

2013

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Khan, Ambreen Afsar; Ellahi, R.; and Usman, Muhammad, "The Effects of Variable Viscosity on the Peristaltic Flow of Non-Newtonian Fluid through a Porous Medium in an Inclined Channel with Slip Boundary Conditions" (2013). *Mathematics Faculty Publications*. 2.

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# THE EFFECTS OF VARIABLE VISCOSITY ON THE PERISTALTIC FLOW OF NON-NEWTONIAN FLUID THROUGH A POROUS MEDIUM IN AN INCLINED CHANNEL WITH SLIP BOUNDARY CONDITIONS

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Original Manuscript Submitted: 4/4/2012; Final Draft Received: 6/11/2012

The present paper investigates the peristaltic motion of an incompressible non-Newtonian fluid with variable viscosity through a porous medium in an inclined symmetric channel under the effect of the slip condition. A long wavelength approximation is used in mathematical modeling. The system of the governing nonlinear partial differential equation has been solved by using the regular perturbation method and the analytical solutions for velocity and pressure rise have been obtained in the form of stream function. In the obtained solution expressions, the long wavelength and low Reynolds number assumptions are utilized. The salient features of pumping and trapping phenomena are discussed explicitly. The flow is investigated in a wave frame of reference moving with velocity of the wave. The features of the flow characteristics are analyzed by plotting the graphs of various values of the physical parameters in detail.

**KEY WORDS:** non-Newtonian fluid, porous medium, nonlinear equations, regular perturbation, variable viscosity, slip condition

## 1. INTRODUCTION

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube, mixing and transporting the fluid in the direction of the wave propagation. This phenomenon is known as peristalsis. The importance of peristaltic flows of non-Newtonian viscous fluids (Ellahi, 2009; Hameed and Nadeem, 2007; Tan and Masuoka, 2005a, b, 2007; Mahomed and Hayat, 2007; Fetecau and Fetecau, 2005; Malik et al., 2011; Dehghan and Shakeri, 2009) has been recognized due to their application such as urine transport from the kidney to the bladder, chyme motion in the gastrointestinal tract, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, transport of spermatozoa, and swallowing food through the esophagus. These flows are extensively studied for different geometries by using various

assumptions such as large wavelength, small amplitude ratio, small wave number, small Deborah number, low Reynolds number and creeping flow, etc. Some relevant studies on the topic can be found from the list of references (Nadeem and Akbar, 2008, 2010) and several references therein. Moreover, the study of fluid flows through porous medium has gained much attention due to its practical applications; for instance, in some pathological situations, the distribution of fatty cholesterol and artery-clogging blood clots in the lumen of coronary artery, the human lung, bile duct, gall bladder with stones, and in small blood vessels can be considered as equivalent to a porous medium. Porous media are also used to transport and store energy in many industrial applications, such as heat pipe, solid matrix heat exchangers, electronic cooling, chemical reactors, beach sand, sandstone, limestone, wood, and rye bread (Vafai, 2011). Furthermore, it is well known that the no-slip condition in polymeric liquids in

## NOMENCLATURE

$A_i$	( $i = 1, 2, 3$ ) Rivlin–Ericksen tensors
$d/dt$	material derivative
$k$	permeability
$\mathbf{S}$	extra stress tensor
$\mathbf{V}$	velocity vector $d/dt$

### Greek Symbols

$\alpha_i$	$\beta_i$ material constants
$\delta$	wave number
$\lambda_i, e_i$	non-Newtonian parameters
$\mu$	dynamic viscosity
$\sigma$	porosity

the presence of high molecular weight is not appropriate at all and mostly fails in many problems like thin film flow on multiple interfaces and rarefied fluid problems. The slip condition not only plays an important role in shear skin, spurt and hysteresis effects but is also equally important in technological applications such as polishing valves of artificial heart and internal cavities (Coleman et al., 1996).

The purpose of the present investigation is to perform a study which can describe the porosity and slip effects on the non-Newtonian peristaltic flow of third order with variable viscosity in an inclined symmetric channel simultaneously. The flow analysis is performed under the assumption of a long wavelength approximation and low Reynolds number. Analytical solutions for axial velocity and pressure gradient in terms of stream function are derived for a small Deborah number by using a regular perturbation method. At the end of the paper, the graphical results against different physical parameters are also presented and discussed. The paper is organized as follows. Section 2 and Section 3 contain the basic equations and formulation of the problem, respectively. Solution of the problem is given in Section 4. Section 5 is devoted to results and discussion, and finally the conclusion is presented in Section 6.

## 2. BASIC EQUATIONS

The basic field equation governing the law of conservation of momentum for non-Newtonian fluid through a porous medium is given by

$$\rho \frac{d\bar{V}}{dt} = -grad p + div \bar{S} - \frac{\mu}{k} \bar{V}, \quad (1)$$

where  $\rho$  is the density,  $\bar{V}$  is the velocity vector,  $\bar{k}$  is the permeability of the porous medium,  $p$  is pressure, and  $d/dt$  denotes the material derivative. The fluid undergoes only isochoric motion; therefore, the law of conservation of mass is defined by

$$div \bar{V} = 0. \quad (2)$$

Due to the complexity of non-Newtonian fluids, there is no single model which describes all properties of non-Newtonian fluids; for instance, stress differences, shear thinning or shear thickening, stress relaxation, elastic effects and memory effects, etc. Among the many models, there is only a third-order model which can only explain the shear thinning and shear thickening properties at the same time even for steady and unidirectional flows. The extra stress tensor  $\bar{S}$  for non-Newtonian third-order fluid can be written as (Yildirim and Sezer, 2010)

$$\begin{aligned} \bar{S} = & \mu \bar{A}_1 + \alpha_1 \bar{A}_2 + \alpha_2 \bar{A}_1^2 + \beta_1 \bar{A}_3 + \beta_2 (\bar{A}_2 \bar{A}_1 \\ & + \bar{A}_1 \bar{A}_2) + \beta_3 (tr \bar{A}_1^2) \bar{A}_1, \end{aligned} \quad (3)$$

where  $\mu$  is the dynamic viscosity,  $\alpha_1, \alpha_2, \beta_1, \beta_2,$  and  $\beta_3$  are the material constants. The Rivlin–Ericksen tensors are defined by

$$\bar{A}_1 = (grad \bar{V}) + grad (\bar{V})^T, \quad (4)$$

$$\begin{aligned} \bar{A}_n = & \frac{d\bar{A}_{n-1}}{dt} + \bar{A}_{n-1} (grad \bar{V}) + grad (\bar{V})^T \bar{A}_{n-1}, \\ & n \geq 2. \end{aligned} \quad (5)$$

## 3. FORMULATION OF THE PROBLEM

Let us consider non-Newtonian third-order fluid with variable viscosity through a porous medium in a uni-

form channel having width  $2a$ , inclined at an angle  $\gamma$  to the horizontal, filled with an incompressible viscous fluid with variable viscosity. We assume an infinite wave train travels with velocity  $c$  along the walls. The geometry of the wall surface is mathematically described by

$$H(\bar{X}, \bar{t}) = a + b \cos \left[ \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) \right]. \quad (6)$$

Here,  $b$  is the wave amplitude,  $\lambda$  is the wave length, and  $\bar{t}$  is the time. The flow becomes steady in the wave frame  $(\bar{x}, \bar{y})$  moving with constant speed  $c$  away from the fixed frame  $(\bar{X}, \bar{Y})$ . If  $(\bar{U}, \bar{V})$  and  $(\bar{u}, \bar{v})$  are the velocity components in fixed and wave frames, respectively, then the relation between them is defined by the following transformations:

$$\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}. \quad (7)$$

We render the governing equations dimensionless by setting

$$\begin{aligned} x &= \frac{\bar{x}}{\lambda}, & y &= \frac{\bar{y}}{a}, & u &= \frac{\bar{u}}{c}, & v &= \frac{\bar{v}}{\delta c}, \\ h &= \frac{\bar{H}}{a}, & S &= \frac{a}{\mu_0 c} \bar{S}(\bar{x}), & \mu(y) &= \frac{\mu(\bar{y})}{\mu_0} \end{aligned} \quad (8)$$

To proceed further, when we nondimensionalize Eqs. (3) and (6), the extra stress tensor and flow geometry in nondimensional form can be written as

$$S = A_1 + \lambda_1 A_2 + \lambda_2 A_1^2 + \varepsilon_1 A_3 + \varepsilon_2 (A_2 A_1 + A_1 A_2) + \varepsilon_3 (tr A_1^2) A_1, \quad (9)$$

$$h(x) = 1 + \phi \cos 2\pi x. \quad (10)$$

Introducing the dimensionless stream function  $\psi(x, y)$  such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (11)$$

Invoking Eqs. (8)–(11) in the equations of momentum and continuity, the dimensionless form of governing Eqs. (1) and (2), after dropping bars for simplicity, leads to the following nondimensional equations:

$$\left. \begin{aligned} & \text{Re} \delta \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \frac{\partial \psi}{\partial y} \right] \\ & = -\frac{\partial p}{\partial x} + \delta \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \\ & - \mu(y) \sigma^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{\text{Re}}{\text{Fr}} \sin \gamma \end{aligned} \right\}, \quad (12)$$

$$\left. \begin{aligned} & \text{Re} \delta^3 \left[ \left( -\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \frac{\partial \psi}{\partial x} \right] \\ & = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y} \\ & + \delta^2 \mu(y) \sigma^2 \frac{\partial \psi}{\partial x} - \delta \frac{\text{Re}}{\text{Fr}} \cos \gamma \end{aligned} \right\}. \quad (13)$$

The following dimensionless quantities are also obtained:

$$\begin{aligned} \delta &= \frac{a}{\lambda}, & \text{Re} &= \frac{\rho c a}{\mu_0}, & \text{Fr} &= \frac{c^2}{g a}, & \sigma^2 &= \frac{a^2}{k}, \\ \lambda_1 &= \frac{\alpha_1 c}{\mu_0 a}, & \lambda_2 &= \frac{\alpha_2 c}{\mu_0 a}, & \varepsilon_1 &= \frac{\beta_1 c^2}{\mu_0 a^2}, & \\ \varepsilon_2 &= \frac{\beta_2 c^2}{\mu_0 a^2}, & \varepsilon_3 &= \frac{\beta_3 c^2}{\mu_0 a^2}, & \phi &= \frac{b}{a}. \end{aligned} \quad (14)$$

where  $\delta$  is the dimensionless wave number,  $\sigma^2$  is the porosity parameter,  $\phi$  is the amplitude ratio or occlusion,  $\text{Re}$  is the Reynolds number, and  $\lambda_i (i = 1, 2)$ , and  $\varepsilon_i (i = 1, 2, 3)$  are non-Newtonian parameters. The compatibility equation which governs the flow in terms of the stream function  $\psi(x, y)$  after eliminating the pressure gradient from Eqs. (12) and (13) is

$$\left. \begin{aligned} & \text{Re} \delta \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \nabla^2 \psi \right] \\ & = \left( \frac{\partial^2}{\partial y^2} - \delta^2 \frac{\partial^2}{\partial x^2} \right) S_{xy} + \delta \frac{\partial^2}{\partial x y} (S_{xx} + S_{yy}) \\ & - \sigma^2 \left[ \frac{\partial}{\partial y} \left( \mu(y) \left( \frac{\partial \psi}{\partial y} + 1 \right) \right) - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right] \end{aligned} \right\}. \quad (15)$$

Under the long wavelength approximation and low Reynolds number (Ebaid, 2008; Nadeem and Akbar, 2009), we obtain

$$\frac{\partial p}{\partial x} = \frac{\partial S_{xy}}{\partial y} - \mu(y) \sigma^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) + \frac{\text{Re}}{\text{Fr}} \sin \gamma, \quad (16)$$

$$\frac{\partial p}{\partial y} = 0, \quad (17)$$

$$S_{xy} = \mu(y) \frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left( \frac{\partial \psi}{\partial y} \right)^3, \quad (18)$$

in which  $\Gamma = \varepsilon_2 + \varepsilon_3$  is the Deborah number. Equation (17) indicates that  $p \neq p(y)$ . Thus, from Eq. (18) we have

$$\begin{aligned} & \frac{\partial^2}{\partial y^2} \left( \mu(y) \frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left( \frac{\partial \psi}{\partial y} \right)^3 \right) - \sigma^2 \left[ \frac{\partial}{\partial y} \right. \\ & \left. \times \left( \mu(y) \left( \frac{\partial \psi}{\partial y} + 1 \right) \right) - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right] = 0. \end{aligned} \quad (19)$$

The boundary conditions in terms of stream function  $\psi$  are defined as

$$\psi = 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \text{at } y = 0, \quad (20)$$

$$\psi = F, \quad \frac{\partial \psi}{\partial y} + \beta S_{xy} = -1 \quad \text{at } y = h(x), \quad (21)$$

where  $\beta = l/a$  is a dimensionless slip parameter and time mean  $\theta$  in the wave frame is defined as

$$\theta = F + 1. \quad (22)$$

In order to seek the effect of variable viscosity on peristaltic flow, we let

$$\mu(y) = e^{-\alpha y} \quad \alpha \ll 1, \quad (23)$$

which by Maclaurin's series can be written as

$$\mu(y) = 1 - \alpha y + O(\alpha^2). \quad (24)$$

The dimensionless pressure rise  $\Delta p$  is defined by

$$\Delta p = \int_0^1 \frac{dp}{dx} dx. \quad (25)$$

In order to solve the present problem, we expand the flow quantities in a power series of the small Deborah number  $\Gamma$  and small viscosity parameter  $\alpha$  as follows:

$$\left. \begin{aligned} \psi &= \psi_0 + \Gamma \psi_1 + \dots \\ F &= F_0 + \Gamma F_1 + \dots \\ p &= p_0 + \Gamma p_1 + \dots \end{aligned} \right\}, \quad (26)$$

where

$$\left. \begin{aligned} \psi_0 &= \psi_{00} + \alpha \psi_{01} + \dots \\ \psi_1 &= \psi_{10} + \alpha \psi_{11} + \dots \\ F_0 &= F_{00} + \alpha F_{01} + \dots \\ F_1 &= F_{10} + \alpha F_{11} + \dots \\ p_0 &= p_{00} + \alpha p_{01} + \dots \\ p_1 &= p_{10} + \alpha p_{11} + \dots \end{aligned} \right\}. \quad (27)$$

When we substitute Eq. (26) into Eqs. (19)–(25) and separate the terms of differential order in  $\Gamma$  and  $\alpha$ , we obtain the following systems of partial differential equations for the stream function and pressure gradients together with boundary conditions:

Case I

$$\frac{\partial^4 \psi_{00}}{\partial y^4} - \sigma^2 \frac{\partial^2 \psi_{00}}{\partial y^2} = 0, \quad (28)$$

$$\frac{dp_{00}}{dx} = \frac{\partial^3 \psi_{00}}{\partial y^3} - \sigma^2 \left( \frac{\partial \psi_{00}}{\partial y} + 1 \right) + \frac{\text{Re}}{\text{Fr}} \sin \gamma, \quad (29)$$

along with the boundary conditions

$$\psi_{00} = 0 = \frac{\partial^2 \psi_{00}}{\partial y^2} \quad \text{at } y = 0, \quad (30)$$

$$\psi_{00} = F_{00}, \quad \frac{\partial \psi_{00}}{\partial y} + \beta \frac{\partial^2 \psi_{00}}{\partial y^2} = -1 \quad \text{on } y = h. \quad (31)$$

Case II

$$\begin{aligned} \frac{\partial^4 \psi_{01}}{\partial y^4} - \sigma^2 \frac{\partial^2 \psi_{01}}{\partial y^2} &= \frac{\partial^2}{\partial y^2} \left( y \frac{\partial^2 \psi_{00}}{\partial y^2} \right) \\ &- \sigma^2 \frac{\partial}{\partial y} \left( y \frac{\partial^2 \psi_{00}}{\partial y^2} \right), \end{aligned} \quad (32)$$

$$\begin{aligned} \frac{dp_{01}}{dx} &= \frac{\partial^3 \psi_{01}}{\partial y^3} - \sigma^2 \frac{\partial \psi_{01}}{\partial y} + \sigma^2 y \frac{\partial \psi_{00}}{\partial y} \\ &- \frac{\partial}{\partial y} \left( y \frac{\partial^2 \psi_{00}}{\partial y^2} \right), \end{aligned} \quad (33)$$

with the corresponding boundary conditions

$$\psi_{01} = 0 = \frac{\partial^2 \psi_{01}}{\partial y^2} \quad \text{at } y = 0, \quad (34)$$

$$\begin{aligned} \psi_{01} = F_{01}, \quad \frac{\partial \psi_{01}}{\partial y} + \beta \frac{\partial^2 \psi_{01}}{\partial y^2} &= \beta y \frac{\partial^2 \psi_{00}}{\partial y^2} \\ \text{at } y = h. \end{aligned} \quad (35)$$

Case III

$$\frac{\partial^4 \psi_{10}}{\partial y^4} - \sigma^2 \frac{\partial^2 \psi_{10}}{\partial y^2} = -6 \frac{\partial}{\partial y} \left[ \left( \frac{\partial^2 \psi_{00}}{\partial y^2} \right)^2 \frac{\partial^3 \psi_{00}}{\partial y^3} \right], \quad (36)$$

$$\frac{dp_{10}}{dx} = \frac{\partial^3 \psi_{10}}{\partial y^3} - \sigma^2 \frac{\partial \psi_{10}}{\partial y} + 6 \left( \frac{\partial^2 \psi_{00}}{\partial y^2} \right)^2 \frac{\partial^3 \psi_{00}}{\partial y^3}, \quad (37)$$

subject to the boundary conditions

$$\psi_{10} = 0 = \frac{\partial^2 \psi_{10}}{\partial y^2} \quad \text{at } y = 0, \quad (38)$$

$$\begin{aligned} \psi_{10} = F_{10}, \quad \frac{\partial \psi_{10}}{\partial y} + \beta \frac{\partial^2 \psi_{10}}{\partial y^2} + 2\beta \left( \frac{\partial^2 \psi_{00}}{\partial y^2} \right)^3 &= 0 \\ \text{at } y = h, \end{aligned} \quad (39)$$

and so forth.

#### 4. SOLUTION OF THE PROBLEM

Solving the above sets of equations by regular perturbation method, we get

$$\begin{aligned} \psi &= \frac{(F\sigma y \cosh[\sigma h] + y(1 + F\sigma^2\beta)) \times \sinh[\sigma h] - (F+h)\sinh[\sigma h]}{h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h]} \\ &+ \alpha [y(p_1 m_1 - p_2 m_2) \\ &+ \frac{(h \sinh[\sigma y] - y \sinh[\sigma h]) [p_1 m_3 + p_2 m_4]}{h(h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h])} \\ &+ p_1 \left( \frac{2\sigma \cosh[\sigma y] - 2 - \sigma y \sinh[\sigma y]}{2\sigma^3} \right) \\ &+ p_2 \left( \frac{y^2 \sigma^2 - 2 \cosh[\sigma y] + 2}{2\sigma^4} \right)] \\ &+ \Gamma \left[ y p_1^3 m_5 + \frac{(h \sinh[\sigma y] - y \sinh[\sigma h]) p_1^3 m_6}{\left( \frac{h(h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h])}{16\sigma^2} \right)} \right] \\ u &= \frac{(F\sigma \cosh[\sigma h] + (1 + F\sigma^2\beta)\sinh[\sigma h]) - (F+h)\sigma \cosh[\sigma y]}{h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h]} \\ &+ \alpha p_1 m_1 - p_2 m_2 \\ &+ \frac{(\sigma h \cosh[\sigma y] - \sinh[\sigma h]) [p_1 m_3 + p_2 m_4]}{h(h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h])} \\ &+ p_1 \left( \frac{2 \sinh[\sigma y] - \sigma y \cosh[\sigma y] - \sinh[\sigma y]}{2\sigma^2} \right) \\ &+ p_2 \left( \frac{2y\sigma - 2 \sinh[\sigma y]}{2\sigma^3} \right) \\ &+ \Gamma p_1^3 m_5 + \frac{\Gamma(h\sigma \cosh[\sigma y] - \sinh[\sigma h]) p_1^3 m_6}{h(h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h])} \\ &+ \Gamma p_1^3 \left( \frac{3 \cosh[3\sigma y] - 12y\sigma \sinh[\sigma y] - 12\sigma \sinh[\sigma y]}{16\sigma} \right) \end{aligned} \tag{40}$$

With the same contrast as given in the case of stream function, the expression of  $p$  for the following equation can be

easily obtained through the routine calculation:

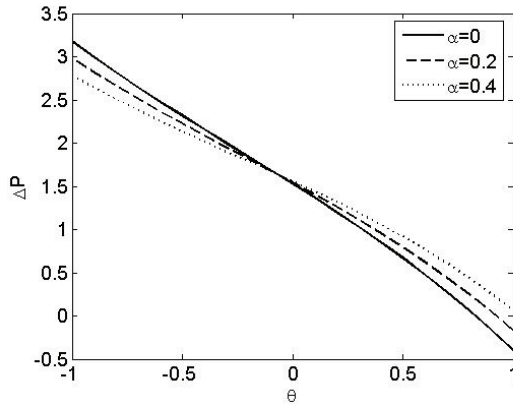
$$\begin{aligned} \frac{dp}{dx} &= \frac{-(F+h)\sigma^3(\cosh[\sigma h] + \beta\sigma \sinh[\sigma h])}{h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h]} \\ &+ \frac{\text{Re}}{\text{Fr}} \sin\gamma + \alpha \left[ \frac{\sigma^2(-p_1 m_1 + p_2 m_2)}{h} \right. \\ &+ \left. \frac{\sigma^2 \sinh[\sigma h] [p_1 m_3 + p_2 m_4]}{h(h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h])} \right] \\ &+ \Gamma \left[ -\frac{p_1^3 \sigma^2 m_5}{h} - \frac{\sigma^2 \sinh[\sigma h] p_1^3 m_6}{h} \right. \\ &\left. \times (h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h]) \right] \end{aligned} \tag{42}$$

where

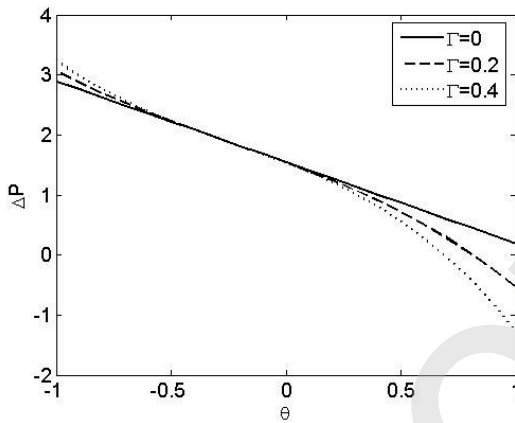
$$\begin{aligned} p_1 &= \frac{(F+h)\sigma^2}{h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h]}, \\ p_2 &= \frac{F\sigma \cosh[\sigma h] + (1 + F\sigma^2\beta)\sinh[\sigma h]}{h\sigma \cosh[\sigma h] + (-1 + h\sigma^2\beta)\sinh[\sigma h]}, \\ m_1 &= \frac{y(2 - 2 \cosh[\sigma h] + \sigma h \sinh[\sigma h])}{2h\sigma^3}, \\ m_2 &= \frac{y\sigma^2(2 - 2 \cosh[\sigma h] + h^2)}{2h\sigma^4}, \\ m_3 &= \frac{(h^2\sigma^2 \cosh[\sigma h] - 2 + 2 \cosh[\sigma h]) - (2h\sigma \sinh[\sigma h] - \beta h^2\sigma^3 \sinh[\sigma h])}{2\sigma^3}, \\ m_4 &= \frac{(2 - 2\beta h\sigma^2 - h^2\sigma^2 - 2 \cosh[\sigma h]) + (2h\sigma \sinh[\sigma h] + 2\beta h\sigma \cosh[\sigma h])}{2\sigma^4}, \\ m_5 &= \frac{12h\sigma \cosh[\sigma h] - \sinh[3\sigma h]}{16\sigma^2}, \\ m_6 &= \frac{(3h\sigma \cosh[3\sigma h] + (17\beta h\sigma^2 - 1) \times \sinh[3\sigma h] - (12h^2\sigma^2 + 48\beta h\sigma^2) \times \sinh[\sigma h] - 12\beta h^2\sigma^3 \cosh[\sigma h])}{16\sigma^2} \end{aligned} \tag{43}$$

#### 5. CONCLUSION

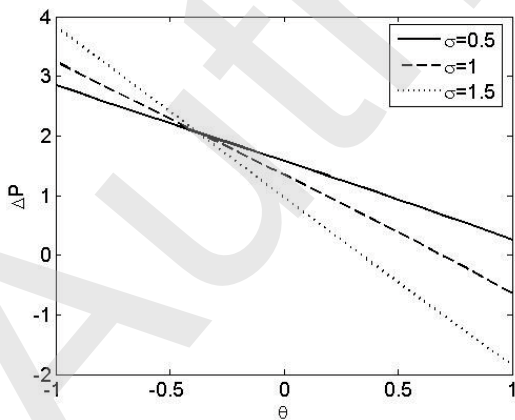
In order to see the variation of parameters the graphical results are included in this section. Figure 1 is the graph of pressure rise  $\Delta P$  versus flow rate  $\theta$  for the case of  $\alpha$ . It is found that  $\theta$  decreases with the increasing value of  $\alpha$  in the pumping region, while it increases with the increasing  $\alpha$  in both free pumping and copumping regions. The variation of pressure rises  $\Delta P$  with  $\theta$  for different values of  $\Gamma$  and  $\sigma$  as depicted in Figs. 2 and 3. It is anticipated



**FIG. 1:** Pressure rise versus flow rate for  $\Gamma = 0.1$ ,  $m = 0.8$ ,  $\sigma = 0.6$ ,  $\phi = 0.3$ ,  $\gamma = 0.5$ ,  $Fr = 0.2$ , and  $\beta = 0.1$ .

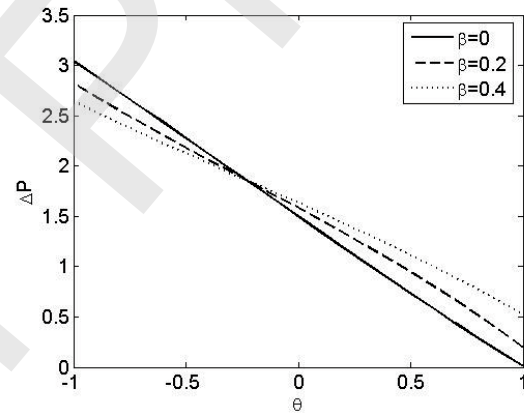


**FIG. 2:** Pressure rise versus flow rate for  $\alpha = 0.2$ ,  $m = 0.8$ ,  $\phi = 0.3$ ,  $\gamma = 0.5$ ,  $Fr = 0.2$ , and  $\beta = 0.1$ .

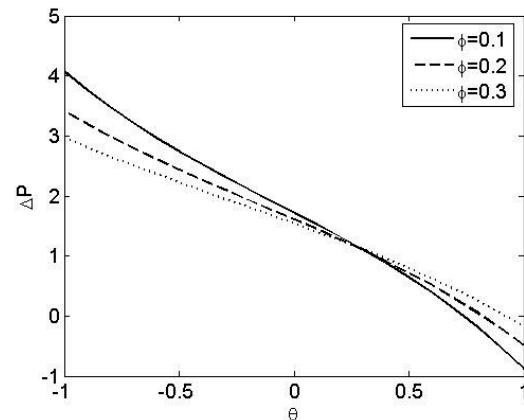


**FIG. 3:** Pressure rise versus flow rate for  $\alpha = 0.2$ ,  $m = 0.8$ ,  $\Gamma = 0.05$ ,  $\phi = 0.3$ ,  $\gamma = 0.5$ ,  $Fr = 0.2$ , and  $\beta = 0.1$ .

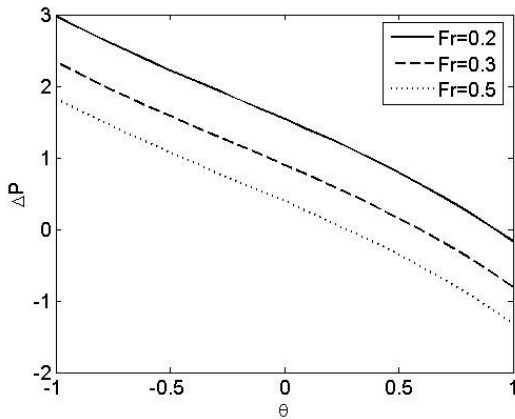
that  $\theta$  decreases with increasing  $\Gamma$  and  $\sigma$  in the pumping region, while it increases with increasing  $\Gamma$  and  $\sigma$  in both free pumping and copumping regions. Figures 4 and 5 are plotted for pressure rise  $\Delta P$  against flow rate for  $\beta$  and  $\phi$ . It is observed that  $\theta$  decreases with increasing  $\beta$  and  $\phi$  in the pumping and free pumping regions, while it increases with increasing  $\beta$  and  $\phi$  in the copumping region. Figures 6 and 7 represent the graph of the pressure versus  $\theta$ . Figure 6 shows that with an increase in  $Fr$  the pumping rate decreases in all the regions. It is observed from Fig. 7 that the pressure rise increases in all the regions with an increase in the inclination angle  $\gamma$ . The longitudinal velocity  $u$  versus  $\gamma$  has been plotted in Figs. 8–10. Figures 8 and 9 show that the velocity at the center of the channel and near the wall represents the opposite be-



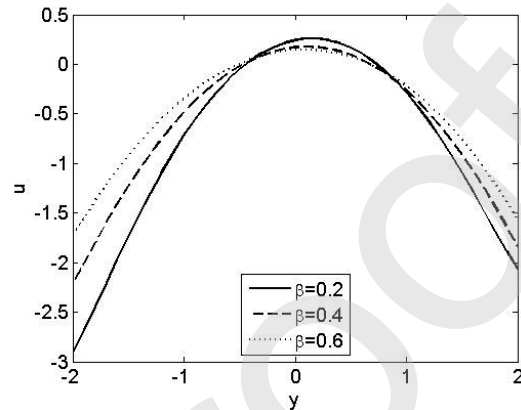
**FIG. 4:** Pressure rise versus flow rate for  $\alpha = 0.2$ ,  $m = 0.8$ ,  $\sigma = 0.6$ ,  $\phi = 0.3$ ,  $\gamma = 0.5$ ,  $Fr = 0.2$ , and  $\Gamma = 0.05$ .



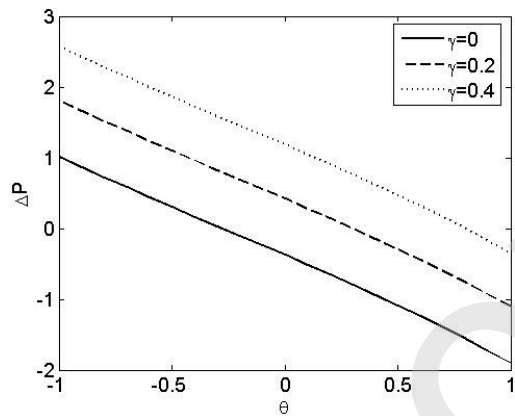
**FIG. 5:** Pressure rise versus flow rate for  $\alpha = 0.2$ ,  $m = 0.8$ ,  $\sigma = 0.6$ ,  $\Gamma = 0.1$ ,  $\gamma = 0.5$ ,  $Fr = 0.2$ , and  $\beta = 0.1$ .



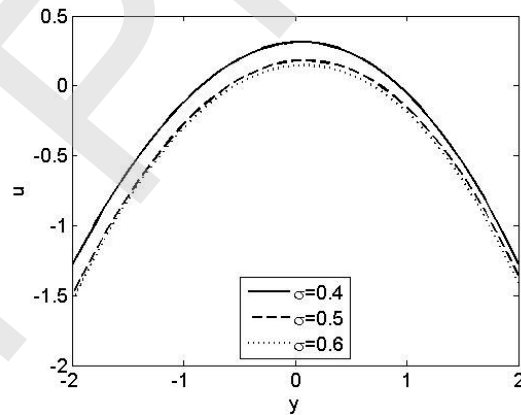
**FIG. 6:** Pressure rise versus flow rate for  $\alpha = 0.2$ ,  $m = 0.8$ ,  $\sigma = 0.6$ ,  $\phi = 0.3$ ,  $\gamma = 0.5$ ,  $\Gamma = 0.1$ , and  $\beta = 0.1$ .



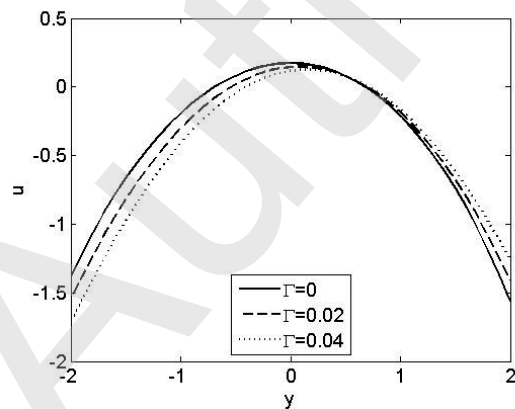
**FIG. 9:** Effect of  $\beta$  on velocity  $u$  when  $\alpha = 0.04$ ,  $\sigma = 0.6$ ,  $x = 1$ ,  $\phi = 0.3$ ,  $\theta = 1$ , and  $\Gamma = 0.02$ .



**FIG. 7:** Pressure rise versus flow rate for  $\alpha = 0.2$ ,  $m = 0.8$ ,  $\sigma = 0.6$ ,  $\phi = 0.3$ ,  $\Gamma = 0.05$ ,  $Fr = 0.2$ , and  $\beta = 0.1$ .



**FIG. 10:** Effect of  $\sigma$  on velocity  $u$  when  $\alpha = 0.04$ ,  $\beta = 0.7$ ,  $x = 1$ ,  $\phi = 0.3$ ,  $\theta = 1$ , and  $\Gamma = 0.02$ .



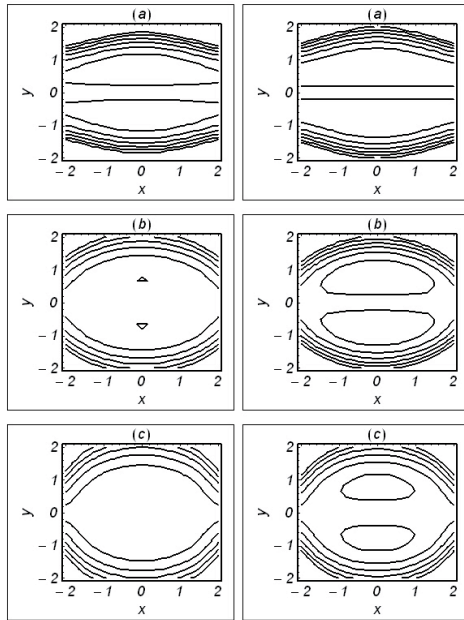
**FIG. 8:** Effect of  $\Gamma$  on velocity  $u$  when  $\alpha = 0.04$ ,  $\sigma = 0.6$ ,  $x = 1$ ,  $\phi = 0.3$ ,  $\theta = 1$ , and  $\beta = 0.7$ .

havior. Further, the velocity at the center of the channel decreases with an increase in  $\beta$  and  $\Gamma$ . It is also observed from Fig. 10 that the velocity profile decreases with an increase in  $\sigma$ . Figures 11–13 examine the trapping phenomenon. It is observed that the size of the trapped bolus increases with an increase in  $\beta$ ,  $\Gamma$ , and  $\sigma$ .

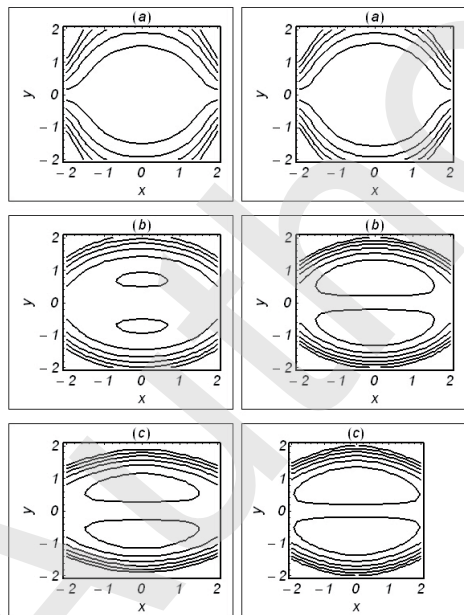
## 6. CONCLUDING REMARKS

The peristaltic flow of non-Newtonian fluid of third order in a porous medium with the slip condition has been studied under the assumptions of long wavelength and low Reynolds number. The governing partial differential equation subjected to their boundary condition has been solved by using the regular perturbation method for small

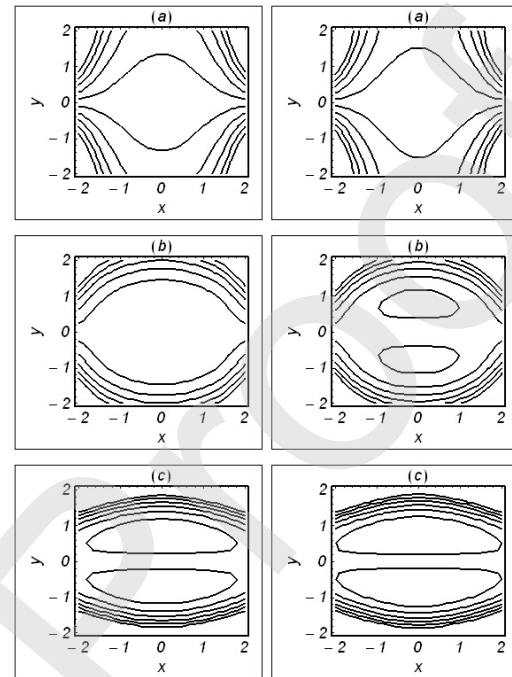




**FIG. 11:** Stream lines for different values of  $\Gamma$ : **(a)**  $\Gamma = 0$ ; **(b)**  $\Gamma = 0.02$ ; and **(c)**  $\Gamma = 0.04$  when  $\beta = 0.7$ ,  $\phi = 0.3$ ,  $\theta = 1$ , and  $\sigma = 0.6$  are fixed. Left panels are for  $\alpha = 0$  and right panels are for  $\alpha = 0.04$ .



**FIG. 12:** Stream lines for different values of  $\beta$ : **(a)**  $\beta = 0.5$ ; **(b)**  $\beta = 1$ ; and **(c)**  $\beta = 1.5$ , when  $\sigma = 0.6$ ,  $\phi = 0.3$ ,  $\theta = 1$ , and  $\Gamma = 0.02$  are fixed. Left panels are for  $\alpha = 0$  and right panels are for  $\alpha = 0.04$ .



**FIG. 13:** Stream lines for different values of  $\sigma$ : **(a)**  $\sigma = 0.4$ ; **(b)**  $\sigma = 0.6$ ; and **(c)**  $\sigma = 0.8$ , when  $\beta = 0.7$ ,  $\phi = 0.3$ ,  $\theta = 1$ , and  $\Gamma = 0.02$  are fixed. Left panels are for  $\alpha = 0$  and right panels are for  $\alpha = 0$  and  $\alpha = 0.04$ .

Deborah number. The expressions for velocity and pressure rise are first obtained in term of stream function; then the analysis has been discussed and presented graphically for the pressure rise and longitudinal velocity. It is noted that an increase in the slip parameter decreases the peristaltic pumping region, whereas the size of the trapped bolus decreases by increasing the slip parameter. Moreover, the trapped bolus increases by increasing the porosity parameter and Deborah number. The results obtained in this paper not only reveal many interesting behaviors for further study on the non-Newtonian nanofluid phenomena but will also be a great source to provide a benchmark in numerical investigations in the future.

## ACKNOWLEDGMENTS

R. Ellahi thanks the Ministry of Science and Technology Pakistan (PCST) for honoring him with the Productive Scientist Award for the year 2011. R. Ellahi is also grateful to the Higher Education Commission Pakistan for awarding him with NRPU.

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