EFFECT OF A HANDS-ON GEOMETRY CURRICULUM ON PREPAREDNESS FOR A FORMAL GEOMETRY COURSE

MASTER'S PROJECT

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by

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CHAPTER I

INTRODUCTION

The Problem

Much controversy surrounds the teaching of formal geometry in our high schools. Student performance is poor, student attitude is bad, and student knowledge is not always increased upon completion of the course. In the United States, 25% of students completing the first year of algebra do not even attempt geometry (Usiskin, 1982).

If we look at all high school students, the results are even more dismal. Based on Usiskin's research, if a sample of 100 graduating seniors was taken, it would show the following:

53 did not complete any type of geometry course. (Of these, 47 did not take a geometry course, and 6 took geometry but dropped the course before the end of the year.)

7 took a nonproof geometry course.

40 took a formal geometry course that included proofs. (Of these, 11 cannot do proofs of any sort, 9 can do only trivial proofs, 7 have

moderate success with proofs, and 13 are successful with proofs.)

Based on Senk's research (1983), a sample of 100 students who had completed a formal geometry course would reveal the following:

28 cannot do proofs of any sort

22 can do only trivial proofs

17 are moderately successful with proofs

33 are successful with proofs

Current research indicates that students need to be at level 4 on the van Hiele scale (CDASSG numbering system) to be able to do proofs (Senk, 1983; Usiskin, 1982; P. van Hiele, 1984b). The results in the preceding paragraphs indicate that most students are not at level 4 even after completion of a formal high school geometry course. If we are going to continue defining success in the formal high school geometry course as the ability to complete nontrivial proofs, then some changes in the curriculum are needed to better prepare the students to write formal proofs.

With the introduction of proficiency testing in the state of Ohio, it is now essential that all students receive some introduction to geometry even if they never take a geometry course. According to the

Ohio Department of Education, High School Proficiency Testing: Fact Sheets, Ninth-Grade Mathematics, sixteen of the forty test items on the ninth-grade proficiency test are designed to measure geometry related outcomes. In the 1993-94 school year, the first twelfth-grade proficiency tests will be given. A draft of the learning outcomes to be tested includes seven geometry topics. Appendix A gives the geometry related outcomes for both tests.

A graduate student at The Ohio State University and staff of the CDASSG project designed tests that measured the knowledge of incoming geometry students. The tests assessed knowledge high school teachers expect students to have prior to entering a formal geometry course. The tests were not identical, but did have 16 questions that were exactly the same. The results of those 16 common items indicate that incoming students do not have the knowledge that the geometry teachers expect. Overall, the mean percentage correct was 62% in the Ohio State study and 54% in the CDASSG Project. The percentage of students unable to correctly answer questions related to the proficiency test outcomes are given below (Usiskin, 1982):

	OSU	CDASSG
Area of a rectangle	20%	28%
Area of a square	48%	54%
Measurement of a right angle	11%	24%

While these results are alarming, they become even more dramatic when we remember that the test was given only to students actually beginning a geometry course. This excludes the 47% of high school students who never even attempt geometry. How high would these percentages be if all high school students were tested (as happens with the proficiency test)? If high schools are to graduate students who have basic knowledge of geometry, some changes in the curriculum need to be made.

Hypothesis

Completion of a one-semester geometry readiness curriculum has no effect on a student's van Hiele level or on the student's knowledge of geometry.

Significance of the Study

Prior to the 1989-90 school year, freshmen at William S. Mason High School enrolled in either General Math I, General Math II, Pre-Algebra, Algebra I, Geometry, or Honors Geometry. A number of problems existed. No clear cut criteria existed for placement in these courses. The Algebra I teacher was discouraged at the slow pace required to meet the needs of the students. Many of the better freshman students were bored by the slow pace. Sophomores who had taken Pre-Algebra as freshmen were upset over the duplication of material between the two courses. In addition, the geometry teachers were frustrated by the poor performance of the geometry students.

In an attempt to address these problems, the lower end of the mathematics curriculum was redesigned over a two-year period (1989-1991). General Math I and II were replaced by a single General Math course in anticipation of state mandates that only one year of general or remedial math will be allowed as credit for high school graduation. The Pre-Algebra course was eliminated at the high school. Students who would normally enroll in Pre-Algebra and then take Algebra I are now taking Algebra I Part I followed by Algebra I Part II, both year-long courses. This move was consistent with the curriculum being offered by other high schools in the area. The pace of the regular Algebra I course was accelerated slightly. Finally, enrollment criteria were established.

The Algebra I Part I and Algebra I Part II courses are the focus of this study. The primary purpose in developing these two courses is to provide a slower paced version of Algebra I for the student who previously took the Pre-Algebra and Algebra I courses. However, the experience of another school district showed that spreading the material over four semesters was not feasible -- only three semesters were needed. In order to keep Algebra I Part II a full-year course like all the other mathematics courses, it was decided to include an introduction to geometry. This gave the lower level students a head start if they chose to enroll in the next course in the sequence -- formal geometry.

The same textbook is used for Algebra I, Algebra I Part I, and the algebra portion of Algebra I Part II. The geometry curriculum for Algebra I Part II is based upon the van Hiele level theory with emphasis on hands-on experiences, manipulatives and computers. At the time this course was designed, there was no textbook available that developed the curriculum using these approaches. As a result, the geometry curriculum was developed using only the research available.

The goal for the geometry portion of the course was twofold. The first goal was to prepare students to be successful in the formal geometry course if they chose to continue their study of mathematics. The second goal was to provide students with the knowledge needed to pass the ninth- and twelfth-grade proficiency tests.

During the 1989-90 school year, the algebra and geometry portions of the Algebra I Part II course were kept separate. The first semester was algebra while the second semester was geometry. The result was a course that was quite difficult for the students during the first semester but was perceived by several of the students as "fun and games" during the second semester. They did not consider it serious work because of the emphasis on manipulatives and group work. To counteract some of this perception, the algebra and geometry were interspersed throughout the course during the 1990-91 school year.

This study was designed to determine whether Algebra I Part II increases students' knowledge of geometry.

Definitions

van Hiele level. In 1957, Pierre van Hiele developed a theory of geometric thought. His theory contends that students progress through a fixed sequence of levels in understanding geometry. Levels cannot be skipped. Using the CDASSG numbering system (Fuys, 1985; Senk, 1983; Usiskin, 1982; P. van Hiele, 1984b.), the levels are defined as follows:

Level 0: Nonfunctional. Student is not operating at the ground or basic level.

Level 1: Recognition/visualization. The student can recognize shapes. This is the basic level of pre-geometric reasoning. Knowledge is obtained exclusively by observation.

Level 2: Analysis. The student can identify properties of figures. The student begins to use reason.

Level 3: Order/Abstraction. The student can logically order figures and relationships. Simple deduction can be followed by the student. The student can follow short proofs but may not be able to write them. This level is the transitional level from informal to formal geometry.

Level 4: Deduction. The student understands the significance of deduction and the roles of postulates, theorems, and proof. Proofs can be written with understanding. This level is needed for success in most high school geometry courses.

Level 5: Rigor. The student can make abstract deductions. Non-Euclidean geometry can be understood.

<u>van Hiele Level Test</u>. A twenty-five question multiple-choice test developed as part of the CDASSG project. The test is comprised of five subtests (one for each van Hiele level), each containing five questions. The test assesses the van Hiele level at which a student is operating. The test is further discussed in Chapter Two.

<u>Cooperative Test - Geometry</u>. A standardized test published by Educational Testing Service. Part A of the test contains forty multiple-choice questions on the content of geometry courses.

<u>CDASSG</u>. Acronym for the Cognitive Development and Achievement in Secondary School Geometry project conducted at the University of Chicago from 1980 to 1982 under the direction of Zalman Usiskin.

Limitations

The sample in this study is not random. All students enrolled in Algebra I and Algebra I Part II at William S. Mason High School during the 1990-91 school year were included. This sample might not be representative of students attending other high schools. Also, the curriculum these students encountered prior to Algebra I or Algebra I Part II might not be comparable to the curricula used in other school systems. Because of these limitations, results from this study can only provide suggestions as to what other school districts might find.

CHAPTER II

REVIEW OF RELATED LITERATURE

Research of the van Hieles

In 1957, Pierre Marie van Hiele and his wife, Dina van Hiele-Geldof, completed companion dissertations. At the time, they were secondary school teachers in the Netherlands with experience in the Montessori method. Shortly after completing her dissertation, Dina was killed in an automobile accident. Since that time, Pierre has continued to write and lecture on what has come to be known as the van Hiele level theory (Usiskin, 1982).

In his dissertation, The Problem of Insight in Connection with School Children's Insight into the Subject Matter of Geometry, Pierre's goal was to study mathematical insight, particularly geometrical insight. He defined insight as the ability of a student to take deliberate action in new learning situations as the result of prior learning. This rational thought had three parts: the forming of structures, the forming of associations, and analysis (P. van Hiele, 1984a).

Dina van Hiele-Geldof's dissertation, The Didactics of Geometry in the Lowest Class of Secondary School, attempted to answer three questions:

1. Is it possible to follow a didactic as a way of presenting material so that the thinking of the child is developed from the lowest level to higher levels in a continuous process?

2. Do twelve year-olds in the first class of secondary school have the potential to reason logically about geometric problems and to what extent can this potential be developed?

3. To what extent is language operative in the transition from one level to the next? (Fuys, 1984, iv)

Dina's method of instruction was to give students concrete material that allowed movement from visual to abstract thinking. She contended that students can move from level one to level two in twenty lessons and from level two to level three in fifty lessons. In her dissertation, and in other writings, she presented specific teaching examples and guidelines (D. van Hiele, 1984a; D. van Hiele, 1984b).

In 1957, Pierre van Hiele presented a paper at a conference in France in which he detailed the levels and the phases within levels of his theory. He pointed out that "understanding mathematics comes down to this: knowing the relationships between theorems that one studies" (P. van Hiele, 1984b, 243). Problems occur in teaching geometry because the teacher knows the relationships among theorems while the student does not even know what a theorem is. Often students do not even understand basic concepts underlying theorems. If material is not presented carefully, students can operate by rote memorization. If relationships are not based on students' prior experiences and are not connected to the real world, they will be forgotten in a very short time and/or the student will have no idea how to apply the relationships in a new situation.

Van Hiele gave five levels of geometric thought. In his original research, they were called levels zero through four. Some researchers in the United States have expanded this scale by remembrance the original levels one through five and adding a new level zero which is used to refer to students who lack basic knowledge of geometry. Using this revised numbering system, level one is the base level. At this level, figures are judged by appearance. At the second level, figures are judged by their properties. At the third level, properties are ordered. At the fourth level, deduction is used. No description of the fifth level was given (Fuys, 1985).

Underlying characteristics of the level theory were also given. First, intrinsic concepts at one level become extrinsic at the next level. For instance, at level one, a student determines the name of a figure by how it looks (i.e., the properties of the figure are intrinsic). However, when the student moves to level two, he becomes aware of those properties and can name them. Second, each level has its own language and symbols. Third, two people at different levels cannot understand each other. It is critical to keep this characteristic in mind when teaching geometry. Finally, progression from one level to the next is accomplished in phases.

In progressing from one level to the next, the first phase is inquiry. In this phase, the student becomes familiar with the topic through the use of examples and nonexamples. The second phase is directed orientation. Through the use of carefully sequenced materials, the student can be led to discover desired relationships. The third phase, explication, occurs when the student becomes conscious of relationships and begins to use the correct technical language. Free orientation is the fourth phase. In this phase, the student applies relationships to a more complex task. For example, once properties of a particular geometric figure are learned, these same properties may be explored for a different figure. The final phase is integration. At this point the student is able to summarize what has been learned. These phases are not strictly sequential. In the study of any new topic, forward and backward movement among phases two, three, and four will occur (P. van Hiele, 1984b; Fuys, 1985).

It was van Hiele's paper that caught the attention of the Soviet Union and led to a complete revamping of that country's geometry curriculum. Since the van Hieles' materials were not available in English, the van Hiele level theory did not receive much attention in the United States until the early 1980s. At that time, three studies exploring the van Hiele level theory received federal funding. These studies were the Oregon Project, the Brooklyn Project, and the CDASSG Project at the University of Chicago.

The Oregon Project

The Oregon Project, directed by William Burger at Oregon State University, was entitled "Using the van Hiele Model to Describe Reasoning Processes in Geometry." In this study, 48 students from kindergarten through grade 12 and one college mathematics major were audiotaped during two 45-minute interviews in which they were asked to do tasks involving triangles and quadrilaterals. Interviews of 14 of the students were analyzed in depth by three reviewers. Qualitative analysis of these interviews implied that a student's thinking about geometric concepts is initially based on visual clues. The interviews also confirmed van Hiele's description and sequence of the levels. However, discreteness of levels was not confirmed, i.e., some students could best be described as in transition from one level to the next. This was especially true between levels two and three. Further, use of formal deduction was nearly absent -- even among geometry and post-geometry students (Fuys, 1985).

Several observations were made from the research (Hoffer, 1981).

1. The van Hiele levels 1, 2, and 3 are useful in describing students' reasoning processes in geometry.
2. No secondary school students were reasoning at level 4. It is suspected that this level of reasoning is rare at this age.

3. It is very likely that the teacher and students are reasoning at different levels. When the teacher writes a definition on the chalkboard (level 3), the student is worrying about all the properties that have been left out of the definition (level 2).
4. The student's view of a concept is often vastly different than what the teacher thinks the student's view is. The concept of triangle means different things to different students. Some students include more shapes than the teacher does. Others strictly limit the number of figures to be included.
5. A year after taking geometry, students may regress to a lower van Hiele level. Responses from post-geometry students were quite similar to responses from pregeometry students except the post-geometry students had a better vocabulary.

Three suggestions for changing the way we teach geometry were made. First, all secondary students should take an informal geometry course. For most students, this would be a full year. For some students, a more formal approach the second semester may be appropriate but many of the traditional topics could be omitted. Second, activities need to be developed that will move students through the van Hiele levels. Very little material exists to help move students from level 1 to level 2 or from level 2 to level 3. Finally, more geometry needs to be taught in elementary and junior high schools. In the Soviet Union, students in grades one to three study the properties of geometric shapes and the relationships among the shapes. In grade four, they begin a semi-deductive study of geometry that continues for the next seven years (Hoffer, 1981).

The Brooklyn Project

From 1980 to 1983, a team at Brooklyn College conducted a study entitled "An Investigation of the van Hiele Model of Thinking in Geometry Among Adolescents." This project considered whether the van Hiele model describes how students learn geometry. The study had four specific objectives. The first objective was to develop and document a working model of the van Hiele levels using several of the van Hieles' writings after translation from Dutch to English. The second objective was to characterize the learning of geometry by sixth- and ninth-graders. The study explored what levels the students were at, whether they could progress to higher levels, and what difficulties they encountered along the way. The third objective was to determine whether teachers could be trained to identify van Hiele levels. The final objective was to analyze textbooks with regard to the van Hiele levels.

To develop a working model of the van Hiele level theory, writings of the van Hieles were reviewed for specific behavioral descriptors and examples. Over 100 passages were identified that related to the levels. In the end, 70 of these passages were used to document what each level meant (Fuys, 1985, 62-78). The passages indicate:

> Thinking at a particular level is more than just knowing content and performing certain geometric processes. It is also being aware of what is expected, planning purposefully to think on a level, and monitoring one's thinking as a problem is solved. (Fuys, 1985, 85)

After the working model was defined, three instructional modules were developed for use with 16 sixth-grade and 16 ninth-grade students from inner city schools. These modules were presented in eight interviews conducted over a three-week period. The purpose of the modules was to assess the students' incoming van Hiele levels and monitor any changes that occurred as students progressed through the modules.

The study concluded that the van Hiele level model provides a reasonable structure for describing the ways students learn geometry. Analysis of the student interviews identified some factors that merited further attention. Those factors were language, misconceptions from prior learning, and learning styles.

The language factors center around student confusion between the mathematical meaning of a word and the way the word is used in everyday conversation. Students have trouble remembering new words or the mathematical meanings of the more common words. In the interviews, students wanted to point and give one-word answers. The impact of language factors on the learning of geometry can be reduced if teachers encourage the use of proper terminology and insist that students give explanations for their answers.

Misconceptions from, and confusion caused by, prior learning were also apparent in the interviews. An example of a misconception occurred when a student insisted that a figure was an angle only if it had a horizontal ray. If there was no horizontal ray, the figure was not an angle. Misconceptions like this occur when the student has not been shown a sufficient variety of examples and nonexamples of the concept. An example of confusion was evident when a student insisted that a square was not a rectangle because a rectangle had to have two congruent long sides and two congruent short sides.

Perceptual difficulties can also lead to misconceptions. Some students can identify figures only if they have a specific orientation. If the figure is not oriented properly, they will turn it to the proper orientation. Other students have difficulties that can be attributed to seeing only a limited range of figures. For example, the only triangle recognized might be an equilateral triangle with one horizontal side. A long, skinny triangle would not be recognized as a triangle.

Learning style problems referred to in this study might have been more properly called attitude problems. Students wanted to be given the correct rule to apply. To them, mathematics was a subject to be memorized and recalled; discovery and reasoning did not play a part in the learning process. "The idea that one could stop and think about a geometry problem, explore it, and find a solution without using a rule was new to many students" (Fuys, 1985, 183). Once students realized explanations, reasons, and justifications were expected, they began to make progress.

The study found that two major factors influencing incoming van Hiele level were the student's ability and prior experience. While many students in the study made good progress through the levels, some students made little or no progress. Some factors that might explain the lack of progress are:

1. Lack of prerequisite knowledge
2. Poor vocabulary or lack of precise language
3. Unresponsiveness to directives and given signals
4. Lack of realization of what was expected of them
5. Lack of experience in reasoning and explaining
6. Insufficient or inappropriate activities to promote progress
7. Insufficient time to assimilate new concepts and experiences
8. Rote learning attitude
9. Not reflective about their own thinking

Another phase of the study worked with eight preservice and five inservice teachers. It was concluded that teachers could be trained to recognize van Hiele levels in student responses and in the review of textbooks.

To determine the van Hiele levels required to understand textbooks, teacher and student books for three commercial K - 8 textbook series were reviewed. It was found that once a topic was introduced, it was reviewed each successive year. The average percent of pages devoted to geometry topics ranged from 4.4% in the first and second grade to 16.2% in the eighth grade. The vast majority of lessons were at level one. Even when material was presented at a higher level, over 90% of the exercises were at level one. Besides the low level of thinking required by textbooks, several other problems were noted. In some lessons, students could easily develop misconceptions because insufficient numbers of nonexamples were given. Misconceptions could also develop because figures were not shown in a variety of orientations. There were very few, if any, questions that required answers in the form of sentences. One-word answers (especially yes/no answers) make it difficult to assess what level the student is at. Such one-word answers make it difficult to determine whether terminology and concepts are really understood or whether the answers are merely lucky guesses or based on how the figure looks. Finally, almost no test questions were included that could not be done with level one thinking or by rote memorization of a formula. In conclusion, "Students will presumably encounter difficulty with a secondary school geometry course at level [3] if they can successfully complete grade 8 with level [1] thinking" (Fuys, 1985, 221).

Six suggestions were made as a result of this research.

1. Teachers should not rely on textbooks when it comes to guiding students through the van Hiele levels. The textbook should be a supplement to other activities and experiments.
2. Students should be encouraged to talk about geometry and helped to develop the language of the subject.
3. Teachers need to be aware of misconceptions students may develop from the lack of visual experiences provided in the textbooks. Both examples and nonexamples are critical.

4. To move from level 1 to level 2, students should be encouraged to test many examples (drawings or manipulatives) to determine if properties are true or false.
5. Students should be required to explain their answers. This will facilitate movement from level 2 to level 3.
6. Tests should include questions that require higher levels of thinking, not just rote memorization.

CDASSG Project

The third major study in the 1980's was led by Usiskin at the University of Chicago and is formally known as the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project. The final report for the project is entitled Van Hiele Levels and Achievement in Secondary School Geometry. "The fundamental purpose of this project is to test the ability of the van Hiele theory to describe and predict the performance of students in secondary school geometry" (Usiskin, 1982, p.8).

The van Hiele level theory has three very appealing properties. First, it is elegant. That is, it has a very simple structure and can be described with very simple statements. One level provides the building blocks for the next level. Second, it is comprehensive. It explains the learning of the entire subject of geometry, explains why students have trouble learning geometry, and suggests what could be done to remove the stumbling blocks. Finally, the theory has wide applicability. It is being used in the Netherlands, the Soviet Union, and the United States. The problem with the theory, as perceived by the CDASSG project, was that these properties (elegance, comprehensiveness, and wide applicability) had led to the acceptance of a theory that had never really been tested. The CDASSG project was designed to substantiate the van Hiele level theory.

The CDASSG project utilized four tests..

1. Van Hiele Level Test. The writings of the van Hieles were examined for passages that described behaviors at each level. From these passages, a 25-question multiple-choice test was developed having five questions at each level. The goal was to have easy questions that would adequately assess each level. Discussion of the grading of this test can be found in Appendix B.
2. Proof Test. Three different versions of a proof test that could be graded holistically were

developed. Each test had six problems. The first problem required students to fill in blanks in a proof that was nearly complete. In the second problem, students were given a statement and asked to draw the figure described by the statement. They were also asked to determine what they would use as the "given" and "prove" if they wanted to prove the statement was true. The final four problems required the students to do complete proofs.

3. Entering Geometry Test. This was a 20-question multiple-choice test developed in the 1970's by a student at The Ohio State University. The goal of this test was to determine the incoming knowledge of geometry students. The test covered geometry material that a student should have studied in junior high school.
4. CAP Test. The Comprehensive Assessment Program (CAP) Geometry Test, published by Scott, Foresman and Co., is a commercially available standardized test whose questions are representative of the geometry curriculum taught today. The only other comparable test, the Cooperative Test - Geometry, published by Educational Testing Service, was

already used at some of the schools in the study. To avoid possible bias due to teacher familiarity with the Cooperative Test, the CAP Test was used instead.

It is necessary to use both the van Hiele Level Test and a standardized test because students can be successful on standardized tests by using memorized definitions and theorems or by applying algebra. Questions on the van Hiele Level Test tend to be more conceptual and require students to do some mental analysis to reach the correct answer.

Study participants were students enrolled in geometry courses at 13 high schools representing a broad socioeconomic range. The students were in grades 7 to 12 with 56% of the students being tenth-graders. During the first week of school, students were given the Entering Geometry Test and the van Hiele Level Test. Three to five weeks before the end of the school year, students were given the van Hiele Level Test again, the Proof Test, and the CAP Test. Nearly 2700 students took one or more of the tests, but only 1596 students took all five tests. The study resulted in fourteen conclusions (Usiskin, 1982).

1. Level 5 either does not exist or cannot be tested. All other levels can be tested.
2. Depending on the grading criteria, 68% to 92% of students could be assigned a van Hiele level.
3. Arbitrary decisions made about the number of correct answers needed for classification to a van Hiele level can affect the level assigned to a student.
4. Students who have the same van Hiele level in the fall, have great variability in their spring van Hiele levels. About one-third of the students stay the same or go down, one-third go up one level, and one-third go up two or more levels. This suggests other factors play a part in the development of understanding in geometry.
5. Van Hiele level is a good predictor of concurrent performance on standardized multiple-choice tests of standard geometry content. Van Hiele level is also a good indicator of concurrent performance on the Proof Test, but performance on the standardized test is a better indicator.
6. A van Hiele level of 3 (using the classical or modified 3-of-5 criterion) or a van Hiele level of 2 (using the classical or modified 4-of-5

criterion) is the dividing line between concurrent success and failure with proofs. Students above these levels are likely to succeed with proofs while students below these levels are likely to fail with proofs.

7. Even in classes that have studied proof during the year, some of the students end the year with van Hiele levels too low to be successful with proofs.
8. A student's fall van Hiele level is a good predictor of spring performance on a standardized multiple-choice test on geometry content. It is not as good a predictor as either the Entering Geometry Test or the spring van Hiele level.
9. In classes that study proof, nearly half the students have fall van Hiele levels that are so low they have less than a 40% chance of succeeding at proofs.
10. Based on van Hiele levels, almost half of the geometry students are placed in courses where their chances of success with proofs are only 50-50.
11. Many students are not learning the basics of geometry in junior high school and are leaving high school without this basic knowledge.

12. Many students leave a geometry course without knowing the basic geometry terminology or ideas.
13. Of all high school students, 60% never study proofs. Only 13% of all high school students are successful with proofs.
14. There are no sex differences in the ability to learn geometry facts or proofs.

In other writings, Wirszup and Hoffer both claim that geometry as it is currently taught is inappropriate for the majority of students. A student needs to be at level 4 to understand proofs but most students are only at level 1. Given this, it is likely that the many students (47%) who never take geometry would not succeed anyway. Unfortunately, Entering Geometry Test results suggest that junior high school teachers do not cover many of the geometry topics assuming students will take geometry in high school (Usiskin, 1982).

The majority of students who take geometry know very little coming into the course. They will have to work very hard to avoid total failure with proofs since nearly half of all geometry students cannot do proofs or can only do trivial proofs even by the end of the geometry course. Few students enter the course with enough knowledge to be relatively assured they will not fail with proofs. Even fewer students enter the course at a high enough level to expect success with proofs. Based on the poor performance on the Proof Test by students who took courses that were supposed to include proofs, it appears that teachers either reduced the time spent on proofs (believing the students were not ready for proofs), or the teachers lowered their expectations regarding proof competence.

Students in some schools were found to know more about geometry at the beginning of the school year than students in other schools know after a full year of studying geometry. Due to the small number of schools (13), the reason for this difference could not be determined. It could be due to socioeconomic factors but could also be related to school size, region of the country, tax base, percentage of students enrolled in geometry, or other factors.

Geometry as it is currently taught is reaching only 30% of all high school students, and a third of those are receiving only a marginal benefit from the course. Tracking allows schools to better match the curriculum to the level of the entering geometry students. In schools with untracked classes, 57% of the students were at a van Hiele level too low to expect success in a proof-oriented course. The percentages for schools with two tracks and three tracks were 48% and 27%, respectively. The study also concluded that offering a non-proof alternative to the standard geometry curriculum could perhaps cut mismatches in half.

Other Related Literature

At the University of Oregon, many freshmen are surveyed each year about their feelings towards mathematics. While the students have a variety of favorite topics, there is almost unanimous agreement that the least favorite topic in high school is geometry. When asked why they disliked geometry, the most common responses were "Had to prove theorems all year long."; "Didn't understand what it was all about."; "Got through the course by memorizing proofs."; "We did more theorems than geometry." From these surveys, classroom observations, and discussions with teachers and students, Hoffer concludes that too many geometry teachers may be putting too great of an emphasis on the writing of proofs. This emphasis uses up class time that might be better spent developing other geometry related skills such as visual skills, verbal skills, drawing skills, logical skills, and applied skills. Also, if formal proofs are started too early in a geometry course, the students may not have reached a "sufficiently high level of mental development to enable them to function adequately at the formal level" (Hoffer, 1981, 17).

Hoffer created a high school geometry course that developed geometric concepts informally (that is, without formal proof) during the first semester. Students studied what they called "fun things," but during that first semester they began using the reasoning needed for formal proofs when explaining why they thought an assertion was true. He suggests that we need to become aware of how students learn geometry so that we can provide them with effective learning experiences (Hoffer, 1981).

Results of the 1977-78 National Assessment of Educational Progress (NAEP) show that students have some knowledge of basic geometric concepts but have too little knowledge of the properties associated with those concepts and the ability to apply those concepts is limited. It is thought that the formal language used in some of the problems may have lowered student performance (Kerr, 1981).

Research cited by Kerr (1981) reports that informal geometry has become a well-established part of the elementary and middle school mathematics curriculum. This provides the opportunity to use the spiraling approach to include increasingly more sophisticated geometry content throughout the curriculum. However, the spiral is interrupted when high school students do not continue the study of geometry. High school geometry is perceived as a difficult course and many high school students and counselors do not believe that the study of geometry serves any real purpose. Even if students continue with high school geometry, the spiral may not continue if the connection between informal and formal geometry is not made.

Crowley points out that language is important in the development and assessment of geometric understanding. Verbalization allows students the opportunity to solidify concepts that might otherwise remain vague or undeveloped. Verbalization also reveals any misconceptions. Initially, there should be little concern with the exact words used by the student. The students should be gradually introduced to standard geometry terminology and encouraged to use it. Teachers should model the correct terminology with particular emphasis on the language related to a certain van Hiele level. At level two, this would be an emphasis on modifiers like "all", "some", "always", "never", etc. The emphasis would be on phrases like "it follows that" and "if ..., then ..." at level three. At level four, "axiom", "postulate", "theorem", "converse", "necessary and sufficient", etc. would be used and their meanings emphasized (Crowley, 1987).

For learning to occur, activities must be matched to the student's van Hiele level. Teacher questioning is the perfect tool for assessing the student's van Hiele level. The student's response to "How do you know that?" reflects the level at which the student is reasoning (Crowley, 1987).

The secondary school geometry curriculum can be improved in a number of ways. First, the excessive emphasis on rigor in the beginning geometry class should be eliminated. Beginning algebra and beginning calculus classes do not emphasize proofs or theorems. The quadratic formula is really a theorem that can be proved but little, if any, emphasis is put on that in a beginning algebra class. Second, teachers should get to the heart of geometry as soon as possible. The Pythagorean theorem has practical applicability and is important to the study of further mathematics, but it takes over 300 pages to reach it in most textbooks. Third, the teaching of geometry should incorporate the techniques of algebra and analytical geometry and not just rely on Euclidean methods. If a proof can be done more simply using algebra, it should be done that way. This helps students see the interrelationship of math courses and help dispel the belief that geometry is an isolated subject unrelated to other mathematics courses. Fourth, geometry should be related to the physical world. Fifth, teachers of geometry need to eliminate the wordiness so often encountered and avoid dwelling on the obvious. Sixth, excessively long and/or difficult proofs should be eliminated or delayed until the student has learned other mathematical techniques that will simplify the proof. Finally, geometry textbooks need to include more problems of intermediate difficulty. Too many of the current problems are extremely easy (Niven, 1987).

Usiskin (1987) suggests four steps that can be taken to increase student performance in geometry. First, an elementary school geometry curriculum by grade level should be specified. There is too much geometry that needs to be learned to wait until high school. Second, students should not be prevented from studying geometry because they are poor at arithmetic or algebra. This is comparable to telling a person that they cannot bowl because they aren't any good at basketball. Third, a significant amount of competence in geometry should be required of all students. Finally, all prospective teachers should be required to study geometry at the college level. Many elementary teachers' only exposure to geometry has been in a high school course (if that). Elementary school teachers are well trained to teach arithmetic but need to be as well trained to teach geometry. High school teachers also need this training. Some high school teachers enjoy teaching geometry while others avoid it at all costs.

Johnson (1989) studied a sample of 1066 students to learn more about van Hiele levels, methods of scoring the van Hiele Test, and geometry achievement. Students were given the Entering Geometry Test and the van Hiele Level Test near the end of the second semester of algebra. The van Hiele Level Test was given again near the end of the first semester of geometry. Near the end of the second semester of geometry, students were given the van Hiele Level Test and the CAP Test. Johnson correlated entering van Hiele levels with success in geometry -- defined as getting 14 or more correct on the CAP Test. Johnson found that the three best criteria for assigning van Hiele levels were the modified 3-of-5, differentiable forced, and forced 3-of-5. Using the forced 3-of-5 criterion, she found the following:

61.5% of level 0 students were unsuccessful
52.8% of level 1 students were unsuccessful
38.1% of level 2 students were unsuccessful
19.4% of level 3 students were unsuccessful
0.0% of level 4 students were unsuccessful

Several articles have suggested activities to be used in the geometry classroom (Crowley, 1987; Dana, 1987: Hoffer, 1981; Kerr, 1981; Prevost, 1985; Shaughnessy and Burger, 1985; Sobel and Maletsky, 1988; D. van Hiele, 1984b). While it is assumed that the specified activities will help students move up the van Hiele levels, no research supporting that contention has been found. Other than Dina van Hiele-Geldof's dissertation, no other research could be found having all the following characteristics:

1. Designed around the van Hiele model.

2. Used on a daily basis over an extended period of time (one or more semesters).
3. Shows students attained a higher van Hiele level at the end of the time period.

CHAPTER III

RESEARCH METHODOLOGY

Using currently available research and the assistance of an outside consultant, William S. Mason High School developed a one-semester geometry readiness curriculum for use in the Algebra I Part II course. The intent of this course was to prepare these students for the formal geometry course that came next in the mathematics curriculum. The curriculum was developed based on the van Hiele model, activities published in professional journals and books, and activities developed in-house. It was included as part of the year-long course Algebra I Part II beginning the second semester of the 1989-90 school year and revised during the summer of 1990. The topics covered in Algebra I Part II are outlined in Appendix C. The course as taught in the 1990-91 school year is the subject of this research.

The study consisted of two groups. The test group included 84 students enrolled in Algebra I Part II (the course including the geometry readiness curriculum). The control group included 47 students enrolled in Algebra I, which followed the normal algebra curriculum. Graphing on the coordinate plane, review of perimeter and area calculations, and the Pythagorean Theorem were the only geometry topics covered.

Both groups took the van Hiele Level Test and the Cooperative Mathematics Test - Geometry - Part A at the beginning and end of the school year. To prevent teacher bias, the teachers were not shown any test scores until after the end of the school year.

Using the van Hiele Level Test, three results were recorded for each student: van Hiele level, weighted sum score, and number of correct responses. See Appendix B for a further discussion of the assignment of van Hiele levels and weighted sum score.

Johnson identified twelve possible scoring methods for the van Hiele Level Test. Under some scoring methods, it may not be possible to assign van Hiele levels to all students. Slightly different levels may be assigned to students depending on which scoring method is used. For further discussion of this, see Appendix B.

For each student, the number of correct responses (out of a possible 40) on The Cooperative Test - Geometry - Part A was recorded.

A total of 131 students took one or more of the tests. Details of the number of students tested is given in Table 1.

Table 1.-- Disposition of Participating Students

Test	Algebra I	Algebra I Part II
Students taking one or more of the tests	47	84
Students taking Coop Test fall and spring	39	58
Students taking van Hiele Level Test fall and spring	35	59
Students taking all four tests	31	54

When results of the van Hiele Level Test were analyzed, all students taking both the fall and spring tests were included. Likewise, when results of the Cooperative Test were analyzed, all students taking both the fall and spring tests were included. No attempts were made to show correlation between the results of the the van Hiele Level Test and the results of the Cooperative Test. Test results for each student are shown in Appendix D.

CHAPTER IV

FINDINGS

Pre-test results for Algebra I and Algebra I Part II classes were compared to determine if the groups possessed similar incoming knowledge. Analogous comparisons were done with the post-test results to determine if the courses affected student achievement. T-tests and nonparametric Wilcoxon Rank Sum tests were used. To determine what changes occurred during the year, pre-test and post-test scores within each course were compared using paired t-tests and nonparametric Wilcoxon Signed Rank tests. Results of the t-tests and Wilcoxon Tests were the same so only the Wilcoxon p-levels are reported. Statistical significance was defined as a two-sided alpha-risk of 0.05. Statistical analysis was done using SAS (Statistical Analysis System, version 6.04, SAS Institute Inc., Cary, NC).

Results of the Cooperative Geometry Test

Ninety-seven students took both the pre- and post-Cooperative Geometry test. Thirty-nine were enrolled in Algebra I and 58 were enrolled in Algebra I Part II. Results are summarized in Table 2.

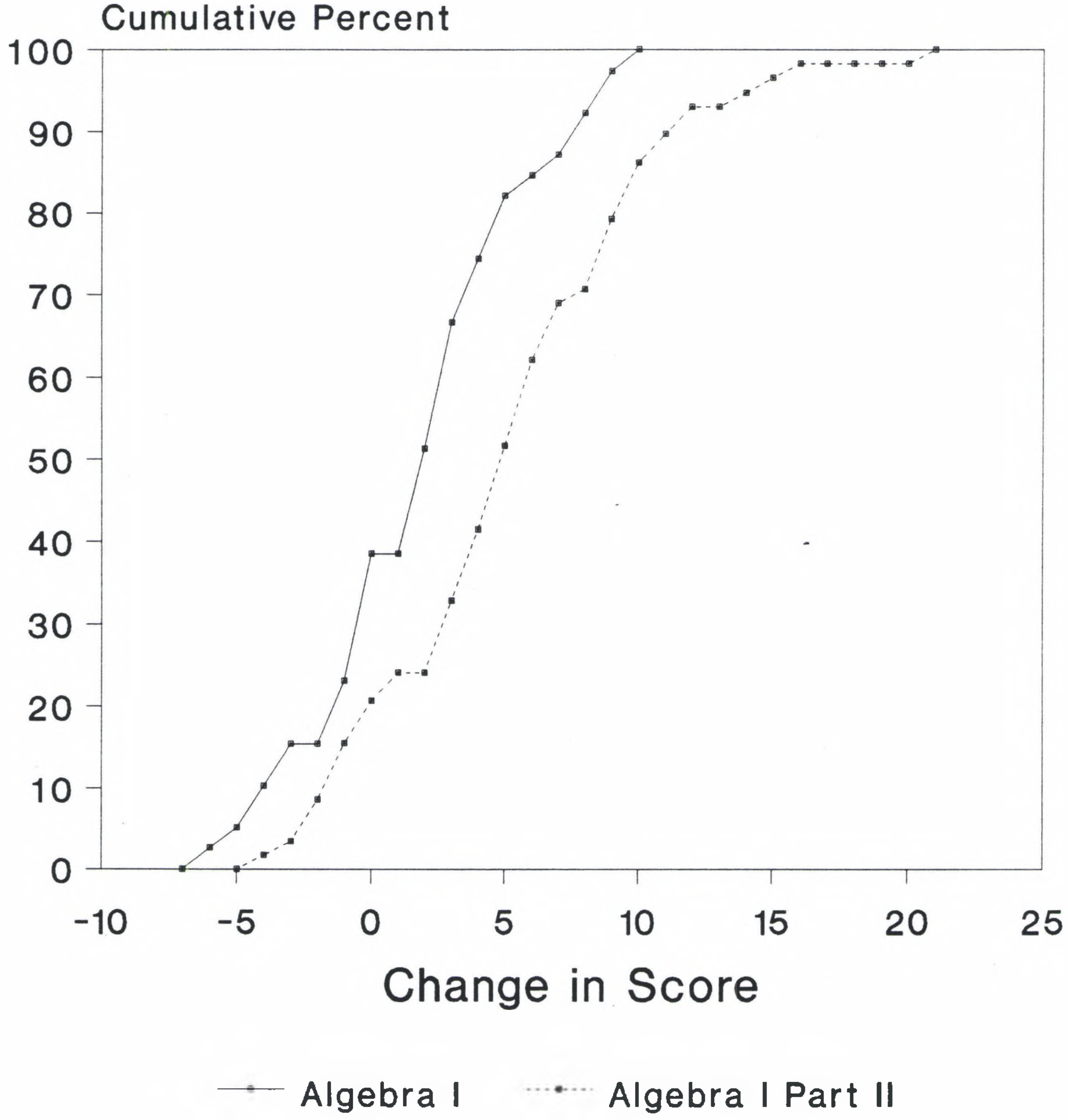
Table 2.--Results of Cooperative Geometry Test

Test	Algebra I (N=39)	Algebra I Part II (N=58)
Pre-Test number correct		
Mean $\pm$ Std dev	12.5 ± 3.7	12.4 ± 3.8
Median	12	12
Range	6 - 21	5 - 20
Post-Test number correct		
Mean $\pm$ Std dev	14.6 ± 3.9	17.9 ± 3.9
Median	14	18
Range	8 - 22	8 - 28

There was no significant difference in pre-test scores between the classes ($p=0.98$). The average number of correct responses was significantly higher on the post-test for both classes (Algebra I, $p=0.002$; Algebra I Part II, $p<0.001$). However, the increase in the Algebra I Part II class was significantly greater than the increase in the Algebra I class ($p=0.001$).

Table 3 and Figure 1 show the extent to which students' scores changed from the pre-test to the

Figure 1
Change in Coop Test Scores



post-test. These emphasize the greater increase in scores in the Algebra I Part II class.

Table 3.--Coop Score Change from Pre-Test to Post-Test

Change	Algebra I (N=39)	Algebra I Part II (N=58)
up 12 or more	0.0%	6.9%
up 9 - 12	7.7	22.4
up 5 - 8	17.9	29.3
up 1 - 4	35.9	20.7
no change	15.4	5.2
down 1 - 4	18.0	15.5
down 5 - 8	5.1	0.0

Results of the van Hiele Test (van Hiele Levels)

The 3-of-5 forced criterion was used to assign a van Hiele level to each student. Differences between the two classes were not significant on either the pre-test ($p=0.36$) or the post-test ($p=0.25$). Although the differences were not significant, it should be noted that the Algebra I Part II students had greater upward shifts in van Hiele levels as evidenced by slightly lower levels on the pre-test and slightly higher levels on the post-test. These shifts are shown graphically in Figures 2 and 3. Figure 2 compares Algebra I levels and Algebra I Part II levels in fall and spring; Figure 3 compares pre-test and post-test levels separately for each course.

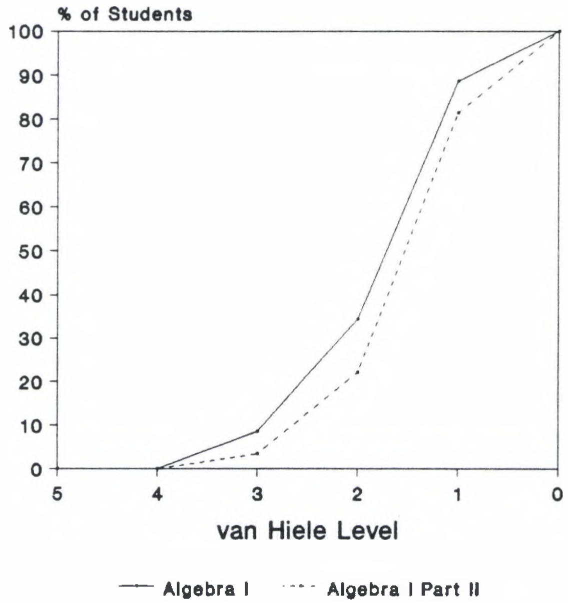
The chi-square test was also used to analyze results from each of the six possible van Hiele scoring criteria used in this project (see Table 4). Regardless of criteria used, there were no significant differences in the pre-test levels. Post-test levels of the Algebra I Part II class were significantly higher only for the modified 4-of-5 and forced 4-of-5 criteria.

Table 4.--Results of Chi-Square Tests Comparing Algebra I and Algebra I Part II van Hiele Levels

Test	Criterion	Chi-Square	P-level
Pre-Test	Conservative 3-of-5	5.34	0.15
	Modified 3-of-5	2.63	0.45
	Forced 3-of-5	2.05	0.36
	Conservative 4-of-5	0.34	0.56
	Modified 4-of-5	0.00	1.00
	Forced 4-of-5	0.01	0.95
Post-Test	Conservative 3-of-5	4.54	0.21
	Modified 3-of-5	2.67	0.26
	Forced 3-of-5	2.73	0.26
	Conservative 4-of-5	4.04	0.13
	Modified 4-of-5	4.96	0.08
	Forced 4-of-5	9.84	0.01

Figure 2

van Hiele Pre-Test Levels



van Hiele Post-Test Levels

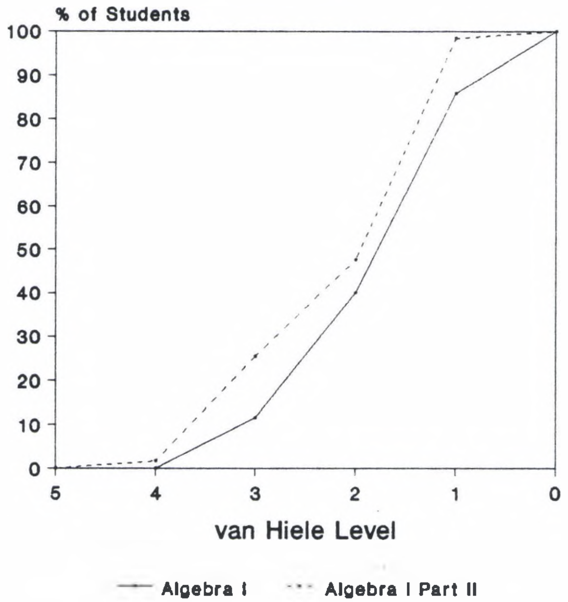
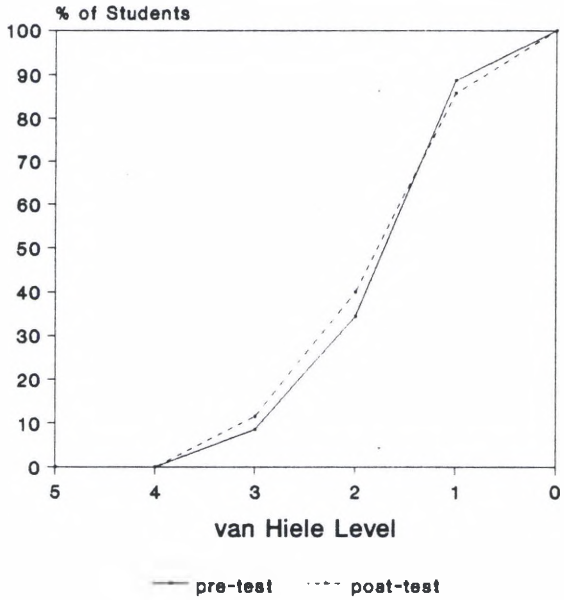


Figure 3

Algebra I van Hiele Level



Algebra I Part II van Hiele Level

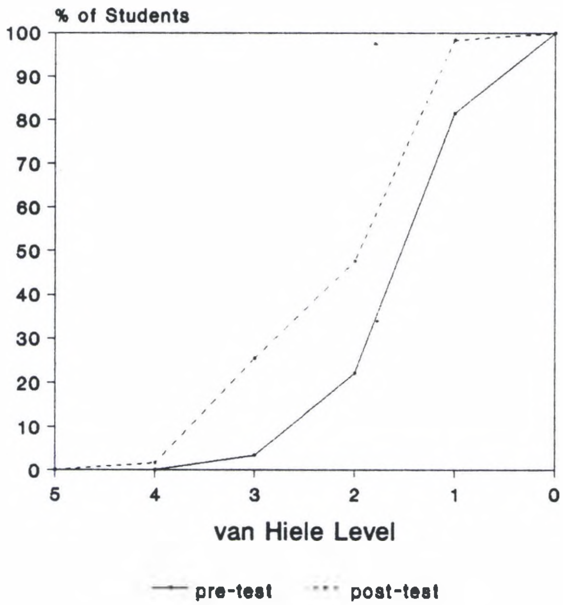


Table 5 shows how the post-test distribution of van Hiele levels in this study compares to the CDASSG Project and Johnson. The Algebra I distribution is quite similar with a slightly higher percentage of students at level 0 and no students at level 4. The Algebra I Part II distribution shows a lower percentage of students at both level 0 and level 4. It should be noted that the percentage of students at level 3 is approximately twice that of any other group.

Table 5.--Comparison of Percentage of Geometry Students at Each van Hiele Level (3-of-5 Forced Criterion)

Level	Johnson	CDASSG	Algebra I	Part II
0	11.5%	9.4%	14.3%	1.7%
1	48.8	46.0	45.7	50.9
2	24.3	28.4	28.6	22.0
3	11.9	12.0	11.4	23.7
4	3.5	3.9	0.0	1.7
No Fit	0.1	0.3	0.0	0.0

(Johnson and CDASSG data from Johnson, 1989, 111)

Table 6 shows the distribution of students' van Hiele levels on the pre- and post-test. While the distribution of Algebra I students stays virtually the same, there is a large decrease in Algebra I Part II students at level 0 and a large increase in students at level 3.

Table 6.--Distribution of van Hiele Levels Using the Forced 3-of-5 Criterion

	Pre-Test		Post-Test	
Level	Algebra I (N=35)	Algebra I Part II (N=59)	Algebra I (N=35)	Algebra I Part II (N=59)
0	11.4%	18.6%	14.3%	1.7%
1	54.3	59.3	45.7	50.9
2	25.7	18.7	28.6	22.0
3	8.6	3.4	11.4	23.7
4	0.0	0.0	0.0	1.7

The CDASSG Project concluded that level 3 (using the 3-of-5 conservative or modified criterion) is the dividing line between failure and success with proof. If we assume that level 3 is also the dividing line with the forced 3-of-5 criterion, then the chances of the Algebra I Part II students being successful with proofs was improved while the Algebra I students were virtually unaffected. At the beginning of the year, 96.6% of the Algebra I Part II students were below level 3. By the end of the year, that percentage dropped to 74.6%. The percentage of Algebra I students below level 3 remained nearly the same - 91.4% at the beginning of the year and 88.6% at the end of the year. However, only one student (in Algebra I Part II) could be described as quite likely to succeed with proof as defined by the CDASSG Project.

Applying Johnson's probabilities that correlate van Hiele level with success on standardized geometry tests (see Chapter Two), 50.5% of the Algebra I Part II students and 48.0% of the Algebra I students in this study would be unsuccessful in geometry based on beginning of the year van Hiele levels. By the end of the year, these numbers would become 40.9% for Algebra I Part II students and 46.0% for Algebra I students. Again, Algebra I Part II students showed improvement while Algebra I students showed little change.

Changes in van Hiele level from pre-test to post-test are shown in Table 7. More Algebra I Part II students reached a higher van Hiele level during the year (55.9% versus 31.4%) while fewer dropped to a lower level (8.5% versus 25.8%).

Table 7.--Change in van Hiele Level
(Forced 3-of-5 Criterion)

Change	Algebra I (N=35)	Algebra I Part II (N=59)
+3	0.0%	1.7%
+2	14.3	16.9
+1	17.1	37.3
0	42.8	35.6
-1	14.3	6.8
-2	8.6	1.7
-3	2.9	0.0

Results of the van Hiele Test (Weighted Sum Score)

As explained in Appendix B, a weighted sum score can be assigned to each van Hiele test. Table 8 shows average weighted sum scores for each course.

Using the 3-of-5 criterion to compare the classes, the average weighted sum score of the Algebra I class was significantly higher on the pre-test ($p=0.01$). On the post-test, there was no significant difference ($p=0.22$). This will be discussed further in the conclusions.

The same analysis using the stricter 4-of-5 criterion gives slightly different results. The difference between courses in pre-test weighted sum

Table 8.--Van Hiele Weighted Sum Scores

	Algebra I (N=35)	Algebra I Part II (N=59)
3-of-5 Criterion		
Pre-Test Avg$\pm$Std dev	5.3$\pm$6.2	2.5$\pm$3.7
Median	3	1
Post-Test Avg$\pm$Std dev	5.9$\pm$7.2	6.0$\pm$6.5
Median	3	5
4-of-5 Criterion		
Pre-Test Avg$\pm$Std dev	2.2$\pm$4.7	1.0$\pm$2.2
Median	1	1
Post-Test Avg$\pm$Std dev	0.9$\pm$1.3	2.7$\pm$3.8
Median	1	1

scores is not statistically significant ($p=0.35$). However, the Algebra I Part II students had significantly higher post-test weighted sum scores ($p=0.001$).

The differences between average scores and median scores in Table 8 merit further comment. The distribution of van Hiele weighted sum scores using the 3-of-5 criterion is highly skewed. As seen in the table, this results in higher averages than medians. However, in the 4-of-5 criterion data, the majority of weighted sum scores are 0 or 1 which causes problems with both averages and medians. Under these circumstances, minor fluctuations of 1-2 points in average scores are obtained spuriously by a few outlying data points. For example, the drop from pre-test to post-test average for Algebra I (2.2 to 0.9) is due to three students who had pre-test weighted sum scores of 17 but post-test weighted sum scores of only 0 or 1. Moreover, when the majority of values are 0 or 1, a shift in median score is unlikely to be observed.

A better indicator in this situation is the change in weighted sum scores. These changes are shown in Tables 9 and 10 for the 3-of-5 criterion and 4-of-5 criterion, respectively. Once again, Algebra I Part II students had greater improvement than Algebra I students.

Changes within class were also analyzed. Under either criterion, there was no significant change in the Algebra I weighted sum scores (3-of-5, $p=0.93$; 4-of-5, $p=0.46$). The Algebra I Part II weighted sum scores were significantly higher on the post-test under either criterion (3-of-5, $p<0.001$; 4-of-5, $p<0.001$). As before, changes across the year were

Table 9.--3-of-5 Weighted Sum Score Change

Change	Algebra I (N=35)	Algebra I Part II (N=59)
up 26 - 30	2.9%	1.7%
up 21 - 25	0.0	3.4
up 16 - 20	8.6	5.1
up 11 - 15	2.9	0.0
up 6 - 10	2.9	10.2
up 1 - 5	14.3	45.8
no change	31.4	23.7
down 1 - 5	22.9	5.1
down 6 - 10	8.6	1.7
down 11 - 15	0.0	3.4
down 16 - 20	5.7	0.0

Table 10.--4-of-5 Weighted Sum Score Change

Change	Algebra I (N=35)	Algebra I Part II (N=59)
up 16 - 20	0.0	3.4
up 11 - 15	0.0	0.0
up 6 - 10	0.0	10.2
up 1 - 5	17.1	37.3
no change	57.1	42.4
down 1 - 5	17.1	5.1
down 6 - 10	0.0	0.0
down 11 - 15	0.0	1.7
down 16 - 20	8.6	0.0

significantly greater in the Algebra I Part II course than with Algebra I (3-of-5, $p=0.003$; 4-of-5, $p<0.001$).

Results of the van Hiele Test (Number of Correct Responses)

Although the results of the van Hiele test are typically used only to assign a van Hiele level, the number of correct responses was also analyzed in this study. Results are shown in Table 11.

Table 11.--Number of Correct van Hiele Test Responses

	Algebra I (N=35)	Algebra I Part II (N=59)
Pre-Test number correct		
Average ± Std dev	9.3±2.1	7.9±2.2
Median	9	8
Range	6 - 15	3 - 17
Post-Test number correct		
Average ±Std dev	9.9±2.5	11.5±2.8
Median	10	11
Range	5 - 16	7 - 19

The number of correct responses on the pre-test was significantly higher in the Algebra I class ($p=0.004$) while the number of correct responses on the post-test was significantly higher in the Algebra I Part II class ($p=0.01$).

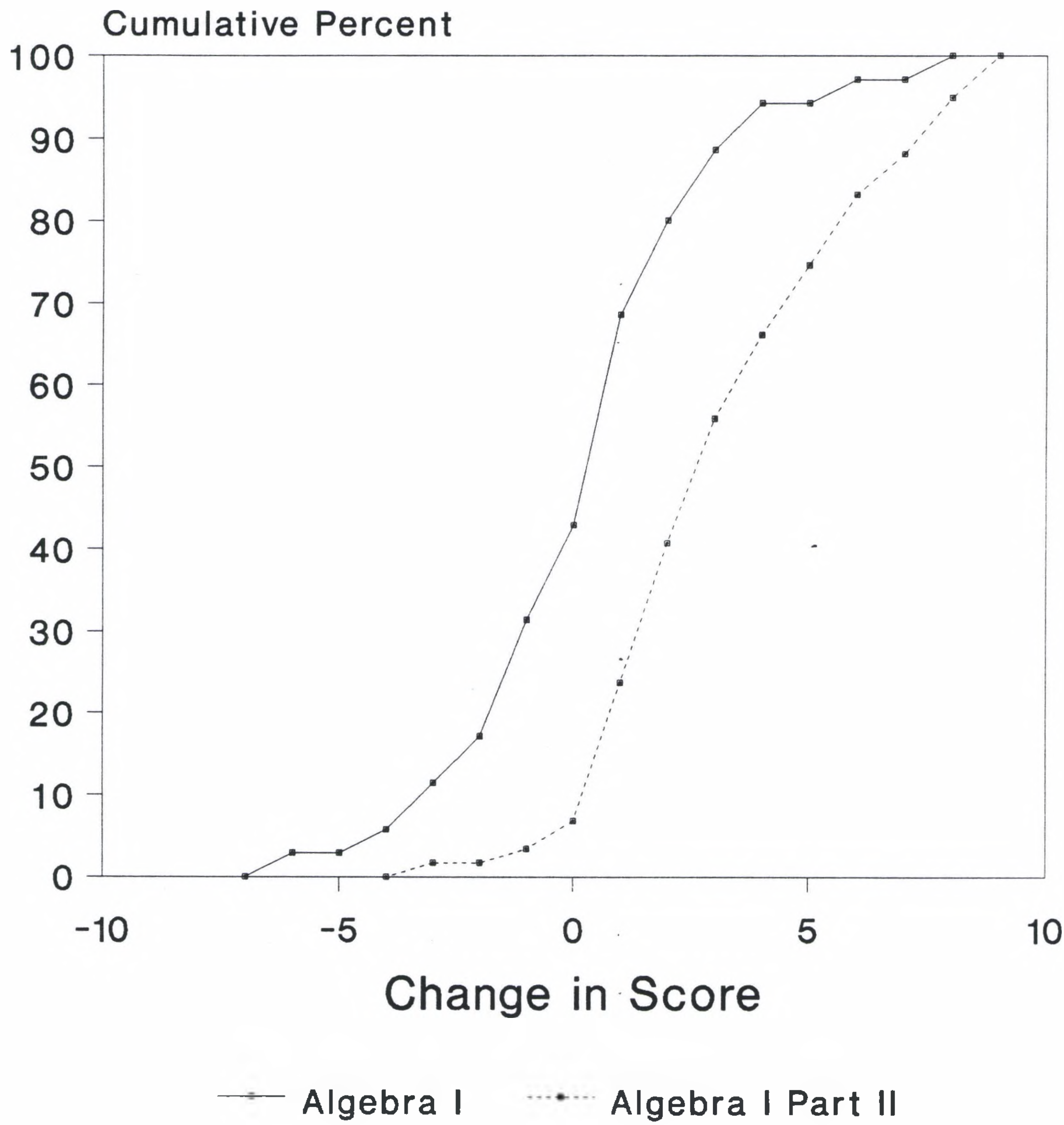
The change in average number of correct responses in the Algebra I class was not significant ($p=0.16$). However, the change in average number of correct responses in the Algebra I Part II class was significant ($p<0.001$).

Table 12 and Figure 4 show the extent to which the number of correct responses changed from the pre-test to the post-test. As before, this emphasizes the increase in learning in the Algebra I Part II class.

Table 12.--Change in Number of Correct Responses from Pre-Test to Post-Test

Change	Algebra I (N=35)	Algebra I Part II (N=59)
up 9 - 12	0.0%	5.1%
up 5 - 8	5.7	28.8
up 1 - 4	51.4	59.3
no change	11.4	3.4
down 1 - 4	28.6	3.4
down 5 - 8	2.9	0.0

Figure 4
Change in van Hiele Scores



CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions About Overall Achievement

The purpose of this research was to determine if one semester of a geometry readiness curriculum (included in Algebra I Part II) would increase students' knowledge of geometry. In reaching the conclusions, two things must be kept in mind. First, the Coop Test and van Hiele Level Test measure two different things. The Coop Test measures geometry achievement; questions in this test can be answered by either rote memorization or the use of algebra. The van Hiele Level Test is intended to measure levels of geometric thought. Answering questions on this test requires mental analysis. Memorization and algebra have little impact on performance on this test.

Second, the control group of Algebra I students were expected to have higher math achievement. Algebra I Part II is half of a two-year sequence designed for students that would benefit from a slower paced version of Algebra I. While not always statistically significant, the Algebra I students did, in fact, score higher on all pre-tests. This supports the in-going hypothesis that Algebra I students would start the year with higher achievement in geometry and higher levels of geometric thought.

By the end of the year, both groups of students had completed the same algebra curriculum. In addition, however, the Algebra I Part II students had completed one semester of a geometry readiness curriculum. Although students in both courses showed increases in geometry comprehension, increases of the Algebra I students were not statistically significant while increases of the Algebra I Part II students were. The significant increases by the Algebra I Part II group were likely due to the geometry emphasis in the curriculum while the smaller, nonsignificant increases of the Algebra I group likely resulted from geometry topics encountered in the typical study of algebra (perimeter, area, Pythagorean theorem, etc.). This indicates the geometry readiness curriculum allowed Algebra I Part II students to surpass their counterparts and complete the year with greater geometry achievement and correspondingly higher levels of geometric thought.

Conclusions About van Hiele Level

Algebra I Part II students showed substantial elevation in van Hiele levels from the beginning to the end of the year while Algebra I students showed little change. Based on previously cited research, many Algebra I Part II students have improved chances of succeeding in geometry. It is estimated that an additional 10% of students will be successful with geometry content and 22% will be successful with proofs after completion of the course. The Algebra I students' likelihood of success or failure with geometry content and proof was virtually unchanged. Again, differences in curriculum are the likely explanation.

Algebra I Part II students were much more likely to attain a higher van Hiele level by year end while Algebra I students were more likely to drop in van Hiele level. Emphasis on geometry is probably the reason for the increase in levels in Algebra I Part II. Lack of a geometry curriculum and/or lack of review of previously learned geometry is the reason for the drop in levels in Algebra I.

Summary of the Conclusions

The curriculum used in Algebra I Part II helped those students gain in geometry content knowledge and level of geometric thought. Some Algebra I students made gains from the limited geometry topics covered in their curriculum, but many appear to have lost knowledge that they had at the beginning of the year.

Recommendations

While improvements observed in the Algebra I Part II class indicate the curriculum was effective, problems were also revealed. First, several Algebra I students regressed to lower van Hiele levels during the year. Second, the majority of students from both groups will enter the formal geometry course at such low van Hiele levels that they will be unlikely to experience success with either geometry content or proofs.

To alleviate the first problem, the geometry curriculum which students encounter prior to the Algebra I course needs to be reviewed on a regular basis. Use of weekly review sheets is one way to maintain both geometry and algebra knowledge.

The second problem, the low van Hiele levels of students entering geometry, requires a significant effort to correct. The following ideas need to be considered:

1. This study confirms that a curriculum based on van Hiele level theory is effective in raising students' van Hiele levels. A carefully structured geometry curriculum needs to be introduced in the elementary and junior high grades so that students enter high school prepared to be successful in the formal geometry course. This curriculum will also give students the necessary knowledge to pass the ninth-grade mathematics proficiency test in the state of Ohio. Teachers need to be aware of the van Hiele level theory and use it to guide the development of curriculum, materials, and an understanding of their students.
2. Changes in the high school curriculum are needed. Until curricula at the lower grades can be revised, formal proofs should not be taught during the first semester of geometry. During the second semester, formal proofs may be appropriate only for some students. Other students may need a full year of a nonproof course. Tracking (or two different geometry courses) may best meet the needs of all geometry students.

Suggestions for Further Research

A follow-up study should be done to determine if the apparent advantage that Algebra I Part II students have over Algebra I students is sustained. Do Algebra I Part II students outperform Algebra I students throughout the geometry course? Or do the students that take Algebra I have inherently higher math abilities that allow them to overcome their slightly lower entering van Hiele levels and enable them to outperform Algebra I Part II students?

Another project is development of appropriate activities for elementary and junior high classrooms and determining if students can be brought to van Hiele level 3 prior to undertaking the high school geometry course.

Conclusion

This study showed that a curriculum can be used to raise van Hiele level of students. Other studies have shown that van Hiele level is related to success with geometry content and proof. If we want all students to be successful in the formal geometry course in high school, then a curriculum based on van Hiele level theory needs to be developed and implemented throughout the elementary and junior high grades.

APPENDIX A

STATE OF OHIO PROFICIENCY TESTING

According to the Ohio Department of Education, High School Proficiency Testing: Fact Sheets, Ninth-Grade Mathematics, sixteen of the forty test items on the ninth-grade proficiency test are designed to measure the following geometry related outcomes.

1. Select and compute with appropriate standard or metric units to measure length, area, volume, angles, weight, capacity, time, temperature, and money. Students will need to know when a particular measurement unit is appropriate and to know approximate measurements of common items.
2. Convert, compare, and compute with common units of measure within the same measurement system.
3. Read the scale on a measurement device to the nearest mark and make interpolations where appropriate. Test questions require students to read facsimiles of devices used to measure length, angles, weight, time and temperature, as well as to use that information to solve problems.
4. Recognize, classify, and use characteristics of lines and simple two-dimensional figures. Students will need to be familiar with concepts such as perpendicular, vertical, and parallel and to be knowledgeable about triangles, quadrilaterals, pentagons, and circles.
5. Find the perimeters (circumference) and areas of polygons (circles). Students will need to know formulas for calculating the area of triangles,

rectangles, and circles. Questions will involve a knowledge of formulas or strategies for finding the perimeter of a polygon and the circumference of a circle. Students will need to know an approximate value of pi is between three and four.

6. Find surface areas and volumes of rectangular solids. Questions will require knowledge of formulas and strategies for finding the surface area and volume of rectangular solids.

A draft of the learning outcomes to be tested on the twelfth-grade proficiency test beginning in 1993-94 includes the following:

The student will:

1. Determine area and volume.
2. Estimate and use measurements.
3. Apply the Pythagorean Theorem.
4. Use deductive reasoning.
5. Describe and apply the properties of similar and congruent figures.
6. Determine slope, mid-point, and distance.
7. Demonstrate an understanding of angles and parallel lines.

APPENDIX B

SCORING OF THE VAN HIELE GEOMETRY TEST

Several methods exist to assign van Hiele levels to students based on the results of the van Hiele Level Test. Each grading method considers two factors (Johnson, 1989):

1. How many questions must be answered correctly to indicate mastery of the subtest?
2. What van Hiele level should be assigned to a student based upon what subtests have been mastered?

Johnson (1989) identifies twelve possible grading methods. They are:

Classical, 3-of-5	Classical, 4-of-5
Modified, 3-of-5	Classical, 4-of-5
Forced, 3-of-5	Forced, 4-of-5
Rasch Modified, 3-of-5	Rasch Modified, 4-of-5

HOW MANY QUESTIONS MUST BE ANSWERED CORRECTLY FOR MASTERY?

In the CDASSG Project, mastery of a subtest was assumed by answering either 3-of-5 or 4-of-5 questions correctly (Usiskin, 1982).

The choice of criterion, given the nature of this test, is based upon whether one wishes to reduce Type I or Type II error. Recall that Type I error refers to a decision made (in this case a student meeting a criterion) when it should not have been made.

P(3 of 5 correct by random guessing) = 0.05792
P(4 of 5 correct by random guessing) = 0.00672

So the 4 of 5 criterion avoids about 5% of cases in which Type I error may be expected to manifest itself. However, consider the probability of Type II error, the probability that a student who is operating at a given level at, let's say, 90% mastery, a strong criterion, will be found by the test to not meet the criterion.

P(less than 3 of 5 correct given 90% chance on each item) = 0.00856
P(less than 4 of 5 correct given 90% chance on each item) = 0.08146

The 3 of 5 criterion avoids about 7% of cases in which Type II error may be expected to appear. If weaker mastery, say 80%, is expected of a student operating at a given level, then it is absolutely necessary to use the 3 of 5 criterion, fir Type II errors with the stricter criterion are much too frequent.

P(less than 3 of 5 correct given 80% chance on each item) = 0.05792
P(less than 4 of 5 correct given 80% chance on each item) = 0.26272

A differentiable scoring method was also suggested by the CDASSG Project. Under this method, 4-of-5 would indicate mastery of the subtests for levels 1 and 2 and 3-of-5 would indicate mastery of the subtests for levels 3, 4, and 5.

The Rasch method assigns a difficulty level index to each subtest item by analyzing the responses of the group being tested. Using this method, the 3-of-5 criterion means the student got the 3 most difficult problems in the subtest correct. Similarly, the 4-of-5 criterion means the student got the 4 most difficult questions in the subtest correct.

WEIGHTED SUM SCORES

Once it is decided how many questions correct are needed to indicate mastery of a subtest, a weighted sum score can be assigned. Mastery of the subtest for level one receives 1 point, level two receives 2 points, level three receives 4 points, level four receives 8 points, and level five receives 16 points.

WHAT VAN HIELE LEVEL SHOULD BE ASSIGNED?

The CDASSG Project looked at three ways of assigning a van Hiele level to a student based on the weighted sum score attained: the classical method, the modified method, and the forced method.

The classical method considers all five of the van Hiele levels and puts special emphasis on the belief that there must be sequential progression through the levels. Levels are assigned as follows:

Level 0 corresponds to a weighted sum of 0

Level 1 corresponds to a weighted sum of 1

Level 2 corresponds to a weighted sum of 3

Level 3 corresponds to a weighted sum of 7

Level 4 corresponds to a weighted sum of 15

Level 5 corresponds to a weighted sum of 31

A student receiving any other weighted sum is said to be a "no fit". In the CDASSG study, this was approximately 30% of the students using the 3-of-5 criterion and approximately 13% using the 4-of-5 criterion on the pre-test.

The modified method is the result of two factors. In 1980, P. van Hiele disavowed belief in the fifth level and had to be reconvinced of the existence of the fourth level (Usisken, 1982). In administering the test, it was found that some of the level 5 subtest items were easier than lower level subtest items. Many student who did not show mastery of the level 3 or 4 subtests did show mastery of the level 5 subtest (Johnson, 1989). As a result, the modified method excludes level 5 from consideration but leaves the sequential progression assumption in place. Weighted sum scores are assigned as detailed above and levels are assigned as follows:

Level 0 corresponds to a weighted sum of 0 or 16

Level 1 corresponds to a weighted sum of 1 or 17

Level 2 corresponds to a weighted sum of 3 or 19

Level 3 corresponds to a weighted sum of 7 or 23

Level 4 corresponds to a weighted sum of 15 or 31

As with the classical method, any other score is considered a "not fit". In the CDASSG study, this was 15% of the students using the 3-of-5 criterion and 8% of the students using the 4-of-5 criterion.

In order to assign levels to all students, the forced method was developed. This method excludes level 5. The sequential progression is considered valid. This method assumes that a student whose responses do not fit the sequence is probably demonstrating random fit rather than a weakness in theory. If more questions or better questions had been used in the test, the student could have been assigned a level under the classical or modified method. A student's forced van Hiele level is determined as follows:

> To determine a student's forced van Hiele assignment, the following procedure is used. First, a criterion is chosen (3-of-5 or 4-of-5) and a student is assigned a modified van Hiele level according to that

> criterion. The responses of those students who do not fit that modified van Hiele level are examined. A student is assigned a level n if (a) the student meets the criterion at levels n and n-1 but perhaps not at one of n-2 or n-3, or (b) the student meets the criterion at level n, all levels below n, but not at level n+1 yet also meets criterion at one higher level (Usiskin, 1982, 34).

This method allows a level to be assigned for any weighted sum score except 10, 12, 16 or 28. In the CDASSG Project, 0.3% of the students were "no fits" using the 3-of-5 criterion and 0.2% of the students were "no fits" using the 4-of-5 criterion. However, forced van Hiele levels were not used in the CDASSG Project because the forced levels assume the theory holds and that was what the Project was attempting to verify.

APPLICATION TO THIS STUDY

In this study, each test was assigned van Hiele levels using six of the scoring methods. The methods used were: classical 3-of-5, modified 3-of-5, forced 3-of-5, classical 4-of-5, modified 4-of-5, and forced 4-of-5. Since this study accepts the validity of the van Hiele theory, the use of the forced levels is acceptable and allows for the assignment of a van Hiele level to all students in the study.

APPENDIX C

TOPICS COVERED IN ALGEBRA I PART II

The following geometry related topics were covered in Algebra I Part II:

I. Basic concepts - labeling and naming
   A. Points
   B. Rays
   C. Lines
   D. Line segments
II. Lines and line segments
   A. Measuring
   B. Constructing a segment from a given segment
   C. Types
      1. Parallel
      2. Perpendicular
III. Angles
   A. Labeling and naming
   B. Classifying
      1. Acute
      2. Right
      3. Obtuse
      4. Straight
   C. Measuring
   D. Constructions
      1. Copying a given angle
      2. Bisecting an angle
   E. Complementary angles
   F. Supplementary angles
IV. Triangles
   A. Shape recognition and properties
   B. Naming and labeling
   C. Classifying
      1. By sides
         a. Scalene
         b. Isosceles
         c. Equilateral
      2. By angles
         a. Acute
         b. Right
         c. Obtuse

D. Constructions
  1. Copying a given triangle
  2. Constructing a triangle given
    a. Three sides
    b. Two sides and the included angle
    c. Two angles and the included side
E. Informal proofs
  1. Sum of the angles of a triangle equal 180
  2. Triangle inequality theorem
F. Congruent triangles
  1. Concept of congruency
  2. Triangle construction from patterns (i.e. SAS, ASA, SSS, AAA, SSA, SAA)
  3. Recognizing congruence patterns that work
    a. Given one triangle
    b. Given two triangles with the same orientation
    c. Given two triangles with different orientation
  4. Identifying corresponding parts
  5. Writing congruence statements
  6. Determining congruency based on given information
G. Overlapping triangles

V. Properties of parallel lines cut by a transversal
  A. Corresponding angles
  B. Alternate interior angles
  C. Alternate exterior angles
  D. Same side interior angles
  E. Vertical angles

VI. Quadrilaterals
  A. Labeling and naming
  B. Concepts of convex and concave
  C. Properties
  D. Interrelationships and hierarchy of the special types of quadrilaterals
  E. Congruent figures
  F. Similar figures

VII. Other polygons
  A. Naming
  B. Regular versus non-regular figures
  C. Properties
    1. Sum of the interior angles
    2. Sum of the exterior angles

        3. Measurement of each interior angle of a regular polygon
        4. Measurement of each exterior angle of a regular polygon
        5. Number of diagonals

VIII. Similar figures
    A. Identifying and naming
    B. Finding missing measurements

IX. Circles
    A. Terminology
        1. Chord
        2. Diameter
        3. Radius
        4. Tangent
        5. Secant
    B. Measurement
        1. Central angles
        2. Inscribed angles

X. Measurement
    A. Perimeter (typical and complicated figures)
    B. Area (typical and complicated figures)
    C. Circumference
    D. Surface area
    E. Volume
    F. Pythagorean theorem
    G. Distance formula

APPENDIX D

DATA LISTINGS

This appendix includes the test scores that were used in the statistical analysis for this project.

NUMBER RIGHT: COOP PRE-TEST VERSUS COOP POST-TEST

COURSE	COOPERATIVE GEOMETRY TEST SCORES PRE-TEST	POST-TEST
Algebra I	6	9
Algebra I	14	14
Algebra I	14	18
Algebra I	20	17
Algebra I	9	18
Algebra I	16	21
Algebra I	14	14
Algebra I	11	21
Algebra I	13	13
Algebra I	14	22
Algebra I	12	11
Algebra I	10	10
Algebra I	14	17
Algebra I	19	13
Algebra I	10	16
Algebra I	15	18
Algebra I	14	18
Algebra I	14	10
Algebra I	8	10
Algebra I	11	13
Algebra I	8	17
Algebra I	14	22
Algebra I	10	14
Algebra I	15	12
Algebra I	11	11
Algebra I	8	11
Algebra I	21	20
Algebra I	17	13
Algebra I	10	15
Algebra I	11	13
Algebra I	11	14
Algebra I	14	21
Algebra I	14	16
Algebra I	9	9
Algebra I	9	8
Algebra I	9	11
Algebra I	12	15
Algebra I	6	11
Algebra I	19	14

NUMBER RIGHT: COOP PRE-TEST VERSUS COOP POST-TEST (continued)

COURSE	COOPERATIVE GEOMETRY TEST SCORES PRE-TEST	POST-TEST
Algebra I Part II	14	24
Algebra I Part II	11	19
Algebra I Part II	16	16
Algebra I Part II	12	17
Algebra I Part II	15	18
Algebra I Part II	14	17
Algebra I Part II	6	10
Algebra I Part II	12	15
Algebra I Part II	18	24
Algebra I Part II	19	22
Algebra I Part II	10	15
Algebra I Part II	19	17
Algebra I Part II	14	12
Algebra I Part II	10	11
Algebra I Part II	12	16
Algebra I Part II	15	19
Algebra I Part II	12	13
Algebra I Part II	12	19
Algebra I Part II	11	27
Algebra I Part II	12	18
Algebra I Part II	10	14
Algebra I Part II	10	21
Algebra I Part II	9	19
Algebra I Part II	15	15
Algebra I Part II	14	13
Algebra I Part II	7	14
Algebra I Part II	20	20
Algebra I Part II	13	22
Algebra I Part II	13	18
Algebra I Part II	16	14
Algebra I Part II	11	20
Algebra I Part II	10	15
Algebra I Part II	15	20
Algebra I Part II	18	17
Algebra I Part II	7	28
Algebra I Part II	12	8
Algebra I Part II	18	17
Algebra I Part II	16	13
Algebra I Part II	20	24
Algebra I Part II	15	21
Algebra I Part II	16	22
Algebra I Part II	9	16
Algebra I Part II	8	19
Algebra I Part II	8	17

NUMBER RIGHT: COOP PRE-TEST VERSUS COOP POST-TEST
(continued)

COURSE	COOPERATIVE GEOMETRY TEST SCORES PRE-TEST	POST-TEST
Algebra I Part II	9	21
Algebra I Part II	11	17
Algebra I Part II	6	21
Algebra I Part II	9	19
Algebra I Part II	12	22
Algebra I Part II	7	19
Algebra I Part II	7	16
Algebra I Part II	13	20
Algebra I Part II	5	19
Algebra I Part II	12	18
Algebra I Part II	9	18
Algebra I Part II	16	19
Algebra I Part II	11	16
Algebra I Part II	17	16

CONSERVATIVE VAN HIELE LEVELS ASSIGNED TO STUDENTS

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I	NF	0	0	0
Algebra I	NF	NF	0	0
Algebra I	NF	NF	0	NF
Algebra I	1	1	1	0
Algebra I	1	1	1	1
Algebra I	NF	NF	1	1
Algebra I	1	NF	0	0
Algebra I	2	1	1	1
Algebra I	NF	1	NF	1
Algebra I	NF	NF	NF	0
Algebra I	3	1	0	1
Algebra I	0	2	0	0
Algebra I	1	2	1	1
Algebra I	1	NF	1	1
Algebra I	2	2	0	1
Algebra I	1	3	1	0
Algebra I	NF	3	0	0
Algebra I	NF	3	2	3
Algebra I	3	3	NF	1
Algebra I	2	0	1	0
Algebra I	2	0	0	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	1	1	0	0
Algebra I	NF	1	NF	1
Algebra I	1	1	1	0
Algebra I	1	1	1	1
Algebra I	NF	1	NF	1
Algebra I	2	NF	1	1
Algebra I	0	NF	0	0
Algebra I	NF	NF	NF	2
Algebra I	1	2	0	0
Algebra I	NF	2	0	0
Algebra I	1	2	1	1
Algebra I	2	2	1	2

CONSERVATIVE VAN HIELE LEVELS ASSIGNED TO STUDENTS
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	0	1	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	NF	1	0
Algebra I Part II	1	NF	1	1
Algebra I Part II	0	3	0	1
Algebra I Part II	1	NF	1	NF
Algebra I Part II	2	3	2	NF
Algebra I Part II	2	NF	1	3
Algebra I Part II	2	5	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	NF	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	NF	NF	0	0
Algebra I Part II	1	NF	0	1
Algebra I Part II	NF	NF	0	1
Algebra I Part II	2	NF	1	1
Algebra I Part II	1	2	0	1
Algebra I Part II	1	2	1	0
Algebra I Part II	NF	NF	1	NF
Algebra I Part II	1	3	1	NF
Algebra I Part II	1	3	1	3
Algebra I Part II	2	3	1	NF
Algebra I Part II	2	3	0	3
Algebra I Part II	2	3	1	3
Algebra I Part II	3	3	3	3
Algebra I Part II	NF	NF	NF	1
Algebra I Part II	NF	NF	0	0
Algebra I Part II	NF	1	0	0
Algebra I Part II	1	NF	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	1	NF	1	1
Algebra I Part II	2	NF	0	NF
Algebra I Part II	0	2	0	1
Algebra I Part II	0	2	0	2
Algebra I Part II	1	2	1	1
Algebra I Part II	1	2	1	1
Algebra I Part II	1	2	0	2
Algebra I Part II	NF	2	0	2
Algebra I Part II	1	3	1	1
Algebra I Part II	2	3	1	3

CONSERVATIVE VAN HIELE LEVELS ASSIGNED TO STUDENTS
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	NF	1	NF	1
Algebra I Part II	0	1	0	1
Algebra I Part II	0	NF	0	1
Algebra I Part II	0	NF	0	NF
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	1	NF	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	NF	1	1
Algebra I Part II	1	NF	1	NF
Algebra I Part II	2	1	0	1
Algebra I Part II	3	NF	0	1
Algebra I Part II	NF	2	NF	2
Algebra I Part II	1	2	1	1
Algebra I Part II	2	2	1	1
Algebra I Part II	1	3	0	1
Algebra I Part II	1	NF	1	3

MODIFIED VAN HIELE LEVELS ASSIGNED TO STUDENTS

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I	NF	0	0	0
Algebra I	1	0	0	0
Algebra I	NF	NF	0	NF
Algebra I	1	1	1	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	1	1	0	0
Algebra I	2	1	1	1
Algebra I	2	1	1	1
Algebra I	2	NF	1	0
Algebra I	3	1	0	1
Algebra I	0	2	0	0
Algebra I	1	2	1	1
Algebra I	1	NF	1	1
Algebra I	2	2	0	1
Algebra I	1	3	1	0
Algebra I	1	3	0	0
Algebra I	NF	3	2	3
Algebra I	3	3	NF	1
Algebra I	2	0	1	0
Algebra I	2	0	0	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	1	1	0	0
Algebra I	NF	1	NF	1
Algebra I	1	1	1	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	2	NF	1	1
Algebra I	0	2	0	0
Algebra I	NF	2	NF	2
Algebra I	1	2	0	0
Algebra I	NF	2	0	0
Algebra I	1	2	1	1
Algebra I	2	2	1	2

MODIFIED VAN HIELE LEVELS ASSIGNED TO STUDENTS
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	0	1	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	NF	1	0
Algebra I Part II	1	2	1	1
Algebra I Part II	0	3	0	1
Algebra I Part II	1	3	1	NF
Algebra I Part II	2	3	2	NF
Algebra I Part II	2	3	1	3
Algebra I Part II	2	4	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	NF	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	NF	NF	0	0
Algebra I Part II	1	NF	0	1
Algebra I Part II	NF	NF	0	1
Algebra I Part II	2	NF	1	1
Algebra I Part II	1	2	0	1
Algebra I Part II	1	2	1	0
Algebra I Part II	1	2	1	2
Algebra I Part II	1	3	1	NF
Algebra I Part II	1	3	1	3
Algebra I Part II	2	3	1	NF
Algebra I Part II	2	3	0	3
Algebra I Part II	2	3	1	3
Algebra I Part II	3	3	3	3
Algebra I Part II	0	NF	0	1
Algebra I Part II	NF	NF	0	0
Algebra I Part II	NF	1	0	0
Algebra I Part II	1	NF	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	1	NF	1	1
Algebra I Part II	2	NF	0	NF
Algebra I Part II	0	2	0	1
Algebra I Part II	0	2	0	2
Algebra I Part II	1	2	1	1
Algebra I Part II	1	2	1	1
Algebra I Part II	1	2	0	2
Algebra I Part II	1	2	0	2
Algebra I Part II	1	3	1	1
Algebra I Part II	2	3	1	3

MODIFIED VAN HIELE LEVELS ASSIGNED TO STUDENTS
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	NF	1	NF	1
Algebra I Part II	0	1	0	1
Algebra I Part II	0	NF	0	1
Algebra I Part II	0	NF	0	NF
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	1	NF	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	NF	1	NF
Algebra I Part II	2	1	0	1
Algebra I Part II	3	NF	0	1
Algebra I Part II	NF	2	NF	2
Algebra I Part II	1	2	1	1
Algebra I Part II	2	2	1	1
Algebra I Part II	1	3	0	1
Algebra I Part II	1	3	1	3

FORCED VAN HIELE LEVELS ASSIGNED TO STUDENTS

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I	0	0	0	0
Algebra I	1	0	0	0
Algebra I	3	0	0	0
Algebra I	1	1	1	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	1	1	0	0
Algebra I	2	1	1	1
Algebra I	2	1	1	1
Algebra I	2	1	1	0
Algebra I	3	1	0	1
Algebra I	0	2	0	0
Algebra I	1	2	1	1
Algebra I	1	2	1	1
Algebra I	2	2	0	1
Algebra I	1	3	1	0
Algebra I	1	3	0	0
Algebra I	2	3	2	3
Algebra I	3	3	0	1
Algebra I	2	0	1	0
Algebra I	2	0	0	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	1	1	0	0
Algebra I	1	1	1	1
Algebra I	1	1	1	0
Algebra I	1	1	1	1
Algebra I	1	1	1	1
Algebra I	2	1	1	1
Algebra I	0	2	0	0
Algebra I	0	2	0	2
Algebra I	1	2	0	0
Algebra I	1	2	0	0
Algebra I	1	2	1	1
Algebra I	2	2	1	2

FORCED VAN HIELE LEVELS ASSIGNED TO STUDENTS
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	0	1	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	0
Algebra I Part II	1	2	1	1
Algebra I Part II	0	3	0	1
Algebra I Part II	1	3	1	0
Algebra I Part II	2	3	2	1
Algebra I Part II	2	3	1	3
Algebra I Part II	2	4	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	0	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	0
Algebra I Part II	1	1	0	1
Algebra I Part II	1	1	0	1
Algebra I Part II	2	1	1	1
Algebra I Part II	1	2	0	1
Algebra I Part II	1	2	1	0
Algebra I Part II	1	2	1	2
Algebra I Part II	1	3	1	3
Algebra I Part II	1	3	1	3
Algebra I Part II	2	3	1	0
Algebra I Part II	2	3	0	3
Algebra I Part II	2	3	1	3
Algebra I Part II	3	3	3	3
Algebra I Part II	0	1	0	1
Algebra I Part II	0	1	0	0
Algebra I Part II	1	1	0	0
Algebra I Part II	1	1	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	1	1	1	1
Algebra I Part II	2	1	0	1
Algebra I Part II	0	2	0	1
Algebra I Part II	0	2	0	2
Algebra I Part II	1	2	1	1
Algebra I Part II	1	2	1	1
Algebra I Part II	1	2	0	2
Algebra I Part II	1	2	0	2
Algebra I Part II	1	3	1	1
Algebra I Part II	2	3	1	3

FORCED VAN HIELE LEVELS ASSIGNED TO STUDENTS
(continued)

	3-OF-5 CRITERION		4-OF-5 CRITERION	
COURSE	PRE-TEST	POST-TEST	PRE-TEST	POST-TEST
Algebra I Part II	0	1	0	1
Algebra I Part II	0	1	0	1
Algebra I Part II	0	1	0	1
Algebra I Part II	0	1	0	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	2	1	0	1
Algebra I Part II	3	1	0	1
Algebra I Part II	0	2	0	2
Algebra I Part II	1	2	1	1
Algebra I Part II	2	2	1	1
Algebra I Part II	1	3	0	1
Algebra I Part II	1	3	1	3

VAN HIELE WEIGHTED SUM SCORES

	3-OF-5 CRITERION		4-OF-5 CRITERION	
COURSE	PRE-TEST	POST-TEST	PRE-TEST	POST-TEST
Algebra I	6	18	0	2
Algebra I	1	27	1	1
Algebra I	1	3	1	1
Algebra I	7	7	2	1
Algebra I	0	3	0	0
Algebra I	7	1	0	1
Algebra I	17	7	0	0
Algebra I	1	1	1	0
Algebra I	3	1	1	1
Algebra I	1	1	1	1
Algebra I	19	9	17	0
Algebra I	1	7	1	0
Algebra I	17	17	1	1
Algebra I	1	17	0	0
Algebra I	4	0	0	0
Algebra I	17	16	0	0
Algebra I	3	3	0	2
Algebra I	11	7	3	7
Algebra I	19	1	17	1
Algebra I	3	3	1	3
Algebra I	3	0	0	0
Algebra I	0	19	0	0
Algebra I	1	1	1	1
Algebra I	5	1	5	1
Algebra I	5	3	0	0
Algebra I	1	1	1	1
Algebra I	2	19	2	3
Algebra I	17	1	17	1
Algebra I	1	3	0	0
Algebra I	1	1	0	0
Algebra I	1	1	1	1
Algebra I	3	5	1	1
Algebra I	1	3	1	1
Algebra I	1	1	1	0
Algebra I	3	0	1	0

VAN HIELE WEIGHTED SUM SCORES
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	3	7	3	5
Algebra I Part II	1	1	1	1
Algebra I Part II	1	19	1	1
Algebra I Part II	1	23	1	20
Algebra I Part II	1	31	1	1
Algebra I Part II	1	5	1	0
Algebra I Part II	0	1	0	0
Algebra I Part II	0	7	0	1
Algebra I Part II	3	23	1	7
Algebra I Part II	3	7	1	7
Algebra I Part II	1	1	1	1
Algebra I Part II	17	19	1	19
Algebra I Part II	5	5	0	0
Algebra I Part II	3	3	1	2
Algebra I Part II	1	1	1	1
Algebra I Part II	1	2	0	0
Algebra I Part II	1	7	1	6
Algebra I Part II	3	7	0	7
Algebra I Part II	1	3	1	0
Algebra I Part II	1	5	0	1
Algebra I Part II	7	7	7	7
Algebra I Part II	1	7	1	7
Algebra I Part II	1	3	0	1
Algebra I Part II	1	1	1	1
Algebra I Part II	3	5	1	1
Algebra I Part II	5	5	0	1
Algebra I Part II	16	5	16	1
Algebra I Part II	17	3	0	3
Algebra I Part II	1	3	1	1
Algebra I Part II	1	3	0	3
Algebra I Part II	1	9	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	3	7	1	7
Algebra I Part II	1	3	1	1
Algebra I Part II	0	3	0	3
Algebra I Part II	1	5	0	0
Algebra I Part II	3	5	0	5
Algebra I Part II	1	7	1	1
Algebra I Part II	9	1	0	0
Algebra I Part II	2	5	0	0
Algebra I Part II	1	1	1	1
Algebra I Part II	0	3	0	1

VAN HIELE WEIGHTED SUM SCORES
(continued)

COURSE	3-OF-5 CRITERION PRE-TEST	3-OF-5 CRITERION POST-TEST	4-OF-5 CRITERION PRE-TEST	4-OF-5 CRITERION POST-TEST
Algebra I Part II	3	3	1	1
Algebra I Part II	7	5	0	1
Algebra I Part II	0	5	0	1
Algebra I Part II	0	5	0	5
Algebra I Part II	1	1	1	1
Algebra I Part II	1	5	1	5
Algebra I Part II	0	1	0	1
Algebra I Part II	1	3	1	1
Algebra I Part II	1	1	1	1
Algebra I Part II	1	1	0	1
Algebra I Part II	2	3	2	3
Algebra I Part II	1	7	0	1
Algebra I Part II	1	23	1	7
Algebra I Part II	2	1	2	1
Algebra I Part II	3	1	0	1
Algebra I Part II	1	5	1	1
Algebra I Part II	1	17	1	1

NUMBER RIGHT: VAN HEILE TEST

COURSE	VAN HIELE TEST SCORES PRE-TEST	POST-TEST
Algebra I	9	12
Algebra I	7	15
Algebra I	8	10
Algebra I	11	12
Algebra I	9	10
Algebra I	13	9
Algebra I	11	11
Algebra I	9	8
Algebra I	10	11
Algebra I	6	9
Algebra I	14	12
Algebra I	10	11
Algebra I	11	10
Algebra I	9	11
Algebra I	9	8
Algebra I	9	7
Algebra I	9	12
Algebra I	15	16
Algebra I	11	11
Algebra I	7	11
Algebra I	10	10
Algebra I	7	11
Algebra I	7	6
Algebra I	10	7
Algebra I	8	9
Algebra I	8	5
Algebra I	8	14
Algebra I	11	12
Algebra I	8	7
Algebra I	6	7
Algebra I	8	8
Algebra I	9	10
Algebra I	10	12
Algebra I	6	8
Algebra I	12	6

NUMBER RIGHT: VAN HEILE TEST
(continued)

COURSE	VAN HIELE TEST SCORES PRE-TEST	POST-TEST
Algebra I Part II	10	14
Algebra I Part II	6	11
Algebra I Part II	7	12
Algebra I Part II	11	14
Algebra I Part II	10	17
Algebra I Part II	8	11
Algebra I Part II	6	7
Algebra I Part II	6	11
Algebra I Part II	10	19
Algebra I Part II	10	16
Algebra I Part II	6	9
Algebra I Part II	11	13
Algebra I Part II	8	11
Algebra I Part II	9	9
Algebra I Part II	9	13
Algebra I Part II	6	7
Algebra I Part II	8	14
Algebra I Part II	9	15
Algebra I Part II	8	10
Algebra I Part II	8	11
Algebra I Part II	17	18
Algebra I Part II	9	16
Algebra I Part II	8	9
Algebra I Part II	8	7
Algebra I Part II	8	12
Algebra I Part II	9	11
Algebra I Part II	8	11
Algebra I Part II	10	13
Algebra I Part II	7	9
Algebra I Part II	4	12
Algebra I Part II	10	13
Algebra I Part II	6	10
Algebra I Part II	8	15
Algebra I Part II	7	11
Algebra I Part II	5	13
Algebra I Part II	7	9
Algebra I Part II	6	15
Algebra I Part II	10	12
Algebra I Part II	12	8
Algebra I Part II	7	12
Algebra I Part II	8	9
Algebra I Part II	7	8
Algebra I Part II	8	9

NUMBER RIGHT: VAN HEILE TEST
(continued)

COURSE	VAN HIELE TEST SCORES PRE-TEST	POST-TEST
Algebra I Part II	10	12
Algebra I Part II	3	12
Algebra I Part II	5	13
Algebra I Part II	7	10
Algebra I Part II	8	13
Algebra I Part II	6	8
Algebra I Part II	6	12
Algebra I Part II	6	8
Algebra I Part II	5	9
Algebra I Part II	9	11
Algebra I Part II	6	14
Algebra I Part II	10	16
Algebra I Part II	7	7
Algebra I Part II	9	10
Algebra I Part II	8	9
Algebra I Part II	9	11

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