A DESCRIPTIVE ANALYSIS OF TEACHERS' OPINIONS REGARDING STRATEGIES USED TO TEACH BASIC ADDITION AND SUBTRACTION FACTS

MASTER'S THESIS

Submitted to the School of Education University of Dayton, in Partial Fulfillment of the Requirements for the Degree Master of Science in Education

by

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V

## DEDICATION

For my parents on their fiftieth wedding anniversary

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#### CHAPTER I

#### INTRODUCTION

#### Purpose for the Study

It is late September. A few elementary teachers are gathered in the school lounge. Some are sipping coffee; others are grading papers. The overall mood is somber. The initial excitement of the new school year has begun to fade.

The third grade teacher stops grading momentarily and states, "How can I teach complex calculations when my students still don't know the basic addition and subtraction facts? Many still use their fingers or make cryptic marks on their paper just to add 4+8. How are they also going to learn all of the multiplication and division facts this year?"

The fifth grade teacher nods in agreement as he responds, "Some of my students are still counting and have calculators hidden in their pockets. In spite of this, the district wants the emphasis at every grade level to be problem solving. How do they solve complex problems if they have not memorized the basic facts?"

The new second grade teacher has been listening intently and attempts to enlighten them, "Oh, the key to teaching the basic facts is understanding, not memorization. My students use counting, manipulatives, and thinking strategies to learn them. Anyway, problem solving is what's important, not computing!"

This scenario illustrates a common lament of elementary

teachers that students have not mastered the basic addition and subtraction facts. It also highlights an underlying debate about how the basic facts are mastered and their relative importance within a mathematics program. This debate is fueled by conflicting theories regarding how students learn as well as a changing emphasis in mathermatics instruction from computation to problem solving (National Council of Teachers of Mathematics, 1989).

There are two contrasting theories which give rise to the different views of how mathematics is learned. One is the absorption theory which is based upon the belief that knowledge is a collection of facts which are learned through memorization. Knowledge is taken from outside the person and impressed upon the mind. According to this theory, children learn by imitating the skills of adults, and what is learned are associations between otherwise unrelated stimuli. These are strengthened by drill or repetition. Within this framework, mathematics is a collection of facts and skills; the basic facts are viewed as isolated pieces of information to be memorized through repetition and learning is passive (Baroody, 1985, 1987).

In contrast, the cognitive theory of learning views knowledge as a relationship of information which the learner joins together from within, in active, meaningful, and organized ways. Within this theoretical construct, mathematics is an interrelated system of processes, ideas,

and principles (Baroody, 1987). According to Brownell (1935) children learn by making these conceptual relations through understanding. Thus, the basic facts are learned initially on a concrete level and eventually on an abstract level with relatedness and understanding being key components (cited in Baroody, 1985).

The National Council of Teachers of Mathematics (NCTM, 1989) clearly espouses to the cognitive theory of learning as evidenced by its delineation of standards for mathematics. The NCTM envisions students to be active participants in the learning process, immersed in finding and making connections of mathematics principles and processes, and engrossed in solving problems using reasoning ability. Further, there is a major shift in emphasis within its curriculum standards from computation to problem solving. The basic addition and subtraction facts are to be learned within this context.

This writer learned the basic addition and subtraction facts primarily through rote memorization. Learning these facts was the major emphasis of mathematics instruction during the primary grades and was viewed as the foundation necessary for more abstract mathematical problem solving. Admittedly, this writer retains some allegiance to this approach but is attempting to teach these basic facts in closer alignment with the current, concept based standards. This writer perceives an inherent tension, created within

this shift from one theoretical approach to another, which gives rise to several questions: How do students learn and master the basic addition and subtraction facts? What role do concrete objects, counting, thinking strategies, and drill play in mastering these facts? Which strategies are successful in helping students recall these fact with automaticity? Is achieving automaticity necessary? How can students solve problems until the basic facts are mastered?

As a result of this theoretical debate as well as the changing emphasis of the curriculum standards, this writer believes there is a broad range of opinions among teachers with regard to the answers to these questions. This descriptive study analyzed the opinions of elementary teachers regarding the various strategies used to teach the basic addition and subtraction facts. The analysis of the results may provide practical considerations regarding the effectiveness of various approaches.

#### Problem Statement

The purpose of this study was to analyze the opinions of elementary teachers regarding strategies used to teach the basic addition and subtraction facts.

#### Assumptions

To conduct this study a Likert-type survey (Best & Kahn, 1989) was used to gather and analyze the opinions of kindergarten through sixth grade teachers toward the instruction of the basic addition and subtraction facts. The writer assumed that the instrument was valid in that it measured the opinions it was intended to measure (Fuchs, 1980). The writer also assumed that the teachers selected to complete this instrument answered it in a way which reflected their personal experience in teaching the basic addition and subtraction facts.

#### Limitations

This study may have several limitations. One limitation may be the sample size of the kindergarten through sixth grade teachers surveyed. Another limitation may be that all of the teachers surveyed are from a limited geographic area within the state of Ohio.

Basic Addition Facts refer to the 100 addition combinations with single digit addends (0+0 to 9+9).

Basic Subtraction Facts refer to the corresponding subtraction facts which are inverses of the basic addition computation facts (0-0 to 18-9).

Thinking Strategies are methods of finding meaningful cognitive relationships based upon mathematical principles, stored rules, or procedures (Baroody, 1985).

<u>Rote Memorization</u> refers to storing information in associative memory through repetition.

<u>Automaticity</u> refers to a state of mastery in which information can be retrieved from memory instantaneously.

<u>Primary Teachers</u> are teachers of kindergarten, first, second, or third grade students.

Intermediate Teachers are teachers of fourth, fifth, or sixth grade students.

#### CHAPTER II

### REVIEW OF THE RELATED LITERATURE

In this chapter, the review of the literature is presented. The chapter is divided into the following sections: the theories of how the basic addition and subtraction facts are mastered, the current curriculum standards regarding these facts, the reasons for mastering the basic addition and subtraction facts, the strategies used to achieve mastery, and a chapter summary.

Theories of How the Basic Addition and Subtraction Facts

#### are Mastered

According to Baroody (1985, 1987), there are two contrasting theories of learning which are responsible for the opposing viewpoints of how students master the basic addition and subtraction facts. These viewpoints differ in terms of how the basic addition and subtraction facts are stored in memory, the role of the student in the learning process, the roles of understanding and drill in mastering these facts, and the expected time required to master these facts.

One viewpoint of how the basic addition and subtraction facts are mastered is based upon the absorption theory (Baroody, 1987). Within this theory of learning, Ashcraft (1985) explains that the basic number facts are learned by strengthening the association between each problem and answer. According to Brownell (1935), these associations are formed mainly through repetition making drill, rather than understanding, the focus of instruction (cited in Baroody, 1985). Baroody (1987) interprets this theory to imply that students should master the basic addition and subtraction facts within the first few years of school with adequate practice.

Baroody (1987) contends that within this theory knowledge is viewed as an accumulation of isolated facts which the learner receives in a passive way through repetition. Knowledge expands as memorization increases the amount of facts stored in memory. "Furthermore, basic facts or habits can be linked together to form more complex facts or habits" (Baroody, 1987, p. 8). Baroody indicates that the absorption model implies that the basic number facts are stored in associative memory as separate, unrelated associations with drill being the primary method of learning them.

Another viewpoint of how the basic addition and subtraction facts are learned is based upon the cognitive theory (Baroody, 1987). Within this theory of learning, Olander (1931) and Carpenter (1985) indicate that children learn the basic number facts "as a system of interrelated experiences" (cited in Baroody, 1987, p. 179). Students begin understanding these number combinations through counting and other informal strategies (Baroody, 1987). Students actively construct relationships among these number facts based upon rules, principles, procedures and arbitrary associations. Mastering the basic addition and subtraction facts is viewed as intimately connected with understanding mathematical principles and relationships (Baroody, 1885). This is viewed as a gradual process wherein some relationships are more difficult to grasp

than others. Therefore, mastering the basic number facts is considered to be a lengthy and intricate process (Baroody, 1987).

Cognitive theorists view knowledge as the insightful connecting of information into "an organized and meaningful whole" (Baroody, 1987, p. 9). Knowledge expands through finding new internal connections and assimilating new information (Baroody, 1987). Baroody further states that the memory stores these relationships which efficiently summarize huge amounts of information, such as the basic number facts. Drill is an important component of basic addition and subtraction fact instruction to help make the rules, and thinking strategies automatic but only after students have learned the mathematical relationships (Baroody, 1987).

In the above section, the writer discussed the theories of how the basic addition and subtraction facts are learned. In the next section, the current curriculum standards relating to these basic number facts are presented

#### Current Curriculum Standards

Regarding the Basic Addition and Subtraction Facts The National Council of Teachers of Mathematics (NCTM, 1989), in its delineation of curriculum standards, clearly espouses to the cognitive theory of learning in most key aspects. The NCTM views the learning process as an active, constructive endeavor which should be reflected in the way mathematics is taught. The NCTM believes that the mathematics curriculum should foster "mathematical insight, reasoning, and problem-solving" (NCTM, 1989, p. 15), rather than rote activities. Students are to be active

participants in discovering mathematical relationships and principles, assimilating "new information and constructing their own meanings" (NCTM, 1989, p. 10). The NCTM acknowledges the current standards reflect a shift in curriculum emphasis:

Traditional teaching emphases on practice in manipulating expressions and practicing algorithms as a precursor to solving problems ignore the fact that knowledge often emerges from the problems. This suggests that instead of the expectation that skill in computation should precede word problems, experience with problems helps develop the ability to compute (NCTM, 1989, p. 9).

The NCTM (1980, 1989) spends a great amount of verbage on deemphasizing computational endeavors within the curriculum in favor of reasoning and experiential problem solving activities. It is the NCTM's view that the basic addition and subtraction facts are learned for the purpose of solving problems and that the basic facts should be learned within problem solving contexts. The NCTM is emphatic that "there should be decreased emphasis on such activities as isolated drill with numbers apart from problem contexts" (NCTM, 1980, p. 7). These basic number facts are to be taught by "helping children develop thinking strategies" which "enable them to understand relationships and to reason mathematically" (NCTM, 1989, p. 44). Calculators and computers are also promoted as useful tools to learn the basic number facts as well as to avoid time consuming lessons on complex calculations in the upper grades (NCTM, 1980, 1989).

There is one aspect of the NCTM's position on teaching the basic number facts which contrasts with Baroody's (1987) assertion that the conceptual approach takes a great deal of time. The NCTM (1989) posits that "strong evidence suggests that conceptual approaches to computation instruction result in good achievement, good retention, and a reduction on the amount of time children need to master computational skills (NCTM, 1989, p. 44). The curriculum standards set forth by the NCTM reflect this belief in that only two of the thirteen curriculum standards for grades kindergarten through fourth address computational skills.

In the above section, the writer discussed the curriculum standards for teaching the basic addition and subtraction facts. In the following section, the reasons for mastering these basic number facts is presented.

#### Reasons for Mastering

the Basic Addition and Subtraction Facts

One reason for mastering the basic addition and subtraction facts may be to calculate more complex algorithms with speed and accuracy. Research shows that many experts agree. Resnick (1983), for instance, indicates that using cumbersome strategies such as counting to compute the basic number facts interferes with higher level computation success such as multiple digit addition and subtraction (cited in Hasselbring, Goin, & Bransford, 1987). Hasselbring et al. state that as the basic number facts reach a level of automaticity students have more cognitive processing capacity left to execute higher level skills. Further, studies

by Suydam and Reys (1978), and Suydam and Dessart (1980) indicate that a "readily accessible knowledge base of the basic math facts" is "a critical component of successful computation" (cited in Goldman, Mertz, & Pellegrino, 1989, p. 481). According to Van Parreren (1978) and Anderson (1982), the reason why knowing the basic number facts fluently is critical for success in more complex calculations is due to cognitive theories which maintain there is a limited capacity in the cognitive processing system (cited in Goldman, et al., 1989). Therefore, the more aspects of a calculation requiring laborious, conscious attention, the more overloaded the processing system becomes. This increases the probability of errors.

Kirby and Becker (1988) conducted a study for the purpose of determining which components contributed to errors in computing complex calculations. A sampling of forty-eight fifth grade students was selected from general education classes. Three groups were formed: a group with arithmetic problems, a group with reading problems, and a control group. A fifty-five problem test was given which consisted of calculations appropriate for this grade level. The responses were analyzed. The study found evidence that learning problems in arithmetic are related to slow executions of operations. Kirby and Becker hypothesize this is due to these students not knowing the basic number facts to the level of automaticity. They cite Biggs and Collis (1982), Case (1985), and Torgeson (1986) who have found that "slow execution of simple tasks would overload working memory and reduce the likelihood that relevant information

was active when it was needed" (cited in Kirby & Becker, 1988, p. 14).

A similar study was conducted by Brumfield and Moore (1985) but with different results. This study was prompted by complaints by third through sixth grade teachers that students were having problems with addition and subtraction algorithms because they did not know their basic facts. Fourth grade students from Title I classis were selected. A forty item test was given consisting of a range of addition and subtraction problems from basic facts to three digit calculations with regrouping. The errors were analyzed and grouped into three categories: basic fact errors, random errors, and procedural errors. In this study random and procedural errors composed the bulk of errors, with only two students making errors relating to basic facts. This writer questions whether the procedural and random errors may have been a result of undue focus on the basic fact component of the complex algorithms resulting in processing overload as previously This possibility may have been overlooked in the postulated. analysis of the results.

Another reason for mastering the basic addition and subtraction facts may be to increase problem solving competency. There are many experts that agree. Suydam and Reys (1978), and Suydam and Dessart (1980) indicate that "a readily accessible knowledge base of basic math facts" is a "critical component of successful problem solving" (cited in Goldman, Mertz & Pelligrino, 1989, p. 481). Further, Hamann and Ashcraft (in press) presented documentation

that a network of stored basic facts and a "body of procedural knowledge" "interact in typical arithmetic problem solving situations" (cited in Ashcraft, 1985, p. 100). Similarly, Cawley, Miller, and School (1987), Fleischner, Nuzum, and Marzola (1987), and Kameenui and Simmons (1990) indicate that problem solving has three major components which include the need for a mathematical knowledge base, application of knowledge in unfamiliar situations, and using analytical skills (cited in Mercer & Miller, 1992). It seems reasonable to infer that the basic addition and subtraction facts are considered an integral part of a knowledge base.

In contrast, other experts do not agree that successful problem solving is dependent upon mastery of the basic number facts. Some experts believe that mastery of the basic number facts is dependent upon successful problem solving. The NCTM (1980, 1989) advises that problem solving should be the driving force in all mathematics instruction. It is not seen as a separate skill but that catalyst which should fuel all mathematical curriculum including teaching the basic number facts. The NCTM envisions students learning the basic addition and subtraction facts by developing thinking strategies, and finding relationships which are grasped within problem solving contexts. Baroody (1987) is in agreement with the NCTM in this matter. Further, Court (1920), Carpenter, Hiebert, and Moser (1981), and Carpenter and Moser (1982, 1983, 1984) found that children can use informal, devised strategies to solve simple word problems prior to receiving formal arithmetic instruction (cited in Baroody, 1987).

These contrasting views seem to be intrinsically linked to varying perceptions of the definition of problem solving. Mercer (1992) reports that after inspecting ten books and several articles, thirty seven different descriptions of problem solving were found with no actual definitions (cited in Mercer & Miller, 1992). However, it can be inferred from the literature that some experts view problem solving as a skill which is dependent upon mastery of the basic number facts; other experts view mastering the basic number facts a result of problem solving

A third reason for mastering the basic addition and subtraction facts may be to construct a framework of arithmetic knowledge. Lazerick (1981) states that implicit within teaching these facts in a systematic, related way is the structure and order of the number system. Baroody (1987) contends that general arithmetic knowledge and basic number fact knowledge are dependent upon each other, with growth in one area prompting growth in the other. Students learn these basic facts by finding the intrinsic relationships and principles underlying these facts such as commutativity, the meaning of 0, the base 10 structure, and other general arithmetic knowledge.

In the above section, the writer examined the reasons for mastering the basic addition and subtraction facts. In the following section, the writer presents strategies to achieve mastery of these basic number facts.

Strategies Used to Achieve Masterry

of the Basic Addition and Subtraction Facts One strategy used to achieve mastery of the basic addition and subtraction facts is to proceed from concrete to semi-concrete methods, and finally to abstract methods (Isenberg & Altizer-Tuning, 1984). Rathmell (1978) indicates that concrete materials aid in concept development. Baroody (1987) agrees that children use concrete objects including their own fingers to figure out sums. Groven and Resnick (1977) observe that children, in time, "spontaneously abandon concrete procedures and invent mental counting procedures for computing sums" (cited in Baroody, 1987, p. 134). Baroody indicates students invent short cuts to compute the basic number facts, eventually using abstract mental procedures based upon underlying principles and relationships. Lazerick (1981) adds that students need to use concrete objects to internalize the concept of conservation of quantity (e.g., two sets of objects when joined together contain the sum of both sets). She states that a child should not begin to memorize the basic number facts until he/she has had sufficient time "to explore the number families using concrete objects" (Lazerick, 1981, p.21). Further, Roberts (1968) suggests that many computational errors may be the result of students attempting to learn the basic number facts through rote memorization without finding meaningful content in them through prior concrete experiences.

Developing thinking strategies regarding underlying

connections and principles is another strategy used to achieve mastery of the basic addition and subtraction facts (Baroody, 1985, 1987). Thiele (1938) and Swenson (1949) found evidence that teaching thinking strategies helped students learn and retain the basic addition facts (cited in Thorton, 1978). Further, Thorton (1978) conducted an experiment with second and fourth grade pupils from two elementary schools to determine the effects of teaching thinking strategies on learning and retaining the basic number facts. The results suggest that teaching thinking strategies facilitates the learning of the basic number facts.

Several thinking strategies are referenced in the literature that may be useful in learning the basic addition and subtraction facts. The thinking strategies include commutativity, adding and subracting 0 or 1, relating number patterns to addition and subraction facts, relating the "doubles" to the symmetry within the environment, relating the "doubles" to the "near doubles", adding or subtracting 9 by regrouping to 10, redistributing to make one addend a 10, and relating subtraction as the inverse of addition.

Commutativity (e.g., 4+7=7+4) is a thinking strategy that helps a child relate approximately half of the basic addition facts to the remaining half excluding, of course, the "doubles" such as 7+7 (Lazerick, 1981; Suydam, 1984). Lazerick (1981) suggests having students use concrete objects to begin to grasp this connection.

Another thinking strategy involves adding 0 to a number or subtracting 0 from a number resulting in the number remaining the same (Lazerick 1981; Suydam, 1984; Greene, 1985). Lazerick (1981) suggests demonstrating this concept by adding nothing to a group of objects; similarly, take nothing away from a group of objects. Lazerick claims students quickly make the connection.

Counting forward by one or counting backward by one is a thinking strategy for the addition facts with one as an addend and the subtraction facts with one as a subtrahend (Lazerick, 1981; Suydam, 1984; Greene, 1985). Greene (1985) states that initially students can effectively use their fingers to understand this strategy. He also suggests using a number line to help students understand this concept.

Relating number patterns such as counting by two's to the basic addition facts with two as an addend and to the basic subtraction facts with two as a subtrahend is an effective thinking strategy (Lazerick, 1981). Lazerick advises that students practice counting by two's beginning with one (e.g.,1,3,5,...) and also beginning with 2 (e.g., 2,4,6...). She suggests using a number line and cubes to aid in understanding this concept. With this strategy students learn, for example, that 6+2 can be readily found by knowing that 8 follows 6 when counting by two's. Similarly, counting backwards by two is a strategy that can be used when two is the subtrahend.

The "doubles" (e.g., 6+6, 7+7) are basic addition facts

easily learned by children (Lazerick, 1981; Baroody, 1987). Baroody believes this may be due to readily available examples within the environment of symmetrical objects. He observes that students develop thinking strategies for the "doubles" by connecting 5+5, for example, to five fingers on each hand or 1+1 to one eye on each side of the face. He also suggests using dice to help reinforce some of the larger "doubles".

Lazerick (1981), Suydam (1984), Greene (1985), and Feinberg (1990) discuss thinking strategies which involve the many addition basic fact combinations in close proximity to the "doubles". One strategy involves those combinations where one addend is one more than the other addend. They can be seen as a "double" plus one or a "double" minus one (e.g., 7+8 is the same as 7+7+1 or 14+1; 7+8 is also the same as 8+8-1 or Feinberg (1990) suggests displaying the "doubles" and 16-1). "near doubles" on a large chart so students can see the relationship. Another strategy related to the "doubles" involves those combinations where one addend is two more than the other addend (e.g., 5+7). Students can find the answer by doubling the number that would fall between the two addends. For instance, 5+7 is the same as 6+6. Greene (1985) recommends using a number line to demonstrate this strategy.

Other thinking strategies involve nine as an addend or as a subtrahend (Lazerick, 1981; Greene, 1985; Feinberg, 1990). One strategy involves nine as an addend. Students can mentally regroup an addition problem with nine as an addend so that the

nine addend becomes a ten (e.g., 9+7=10+6=16). Regrouping in this way makes the sum readily apparent. Lazerick (1981) recommends that students initially use tile squares to understand redistributing the addends. Another strategy involves nine as a subtrahend. Students can learn to add one to the ones column of the minuend to find the answer (e.g., 16-9=16-10+1=6+1=7). Greene (1985) recommends using the base ten blocks to demonstrate this regrouping. Further, Feinberg (1990) suggests having the students analyze the "nine" addition facts as well as the "nine" subtraction facts to discover, for themselves, the patterns that emerge between the problems and solutions.

Thompson and Van de Walle (1984) suggest another thinking strategy which involves addends close to ten with sums more than ten. Again, regrouping so that one addend becomes ten is proposed. For example, 8+5 is regrouped to 10+3 in order for the sum of thirteen to be readily apparent. Thompson and Van de Walle suggest using counters until students become familiar with the regrouping of addends.

Teaching that subtraction is the inverse of addition is an important thinking strategy for learning subtraction (Baroody, 1987; Greene, 1987). Greene points out that many texts treat addition and subtraction as seperate skills and teachers must intercede to help students make the connection. He suggests that initially this could be done by using concrete objects for visual representations.

Drill is another strategy used to achieve mastery of the basic addition and subtraction fact. Experts have various opinions and supportive research about the use, extent, and effects of drill. Experts seem to agree that drill is a necessary component in achieving mastery of the basic number facts (Ashcraft, 1985; Baroody, 1987; Ashcraft & Christy, 1995).

May (1984) and Usnick (1991) indicate children should drill a grouping of basic number facts only after they have demonstrated understanding of these facts. They state that only a few facts should be drilled at a time and that a quick response time is a key element. Further, students should not be allowed to employ counting or other "figuring out" strategies during drill. This would only improve the counting activity rather than help the student memorize the basic number facts. This is supported by Brownell and Chazal (1935) who conducted a study to determine the effects of drill upon third grade students. They found that drill did increase the efficiency of computation of the basic addition facts on the timed tests. However, they found that students who had previously employed counting or other immature strategies had become more proficient at these immature strategies rather than employing more mature strategies. Hasselbring, Goin, and Bransford (1987) amplify on this theme by stating that the most critical part of drill and practice is the challenge time. They indicate that when challenge times are reduced to about one second, students "tend not to revert to counting" (Hasselbring et al., 1987, p. 32).

Goldman, Mertz, and Pellegrino (1989) have a somewhat different view. They conducted a study of twenty-seven third and fourth grade students in need of increasing their response time for the basic addition facts. They conducted this study to determine the effect of practice on response time and strategy choice for operations. They concluded that practice increased the response time and also effected a shift to more efficient counting strategies and more direct retrieval responses. Further, they hypothesize that slow retrieval time may be a result of a student adopting a counting or similar strategy in response to a high error rate due to having moved prematurely to direct retrieval responses. Goldman et al. conclude that their research supported Ashcraft (1987) and Siegler (1987) in that drill increased the strength of associations between the addends and answers, increasing speed and affecting strategy choice. One can infer from this study that students progress to more efficient strategy choices as confidence in their answer choices increase. Therefore, when drill increases the speed of a counting strategy, this can be viewed as a step toward direct retrieval responses.

A variety of drill methods are found in the literature. These methods include flash cards, computer drill-type programs, games, timed tests and the graphing of results, and peer tutoring.

Using flash cards is one method of providing drill to students. Alexander (1986) suggests flashcards can be easily

made with the problems on the front and the answers written on the back, if they cannot be bought. She indicates that students should be allowed no more than three seconds to respond to the problem. If the answer is correct, she suggests putting it in the "go" pile. If it is incorrect or not answered within three seconds, the problem and answer should be repeated to the student and placed in the "stop" pile for further drill. She suggests the basic number facts should be practiced in small, systematic groupings, and that one grouping should be mastered before attempting another grouping. Similarly, Fuson and Brinko (1985) suggest having students study ten flash cards at a time. Further, they advise giving a timed test for those ten facts after each practice session. They indicate that the problems selected can be based on the results of a pretest. They suggest that students decide when the selected flash cards have been mastered and new flash cards can then replace them. Nelson and Clark (1991) indicate that a variation of traditional flash cards is a homemade calculator which can easily be made from a can and manilla tag board. The answers to the addition or subtraction basic facts are revealed on this homemade device, and it can be used for drill in a similar manner as traditional flash cards.

Using computer drill programs is another method of providing drill for the basic addition and subtraction facts. Computer Assisted Instruction (CAI), for example, was pioneered by Suppes (Suppes, Jerman, & Brian, 1968 as cited in Carrier, Post, &

Heck, 1985). Edwards, Norton, Taylor, Weiss, and Van Dusseldorp (1975) had variable results when CAI was substituted for traditional mathematics instruction (cited in Carrier et al., 1985). When Carrier et al. researched the effects of computer drill and practice programs as compared to a worksheet practice method, they also had variable results. They did find the computer programs required less instruction and held the fourth grade students' interest more than the worksheets did. There was, however, no significant achievement and retention gains of the basic facts by one method over the other. Similarly, Fuson and Brinko (1985) found that second, third, and fourth graders received equivalent learning from flash card and microcomputer conditions. They did find that when one condition was exchanged for the other, there was an increase in effort and motivation pointing to a need for variety in drill activities. Fuson and Brinko do list advantages of some computer drill programs to include rapid feedback and correction of errors, recording of errors and response speed, and the capacity to individualize drill programs.

Although Hasselbring, Goin, and Bransford (1987) agree that computers can help supply drill and practice of the basic number facts, they caution teachers to select software that can be individualized for students and that provide time restraints for responses. Further, Hativa (1988) observes that higher achieving students benefit more from computer drill programs with time constraints for responses. She believes

that individual learning styles should be examined prior to selecting drill methods for students.

Playing games is a third method of providing drill for the basic addition and subtraction facts. Nelson and Whitaker (1983) applaud the use of games to practice the basic number facts since students enjoy them and, thus, are motivated to learn from them. They caution teachers to discriminate between maintenance games and teaching games depending on the purpose of the activity. Maintenance games are designed to provide drill for basic facts students already know but need to practice. These games usually consist of numbers or equations on various cards or game boards. Teaching games are designed to help students learn the facts or concepts. They consist of concrete or picture type models. Nelson and Whitaker (1983), May (1984), Feinberg (1990), and Usnick (1991) suggest a variety of games that can be created or adapted to drill the basic addition and subtraction facts. Suggestions include variations of bingo, tic-tac-toe, baseball, war, solitaire, and many others. Usnick references books by Golick (1973), and Morehead and Mott-Smith (1963, 1977) that have many game ideas to provide practice for the basic number facts.

A fourth method of drill for the basic number facts is the use of timed tests. Greene (1985) believes that daily timed tests on the basic number facts increases motivation. He suggests that students set goals regarding time completion and error rate. Further, students can monitor their own progress

by graphing the results. Miller (1983) also applauds the use of timed tests but suggests a set time of 120 seconds for twentyfour random basic number facts. He concurs that students graphing their own results motivates them by giving a visual representation of their own progress. Usnick (1991) suggests only putting four or five number facts on a timed test but repeating these facts randomly twenty to twenty-five times in order to increase exposure to these facts for reinforcement. Mercer and Miller (1992) suggest that factors such as age level, academic skill, and motor ability affect the time given for a test. They do indicate that for most students "a rate of 40 to 60 correct digits per minute with two or fewer errors is appropriate" to determine if mastery has occured (Mercer & Miller, 1992, p. 23).

Peer tutoring is another method of providing drill for the basic addition and subtraction facts. Dineen, Clark, and Risley (1977), Allen and Boraks (1978), Parson and Heward (1978), and Heward, Heron, and Cooke (1982) are cited by Alexander (1986) as having found that peer tutoring provides an efficient and effective way to provide individual instruction in mathematics as well as other subjects. Alexander (1986) developed a program to drill the basic math facts. Pretests were given to determine the students' levels. Students were then paired with someone of a similar skill level. Each student took turns practicing a set of flash cards with the other student. Facts were drilled to the mastery level before moving to a new set of facts.

Progress was charted daily in individual folders. Alexander found this method effective, efficient, and enjoyable to the students. Similarly, Fantuzzo, Polite, and Grayson (1990) found that a peer tutoring strategy not only increased performance of basic math operations but also had a positive effect on student attendance.

In the above section, the writer examined the strategies used to achieve mastery of the basic addition and subtraction facts. In the following section, the writer presents a summary of this chapter.

#### Summary

This chapter presented a review of the literature regarding the theories of how the basic addition and subtraction facts are mastered, the current curriculum standards, the reasons for mastering these facts, and the strategies used to achieve mastery.

A review of the literature revealed there are two theories of learning which spawn contrasting views as to how the basic addition and subtraction facts are mastered. According to the absorption theory, the primary mode of learning these facts is repetition which leads to memorization. According to the cognitive theory of learning, the basic number facts are learned by making cognitive connections between various facts based upon underlying principles, rules, and procedures. Further, drill activities are to be initiated only after students learn these cognitive connections.

The current curriculum standards are set forth by the National Council of Teachers of Mathematics (NCTM) which clearly espouses to the cognitive theory of learning in most respects. The literature revealed there has been a shift in the curriculum standards from drill and computation being the primary emphasis to problem solving being the primary focus. The NCTM is emphatic that the basic number facts should be learned within problem solving contexts, that thinking strategies should be promoted, and isolated drill activities should be minimized.

Various reasons for mastering the basic addition and subtraction facts were presented in the literature. These reasons included increasing the speed and accuracy of more complex computation, and constructing a framework of arithmetic knowledge based upon underlying principles, rules, and procedures. Some experts also cited more competent problem solving as a reason for mastering the basic number facts; however, other experts did not see computation as a separate skill apart from problem solving. Instead, some experts believe computation is learned through problem solving.

Finally, numerous strategies were presented which help students learn the basic addition and subtraction facts. These strategies included proceeding from concrete methods to abstract methods. Further, thinking strategies were presented which help students make cognitive connections regarding the basic number facts. Drill strategies were also examined as methods to help students master the basic addition and subtraction facts.

# CHAPTER III PROCEDURE

#### Subjects

The subjects chosen for this study consisted of seventy kindergarten through sixth grade teachers from six public elementary schools located in an urban school district. These subjects were a nonprobability sampling of teachers who teach mathematics for at least one period per day (Best & Kahn, 1989). Sixty-three percent of the sampling teach kindergarten, first, second, or third grade students; thirty-seven percent of the sampling teach fourth, fifth, or sixth grade students. Thirty-nine percent of the sampling have ten or less years of experience; sixty-one percent of the sampling have more than ten years of experience. While all of the teachers surveyed have a bachelor's degree, thirty-two percent have earned a master's degree. The majority of teachers (ninetythree percent) are female with only seven percent of the sampling being male.

#### Setting

Schools. The schools selected for this study came from the same urban school district located in the midwest. There are thirty-three elementary schools from which six were selected. Each elementary school varies in student population. The elementary school enrollment for the 1996-97 school year was approximately sixteen thousand with approximately sixty-seven percent minority. African Americans compose the majority of the minority population of this urban school district

Community. The community where this study took place is

located within the city limits of a mid-size city in the midwest. There is a large African American and Appalachian American population within the city limits. Manufacturing and technology are major industries for this community.

#### Data Collection

Construction of the Data Collecting Instrument. The instrument was constructed by the writer using information gathered from the review of the literature establishing content validity (Fuchs, 1980). The instrument used was a Likert-type survey (see Appendix A). A brief explanation of the purpose of the study was included in the instrument. Also, an explanation of the terms: basic addition and subtraction facts, thinking strategies, automaticity, and rote memorization, were included in the instrument. The instrument presented a list of statements that each respondent answered based on his/her opinions of how the basic addition and subtraction facts are learned.

The following topics were used: demographics, the principles of theoretical approaches, reasons for mastery, level of expected mastery per grade level, current level of mastery, characteristics of strategies, and the effectiveness of various strategies. The instrument was field tested by several elementary teachers prior to use.

Administration of the Data Collecting Instrument. The instrument was reviewed and comments were returned to the writer by February 10, 1998. All necessary revisions were made, and the instrument was hand-delivered to the schools during February.
This writer met with the staff of two of the schools and included a cover letter with the surveys for the remaining schools (see Appendix B). Surveys were collected within a few days of delivery and a candy incentive was given. Letters of appreciation were also mailed (see Appendix C).

One hundred eleven surveys were distributed and seventy were completed and returned. The return rate was sixty-three percent. A few kindergarten teachers complained that many parts of the survey were not deemed relevant by them. Also, one sixth grade teacher stated the survey was not relevant at her grade level since all calculations in her class are done on calculators.

#### CHAPTER IV

#### RESULTS

#### Presentation of Results

The results of the Likert-type instrument are presented and analyzed in this chapter. Five tables are used to present the results. Table One presents the responses of all teachers regarding their perceptions of how the basic addition and subtraction facts are learned. Table Two presents the responses to the survey of all primary teachers who teach kindergarten, first, second, or third grade. Table Three presents the responses of all intermediate teachers surveyed who teach fourth, fifth, or sixth grade. Tables Four and Five present the responses to the survey of teachers with ten or less years of experience and with more than ten years of experience, respectively.

The tables present the thirty-one survey statements in paraphrased form. The complete statements can be found in Appendix A. The total number of responses to each item is also displayed (see column <u>n</u>). Totals vary due to some statements not being completed by every respondent. The tables also display the responses to the rating options for each survey statement in percentages which are rounded to the nearest whole percent. Due to rounding, the total percentage for each survey statement may not equal one hundred percent. An explanation of the rating code is as follows: SA means strongly agree, GA means generally agree, U means undecided, GD means generally disagree, SD means strongly disagree, and NU means never used.

### Table 1

## Responses of All Teachers to the Survey Regarding Teaching the Basic Addition

### and Subtraction Facts

		Responses			Rat	ings		
	Survey statements	<u>n</u>	SA	GA	U	GD	SD	NU
1.	Rote memorization through drill	70	24	49	6	10	11	NA
2.	Stored as isolated pieces of data	68	9	40	21	18	13	NA
3.	Form cognitive relationships	67	18	46	13	19	3	NA
4.	Stored in an interconnected fashion	69	17	43	22	13	3	NA
5.	Enough emphasis for grade level	70	13	27	21	27	11	NA
6.	Problem solving is main emphasis	70	29	46	19	7	0	NA
7.	Computation is main emphasis	70	3	13	24	46	14	NA
8.	Speed and accuracy of calculations	70	47	26	9	10	9	NA
9.	Problem solving competency	70	39	37	7	11	6	NA
10.	Framework of arithmetic knowledge	68	26	38	22	7	6	NA
11.	Accomplish grade level objectives	70	29	36	17	13	6	NA
12.	Majority of students have mastered	69	4	36	13	26	20	NA
13.	Proceed from concrete to abstract	70	41	49	7	1	1	0
14.	Develop thinking strategies	70	34	47	10	7	0	1
15.	Principle of commutativity	70	39	56	1	0	0	4
16.	Relate to two's and three's patterns	s 70	10	39	27	10	3	11
17.	Relate to the "doubles"	70	19	49	17	9	0	7
18.	Redistribute to ten	70	16	40	17	16	1	10
19.	Arithmetic rule regarding zero	69	30	48	12	3	1	6
20.	Counting forward or backward by one	69	39	46	9	1	1	3
21.	Subtraction as inverse of addition	69	35	48	7	3	1	6

### Table 1 (continued)

Responses of All Teachers to the Survey Regarding Teaching the Basic Addition and Subtraction Facts

		Responses			Rat			-	
	Survey statements	n	SA	GA	U	GD	SD	NU	
22.	Drill strategies for repetition	69	45	35	12	4	3	1	
23.	Flash card drill	69	43	35	13	1	3	4	
24.	Computer drill practice programs	69	28	41	20	0	3	9	
25.	Playing games for drill	69	48	45	6	0	0	1	
26.	Using timed tests for drill	68	29	44	13	4	1	7	
27.	Graphing results of timed tests	69	17	42	23	4	1	12	
28.	Peer tutoring for drill	69	19	64	13	1	0	3	
29.	Thinking strategies only for master	y 68	6	24	19	38	10	3	
30.	Drill strategies only for mastery	68	9	28	16	32	12	3	
31.	Both strategies for mastery	68	43	44	9	0	1	3	

<u>Note.</u> Refer to Appendix B for complete survey statements. Ratings are expressed as percentages rounded to the nearest whole percent; thus, totals may not equal one hundred percent. Number of responses ( $\underline{n}$ ) may differ due to skipped responses by teachers on some survey statements.

### Table 2

Responses of Primary Teachers to the Survey Regarding Teaching the Basic Addition

and Subtraction Facts

		Responses			Rat	ings			
	Survey statements	n	SA	GA	U	GD	SD	NU	
1.	Rote memorization through drill	44	25	41	7	11	16	NA	-
2.	Stored as isolated pieces of data	42	12	38	17	17	17	NA	
3.	Form cognitive relationships	42	21	45	14	19	0	NA	
4.	Stored in an interconnected fashion	43	19	44	16	16	5	NA	
5.	Enough emphasis for grade level	44	20	27	14	27	11	NA	
6.	Problem solving is main emphasis	44	30	43	16	11	0	NA	
7.	Computation is main emphasis	44	2	7	30	48	14	NA	
8.	Speed and accuracy of calculations	44	39	20	14	14	14	NA	
9.	Problem solving competency	44	25	41	11	14	9	NA	
10.	Framework of arithmetic knowledge	43	26	33	26	7	9	NA	
11.	Accomplish grade level objectives	44	14	41	18	18	9	NA	
12.	Majority of students have mastered	43	2	30	14	28	26	NA	
13.	Proceed from concrete to abstract	44	48	48	2	0	2	0	
14.	Develop thinking strategies	44	39	41	14	5	0	2	
15.	Principle of commutativity	44	32	59	2	0	0	7	
16.	Relate to two's and three's patterns	s 44	7	39	23	11	2	18	
17.	Relate to the "doubles"	44	20	52	9	7	0	11	
18.	Redistribute to ten	44	16	36	16	18	0	14	
19.	Arithmetic rule regarding zero	43	37	42	7	5	0	9	
20.	Counting forward or backward by one	43	42	44	7	2	0	5	
21.	Subtraction as inverse of addition	43	33	44	9	2	2	9	

### Table 2 (continued)

Responses of Primary Teachers to the Survey Regarding Teaching the Basic Addition

### and Subtraction Facts

		Responses			Ratings				
	Survey statements	<u>n</u>	SA	GA	U	GD	SD	NU	
22.	Drill strategies for repetition	43	42	33	14	5	5	2	
23.	Flash card drill	43	37	35	16	2	5	5	
24.	Computer drill practice programs	43	21	44	21	0	2	12	
25.	Playing games for drill	43	44	49	5	0	0	2	
26.	Using timed tests for drill	43	26	40	16	5	2	12	
27.	Graphing results of timed tests	43	12	44	26	2	0	16	
28.	Peer tutoring for drill	43	19	67	7	2	0	5	
29.	Thinking strategies only for master	y 43	5	28	21	30	12	5	
30.	Drill strategies only for mastery	43	12	26	12	35	12	5	
31.	Both strategies for mastery	43	37	49	9	0	0	5	

<u>Note.</u> Refer to Appendix B for complete survey statements. Ratings are expressed as percentages rounded to the nearest whole percent; thus, totals may not equal one hundred percent. Number of responses ( $\underline{n}$ ) may differ due to skipped responses by teachers on some survey statements.

### Table 3

### Responses of Intermediate Teachers to the Survey Regarding Teaching the Basic

### Addition and Subtraction Facts

		Responses			Rat	ings			
	Survey statements	<u>n</u>	SA	GA	U	GD	SD	NU	
1.	Rote memorization through drill	26	23	62	4	8	4	NA	
2.	Stored as isolated pieces of data	26	4	42	27	19	8	NA	
3.	Form cognitive relationships	25	12	48	12	20	8	NA	
4.	Stored in an interconnected fashion	26	15	42	31	8	4	NA	
5.	Enough emphasis for grade level	26	0	27	35	27	12	NA	
6.	Problem solving is main emphasis	26	27	50	23	0	0	NA	
7.	Computation is main emphasis	26	4	23	15	42	15	NA	
8.	Speed and accuracy of calculations	26	62	35	0	4	0	NA	
9.	Problem solving competency	26	62	31	0	8	0	NA	
10.	Framework of arithmetic knowledge	25	28	48	16	8	0	NA	
11.	Accomplish grade level objectives	26	54	27	15	4	0	NA	
12.	Majority of students have mastered	26	8	46	12	23	12	NA	
13.	Proceed from concrete to abstract	26	31	50	15	4	0	0	
14.	Develop thinking strategies	26	27	58	4	12	0	0	
15.	Principle of commutativity	26	50	50	0	0	0	0	
16.	Relate to two's and three's pattern	s 26	15	38	35	8	4	0	
17.	Relate to the "doubles"	26	15	42	31	12	0	0	
18.	Redistribute to ten	26	15	46	19	12	4	4	
19.	Arithmetic rule regarding zero	26	19	58	19	0	4	0	
20.	Counting forward or backward by one	26	35	50	12	0	4	0	
21.	Subtraction as inverse of addition	26	38	54	4	4	0	0	

### Table 3 (continued)

#### Responses of Intermediate Teachers to the Survey Regarding Teaching the Basic

### Addition and Subtraction Facts

		Responses			Rati	ings		
	Survey statements	<u>n</u>	SA	GA	U	GD	SD	NU
22.	Drill strategies for repetition	26	50	38	8	4	0	0
23.	Flash card drill	26	54	35	8	0	0	4
24.	Computer drill practice programs	26	38	35	19	0	4	4
25.	Playing games for drill	26	54	38	8	0	0	0
26.	Using timed tests for drill	25	36	52	8	4	0	0
27.	Graphing results of timed tests	26	27	38	19	8	4	4
28.	Peer tutoring for drill	26	19	58	19	4	0	0
29.	Thinking strategies only for master	y 25	8	16	16	52	8	0
30.	Drill strategies only for mastery	25	4	32	24	28	12	0
31.	Both strategies for mastery	25	52	36	8	0	4	0

<u>Note.</u> Refer to Appendix B for complete survey statements. Ratings are expressed as percentages rounded to the nearest whole percent; thus, totals may not equal one hundred percent. Number of responses ( $\underline{n}$ ) may differ due to skipped responses by teachers on some survey statements.

### Table 4

Responses of Teachers with Ten or Less Years Experience to the Survey Regarding

Teaching t	he	Basic	Addition	and	Subtraction	Facts

		Responses			Rat	ings			
	Survey statements	<u>n</u>	SA	GA	U	ĞD	SD	NU	_
1.	Rote memorization through drill	27	15	48	7	15	15	NA	
2.	Stored as isolated pieces of data	27	4	30	26	22	19	NA	
3.	Form cognitive relationships	27	15	41	15	22	7	NA	
4.	Stored in an interconnected fashion	27	26	26	22	15	11	NA	
5.	Enough emphasis for grade level	27	0	33	11	41	15	NA	
6.	Problem solving is main emphasis	27	30	48	15	7	0	NA	
7.	Computation is main emphasis	27	4	11	33	33	19	NA	
8.	Speed and accuracy of calculations	27	41	30	0	15	15	NA	
9.	Problem solving competency	27	52	19	0	19	11	NA	
10.	Framework of arithmetic knowledge	27	30	30	26	4	11	NA	
11.	Accomplish grade level objectives	27	30	26	22	11	11	NA	
12.	Majority of students have mastered	27	7	26	15	26	26	NA	
13.	Proceed from concrete to abstract	27	52	37	7	0	4	0	
14.	Develop thinking strategies	27	37	41	11	7	0	4	
15.	Principle of commutativity	27	26	63	4	0	0	7	
16.	Relate to two's and three's pattern	s 27	7	26	26	15	4	22	
17.	Relate to the "doubles"	27	11	41	19	15	0	15	
18.	Redistribute to ten	27	19	26	19	15	4	19	
19.	Arithmetic rule regarding zero	27	37	41	7	4	0	11	
20.	Counting forward or backward by one	27	41	52	4	0	0	4	
21.	Subtraction as inverse of addition	27	26	56	0	4	4	11	

#### Table 4 (continued)

Responses of Teachers with Ten or Less Years of Experience to the Survey Regarding Teaching the Basic Addition and Subtraction Facts

		Responses			Rati	ings		
	Survey statements	<u>n</u>	SA	GA	U	GD	SD	NU
22.	Drill strategies for repetition	27	33	41	15	4	7	0
23.	Flash card drill	27	37	30	22	0	7	4
24.	Computer drill practice programs	27	22	37	26	0	4	11
25.	Playing games for drill	27	52	37	7	0	0	4
26.	Using timed tests for drill	26	19	42	19	4	4	12
27.	Graphing results of timed tests	27	22	26	22	7	4	19
28.	Peer tutoring for drill	27	19	48	30	0	0	4
29.	Thinking strategies only for master	y 27	0	19	26	30	22	4
30.	Drill strategies only for mastery	27	4	22	19	30	22	4
31.	Both strategies for mastery	27	37	44	11	0	4	4

<u>Note.</u> Refer to Appendix B for complete survey statements. Ratings are expressed as percentages rounded to the nearest whole percent; thus, totals may not equal one hundred percent. Number of responses (<u>n</u>) may differ due to skipped responses by teachers on some survey statements.

### Table 5

# Responses of Teachers with Over Ten Years Experience to the Survey Regarding Teaching the Basic Addition and Subtraction Facts

		Responses		Rati	inas				
	Survey statements	n	SA	GA	U	GD	SD	NÜ	
1.	Rote memorization through drill	43	30	49	5	7	9	NA	
2.	Stored as isolated pieces of data	41	12	46	17	15	10	NA	
3.	Form cognitive relationships	40	20	50	13	18	0	NA	
4.	Stored in an interconnected fashion	42	12	55	21	12	0	NA	
5.	Enough emphasis for grade level	43	21	23	28	19	9	NA	
6.	Problem solving is main emphasis	43	28	44	21	7	0	NA	
7.	Computation is main emphasis	43	2	14	19	53	12	NA	
8.	Speed and accuracy of calculations	43	51	23	14	7	5	NA	
9.	Problem solving competency	43	30	49	12	7	2	NA	
10.	Framework of arithmetic knowledge	41	24	44	20	10	2	NA	
11.	Accomplish grade level objectives	43	28	42	14	14	2	NA	
12.	Majority of students have mastered	42	2	43	12	26	17	NA	
13.	Proceed from concrete to abstract	43	35	56	7	2	0	0	
14.	Develop thinking strategies	43	33	51	9	7	0	0	
15.	Principle of commutativity	43	47	51	0	0	0	2	
16.	Relate to two's and three's pattern	s 43	12	47	28	7	2	5	
17.	Relate to the "doubles"	43	23	53	16	5	0	2	
18.	Redistribute to ten	43	14	49	16	16	0	5	
19.	Arithmetic rule regarding zero	42	26	52	14	2	2	2	
20.	Counting forward or backward by one	42	38	43	12	2	2	2	
21.	Subtraction as inverse of addition	42	40	43	12	2	0	2	

### Table 5 (continued)

Responses of Teachers with Over Ten Years Experience to the Survey Regarding Teaching the Basic Addition and Subtraction Facts

	Responses			Rati	ings				
	Survey statements	n	SA	GA	U	ĞD	SD	NU	
22.	Drill strategies for repetition	42	52	31	10	5	0	2	
23.	Flash card drill	42	48	38	7	2	0	5	
24.	Computer drill practice programs	42	31	43	17	0	2	7	
25.	Playing games for drill	42	45	50	5	0	0	0	
26.	Using timed tests for drill	42	36	45	10	5	0	5	
27.	Graphing results of timed tests	42	14	52	24	2	0	7	
28.	Peer tutoring for drill	42	19	74	2	2	0	2	
29.	Thinking strategies only for master	y 41	10	27	15	44	2	2	
30.	Drill strategies only for mastery	41	12	32	15	34	5	2	
31.	Both strategies for mastery	41	46	44	7	0	0	2	

<u>Note.</u> Refer to Appendix B for complete survey statements. Ratings are expressed as percentages rounded to the nearest whole percent; thus, totals may not equal one hundred percent. Number of responses ( $\underline{n}$ ) may differ due to skipped responses by teachers on some survey statements.

The results of the responses to each survey statement are presented in sequential order. The presentation of the results will include the responses of the total sampling as well as the aforementioned subgroupings of this sampling.

Survey statement one indicates that students primarily learn the basic addition and subtraction facts by rote memorization through repetition. Seventy-three percent of the total respondents either strongly or generally agreed and twenty-one percent strongly or generally disagreed. Six percent were undecided. The intermediate teachers' responses showed an agreement rating of eighty-five percent and a twelve percent disagreement response. The primary teachers had a sixty-six percent agreement response and a twenty-seven percent disagreement response. Teachers with ten or less years of experience had a sixty-three percent agreement rating and a thirty percent disagreement rating. Teachers with more experience had a seventy-nine percent agreement response and a sixteen percent disagreement response. The subgroupings had similar undecided ratings.

Survey statement two indicates that the basic number facts are stored as isolated pieces of information in associative memory. Forty-nine percent of the total respondents strongly or generally agreed whereas thirty-one percent strongly or generally disagreed. Twenty-one percent were undecided. The primary and intermediate teachers had similar agreement and disagreement responses. The primary teachers had a seventeen

percent undecided rating as compared to a twenty-seven percent undecided rating by the intermediate teachers. The teachers with ten or less years of experience had a thirty-four percent agreement response and a forty-one percent disagreement response. Teachers with more experience had a fifty-eight percent agreement response and a twenty-five percent disagreement response.

Survey statement three indicates that students primarily learn the basic number facts by forming cognitive relationships through developing thinking strategies. Sixty-four percent of the total respondents agreed, twenty-two percent disagreed, and thirteen percent were undecided. There were only slight variations between the responses of primary and intermediate teachers. The strongest agreement response was from teachers with over ten years experience. They had a seventy percent agreement response and an eighteen percent disagreement response. Teachers with less experience had a fifty-six percent agreement rating and a twenty-nine percent disagreement rating.

Survey statement four indicates the basic number facts are stored in memory in an interconnected, weblike fashion. Sixty percent of the total respondents agreed, sixteen percent disagreed, and twenty-two percent were undecided. There were only slight differences in responses between primary and intermediate teachers. The intermediate teachers did have the highest undecided response of thiry-one percent. The primary teachers had a sixteen percent undecided response. Teachers with over ten years experience had an agreement response

of sixty-seven percent which contrasts with an agreement response of fifty-two percent for the teachers with less years of experience. The teachers with less years of experience had the highest disagreement response which was twenty-six percent as compared to twelve percent for teachers with more experience.

Survey statement five indicates that the district's curriculum placed enough emphasis on learning and reinforcing the basic addition and subtraction facts for the grade level currently taught. Forty percent of the total respondents agreed there was enough emphasis placed and thirty-nine percent disagreed, with twenty-one percent undecided. Primary teachers had a forty-seven percent agreement response to this statement and a thirty-nine percent disagreement response. Intermediate teachers had a twenty-seven percent agreement response and a thirty-nine percent disagreement response. Intermediate teachers were more undecided than the primary teachers on this issue with thirty-five percent undecided versus fourteen percent. The teachers with ten or less years of experience voiced a thirty-three percent agreement response, a fifty-six disagreement response, and an eleven percent undecided response. Teachers with more experience had a forty-four percent agreement response, a twenty-eight percent disagreement response, and a twenty-eight percent undecided response.

Survey statement six indicates that problem solving is the main emphasis of the curriculum objectives for the grade level taught. The total number of responses reflect that

seventy-five percent of the teachers agreed, seven percent disagreed, and nineteen percent were undecided. The responses of the subgroupings were similar in percentage to the total response ratings.

Survey statement seven indicates that computation is the main emphasis of the curriculum objectives for the grade level taught. Sixty percent of the total respondents disagreed with this statement, twenty-six percent agreed, and twenty-four percent were undecided. Of the primary teachers, sixty-two percent disagreed, nine percent agreed, and thirty percent were undecided. In contrast, while fifty-seven percent of the intermediate teachers disagreed, twenty-seven percent agreed and fifteen percent were undecided. Teachers with ten or less years of expperience had a fifty-two percent disagreement response and a thirty-three percent undecided response. Teachers with more experience had a sixty-five percent disagreement response and a nineteen percent undecided rating. The agreement responses were similar.

Survey statement eight indicates that students taught at the current grade level need to master the basic number facts in order to calculate more complex algorithms with speed and accuracy. Seventy-three percent of the total agreed with forty-seven percent strongly agreeing. Nineteen percent disagreed and nine percent remained undecided. Ninety-seven percent of the intermediate teachers agreed with this statement as contrasted to fifty-nine percent of the primary teachers.

Four percent of the intermediate teachers disagreed whereas twenty-eight percent of the primary teachers disagreed. Fourteen percent of the primary teachers were undecided whereas none of the intermediate teachers were undecided. Although agreement responses varied only slightly between the subgroups based upon years of experience, teachers with ten or less years of experience had a thirty percent disagreement rate as contrasted to teachers with more than ten years of experience who had a twelve percent disagreement rate. While fourteen percent of the teachers with more than ten years experience were undecided, none of the teachers with less experience were undecided.

Survey statement nine indicates that students taught at the current grade level need to master the basic number facts in order to increase problem solving competency. Seventy-six percent of the total agreed, seventeen percent disagreed, and seven percent were undecided. Ninety-three percent of the intermediate teachers agreed with sixty-two percent citing strong agreement. Only sixty-six percent of the primary teachers agreed with twenty-five percent citing strong agreement. Only eight percent of the intermediate teachers disagreed, whereas twentythree percent of the primary teachers disagreed. While there was little variance in the agreement responses based upon years of experience, teachers with ten or less years of experience had a thirty percent disagreement response as contrasted to teachers with more experience who had a nine percent disagreement response. Further, twelve percent of the teachers with

more than ten years of experience were undecided in contrast to zero percent of the teachers with less experience.

Survey statement ten indicates that students currently being taught need to master the basic number facts in order to construct a framework of arithmetic knowledge. Sixty-four percent of the total responses were in agreement, thirteen percent were in disagreement, and twenty-two percent were undecided. Seventy-six percent of the intermediate teachers agreed with only eight percent disagreement. Fifty-nine percent of the primary teachers agreed with sixteen percent disagreement. There were only slight variations in responses based upon years of experience.

Survey statement eleven indicates that students per current grade level need to master the basic number facts in order to accomplish the grade level curriculum objectives. Sixty-five percent of the total respondents agreed, nineteen percent disagreed, and seventeen percent were undecided. Eighty-one percent of the intermediate teachers agreed with only four percent in disagreement. Fifty-four percent were in strong agreement. In contrast, fifty-five percent of the primary teachers were in agreement and twenty-seven percent disagreed. While teachers with over ten years experience had a seventy percent agreement response, teachers with ten or less years experience had a fifty-six percent agreement response. There was only a slight variance in the percentage of disagreement responses between the work experienced based subgroups.

Survey statement twelve indicates that the majority of students currently taught know the basic addition and subtraction facts to the level of automaticity. Forty percent of the total respondents cited agreement, forty-six percent cited diagreement, and thirteen percent were undecided. Only thirty-two percent of the primary teachers agreed while fifty-four percent disagreed. In contrast, fifty-four percent of the intermediate teachers agreed while thirty-five percent disagreed. Teachers with over ten years experience had a forty-five percent agreement response while teachers with less experience had a thirty-three percent agreement response. The disagreement responses varied only slightly for the work experienced based subgroups.

Survey statement thirteen indicates to teach the basic number facts by proceeding from concrete to abstract methods. Ninety percent of the total respondents agreed with only two percent disagreeing. All subgroups showed similar percentages of agreement and disagreement.

Survey statement fourteen indicates that developing thinking strategies is effective in teaching the basic number facts. Eighty-one percent of the total respondents agreed, seven percent disagreed, ten percent were undecided, and one percent never used this strategy. The response percentages of the subgroups vary only slightly from that of the total group.

Survey statement fifteen indicates that the principle of commutativity is an effective thinking strategy in learning the basic number facts. Ninety-five percent of the total

respondents agreed, zero percent disagreed, one percent were undecided, and four percent never used it. The responses of the subgroups were similar to those of the total group.

Survey statement sixteen indicates that an effective thinking strategy is relating the basic number facts to the two's and three's patterns. Forty-nine percent of the total responded they agreed, thirteen percent disagreed, eleven percent never used it, and twenty-seven percent were undecided. Primary and intermediate teachers had similar percentages of agreement and disagreement as the total group. Their percentages of undecided responses were twenty-three and thirty-five, respectively. Teachers with ten or less years of experience had thirty-three percent in agreement, nineteen percent disagreeing, and twenty-two percent never using it. In contrast, teachers with over ten years of experience had fifty-nine percent in agreement, nine percent disagreeing and five percent never using it. Both groups had similar percentages of undecided responses.

Survey statement seventeen indicates that relating the "doubles" to the relevant basic addition facts is an effective thinking strategy. Sixty-eight percent of the total respondents agreed, nine percent disagreed, seven percent never used it, and seventeen percent were undecided. Seventy-two percent of the primary teachers agreed in contrast to agreement by fiftyseven percent of the intermediate teachers. While only nine percent of the primary teachers were undecided, thirty-one

percent of the intermediate teachers were undecided. There was only a slight difference in their disagreement responses. Seventy-six percent of the teachers with over ten years experience agreed, while only fifty-two percent of those with less experience agreed. Teachers with more than ten years experience had only five percent of the responses disagree, while those with less experience had fifteen percent of the responses disagree. The undecided response rates were similar.

Survey statement eighteen indicates that redistributing to ten is an effective thinking strategy for relevant basic addition and subtraction facts. Fifty-six percent of the total respondents agreed, seventeen percent disagreed, ten percent never used it and seventeen percent were undecided. The response percentages of primary and intermediate teachers were similar to the total group. There is a difference in responses between teachers with ten or less years of experience and those with more than ten years of experience. Those with ten or less years had forty-five percent in agreement and nineteen percent never using it, whereas those with more than ten years had sixty-three percent in agreement and five percent never using it. The percentages of disagreement were similar.

Survey statement nineteen indicates that teaching the rule that when adding or subtracting zero, the number remains the same is an effective thinking strategy. The total group of respondents had seventy-eight percent agreement, four percent disagreement, six percent never using it, and twelve percent

undecided. The responses of the subgroupings were similar to the total group. There was one noteworthy difference in that only seven percent of the primary teachers were undecided, whereas nineteen percent of the intermediate teachers were undecided.

Survey statement twenty indicates that counting forward by one or backward by one is an effective thinking strategy for the basic addition and subtraction facts with one as an addend or subtrahend. Eighty-five percent of the total responses were in agreement, two percent disagreed, three percent never used it, and nine percent were undecided. The responses of the primary and intermediate teachers were similar to the total group. Teachers with ten or less years experience had a ninetythree percent agreement response with four percent undecided, while teachers with more experience had an eighty-one percent agreement response with twelve percent undecided. The other responses for these subgroups were similar.

Survey statement twenty-one indicates that an effective thinking strategy is teaching that subtraction is the inverse of addition. Eighty-three percent of the total responses were in agreement, four percent disagreed, six percent never used it, and seven percent were undecided. There is a difference in response between primary and intermediate teachers. Seventyseven percent of the primary teachers agreed while ninety-one percent of the intermediate teachers agreed. The other responses had only slight differences. The subgroupings based on years

of work experience had similar responses as the total grouping.

Survey statement twenty-two indicates drill/practice strategies involving repetition are effective in teaching the basic number facts. Eighty percent of the total respondents agreed, seven percent disagreed, one percent never used it, and twelve percent were undecided. Primary teachers had a seventy-five percent agreement response while intermediate teachers had an eighty-eight percent agreement response. Other responses for these subgroups had only slight variance. Teachers with ten or less years experience had a seventy-four percent agreement response while teachers with more experience had an eighty-three percent agreement response. The other responses for these groups had only slight differences.

Survey statement twenty-three cites flash card drill as an effective drill strategy. Seventy-eight percent of the total respondents agreed, four percent disagreed, four percent never used it, and thirteen percent were undecided. The primary teachers had a seventy-two percent agreement response as compared to an eighty-nine percent agreement response for the intermediate teachers. There were only slight differences in percentages when comparing the other ratings of these subgroups. Teachers with ten or less years of experience had a sixty-seven percent agreement response and a twenty-two percent undecided response. Teachers with more experience had an eighty-six percent agreement response and a seven percent undecided response. There were only slight differences between the other responses for these

subgroups.

Survey statement twenty-four cites computer drill programs as effective drill strategies in teaching the basic number facts. The total responses indicated sixty-seven percent agreement, three percent disagreement, nine percent never using it, and twenty percent undecided. The differences in responses between the primary and intermediate teachers were minimal. Teachers with ten or less years of experience had fifty-nine percent of the responses agree with twenty-six percent undecided. Teachers with more experience had seventy-four percent agreement with seventeen percent undecided. There was minimal difference between the other responses for these subgroups.

Survey statement twenty-five indicates that playing games are effective drill strategies. Ninety-three percent of the total sampling agreed, zero percent disagreed, one percent never used it, and six percent were undecided. There were only minimal differences in the responses amongst the subgroups.

Survey statement twenty-six indicates that using timed tests is an effective drill strategy. The responses for the total group show seventy-three percent agreement, five percent disagreement, seven percent as never used, and thirteen percent undecided. Primary teachers had sixty-six percent in agreement, and twelve percent never using it as contrasted to the intermediate teachers with eighty-eight percent in agreement and zero percent never using it. There were minimal differences amongst the other ratings for these subgroups. While sixty-

one percent of teachers with ten or less years of experience were in agreement and nineteen percent were undecided, eightyone percent of the teachers with more experience were in agreement with ten percent being undecided. There were only slight differences in the other ratings between these subgroups.

Survey statement twenty-seven indicates that graphing results of timed tests is an effective drill strategy. Fiftynine percent of the total sampling agreed, five percent disagreed, twelve percent never used it, and twenty-three percent were undecided. The subgroups showed similar responses as the total group in all but the category of never used. Primary teachers had sixteen percent in this category whereas intermediate teachers had only four percent. Teachers with ten or less years of experience had nineteen percent in this category and teachers with more experience had seven percent.

Survey statement twenty-eight indicates that peer tutoring is an effective drill strategy. The total sampling had eightythree percent agreement, one percent disagreement, three percent as never used, and thirteen percent as undecided. The primary teachers had an eighty-six percent agreement response and a seven percent undecided response. The intermediate teachers had a seventy-seven percent agreement response and a nineteen percent undecided response. The other categories had similar responses for these subgroups. Teachers with ten or less years of experience had a sixty-seven percent agreement response and a thirty percent undecided rating while the teachers with more

experience had a ninety-three percent agreement response and a two percent undecided rating. The other categories for these two subgroups had similar ratings.

Survey statement twenty-nine indicates that thinking strategies alone help students master the basic addition and subtraction facts. Thirty percent of the total sampling agreed, forty-eight percent disagreed, nineteen percent were undecided, and three percent never used it. The primary teachers had a thirty-three percent agreement response and a forty-two percent disagreement response, whereas the intermediate teachers had a twenty-four percent agreement response and a sixty percent disagreement response. The other categories had similar responses for both subgroups. Teachers with ten or less years of teaching experience had a nineteen percent agreement response, a fifty-two percent disagreement response, and a twenty-six percent undecided response. Teachers with more experience had a thirty-seven percent agreement response, a forty-six percent disagreement response, and a fifteen percent undecided response. The other category had similar responses for both subgroups.

Survey statement thirty indicates that drill strategies alone help students master the basic number facts. The total group had thirty-seven percent agreement, a forty-four percent disagreement response, a three percent never used response, and a sixteen percent undecided response. While twenty-four percent of the intermediate teachers were undecided in contrast to twelve percent of the primary teachers being undecided, the

other categories had similar responses for both subgroups. Teachers with ten or less years of experience had a twenty-six percent agreement response and a fifty-two percent disagreement response. In contrast, teachers with more experience had a forty-four percent agreement response and a thirty-nine percent disagreement response. The other categories had similar responses for both subgroups.

Finally, survey statement thirty-one indicates that students master the basic number facts by developing thinking strategies and using drill strategies. The total group had an eighty-seven percent agreement response, a one percent disagreement response, a three percent never used response, and a nine percent undecided response. The responses of primary and intermediate teachers were similar. Teachers with less than ten years experience had an eighty-one percent agreement response while teachers with more experience had a ninety percent agreement response. The other categories had similar responses for both subgroups.

In the above section, the writer presented the results of the survey. In the following section, the writer discusses the results and relates them to the literature.

### Discussion of the Results

The results of the survey regarding teachers' opinions of how the basic addition and subtraction facts are mastered will be discussed by analyzing the results of related survey statements and the related literature. The survey statements are grouped as follows: learning theories, curriculum emphasis,

reasons for mastery, and strategies used to teach the basic addition and subtraction facts.

The literature has indicated there are two contrasting theories of learning which give rise to differing views of how the basic number facts are mastered. The absorption theory presents a model wherein the student relies upon drill to commit these facts to memory. The facts are stored in associative memory as isolated pieces of information. In contrast, the cognitive theory presents a model wherein the learner actively relates the basic number facts together in various ways based upon mathematical rules, principles, and relationships. The facts are stored in memory in an interconnected, weblike fashion. Drill is only a component after the student has grasped these relationships (e.g., Ashcraft, 1985; Baroody, 1985, 1987).

In survey statements one through four this writer sought to ascertain teachers' opinions regarding these theories of learning the basic number facts. An analysis of the data indicates teachers as a whole were divided as to whether students learned these facts primarily through rote memorization or primarily through forming cognitive relationships. They were also divided as to how the basic number facts are stored in memory. The subgroupings also followed this trend except for the intermediate teachers. They favored rote memorization as the primary method of learning these facts by fifteen percent. However, they had eleven percent more teachers believe these facts are stored in an interconnected fashion in memory which

is inconsistent with the aforementioned literature. Also, there is a range of thirteen to thirty-one percent undecided responses of the various groupings for survey items two through four.

It can be inferred that this sampling of teachers is divided as to how students learn and store the basic addition and subtraction facts. Due to the large amount of undecided responses, it can also be inferred that many teachers are not certain how these number facts are learned and stored in memory.

The NCTM (1980, 1989) has been shown to be in agreement with the cognitive theory of learning in most respects. It has also been shown that its curriculum standards reflect this and that problem solving, at all grade levels, is the driving force. Computation is to be learned within problem solving contexts with minimal emphasis on drill. This writer has examined the curriculum guide from the distict where this survey took place. It is reflective of the NCTM's standards.

Survey statements five through seven address the curriculum emphases of the district from which this sampling was drawn. The teachers as a whole were divided as to whether or not there was enough emphasis on the basic addition and subtraction facts at their grade level. Intermediate teachers had the lowest agreement response that there was enough emphasis at their grade level which was twenty-seven percent. The teachers with less than ten years of experience had the highest percentage of disagreement that there was enough emphasis with fifty-six percent. In contrast, teachers with more experience had the

lowest disagreement percentage which was twenty-eight percent. Regardless of grouping, the majority of teachers agreed problem solving is the main emphasis of the curriculum at their grade level and disagreed that computation is the main emphasis. There is a fourteen to thirty-five percent range of response as undecided for these three questions among the various groups.

It can be concluded that the majority of teachers believe that problem solving is the main emphasis of the curriculum and disagree that computation is the main emphasis. It can also be concluded that a large portion of these teachers do not believe there is enough emphasis on the basic addition and subtraction facts at their grade level. It is also noteworthy that teachers with ten or less years of experience expressed the most dissatisfaction with this. Further, it can be inferred that many teachers are undecided about the curriculum emphases.

It was presented in the literature that there are three main reasons why students need to know the basic addition and subtraction facts to the level of automaticity. These reasons are calculating more complex calculations with speed and accuracy, increasing problem solving competency, and constructing a framework of arithmetic knowledge (e.g., Anderson, 1982; Baroody, 1987; Resnick, 1983; Suydam & Dessart, 1980). Some experts did not agree that mastery of these number facts was necessary for problem solving competency (e.g., Baroody, 1987; NCTM, 1980, 1989). It was shown, however, that problem solving was defined differently by different experts which may have

accounted for at least some of the disagreement (Mercer, 1982 as cited in Mercer & Miller, 1992). Also, the contrasting theoretical constructs accounted for some of the difference of opinion (Baroody, 1987).

Survey statements eight through ten delineate the aforementioned reasons for mastering the basic addition and subtraction facts. This writer also queried in survey statement eleven as to whether or not students needed to master the basic number facts in order to accomplish the grade level mathematics objectives. In survey statement twelve, teachers' opinions were sought regarding whether or not the majority of students at their grade level had mastered these basic number facts.

The majority of teachers agreed that students needed to master the basic addition and subtraction facts in order to calculate more complex algorithms with speed and accuracy and to increase problem solving competency. The undecided responses were minimal. While the majority of teachers agreed that mastery of these facts was also necessary to build a framework of arithmetic knowledge, the percentage was weaker than the previously mentioned areas and the undecided response was stronger at twenty-two percent. The responses of intermediate teachers were decidedly stronger than the primary teachers' responses to these three survey statements by between thirty and forty percent. The primary teachers had higher percentages of undecided responses. The subgroups based upon years of experience had minimal differences of response. The

majority of teachers also agreed that students needed to master the basic addition and subtraction facts in order to accomplish the grade level objectives. Again, intermediate teachers had a stronger agreement response than the primary teachers by approximately twenty-five percent. Teachers with more than ten years of experience also had a stronger agreement response by approximately fifteen percent. Teachers were divided as to whether or not the majority of their students had mastered the basic number facts. The majority of primary teachers disagreed that the majority of their students had mastered them. While the majority of intermediate teachers agreed the majority of their students had mastered them, it was a small agreement majority response of fifty-four percent. Teachers with more than ten years of experience had a divided response to this survey statement. However, teachers with less experience had approximately twenty percent more responses disagreeing with this statement than agreeing.

It can be concluded that the majority of teachers believe that students need to master the basic addition and subtraction facts in order to calculate more complex algorithms with speed and accuracy, to increase problem solving competency, and to build a framework of arithmetic knowledge. Intermediate teachers, as might be expected, had the strongest agreement response to these statements. Similarly, it can be concluded that teachers agreed that students needed to master the basic number facts to accomplish the grade level objectives.

Intermediate teachers and teachers with more than ten years of experience had the strongest agreement responses. It can also be concluded that although teachers agreed mastery of the number facts is important, a large portion of the teachers disagreed that the majority of their students have mastered them. This was true for both primary and intermediate teachers. It can be inferred from this that a large portion of this sampling of teachers is dissatisfied with the level of mastery of the number facts by the majority of their students.

It has been established in the literature that strategies used to help students master the basic addition and subtraction facts include proceeding from concrete to abstract methods, thinking strategies based upon mathematical rules, principles, and relationships, and drill strategies. Numerous experts were presented in Chapter II regarding these various strategies. Further, while the absorption model relies heavily on the drill strategies, the cognitive model suggests that thinking strategies should be followed by drill strategies (e.g., Ashcraft, 1985; Baroody, 1985, 1987). The NCTM (1980, 1989) places its main emphasis on promoting thinking strategies and deemphasizing drill strategies.

Survey statements thirteen through twenty-one address a variety of thinking strategies. Survey statements twenty-two through twenty-eight deal with drill strategies. The last three survey statements, items twenty-nine through thirty-one, seek teachers' opinions regarding whether thinking strategies, drill

strategies, or a combination of both are needed to achieve mastery of the basic addition and subtraction facts.

A large majority of the teachers believed thinking strategies were important to mastering the basic number facts. This was true regardless of the subgroup analyzed. The largest percentages of agreement responses from the total sampling were for proceeding from concrete to abstract methods, using commutativity, using the rule regarding zero, counting forward and backward by one and seeing subtraction as the inverse of addition. The least favored thinking strategies were relating to number patterns, relating to the "doubles", and redistributing to ten. The least favored strategies had many undecided and never used responses rather than disagreed. The above statements held true for the subgroups in most respects. The subgroup with ten or more years experience included the "doubles" as a favored strategy.

A large majority of the teachers also believed drill strategies were important in mastering the basic number facts. The majority of teachers cited agreement for all of the drill strategies presented with playing games being the most favored. The least favored, although still meriting a majority of agreement, was graphing the results of timed tests. This rendered the largest undecided and never used responses. Computer drill programs also had a rather large undecided response. Primary teachers favored peer tutoring more than intermediate teachers. These groups had similar agreement percentages toward

playing games. The intermediate teachers, however, favored flash cards, computer drill, and timed tests more than the primary teachers. The intermediate teachers also had lower percentages of undecided and never used responses. The teachers with more than ten years experience had a higher majority of agreed responses on all of the drill strategies and fewer undecided and never used responses.

A majority of teachers cited agreement that both thinking strategies and drill strategies are necessary for students to master the addition and subtraction facts. The percentages of the subgroups were similar to the total sampling.

It can be concluded that teachers favor a variety of thinking and drill strategies. Further, based upon the percentage of undecided responses and never used responses, inservices regarding these strategies may be beneficial especially for primary teachers and those with less than ten years of experience. It can further be concluded that the majority of teachers believe both thinking strategies and drill strategies are necessary for mastering the basic addition and subtraction facts.

In summary, the discussion of results indicated that teachers were divided regarding how students learn and store the basic number facts in memory. The majority of teachers agreed that problem solving was the main emphasis of the curriculum rather than computation. They also voiced a strong agreement that mastery of the basic number facts was necessary

to calculate complex algorithms, to increase problem solving competencey, to construct a framework of arithmetic knowledge, and to accomplish grade level mathematics objectives. While not a majority, there was a large portion of teachers who did not believe there is enough emphasis in the curriculum on mastering the basic number facts and that a majority of their students had not mastered them. Teachers voiced strong agreement that a wide variety of thinking and drill strategies were important. Teachers expressed strong agreement that students need both thinking strategies and drill strategies to master the basic addition and subtraction facts.
#### CHAPTER V

### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

#### Summary

It is a common lament of elementary teachers that many students have not mastered the basic addition and subtraction facts. Further, there has been a shift in emphasis within the curriculum from computation to problem solving at every grade level. Teachers have debated how this shift in emphasis has affected the learning of the basic number facts to the mastery level. This writer observed an inherent tension, created within this shift from one theoretical approach to another, which prompted several guestions: How do students learn and master the basic addition and subtraction facts? What role do concrete objects, counting, thinking strategies, and drill play in mastering these facts? Which strategies are successful in helping students recall these facts with automaticity? How can students solve problems until the basic computation facts are mastered?

The purpose of this study was to analyze the opinions of elementary teachers regarding strategies used to teach the basic addition and subtraction facts. A Likert-type survey was constructed to gather the opinions of kindergarten through sixth grade teachers. The following topics were used: demographics, the principles of theoretical approaches, reasons for mastery, level of expected mastery per grade level, current level of mastery, characteristics of strategies, and the effectiveness of various strategies. The instrument was constructed by the writer using information gathered from a review of the literature establishing content validity.

A review of the literature revealed there are two theories of learning which spawn contrasting views as to how the basic addition and subtraction facts are mastered. According to the absorption theory, the primary mode of learning these facts is repetition which leads to memorization. According to the cognitive theory of learning, the basic number facts are learned by making cognitive connections between various facts based upon underlying principles, rules, and procedures. Further, drill activities are to be initiated only after students learn these cognitive connections.

The current curriculum standards, as set forth by the National Council of Teachers of Mathematics (NCTM), clearly reflect the cognitive theory of learning in most respects. The literature revealed there has been a shift in the curriculum standards from a drill and computation emphasis to a problem solving emphasis. The NCTM is emphatic that the basic number facts should be learned within problem solving contexts, that thinking strategies should be promoted, and that isolated drill activities should be minimized.

Various reasons for mastering the basic addition and subtraction facts were presented in the literature. These reasons included increasing the speed and accuracy of more complex calculations, and constructing a framework of arithmetic

knowledge based upon underlying principles, rules, and procedures. Some experts also cited more competent problem solving as a reason for mastering the basic number facts; however, other experts did not see computation as a separate skill apart from problem solving. Instead, some experts believed computation is learned through problem solving.

Numerous strategies were presented in the literature which help students learn the basic addition and subtraction facts. These strategies included proceeding from concrete methods to abstract methods. Further, thinking strategies were presented which help students make cognitive connections regarding the basic number facts. Drill strategies were also examined as methods to help students master the basic addition and subtraction facts.

The Likert-style survey was constructed based upon the review of the literature. It consisted of thirty-one survey statements with rating options ranging from strongly agree to strongly disagree as well as the options of never used and undecided. The subjects chosen for this study consisted of seventy kindergarten through sixth grade teachers from six public elementary schools located in an urban school district within the midwest. This was a nonprobability sampling of teachers who teach mathematics for at least one period per day.

The results of the survey were presented and analyzed. Five tables were used to display the opinions of the total sampling, the primary teachers, the intermediate teachers, the

teachers with ten or less years of experience, and the teachers with more than ten years of experience. The discussion of the results was related to the review of the literature.

The results of the survey statements relating to the theories of how the basic addition facts are learned and stored in memory were divided for the total group as well as for most of the subgroups. The subgrouping consisting of the intermediate teachers did favor rote memorization as the primary method of learning these facts by fifteen percent. However, they had eleven percent more teachers believe these facts are stored in an interconnected fashion in memory which is inconsistent with the literature. Also, there was a range of thirteen to thirty-one percent undecided responses for three of the four survey statements relating to this topic.

The results of the survey statements which addressed the curriculum emphases of the district from which this sampling was drawn were presented. They indicated that teachers as a whole were divided as to whether or not there was enough emphasis on the basic addition and subtraction facts at their grade level. Intermediate teachers had the lowest agreement response which was twenty-seven percent. The teachers with less than ten years of experience had the highest percentage of disagreement that there was enough emphasis with fifty-six percent. In contrast, teachers with more experience had the lowest disagreement percentage which was twenty-eight percent. Regardless of grouping, the majority of teachers agreed problem solving is

the main emphasis of the curriculum at their grade level rather than computation. There was a fourteen to thirty-five percent range of response as undecided for the survey statements relating to this topic.

The results of the survey statements regarding the reasons for mastering the basic addition and subtraction facts were presented. Further, the results of the survey statement regarding whether or not a majority of students at the present grade level had mastered these facts were also presented. The majority of teachers agreed that students needed to master the basic addition and subtraction facts in order to calculate more complex algorithms with speed and accuracy and to increase problem solving competency. While the majority of teachers agreed that mastery of these facts was also necessary to build a framework of arithmetic knowledge, the percentage was weaker than the previously mentioned areas and the undecided response was stronger at twenty-two percent. The intermediate teachers' responses were decidedly stronger than the primary teachers' responses by between thirty and forty percent. The primary teachers had a higher percentage of undecided responses. The subgroups based upon years of experience had minimal differences of response. The majority of teachers also agreed that mastery of the basic number facts was necessary to accomplish grade level objectives. Intermediate teachers and those with more than ten years of experience had the strongest agreement percentages. Teachers were divided as to whether or not

the majority of their students had mastered the basic addition and subtraction facts. A large portion of each grouping disagreed that the majority of their students had mastered them.

The results of the survey statements regarding the effectiveness of numerous thinking strategies and drill strategies were presented. Further, the results of the survey statements regarding whether thinking strategies, drill strategies, or a combination of both were needed to achieve mastery of the basic addition and subtraction facts were presented.

The results indicated that a large majority of the teachers believed thinking strategies were important to mastering the basic number facts. The largest percentages of agreement from the total sampling were for proceeding from concrete to abstract methods, using commutativity, using the rule regarding zero, counting forward and backward by one, and seeing subtraction as the inverse of addition. The least favored thinking strategies were relating to number patterns, relating to the "doubles", and redistributing to ten. The least favored thinking strategies had many undecided and never used responses rather than disagreed. The subgroups' responses were reflective of the total in most respects. The subgroup with ten or more years of experience included the "doubles" as a favored strategy.

The results also indicated that a large majority of the teachers also believed that drill strategies were important in mastering the basic number facts. All drill strategies

received a majority of agreement responses with playing games as the most favored, and graphing the results of timed tests as the least favored. Graphing timed tests rendered the largest percentages of undecided and never used responses. Computer drill programs also had a rather large undecided response. Primary teachers favored peer tutoring more than intermediate teachers. The intermediate teachers favored flash cards, computer drill, and timed tests more than the primary teachers. Teachers with more than ten years experience had a higher majority of agreed responses on all of the drill strategies and fewer undecided and never used responses than the teachers with less experience.

The results indicated that a majority of teachers cited agreement that both thinking strategies and drill strategies are necessary for students to master the basic addition and subtraction facts. The responses of the subgroupings were similar to the total sampling.

#### Conclusions

As a result of this study, several conclusions are made. These conclusions are based upon the aforementioned results in relationship to the review of the literature.

It can be concluded that this sampling of teachers was divided as to how students learn the basic addition and subtraction facts and how these facts are stored in memory. While opinions were divided between support for the absorption model and support for the cognitive model, it can also be

concluded that a large portion of the respondents were undecided as to how these basic number facts are learned and stored in memory.

Further, it can be concluded that the majority of teachers believe that problem solving is the main emphasis of the curriculum and disagree that computation is the main emphasis. While this is reflective of the NCTM's curriculum standards, a large portion of these teachers did not believe there is enough emphasis on the basic addition and subtraction facts at their grade level. Based upon the large portion of undecided responses, it can also be concluded that many teachers are undecided about the curriculum emphases.

This writer also concludes that the majority of the teachers believe that students need to master the basic addition and subtraction facts to calculate more complex algorithms with speed and accuracy, to increase problem solving competency, to build a framework of arithmetic knowledge, and to accomplish the grade level objectives. These opinions support the majority of research which has been presented regarding reasons for learning the basic number facts to the level of automaticity. Although teachers agreed that mastery of the basic number facts is important, it can be concluded that a large portion of teachers disagree that a majority of their students have mastered them. This was true for both primary and intermediate teachers. It can be inferred that many teachers are dissatisfied with the level of mastery of the basic addition and subtraction facts

by the majority of their students.

Further, it can be concluded that teachers favor a wide range of thinking and drill strategies. The most favored thinking strategies include proceeding from concrete to abstract methods, commutativity, the rule regarding zero, counting by one, and seeing subtraction as the inverse of addition. The most favored drill strategy for the total group was playing games but all of the drill strategies elicited a majority of agreement responses. Based upon the percentages of undecided and never used responses, it can be inferred that a portion of the teachers are not familiar with all of the thinking and drill strategies which were presented in the literature. Finally, it can be concluded that the majority of teachers believe that teaching thinking strategies followed by drill strategies is effective in helping students master the basic addition and subtraction facts. This opinion is in alignment with the cognitive construct for learning the basic number facts.

#### Recommendations

Based upon the aforementioned results and conclusions, the writer makes the following recommendations.

The writer recommends that school districts provide inservice training to elementary teachers regarding how students learn and master the basic computation facts. Further, these inservices should provide information regarding the numerous thinking strategies and drill strategies that can be utilized.

It is further recommended that colleges and universities

provide the aforementioned training as part of the required course work for students majoring in elementary education.

It is also recommended that similar studies be done with larger random samples to further study this topic and elicit more comprehensive results and conclusions.

It is recommended that the mathematics curriculum standards for the elementary level be reexamined to determine if enough time is being allocated within the curriculum for the thinking and drill components necessary to teach the basic computation facts to the level of mastery.

Finally, it is recommended that the mathematics curriculum for the elementary level be revised to reflect the opinions of teachers that more emphasis on mastering the basic addition and subtraction facts is needed. This recommendation is also buttressed by the cognitive theorists in the literature who indicate that developing thinking strategies reinforced by drill takes time.

#### APPENDIX A

# SURVEY OF STRATEGIES USED TO TEACH THE BASIC ADDITION AND SUBTRACTION COMPUTATION FACTS

The purpose of this survey is to collect teachers' opinions regarding how students learn the basic addition and subtraction facts and the strategies used to teach and reinforce them.

## Background Information

Please complete the following information.

Sex: Male\_\_\_\_ Female\_\_\_\_

Mathematics Language Arts

### Survey of Teachers' Opinions

## Definitions of Terms:

Basic Addition and Subtraction Facts refer to the 100 addition combinations with single digit addends (0+0 to 9+9) and the corresponding subtraction combinations (0-0 to 18-9).

<u>Rote Memorization</u> refers to storing information in associative memory through repetition.

<u>Automaticity</u> refers to a state of mastery in which information can be retrieved from memory instantaneously.

<u>Thinking Strategies</u> are methods of finding meaningful cognitive relationships based upon mathematical principles, stored rules, or procedures (Baroody, 1985).

## **Directions:**

Please read the following statements about the basic addition and subtraction facts and circle the number that best describes your opinion. It is important that all statements are answered. Rating Scale:

5=Strongly Agree	4=Generally Agree	3=Undecided
2=Generally Disagree	1=Strongly Disagree	

## Learning Theories

1. Students primarily learn the basic addition and subtraction facts by rote memorization accomplished through repetition.

#### 5 4 3 2 1

2. The basic addition and subtraction facts are stored as isolated pieces of information in associative memory.

5 4 3 2 1

3. Students primarily learn the basic addition and subtraction facts by developing thinking strategies which form cognitive relationships between these facts.

## 5 4 3 2 1

4. These basic facts are stored in memory in an interconnected weblike fashion.

5 4 3 2 1

### Curriculum Emphasis

5. The district's curriculum objectives place enough emphasis on learning and reinforcing the basic addition and subtraction basic facts for the grade level that I teach.

## 5 4 3 2 1

6. Mathematical problem solving is the main emphasis of the district's curriculum objectives for the grade level that I teach.

5 4 3 2 1

7. Computation is the main emphasis of the district's curriculum objectives for the grade level that I teach.

5 4 3 2 1

# Reasons for Mastery

8. The students that I teach need to know the basic addition and subtraction facts at the level of automaticity in order to calculate more complex algorithms with speed and accuracy.

# 5 4 3 2 1

9. The students that I teach need to know the basic addition and subtraction facts at the level of automaticity in order to increase problem solving competency.

5 4 3 2 1

10. The students that I teach need to know the basic addition and subtraction facts at the level of automaticity in order to construct a framework of arithmetic knowledge based upon the intrinsic connections and principles underlying these facts (Baroody, 1987).

5 4 3 2 1

11. The students that I teach need to know the basic addition and subtraction facts at the level of automaticity in order to accomplish the mathematics curriculum objectives for their grade level.

5 4 3 2 1

12. The majority of the students that I teach know the basic addition and subtraction facts at the level of automaticity.

5 4 3 2 1

# Strategies Used to Teach the Basic Addition and Subtraction Facts Directions:

Please rate the following strategies based upon your personal teaching experience. A strategy is to be considered effective if it helped a majority of your students make significant gains toward achieving automaticity of the basic addition and subtraction facts.

## Rating Scale:

5=Strongly Agree	4=Generally Agree	3=Undecided
2=Generally Disagree	1=Strongly Disagree	0=Never Used

13. One effective strategy is to proceed from concrete to semi-concrete methods, and finally to abstract methods when teaching the basic addition and subtraction facts.

5 4 3 2 1 0

14. Developing thinking strategies regarding underlying relationships and principles is very effective in teaching the basic addition and subtraction facts.

5 4 3 2 1 0

15. One effective thinking strategy for learning the basic addition facts is relating approximately half of these facts to the other half through the principle of commutativity (e.g., 2+7=9 and 7+2=9).

5 4 3 2 1 0

16. Relating the addition and subtraction basic facts which involve two or three as an addend or subtrahend to the counting by two's and three's number patterns is an effective thinking strategy for learning these facts (e.g.,5+2=7 is related to the 1,3,5,7...pattern).

5 4 3 2 1 0

17. Relating the relevant addition basic facts to the doubles (2+2,3+3,etc.) is an effective thinking strategy for learning these facts (e.g., 6+7 is the same as double 6 plus 1 or 12+1; 5+7 is the same as 6+6 or 12).

5 4 3 2 1 0

18. Relating the relevant addition and subtraction basic facts to adding 10 to a number or subtracting 10 from a number is an effective thinking strategy for learning these facts(e.g., 9+7

is the same as 10+7 minus 1; 16-9 is the same as 16-10 plus 1).

## 5 4 3 2 1 0

19. Teaching the arithematic rule that a number with 0 as an addend or subtrahend remains the same develops an effective thinking strategy for learning the addition and subtraction basic facts with 0 as an addend or subtrahend.

5 4 3 2 1 0

20. Teaching that adding one to a number is the same as counting forwards by one and that subtracting one from a number is the same as counting backwards by one develops an effective thinking strategy for learning the basic addition and subtraction facts with one as an addend or subtrahend.

5 4 3 2 1 0

21. Teaching that subtraction is the inverse of addition develops an effective thinking strategy for learning the basic subtraction facts (e.g., 15-7 can be solved by thinking 7 plus what equals 15).

5 4 3 2 1 **0** 

22. Drill/practice strategies involving repetition are very effective in teaching the basic addition and subtraction facts.

23. Flash card drill is an effective strategy in teaching the basic addition and subtraction facts.

3

5

4

5 4 3 2 1 0

2

1

0

24. Computer Assisted Instruction (CAI) is an effective strategy in teaching the basic addition and subtraction facts.

5 4 3 2 1 0

25. Playing games to provide drill/practice of the basic addition and subtraction facts is an effective strategy.

26. Using timed tests to provide drill/practice of the basic addition and subtraction facts is an effective teaching strategy.

## 5 4 3 2 1 0

27. Having students graph the results of timed tests of the basic addition and subtraction facts is an effective teaching strategy.

## 5 4 3 2 1 0

28. Peer-tutoring is an effective strategy in teaching the basic addition and subtraction facts.

5 4 3 2 1 0

29. Thinking strategies alone help students know the basic addition and subtraction facts to the level of automaticity.

5 4 3 2 1 0

30. Drill/practice strategies alone help students know the basic addition and subtraction facts to the level of automaticity.

5 4 3 2 1 0

31. Teaching thinking strategies followed by drill/practice strategies is very effective in helping students learn the basic addition and subtraction facts to the level of automaticity.

5 4 3 2 1 0

APPENDIX B

(Name and address of writer)

(Date)

(Address of school)

Dear Teacher:

I am a teacher at (name of school). As part of my course work at the University of Dayton, I am conducting a study of teachers' opinions regarding strategies used to teach the basic addition and subtraction facts. The accompanying survey is designed to collect teachers' opinions regarding how these basic facts are learned, effective strategies to achieve mastery, and their relative place within a mathematics curriculum. I am requesting all regular kindergarten through sixth grade teachers, who teach mathematics as part of their curriculum, to complete this survey.

Your opinions are very crucial to this study. Realizing your time is extremely valuable, I have constructed this survey to take approximately seven minutes to complete. Further, all opinions will remain anonymous. Thank you in advance for your time and effort in completing this survey. Please return the survey to the folder in the office by Wednesday, February 25, 1998.

Thank you for your help.

Sincerely,

Diane S. Douglas

#### APPENDIX C

(Name and address of writer)

(Date)

(Name of principal and address of school)

Dear (Name of principal) and Teachers:

I wish to thank you for your efforts in completing my survey as requested. Your opinions are crucial to the study I am conducting, and the results may prove to be interesting. Again, I appreciate your cooperation in this matter.

Sincerely,

Diane S. Douglas

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