaneous-emission rate to noncavity modes.

systems. They have found some of their most interesting results in the weak-field limit. Our results are similar in nature to those but differ in the details. Further, we provide an illuminating argument why the effects are most prominent in the weak-field limit.

Experimental work in this area has been pioneered by Turchette *et al.* [18], who drove an atom-cavity system with squeezed light and observed phase-dependent spectra, and the recent work by Lu *et al.* [19], who have observed the effects of squeezing directly in such phase-dependent spectra. For an overview of the interaction of squeezed light with atoms, we refer the reader to two recent reviews by Parkins [20] and Dalton *et al.* [21].

In Sec. II we examine the physical system under consideration. The transmitted spectrum is calculated and discussed in Sec. III. The spectrum of the fluorescent light is considered in Sec. IV. In Sec. V we consider the physical explanation of the anomalous spectra we see, and we conclude in Sec. VI.

II. PHYSICAL SYSTEM

We consider a single two-level atom inside an optical cavity, which also contains a material with a $\chi^{(2)}$ nonlinearity. The atom and cavity are assumed to be resonant at ω and the system is driven by light at 2ω . The system is shown in Fig. 1. The interaction of this driving field with the nonlinear material produces light at the subharmonic ω . This light consists of correlated pairs of photons, or quadrature squeezed light. In the limit of weak driving fields, these correlated pairs are created in the cavity and eventually two photons leave the cavity through the end mirror or as fluorescence out the side before the next pair is generated. Hence we may view the system as an atom-cavity system driven by the occasional pair of correlated photons. In the language of squeezed light, we are interested in the limit $N \rightarrow 0$. As N is increased, the effects we consider here vanish. We wish to understand these effects in terms of photon correlations rather than the usual effects of quadrature squeezed light, where typically the largest nonclassical effects are seen in the large- N limit. The system is described by a master equation in Lindblad form,

$$\dot{\rho} = -i\hbar[H, \rho] + \mathcal{L}_{\text{diss}}\rho \equiv \mathcal{L}\rho, \quad (1)$$

where the system Hamiltonian is

$$H = i\hbar F(a^{\dagger 2} - a^2) + i\hbar g(a^{\dagger}\sigma_- - a\sigma_+) + \hbar\omega(a^{\dagger}a + \frac{1}{2}\sigma_z). \quad (2)$$

Here, $g = \mu(\omega_0/\hbar\epsilon_0 V)^{1/2}$ is the usual Jaynes-Cummings atom-field coupling in the rotating wave and dipole approximations. The cavity-mode volume is V , and the atomic dipole matrix element connecting ground and excited states is μ_d . The effective two-photon driving field F is proportional to the intensity $I_{\text{in}}(2\omega_0)$ of a driving field at twice the resonant frequency of the atom (and resonant cavity) and the $\chi^{(2)}$ of the nonlinear crystal in the cavity, as

$$F = -i\kappa_{\text{in}}\left(\frac{\mathcal{F}}{\pi}\right)\sqrt{\frac{\epsilon_0 VT}{\hbar\omega}}e^{i\phi}\chi^{(2)}I_{\text{in}}(2\omega). \quad (3)$$

The cavity finesse is \mathcal{F} , and T and ϕ are the intensity transmission coefficient and phase change at the input mirror. We also have $\kappa_{\text{in}} = cT/L$ as the cavity field loss rate through the input mirror. The transmission of the input mirror is taken to be vanishingly small, with a large $I_{\text{in}}(2\omega_0)$ so that F is finite. Hence we effectively consider a single-ended cavity. The dissipative Liouvillian describing loss due to the leaky end mirror and spontaneous emission out the side of the cavity is

$$\begin{aligned} L_{\text{diss}}\rho = & \frac{\gamma}{2}(2\sigma_- \rho \sigma_+ - \sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_-) \\ & + \kappa(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a). \end{aligned} \quad (4)$$

Here γ is the spontaneous-emission rate to all modes other than the privileged cavity mode, hereafter referred to as the vacuum modes. The field decay rate of the cavity at the output mirror is κ . As we are working in the weak driving field limit, we only consider states of the system with up to two quanta, i.e.,

$$|0-\rangle, |0+\rangle, |1-\rangle, |1+\rangle, |2-\rangle. \quad (5)$$

Here, the first index corresponds to the excitation of the field (n =number of quanta) and the second index denotes the number of energy quanta in the atoms (+ for ground state and - for excited state). In this basis set we have the following equations for density-matrix elements:

$$\dot{\rho}_{0,-;0,-} = \gamma\rho_{0,+;0,+} + 2\kappa\rho_{1,-;1,-} - 2\sqrt{2}F\rho_{0,-;2,-}, \quad (6a)$$

$$\dot{\rho}_{0,+;0,+} = -\gamma\rho_{0,+;0,+} + 2\kappa\rho_{1,+;1,+} - 2g\rho_{0,+;1,-}, \quad (6b)$$

$$\dot{\rho}_{1,-;1,-} = \gamma\rho_{1,+;1,+} - 2\kappa\rho_{1,-;1,-} + 4\kappa\rho_{2,-;2,-} + 2g\rho_{0,+;1,-}, \quad (6c)$$

$$\dot{\rho}_{1,+;1,+} = -(\gamma + 2\kappa)\rho_{1,+;1,+} - 2\sqrt{2}g\rho_{1,+;2,-}, \quad (6d)$$

$$\dot{\rho}_{2,-;2,-} = -4\kappa\rho_{2,-;2,-} + 2\sqrt{2}F\rho_{0,-;2,-} + 2\sqrt{2}g\rho_{1,+;2,-}, \quad (6e)$$

$$\dot{\rho}_{0,-;1,+} = -(\gamma/2 + \kappa)\rho_{0,-;1,+} - \sqrt{2}g\rho_{0,-;2,-} - \sqrt{2}F\rho_{1,+;2,-}, \quad (6f)$$

$$\begin{aligned} \dot{\rho}_{0,-;2,-} = & -2\kappa\rho_{0,-;2,-} + 2\sqrt{2}F(\rho_{0,-;0,-} - \rho_{2,-;2,-}) \\ & + \sqrt{2}g\rho_{0,-;1,+} + \sqrt{2}F\rho_{1,+;2,-}, \end{aligned} \quad (6g)$$

$$\begin{aligned} \dot{\rho}_{0,+;1,-} = & -(\gamma/2 + \kappa)\rho_{0,+;1,-} - g(\rho_{1,-;1,-} - \rho_{0,+;0,+}) \\ & + 2\sqrt{2}\kappa\rho_{1,+;2,-}, \end{aligned} \quad (6h)$$

$$\begin{aligned} \dot{\rho}_{1,+;2,-} = & -(\gamma/2 + 3\kappa)\rho_{1,+;2,-} + \sqrt{2}F\rho_{0,-;1,+} \\ & - \sqrt{2}g(\rho_{2,-;2,-} - \rho_{1,+;1,+}). \end{aligned} \quad (6i)$$

The other density-matrix elements are not driven by the external field and couple only to themselves, hence if they are initially zero, they remain zero for all time. In the weak-field limit, one might expect that the population of the ground state is of order unity. With this in mind, examine Eq. (6g). Here we see that by taking $\rho_{0,-;0,-} \approx 1$ and $\rho_{0,-;0,-} \gg \rho_{2,-;2,-}$, we have

$$\begin{aligned} \dot{\rho}_{0,-;2,-} = & -2\kappa\rho_{0,-;2,-} + 2\sqrt{2}F + \sqrt{2}g\rho_{0,-;1,+} \\ & + \sqrt{2}F\rho_{1,+;2,-}. \end{aligned} \quad (7)$$

Here we see that $\rho_{0,-;2,-}$ is driven by a term of order F . This leads us to propose the $\rho_{0,-;1,+}$ and $\rho_{0,-;2,-}$ scale as F in the weak-field limit. Carrying this process out in a self-consistent matter, we arrive at the scalings

$$\rho_{0,-;0,-} \approx 1, \quad (8a)$$

$$\rho_{0,+;0,+} \approx F^2, \quad (8b)$$

$$\rho_{1,-;1,-} \approx F^2, \quad (8c)$$

$$\rho_{1,+;1,+} \approx F^2, \quad (8d)$$

$$\rho_{2,-;2,-} \approx F^2, \quad (8e)$$

$$\rho_{0,-;1,+} \approx F, \quad (8f)$$

$$\rho_{0,-;2,-} \approx F, \quad (8g)$$

$$\rho_{0,+;1,-} \approx F^2, \quad (8h)$$

$$\rho_{1,+;2,-} \approx F^2. \quad (8i)$$

These scalings make sense physically, as $\rho_{2,-;2,-}$ is a population driven by F in the Hamiltonian and is then proportional to F^2 to first order. The Jaynes-Cummings coupling g then couples $\rho_{2,-;2,-}$ to $\rho_{1,+;1,+}$, so both two-photon state populations scale as F^2 . Spontaneous emission and cavity decay are then responsible for coupling the two-photon states to the one-photon states, making $\rho_{0,+;0,+}$ and $\rho_{1,-;1,-}$ of order F^2 . The coherences $\rho_{0,-;2,-}$ and $\rho_{0,-;1,+}$ are driven directly by F . Finally the coherence $\rho_{1,+;2,-}$ is driven by the

population of the two-photon states, and hence is proportional to F^2 . Keeping terms to lowest order in F , the relevant equations become

$$\dot{\rho}_{0,-;0,-} = 0, \quad (9a)$$

$$\dot{\rho}_{0,+;0,+} = -\gamma\rho_{0,+;0,+} + 2\kappa\rho_{1,+;1,+} - 2g\rho_{0,+;1,-}, \quad (9b)$$

$$\dot{\rho}_{1,-;1,-} = \gamma\rho_{1,+;1,+} - 2\kappa\rho_{1,-;1,-} + 4\kappa\rho_{2,-;2,-} + 2g\rho_{0,+;1,-}, \quad (9c)$$

$$\dot{\rho}_{1,+;1,+} = -(\gamma + 2\kappa)\rho_{1,+;1,+} - 2\sqrt{2}g\rho_{1,+;2,-}, \quad (9d)$$

$$\dot{\rho}_{2,-;2,-} = -4\kappa\rho_{2,-;2,-} + \sqrt{2}F\rho_{0,-;2,-} + 2\sqrt{2}g\rho_{1,+;2,-}, \quad (9e)$$

$$\dot{\rho}_{0,-;1,+} = -(\gamma/2 + \kappa)\rho_{0,-;1,+} - \sqrt{2}g\rho_{0,-;2,-} - \sqrt{2}F\rho_{1,+;2,-}, \quad (9f)$$

$$\dot{\rho}_{0,-;2,-} = -2\kappa\rho_{0,-;2,-} + \sqrt{2}F + \sqrt{2}g\rho_{0,-;1,+}, \quad (9g)$$

$$\begin{aligned} \dot{\rho}_{0,+;1,-} = & -(\gamma/2 + \kappa)\rho_{0,+;1,-} - 2g(\rho_{1,-;1,-} - \rho_{0,+;0,+}) \\ & + 2\sqrt{2}\kappa\rho_{1,+;2,-}, \end{aligned} \quad (9h)$$

$$\begin{aligned} \dot{\rho}_{1,+;2,-} = & -(\gamma/2 + 3\kappa)\rho_{1,+;2,-} + \sqrt{2}F\rho_{0,-;1,+} \\ & - \sqrt{2}g(\rho_{2,-;2,-} - \rho_{1,+;1,+}). \end{aligned} \quad (9i)$$

In what follows, these equations are numerically solved for the steady-state density-matrix elements of the system. We note here that $\langle a \rangle_{ss} = \rho_{0,-;1,-} + \rho_{1,-;2,-} = 0$, but $\langle a^\dagger a \rangle_{ss} = \rho_{1,-;1,-} + \rho_{1,+;1,+} + 2\rho_{2,-;2,-} \approx F^2$. These results hold in the weak-field limit, but the mean intracavity field is also zero for arbitrary driving field states.

III. OPTICAL SPECTRUM OF THE TRANSMITTED LIGHT

We now turn our attention to a calculation of the spectrum of squeezing and to the incoherent spectrum; we consider both transmitted and fluorescent light fields. For the transmitted spectrum, in a rotating frame such that $\omega = 0$ corresponds to the simultaneous cavity and atomic resonances, we have

$$\begin{aligned} I_{tr}(\omega) &= \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle a^\dagger(0) a(\tau) \rangle \\ &= 2 \operatorname{Re} \int_0^{\infty} d\tau e^{i\omega\tau} \langle a^\dagger(0) a(\tau) \rangle \\ &= 2\pi \langle a \rangle_{ss} \langle a^\dagger \rangle_{ss} \delta(\omega) \\ &\quad + 2 \operatorname{Re} \int_0^{\infty} d\tau e^{i\omega\tau} \langle \Delta a^\dagger(0) \Delta a(\tau) \rangle. \end{aligned} \quad (10)$$

The first term is due to elastic scattering and is zero here, as $\langle a \rangle_{ss} = 0$, and Re denotes the real part. The second term is the incoherent, or inelastic spectrum, and is due to two-photon scattering events. For an optical system driven at ω by a

field of strength E , the coherent spectrum is usually proportional to the driving intensity E^2 and the incoherent spectrum is proportional to the square of the intensity or E^4 . Here, however, the external classical driving field produces pairs of photons via the $\chi^{(2)}$ nonlinearity of the intracavity crystal. Thus there are no single-photon scattering events and no coherent scattering spike, and the height of the incoherent spectrum depends linearly on $I_{\text{in}}(2\omega_0) \propto F^2$.

By the quantum regression theorem we have

$$\begin{aligned} \langle a^\dagger(0) a(\tau) \rangle &= \text{tr}\{a(0) A(\tau)\} \\ &= \sum_{i,n} \sqrt{n+1} \langle i, n+1 | A(\tau) | i, n \rangle, \end{aligned} \quad (11)$$

where $A(0) = \rho_{SS} a^\dagger$ and $\dot{A} = \mathcal{L}A$. The resulting equations can be written in the form

$$\frac{d\vec{A}}{dt} = \vec{M}\vec{A} \quad (12)$$

with

$$\vec{A} = \begin{pmatrix} A_{0,-;0,+} \\ A_{0,-;1,-} \\ A_{1,-;0,-} \\ A_{1,+;0,+} \\ A_{2,-;1,-} \\ A_{0,+;0,-} \\ A_{1,+;1,-} \\ A_{2,-;0,+} \end{pmatrix} \quad (13)$$

with the notation $A_{n,\pm;m,\pm} \equiv \langle n, \pm | A | m, \pm \rangle$ and initial conditions

$$A_{n,\pm;m,\pm}(0) = \langle n, \pm | a^\dagger \rho_{SS} | m, \pm \rangle = \sqrt{n} \langle n-1, \pm | \rho_{SS} | m, \pm \rangle. \quad (14)$$

The matrix M is given as

$$M = \begin{pmatrix} -\gamma/2 & -g & 0 & 0 & 0 & 0 & 0 & 0 \\ g & -\kappa & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}F & -\kappa & \gamma & 2\sqrt{2}\kappa & g & 0 & 0 \\ 0 & 0 & 0 & -(\gamma+\kappa) & 0 & 0 & -g & -\sqrt{2}g \\ 0 & 0 & 0 & 0 & -3\kappa & 0 & \sqrt{2}g & g \\ 0 & 0 & -g & 0 & 0 & -\gamma/2 & 2\kappa & 0 \\ 0 & 0 & 0 & g & -\sqrt{2}g & 0 & -(\gamma/2+2\kappa) & 0 \\ \sqrt{2}F & 0 & 0 & \sqrt{2}g & -g & 0 & 0 & -(\gamma/2+2\kappa) \end{pmatrix}. \quad (15)$$

After taking the Fourier transform of the above equations, we have

$$\vec{\tilde{A}}(\omega) = \{\vec{M} - i\omega\vec{I}\}^{-1} \vec{A}(0) \quad (16)$$

with $\vec{\tilde{A}}(\omega)$ composed of the Fourier transform of $\vec{A}(\tau)$ and then we can easily form the spectrum

$$I_{\text{tr}}(\omega) = \sum_{i,n} \sqrt{n+1} \langle i, n+1 | \text{Re} \vec{\tilde{A}}(\omega) | i, n \rangle. \quad (17)$$

We will also be interested in the spectrum of squeezing, defined as

$$\begin{aligned} S(\omega, \theta) &= \int_{-\infty}^{\infty} d\tau \cos \omega \tau \text{Re}[\langle \Delta a^\dagger(\tau) \Delta a(0) \rangle \\ &+ e^{2i\theta} \langle \Delta a^\dagger(\tau) \Delta a^\dagger(0) \rangle]. \end{aligned} \quad (18)$$

Adding two spectra of squeezing, with phase angles θ and $\theta + \pi/2$, we obtain the following relationship between the incoherent spectrum and the spectrum of squeezing [22]:

$$I_{\text{inc}}(\omega) \propto [S(\omega, \theta) + S(\omega, \theta + \pi/2)]. \quad (19)$$

For fields whose fluctuations can be described by a classical stochastic process, both $S(\omega, \theta)$ and $S(\omega, \theta + \pi/2)$ must be positive. As noted above, for a squeezed quantum field, one of these spectra is negative over some range of frequencies, for appropriate choice of the phase θ . To calculate the second term in Eq. (18) we must use the quantum regression theorem for

$$\begin{aligned} \langle \Delta a^\dagger(0) \Delta a^\dagger(\tau) \rangle &= \text{tr}\{a^\dagger(0) B(\tau)\} \\ &= \sum_{i,n} \sqrt{n} \langle i, n-1 | B(\tau) | i, n \rangle, \end{aligned} \quad (20)$$

where $B(0) = \rho_{SS} a^\dagger$ and $\dot{B} = \mathcal{L}B$ and the nonzero elements of interest are

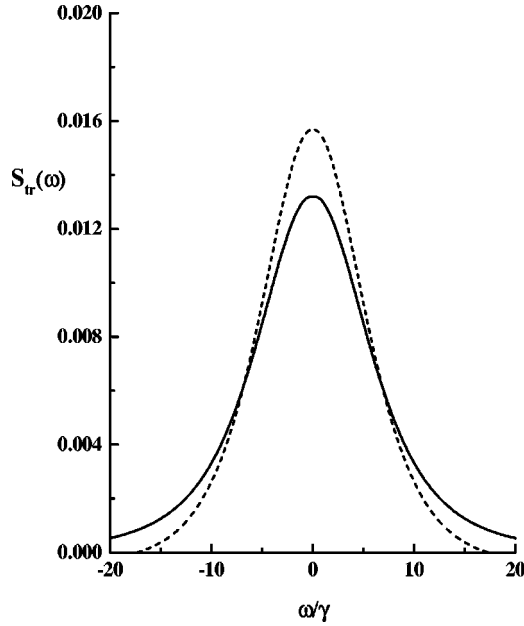


FIG. 2. Spectrum of the transmitted light for $\kappa/\gamma=10.0$ and $g/\gamma=0.1$. The dotted line is the spectrum of squeezing for the in-phase quadrature.

$$\vec{B} = \begin{pmatrix} B_{0,-;0,+} \\ B_{0,-;1,-} \\ B_{1,+;0,+} \\ B_{2,-;1,-} \\ B_{1,+;1,-} \\ B_{2,-;0,+} \\ B_{0,+;0,-} \\ B_{1,-;0,-} \end{pmatrix}. \quad (21)$$

The relevant equations are

$$\frac{d\vec{B}}{dt} = \vec{M}\vec{B}. \quad (22)$$

The second term dominates, and is proportional to F . The spectrum of squeezing that is plotted is $S(\omega, 0)$. The spectrum of squeezing for the quadrature $\pi/2$ out of phase, $S(\omega, \pi/2)$, is equal and opposite in sign to order F . In the system under consideration here we find that

$$S(\omega, \theta) = -S(\omega, \theta + \pi/2). \quad (23)$$

To first order in F , these two contributions to the incoherent spectrum cancel, but they differ in terms of order F^2 . This means that the incoherent spectrum is formed by the subtraction of two quantities in the presence of squeezing.

We now turn to results for the incoherent spectrum. In Figs. 2–4, we plot the incoherent spectrum for $\kappa/\gamma=10.0$ and various values of atom-field coupling g . In all the figures, the solid line is the incoherent spectrum and the dotted line is the spectrum of squeezing for the quadrature in phase

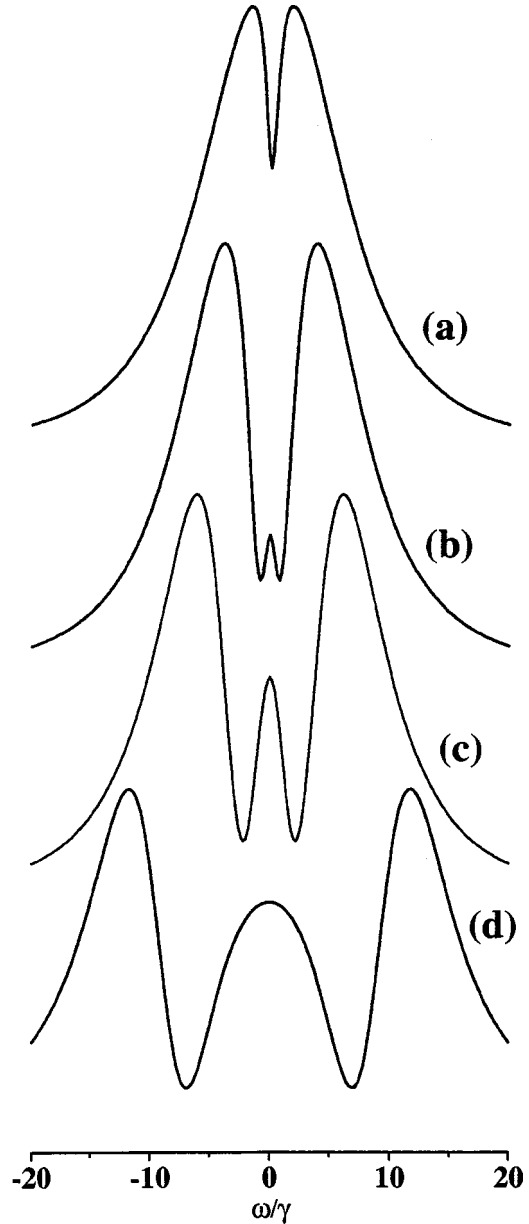


FIG. 3. Spectrum of the transmitted light for $\kappa/\gamma=10.0$ and (a) $g/\gamma=0.3$, (b) $g/\gamma=3.0$, (c) $g/\gamma=5.0$, (d) $g/\gamma=10.0$. The dimensionless y axis is deleted for clarity.

with the driving field. In Fig. 2, for $g/\gamma=0.1$, the spectrum is essentially the square of a Lorentzian of linewidth $\kappa(1 + 2g^2/\kappa\gamma)$. The linewidth of the spectrum is approximately $0.64\kappa(1 + 2g^2/\kappa\gamma)$. In Fig. 3, we see a sequence of plots. For $g/\gamma=0.3$, a hole appears in the spectrum, which deepens with further increases in g . As g/γ is increased to 3.0, a small bump appears inside the hole. Increasing g/γ to 5.0 leads to the bump inside the hole increasing in size. As g/γ is increased to $g/\gamma=10.0$, the spectrum appears to have a double dip in it. These dips appear near $\omega = \pm g$. We note that this is not a hole due to absorption of energy emitted out the side of the cavity, as it persists in the limit $\gamma \rightarrow 0$. Not only do the holes persist for small γ , $I_{\text{inc}}(\omega=0.0)=0.0$ if $\gamma=0.0$. As one increases g/γ to 50.0, a double-peaked struc-

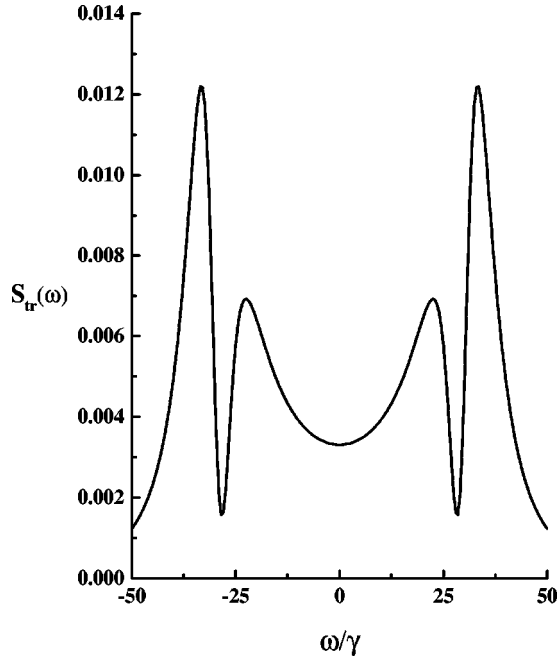


FIG. 4. Spectrum of the transmitted light for $\kappa/\gamma=10.0$ and $g/\gamma=50.0$.

ture appears as in vacuum-Rabi splitting, as shown in Fig. 4. Each vacuum-Rabi peak has a hole in it, however. These holes are deepened if we decrease γ relative to κ and g . Again this points out that these holes are not just due to fluorescence out of the side of the cavity, as the depth of the holes are maximized for $\gamma=0.0$. In the good-cavity limit, $\kappa/\gamma \ll 1$, we find for small g/γ a subnatural width single-peaked spectrum (Fig. 5), which evolves into a vacuum-Rabi doublet with no holes for large g/γ (Fig. 6). Hence in the good-cavity limit, the anomalous effects disappear.

IV. OPTICAL SPECTRUM OF THE FLUORESCENT LIGHT

In this section, we consider the incoherent part of the fluorescent spectrum, given by

$$\begin{aligned}
 I_{\text{fl}}(\omega) &= \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle \sigma_+(0) \sigma_-(\tau) \rangle \\
 &= 2 \operatorname{Re} \int_0^{\infty} d\tau e^{i\omega\tau} \langle \sigma_+(0) \sigma_-(\tau) \rangle \\
 &= 2\pi \langle \sigma_+ \rangle_{ss} \langle \sigma_- \rangle_{ss} \delta(\omega) \\
 &\quad + 2 \operatorname{Re} \int_0^{\infty} d\tau e^{i\omega\tau} \langle \Delta \sigma_+(0) \Delta \sigma_-(\tau) \rangle. \quad (24)
 \end{aligned}$$

Again here there is no coherent or elastic scattering leading to a δ -function component of the spectrum at resonance. By the quantum regression theorem we have

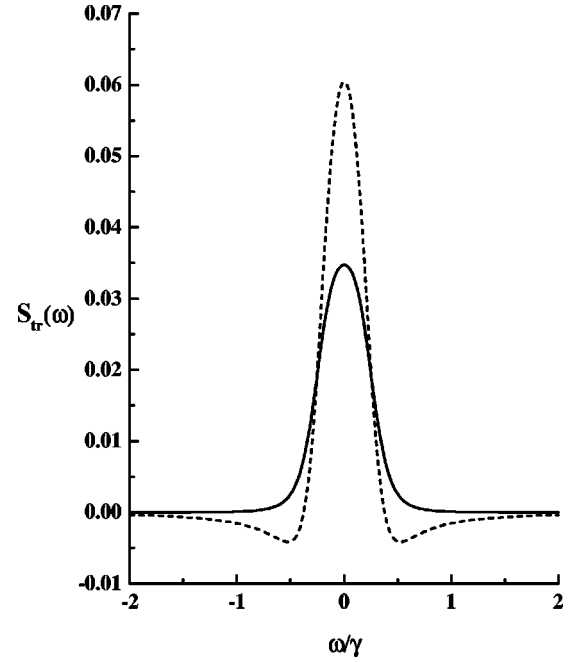


FIG. 5. Spectrum of the transmitted light for $\kappa/\gamma=0.1$ and $g/\gamma=0.1$. The dotted line is the spectrum of squeezing for the in-phase quadrature.

$$\langle \sigma_+ \sigma_-(\tau) \rangle = \operatorname{tr} \{ \sigma_-(0) C(\tau) \} = \sum_n \langle +, n | C(\tau) | -, n \rangle, \quad (25)$$

where $C(0) = \rho_{SS} \sigma_+$ and $\dot{C} = \mathcal{L}A$. Hence we can write the incoherent spectrum as

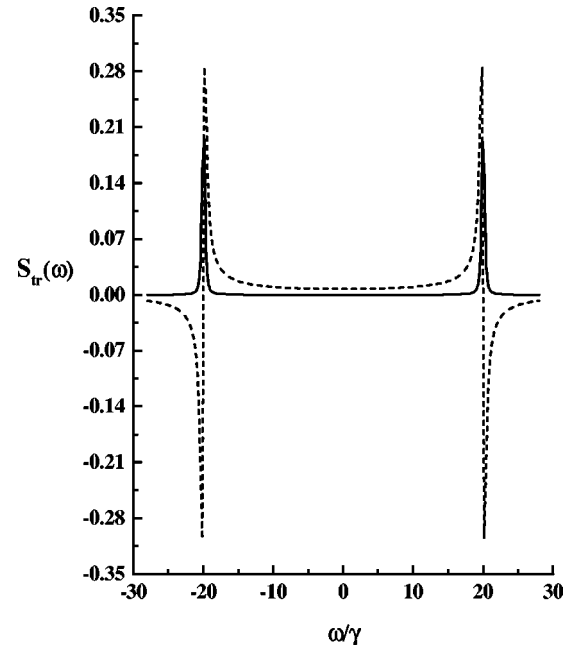


FIG. 6. Spectrum of the transmitted light for $\kappa/\gamma=0.1$ and $g/\gamma=20.0$. The dotted line is the spectrum of squeezing for the in-phase quadrature.

$$I_{\text{fl}}(\omega) = \sum_{i,n} \sqrt{n+1} \langle i, n+1 | \text{Re} \tilde{A}(\omega) | i, n \rangle. \quad (26)$$

The resulting equations can be written in the form

$$\frac{d\vec{C}}{dt} = \vec{M}\vec{C} \quad (27)$$

with

$$\vec{C} = \begin{pmatrix} C_{0,-;0,+} \\ C_{0,-;1,-} \\ C_{0,+;0,-} \\ C_{1,+;1,-} \\ C_{1,-;0,-} \\ C_{1,+;0,+} \\ C_{2,-;1,-} \\ C_{2,-;0,+} \end{pmatrix} \quad (28)$$

with the notation $C_{n,\pm;m,\pm} \equiv \langle n, \pm | C | m, \pm \rangle$ and initial conditions

$$C_{n,-;m,\pm}(0) = \langle n, \pm | \sigma_+ \rho_{ss} | m, \pm \rangle = \langle n, + | \rho_{ss} | m, \pm \rangle, \quad (29)$$

$$C_{n,+;m,\pm} = 0. \quad (30)$$

After taking the Fourier transform of the above equations, we have

$$\vec{C}(\omega) = \{\vec{M} - i\omega\vec{I}\}^{-1} \vec{C}(0) \quad (31)$$

with $\vec{C}(\omega)$ composed of the Fourier transform of $\vec{C}(\tau)$ and then we can easily form the fluorescent spectrum

$$S(\omega) = \sum_n \langle +, n | \text{Re} \vec{C}(\omega) | -, n \rangle. \quad (32)$$

As before, we will be interested in the spectrum of squeezing of the fluorescent light,

$$S_{\text{fl}}(\omega, \theta) = \int_{-\infty}^{\infty} d\tau \cos \omega \tau \text{Re}[\langle \Delta \sigma_+(\tau) \Delta \sigma_-(0) \rangle + e^{2i\theta} \langle \Delta \sigma_+(\tau) \Delta \sigma_+(0) \rangle]. \quad (33)$$

To calculate the second term in the above equation, we must use the quantum regression theorem for

$$\begin{aligned} \langle \Delta \sigma_+(0) \Delta \sigma_+(\tau) \rangle &= \text{tr}\{\sigma_+(0) D(\tau)\} \\ &= \sum_n \langle +, n | D(\tau) | -, n \rangle, \end{aligned} \quad (34)$$

where $D(0) = \rho_{ss} \sigma_+$ and $\dot{D} = \mathcal{L}D$ and the nonzero elements of interest are

$$\vec{D} = \begin{pmatrix} D_{0,-;0,+} \\ D_{0,-;1,-} \\ D_{1,+;0,+} \\ D_{2,-;1,-} \\ D_{1,+;1,-} \\ D_{2,-;0,+} \\ D_{0,+;0,-} \\ D_{1,-;0,-} \end{pmatrix}. \quad (35)$$

The relevant equations are

$$\frac{d\vec{D}}{dt} = \vec{M}\vec{D}. \quad (36)$$

In the case of the fluorescent light, in the bad-cavity limit ($\kappa/\gamma \gg 1$), we have a single-peaked structure with no holes for $g/\gamma \ll 1$, as in Fig. 7(a). Keeping $\kappa/\gamma \gg 1$ and with $g/\gamma \gg 1$, we have a vacuum-Rabi doublet with no holes as in Fig. 7(b). So for the fluorescent light there are no anomalous effects in the spectra in the bad-cavity limit. In Fig. 7(c), we let $\kappa/\gamma \ll 1$, and there are no holes for $g/\gamma \ll 1$. For $\kappa/\gamma \ll 1$, and with $g/\gamma \gg 1$, we see a vacuum-Rabi doublet with holes in Fig. 7(d). The holes are deepened as one goes further into the good-cavity limit, and $I_{\text{inc}}(\omega=0.0) = 0.0$ for $\kappa=0.0$. So it is in the good-cavity limit that we see anomalous effects in the spectra for the fluorescent light from this system. Recall that it was the bad-cavity limit that admitted anomalous behavior for the transmitted light.

V. PHYSICAL INTERPRETATION

So far we have seen incoherent spectra with a subnatural linewidth and also ones with spectral holes. Similar types of spectra have been predicted for a single two-level atom in a microcavity driven by a weak external field resonant with the atom and the cavity [22]. That is essentially the system we consider in the present paper, but without the $\chi^{(2)}$ crystal, and driven at ω_0 and not $2\omega_0$. In that case the spectrum of squeezing was proportional to E^2 , where E is the strength of the driving field at ω_0 and the incoherent spectrum is proportional to E^4 . In particular, in the bad-cavity limit of the previous system, where $\kappa \gg g, \gamma$, it was found there that the incoherent spectrum of the transmitted light was a Lorentzian squared. The Lorentzian had a linewidth of $\delta = \gamma(1 + 2g^2/\kappa\gamma)$, which is the cavity-enhanced spontaneous-emission rate. As the spectrum is the square of that Lorentzian, the linewidth is about $\Delta\omega \approx 0.64\delta$. This result is also obtained for the incoherent spectrum of a driven two-level atom in free space, i.e., resonance fluorescence. There, for weak driving fields $\delta = \gamma$ and a subnatural linewidth results from the squared Lorentzian, as first noted by Mollow [23]. For that same system in the strong-coupling limit, for $g \gg \kappa, \gamma$ a vacuum-Rabi doublet was found, each peak being a squared Lorentzian with $\delta = (2\kappa + \gamma)/2$. In the good-cavity limit of that driven atom-cavity system, $\kappa \ll g \ll \gamma$, a single-peaked structure with a hole appeared as the incoherent spec-

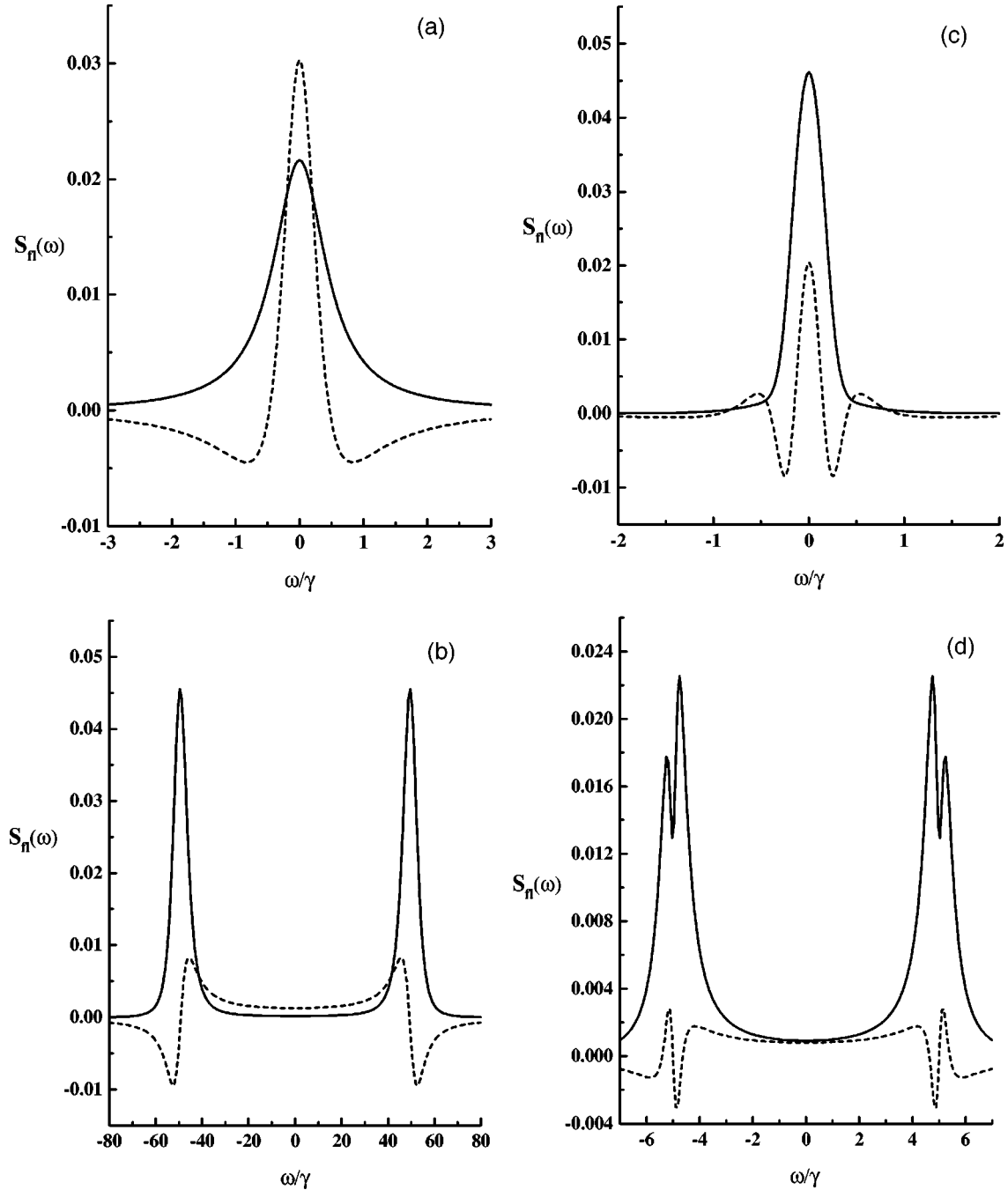


FIG. 7. (a) Spectrum of the fluorescent light for $\kappa/\gamma=10.0$ and $g/\gamma=3.0$. The dotted line is the spectrum of squeezing (scaled down by a factor of 10) for the in-phase quadrature. (b) Spectrum of the fluorescent light for $\kappa/\gamma=10.0$ and $g/\gamma=50.0$. The dotted line is the spectrum of squeezing (scaled down by a factor of 10) for the in-phase quadrature. (c) Spectrum of the fluorescent light for $\kappa/\gamma=0.1$ and $g/\gamma=0.3$. The dotted line is the spectrum of squeezing (scaled down by a factor of 10) for the in-phase quadrature. (d) Spectrum of the fluorescent light for $\kappa/\gamma=0.1$ and $g/\gamma=10.0$. The dotted line is the spectrum of squeezing (scaled down by a factor of 10) for the in-phase quadrature.

trum. Again the depth of that hole reached zero as $\gamma \rightarrow 0$ and so does not represent a loss of photons at line center due to absorption and emission out the side. These phenomena were referred to as squeezing-induced linewidth narrowing (SILN) and squeezing-induced spectral holes (SISH). Recall that the incoherent spectrum is the sum of two squeezing spectra $\pi/2$ out of phase with one another. In the case of resonance fluorescence, the two spectra of squeezing were both single-

peaked functions, and were equal and opposite to order E^2 . Keeping terms to order E^4 , we found there that the two spectra of squeezing were both Lorentzians, but one was negative (indicating squeezing) and the other positive. Hence the Lorentzian squared was formed from the subtraction of two Lorentzians, one with a linewidth of $\gamma/2$ from which is subtracted one with a larger width. Spectral holes were shown in [22] to arise in a similar manner, when the incoherent spec-

trum is the subtraction of two Lorentzian-like structures which are equal at line center but differ in the wings. These holes are only nonclassical if the two squeezing spectra are single-peaked structures, as discussed in [22].

As the subtraction results from one of the spectra of squeezing being negative, indicating fluctuations in that quadrature below the vacuum noise level, it was inferred that these narrowings and holes result from the fact that the light emitted in the incoherent spectrum is squeezed. At the time, theoretical investigations had shown that shining squeezed light on an optical system could reduce the effective linewidth of spontaneous emission from that system by altering the vacuum fluctuations to which the unstable excited state is coupled. So it was proposed that the narrowings/holes seen in the incoherent spectra resulted from the fact that the radiation reaction force on the optical system was squeezed instead of the vacuum fluctuations. The amount of squeezing is vanishingly small in the weak-field limit, and in retrospect it seems odd that a vanishingly small amount of squeezing could result in a 33% reduction in linewidth. Further, the effects of spectral holes and narrowings go away as the driving field strength is increased and the amount of squeezing increases. Another example of this is the optical parametric oscillator (OPO). The output spectrum of that device is a Lorentzian squared for weak pumping fields, with $\delta = \kappa$. The OPO produces a vanishingly small amount of squeezing in that limit. It is a good source of squeezed light at higher pump fields, with large amounts of squeezing produced just below the oscillation threshold. But the linewidth is not narrow in that instance. From Figs. 2–4, we see that in the system under consideration here the physics is probably more complicated. The spectra of squeezing are complex structures that do not yield themselves to the type of interpretation suggested in [22].

We now consider another possible mechanism for holes and narrowing to appear in incoherent spectra. Recall that the incoherent spectrum results from a nonlinear scattering process involving two or more photons. The effect is most evident for weak driving fields, where two photons are emitted from the $\chi^{(2)}$ crystal and interact with the atom-cavity system. After several cavity and/or spontaneous-emission lifetimes, the interaction is completed by the emission of two photons. This can happen via emission of two photons into the cavity mode, one into the cavity mode and one out the side of the cavity, or both out the side of the cavity. In the weak-field limit $F \ll g$, κ , and γ , the next pair of photons from the nonlinear crystal arises long after the previous two-photon scattering process is completed. The two emitted photons are highly correlated, as their frequencies must satisfy energy conservation $\omega_1 + \omega_2 = 2\omega_0$, which requires that the two photons be emitted at frequencies $\omega_0 \pm \delta\omega$. The emitted photons momenta must similarly satisfy conservation of momentum as $\vec{k}_1 + \vec{k}_2 = 2\vec{k}_0$. Single-photon scattering events lead to the δ -function component of the spectrum, the elastic, or coherent scattering. There is no contribution to that in our system, but there may be in other nonlinear optical systems. So the thought occurs that perhaps the root cause of the anomalous effects (holes and narrowings) is due to quantum interference between various indistinguishable

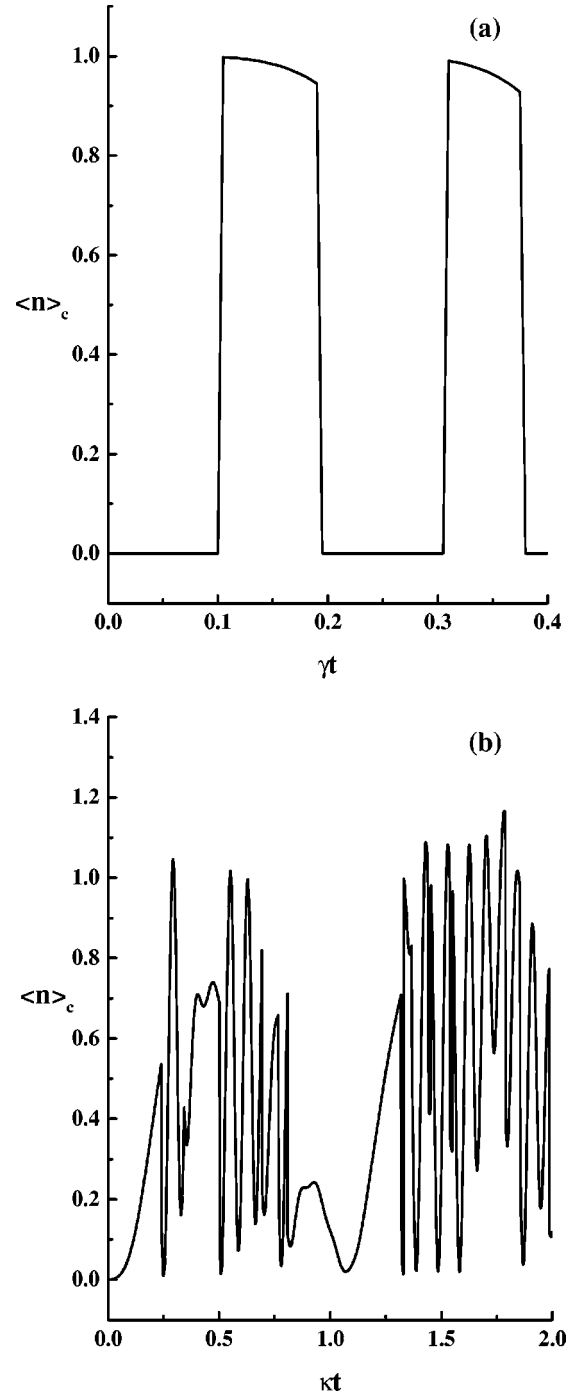


FIG. 8. (a) Conditioned mean intracavity photon number for $g/\gamma = 1.0$, $\kappa/\gamma = 10.0$, and $F/\gamma = 0.1$. (b) Conditioned mean intracavity photon number for $g/\gamma = 40.0$, $\kappa/\gamma = 10.0$, and $F/\gamma = 1.0$.

emission pathways, akin to similar effects in absorption (e.g., electromagnetically induced transparency) and spontaneous emission from a given initial unstable state.

In the case of resonance fluorescence, it has recently been shown [24] that the Lorentzian squared results from quantum interference as the probability to obtain a photon in mode k can be written (in the weak-field limit) as

$$|c_k|^2 = \left| \sum_{k'} c_{b1k1k'} + \sum_{k'} c_{b1k'1k} \right|^2. \quad (37)$$

The first term c_{b1k} is the probability amplitude for scattering one photon into mode k , the coherent scattering. This piece gives rise to a δ function, the so-called coherent spike, to the spectrum. The other two terms are two-photon scattering terms, where a pair of photons has been scattered, one into mode k and one into mode k' . There are two ways for the two-photon scattering to happen. The k photon may come before or after the k' photon, as represented by $c_{b1k1k'}$ and $c_{b1k'1k}$. If these two probability amplitudes are nonzero over an overlapping range of k' modes, then there is a cross term that results in Eq. (37). We propose that this mechanism is also responsible for the anomalous spectra observed in the case of the two-level atom inside a weakly driven optical parametric oscillator, and indeed for all weakly driven nonlinear optical systems.

It is instructive to look at quantum trajectories for this system. In this case we describe the system by a conditioned wave function, a non-Hermitian Hamiltonian, and associated collapse processes. These are given by

$$|\psi_c(t)\rangle = \sum_{n=0}^{\infty} \alpha_{g,n}(t) e^{-iE_{g,n}t} |g,n\rangle + \alpha_{e,n}(t) e^{-iE_{e,n}t} |e,n\rangle, \quad (38)$$

$$H_D = -i\kappa a^\dagger a + -i\gamma/2\sigma_+ + i\hbar F(a^{\dagger 2} - a^2) + i\hbar g(a^\dagger \sigma_- - a\sigma_+), \quad (39)$$

where we also have collapse operators $C_{\text{cav}} = \sqrt{\kappa}a$ and $C_{\text{spont. em.}} = \sqrt{\gamma/2}\sigma_-$.

In Fig. 8(a), we plot $\langle \psi_{\text{cond}} | a^\dagger a | \psi_{\text{cond}} \rangle$ as a function of time, in a case where $\gamma \ll \kappa$. The system is in steady state, and then a photon emission occurs out the front of the cavity. The conditioned photon number rises to unity [25]. This is because we know that photons are created in pairs in this system, and detection of one outside the cavity means that one must remain. We know that the first photon detected is at $\omega_0 + \delta\omega$ and that the photon that remains inside the cavity is of frequency $\omega_0 - \delta\omega$. However, we are unsure what the value of $\delta\omega$ is. In particular, is $\delta\omega$ greater than or less than zero? In other words, is the first emission event the photon that falls to the right or left of the resonant frequency in the incoherent spectra? It is this indistinguishability that leads to the spectra we present. We should expect different results for the fluorescent spectrum in this case. When $\gamma \ll \kappa$, it is most likely that the two photons will exit the system through the cavity mirror. Occasionally, one leaves via the cavity mirror and one is emitted out the side of the cavity. Even more rare in this limit is two photons scattered out the side of the cavity. There is no narrowing or hole in the fluorescent spectrum. This is because there is no quantum interference in this case. The photon detected in fluorescence is most probably associated with another photon emitted out the cavity mirror. These photons are distinguishable in the sense that we know

which direction they have been emitted into, and hence no interference. We see something similar in the limit where $\gamma \gg \kappa$, where we see anomalous spectra in fluorescence (where pairs of photons are most likely emitted) and not in transmission (where a transmitted photon is most likely paired with a fluorescent photon). This lends credence to our proposal that quantum interference is responsible for the spectral narrowing and holes.

At higher driving field strengths, there are more terms in Eq. (37), which are added and then squared to get the probability of obtaining the photon at a given k . The relative phase of the complex amplitudes is such that the size of the cross, or ‘‘interference’’ terms, becomes smaller. If, for example, we have two two-photon scattering events within a cavity lifetime, the two photons detected in the output of the cavity may or may not have been correlated before they were scattered. This type of behavior can be seen in Fig. 8(b), where we plot $\langle \psi_{\text{cond}} | a^\dagger a | \psi_{\text{cond}} \rangle$ as a function of time for larger driving fields. This will tend to reduce the size of the effect. Inasmuch as we are considering a system driven by very weakly squeezed light, if one drives an optical system with a weakly squeezed field with no coherent component, or a weakly squeezed vacuum, similar effects should be obtained. This is indeed the case as shown by the work of Swain *et al.* [17]. We then conclude that these types of anomalous spectra in weakly driven nonlinear optical systems are indeed due to the type of quantum interference in the manner of Eq. (37). We note that Hegerfeldt *et al.* [26] have seen interference effects in spectra involving squeezed light. There, however, the interference was between the squeezed vacuum field and the fluorescence.

VI. CONCLUSION

We have shown that the transmitted and fluorescent incoherent spectra of a two-level atom in a weakly driven optical parametric oscillator can exhibit spectral holes and spectral narrowing. These types of phenomena have been predicted for other nonlinear optical systems, but the previous description of why they occur has been found lacking. We propose an alternative mechanism for these effects, based on recent work on resonance fluorescence. Further work on this system and others should lead to a better understanding of such anomalous spectra, and indeed the difference between a spontaneous-emission spectrum for a system prepared in a particular unstable state and the driven type of spectra that we consider here.

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