EXERGY ANALYSIS FOR INDUSTRIAL ENERGY ASSESSMENT

Thesis

Submitted to

The Department of Mechanical and Aerospace Engineering

School of Engineering

UNIVERSITY OF DAYTON

In partial fulfillment of the requirements for

The degree

Master of Science in Mechanical Engineering

by

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UNIVERSITY OF DAYTON

Dayton, Ohio

August, 2000

EXERGY ANALYSIS FOR INDUSTRIAL ENERGY ASSESSMENT

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ABSTRACT

EXERGY ANALYSIS FOR INDUSTRIAL ENERGY ASSESSMENT

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University of Dayton, 2000

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Traditional approaches to improving energy efficiency often focus on leak-plugging and housekeeping efforts to avoid wasting energy. In many cases however, the way energy is used, rather than the amount used, is the primary cause of inefficiency. Exergy analysis provides a thermodynamic basis for optimizing energy systems. By considering the quality of energy as well as the quantity, the true thermodynamic efficiencies can be quantified and proper priorities for improvement projects can be established. In spite of its value for optimizing thermodynamic systems, exergy analysis has seen only limited use as a practical tool for industrial energy assessment.

In this thesis, the energy and exergy efficiencies of two common industrial processes, air compression and aluminum melting, are developed. Methods for improving the processes are quantitatively discussed. Finally, an application of exergy analysis to analyzing energy information in industrial plants is described.

This analysis suggests that in many cases, energy efficiency is an incomplete characterization of the true efficiency of an energy system. In contrast, exergy analysis reveals how and where to make improvements. In addition, the exergy approach can provide information needed to decide whether to improve an existing process or to re-design it. Thus, exergy analysis appears to be an essential component of effective resource utilization in the industrial environment.

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NOMENCLATURE

C_p	Constant pressure specific heat (Btu/lbm-°R)
ρ	Density (lbm/ft^3)
E	Energy (Btu)
$\dot{E}$	Energy rate (Btu/hr)
G	Gibbs function (Btu)
h	Enthalpy (Btu/lbm)
$\bar{h}$	Enthalpy flow (Btu/lbmol)
$\bar{h}^o$	Enthalpy at standard conditions of 77°F, 1 atm (Btu/lbmol)
$\bar{h}_f^o$	Enthalpy of formation at standard conditions of 77°F, 1 atm (Btu/lbmol)
H	Enthalpy (Btu)
$\dot{H}$	Enthalpy flow (Btu/hr)
m	Mass (lbm)
k_f	Thermal conductivity of a fluid (Btu/hr-ft-°F)
k_w	Thermal conductivity of a solid (Btu/hr-ft-°F)
$\dot{m}$	Mass flow rate (lbm/hr)
M	Molar mass (lbm/lbmol)
η_1	First Law Efficiency, energy efficiency
η_2	Second Law Efficiency, exergetic efficiency
N	Molar coefficient (mol)
$\dot{N}$	Molar flow rate (mol/hr)
P	Pressure (psig, psia)
Q	Heat (Btu)
$\dot{Q}$	Heat flow (Btu/hr)
R	Universal gas constant (1.986 Btu/lbmol-°R)
s	Entropy (Btu/lbm-°F)
$\bar{s}$	Entropy (Btu/lbmol-°R)
$\bar{s}^o$	Absolute entropy at specific temperature and pressure (Btu/lbmol-°R)
T	Temperature (°R, °F)
U	Internal energy (Btu)
v	Specific volume (ft^3/lbm)
W	Work (Btu)
$\dot{W}$	Rate of work production or consumption (Btu/hr)
X	Exergy (Btu)
$\dot{X}$	Rate of exergy production or consumption (Btu/hr)
z	Gas compressibility factor

CHAPTER I

INTRODUCTION

In the past, generating pollution seemed necessary for progress, and expending effort and capital to decrease or remediate pollution was thought to negatively impact the business. Today, however, we find that the companies that are profitable and growing are most often the ones who actively work to reduce their environmental impact. Pollution is waste. Reducing or eliminating pollution reduces or eliminates waste, and improves the bottom line. This approach makes good business sense from a marketing standpoint as well. Environmental consciousness among consumers has prompted companies to market their reduced environmental impact as a means of differentiating themselves from their competitors.

To eliminate waste, companies strive to use resources effectively. A primary resource for any company is energy. This thesis describes the exergy method for analyzing energy systems, and contrasts it to traditional energy efficiency methods. Case studies are utilized to illustrate the concepts. Two common industrial energy-using processes are investigated: air compression and aluminum melting. Finally, an example of using exergy analysis to examine industrial electricity use is presented.

Energy versus Exergy

The First Law of Thermodynamics tells us that energy is conserved. It may be converted from one form to another, but energy is never destroyed or lost. But intuitively, we know that something is lost when a tank full of gasoline is combusted or a meal consumed. That something is exergy - the capacity to do work. When we say that we should "conserve energy", we really mean that we should not waste the capacity of energy to do work.

In the U.S., the capacity to do work has been called “availability”; internationally, it is often called “exergy”. The exergy of a system or resource is the maximum work that could be produced as the system comes into equilibrium with the environment. The capacity to do work is a result of an energy resource’s non-equilibrium with the environment. It is different than energy content and is specific to a particular type of energy resource, i.e. heat, electricity, etc., and its application.

Exergy Content of Energy Forms

The exergy approach to energy systems analysis stems from the Second Law of Thermodynamics and is necessarily linked to the concept of *work*. Work is a form of energy that crosses a system boundary in the form of organized motion. A thermodynamic definition of work is a force F acting through a distance x. A classical definition is: “work is done by a system if the sole effect on the surroundings could be the raising of a weight.”[1]

Mechanical energy - falling water, rotating shafts, wind - could be completely converted to the raising of a weight, in the absence of losses or *irreversibilities* like friction. Electricity could be completely converted to the raising of a weight by a reversible electric motor. Thus, the quantity of exergy in work is equal to its energy.

Thermal energy - from rejected process heat, electric heating elements, furnaces - can be converted to the "raising of a weight" only through the use of a heat engine. In 1824, Sadi Carnot showed that the maximum fraction of heat that can be converted to work is:

$$\eta_{th} = \left(1 - \frac{Q_{out}}{Q_{in}}\right) = \left(1 - \frac{T_{out}}{T_{in}}\right) \qquad \text{Equation (1)}$$

where: Q_{in} = heat supplied to a heat engine from a high-temperature source
Q_{out} = heat rejected from a heat engine to a low-temperature source
T_{in} = temperature at which heat is supplied to a reversible heat engine
T_{out} = temperature at which heat is rejected from a reversible heat engine

The derivation for Equation (1) is as follows. A reversible heat engine is shown in Figure 1. This engine produces work by taking energy from a high-temperature source and rejecting some energy to a low-temperature sink.

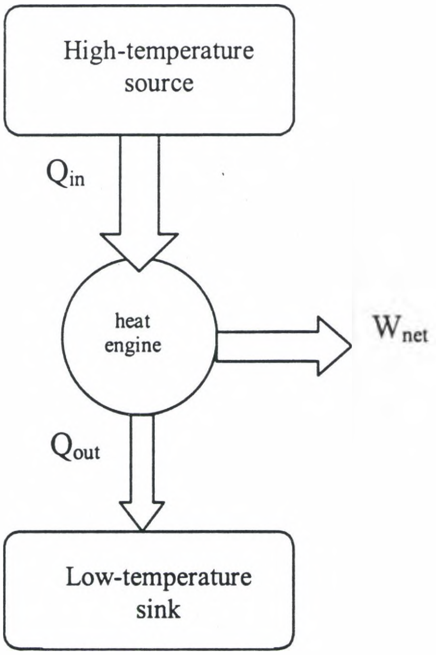

Figure 1. General diagram of a heat engine, with energy flows.

A First Law energy balance for the heat engine, with steady-state operation, is:

$$Q_{in} - Q_{out} - W_{net} = 0 \qquad \text{Equation (2)}$$

Thus, the useful work output $W_{net} = Q_{in} - Q_{out}$. The efficiency of this heat engine is:

$$\eta_{th} = \frac{\text{useful energy output}}{\text{valued energy input}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \qquad \text{Equation (3)}$$

The Carnot Principles^{2} state that 1.) the efficiency of an irreversible (real) heat engine is less than the efficiency of a reversible (theoretical, no losses) heat engine operating between the same two thermal reservoirs, and 2.) the efficiencies of all reversible heat engines operating between the same two reservoirs are the same. This second principle implies that, for reversible (ideal) heat engines, the efficiency is dependent only on the reservoir temperatures. Thus, the above equation can be restated in absolute temperatures as in Equation (1).

A reversible heat engine has no losses. It is the ideal with which we compare actual thermal processes. In other words, given specific heat source and sink temperatures T_{in} and T_{out} , the upper limit of available work is the work output of a reversible (Carnot) heat engine W_{rev}.

$$W_{rev} = Q\left(1 - \frac{T_{out}}{T_{in}}\right) \qquad \text{Equation (4)}$$

Exergy is defined as the reversible (maximum) work available as a system comes into equilibrium with the environment. Thus, with the environment at temperature T_o and the source reservoir at temperature T_Q, the exergy X available in a quantity of heat Q is equal to the work output of a reversible heat engine:

$$X = Q\left(1 - \frac{T_o}{T_Q}\right) \qquad \text{Equation (5)}$$

Since heat spontaneously flows only from higher temperatures to lower temperatures, T_Q is larger than T_o, and the exergy content of the quantity of thermal energy is less than its energy content.

Chemical energy - from natural gas, coal, biofuels, etc. - is primarily used for combustion-derived heat. Like mechanical forms of energy, the exergy content of chemical forms is equal to the energy content due to their highly-ordered molecular structure. Unfortunately, a considerable amount of the chemical exergy content of the fuel is destroyed during combustion, and more is lost when the thermal energy is used. In Chapter II, an expression is derived for chemical exergy of fuels in combustion. Non-combustion energy conversion processes, such as solar photovoltaics and fuel cells, are not constrained in the same way, and thus can have higher exergetic efficiencies. These are not considered in this thesis. We also do not consider nuclear energy conversion, since this is not typically used in the industrial setting.

These concepts imply that energy resources have quality, as well as quantity. For example, the entire energy content of 1 kWh of electricity is available as exergy, but a stream of hot gas with a heat content of 1 kWh contains less than 1 kWh of exergy. As a result, a quantity or flow of electrical energy is more valuable, and more costly, than a quantity or flow of hot gases with the same energy content. Energy conservation methods based on the First Law of Thermodynamics treat all energy resources as equivalent.

By contrast, exergy analysis recognizes that energy resources have differing qualities. When optimizing systems using exergy methods, the exergy or quality of energy supplied to a process is matched with the actual needs of the process. In this way, the exergy used to accomplish the task can be minimized and costs reduced.

Efficiencies

The traditional measure of efficiency, called First Law Efficiency or energy efficiency, is:

$$\eta_I = \frac{\text{useful energy output}}{\text{valued energy input}} \qquad \text{Equation (6)}$$

Since the First Law efficiency considers only energy, it makes no distinction between the qualities of energy input and output. Thus, 1 kWh of electricity is regarded the same as 1 kWh in a hot gas stream. In reality, the capacity to do work of the electricity is much greater than that of the hot gas. Thus, a more appropriate way to evaluate the efficiency of a process would be to consider the exergy content of resources. The exergy method calculates the efficiency of a process as:

$$\eta_2 = \frac{\text{useful exergy output}}{\text{valued exergy input}} \text{ or } \frac{\text{exergy in products}}{\text{exergy in reactants}} \qquad \text{Equation (7)}$$

Thus, exergy efficiency captures the true value, both quality and quantity, of the resources and is a much better measure of process efficiency.

The Exergy Approach

In applying exergy analysis, these general steps are followed:

- Determine the desired result or output of the process.
- Define the boundary conditions: describe how the process interacts with the environment.
- Establish an ideal with which to compare the process or system. Determine the theoretically reversible, or best possible way to accomplish the task.
- Determine the actual resource inputs to the process or system.
- Calculate the process efficiency, η_2.
- Examine process inputs and outputs. Determine how the process could be improved.

The following questions should also be considered:

- If the process were totally redesigned, what would be different and how would its resource efficiency be improved?
- What are the economic implications of improving or redesigning the process?

CHAPTER II

THERMODYNAMICS OF ENERGY ANALYSIS

In subsequent chapters, we examine several thermodynamic processes. This chapter contains derivations of fundamental concepts that will be used to describe these processes. In addition, we state the assumptions made in our models, and verify the use of these assumptions.

Gibbs Relations

Consider a fixed mass of compressible fluid, contained in a rigid cylinder with a sealed, movable piston, as shown in Figure 2. We define the fixed mass of fluid as the "system".

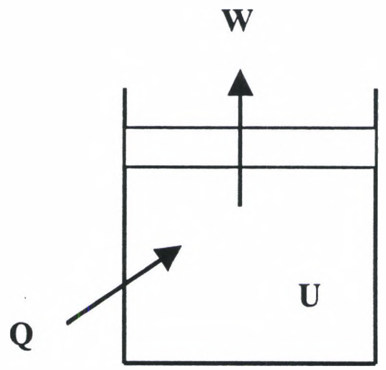

Figure 2. Rigid cylinder with movable piston, containing compressible gas.

Adding an incremental quantity of heat δQ will cause the fluid to expand. The system volume will increase, raising the piston. Since the piston has mass, force must be exerted by the system to raise the piston: work δW is done. The net change in energy of the system is the change in internal energy dU. A First Law balance for this system states:

$$\delta Q - \delta W = dU \qquad \text{Equation (8)}$$

The Clausius definition[2] of the entropy change for a reversible system states that

$$\delta Q = TdS \qquad \text{Equation (9)}$$

where S represents entropy and T represents temperature. The boundary work of moving the piston is:

$$\delta W = PdV \qquad \text{Equation (10)}$$

Thus, Equation (8) can be restated as the first Gibbs relation:

$$\begin{aligned} TdS - PdV &= dU \\ TdS &= dU + PdV \end{aligned} \qquad \text{Equation (11)}$$

But the definition of enthalpy is H = U + PV and dH = dU + PdV + VdP. Substituting this into Equation (11), the second Gibbs relation is obtained:

$$TdS = dH - VdP \qquad \text{Equation (12)}$$

Equation (12) will be used in Chapter III in analytical formulations for work.

Entropy Change

Expressing Equation (12) on a per-mass basis and dividing through by T results in:

$$ds = \frac{dh}{T} - \frac{vdP}{T} \qquad \text{Equation (13)}$$

With ideal gases, enthalpy is a function of temperature alone and dh = C_pdT. Also, the ideal gas law holds so that $v = \frac{RT}{P}$, where R is the specific gas constant. Inserting both of these relations into the above equation results in:

$$ds = \frac{C_p dT}{T} - \frac{RdP}{P} \qquad \text{Equation (14)}$$

Integrating between states 1 and 2 with constant C_p and R gives:

$$\begin{aligned} \int_1^2 ds &= \int_1^2 \frac{C_p dT}{T} - \frac{RdP}{P} \\ s_2 - s_1 &= C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \end{aligned} \qquad \text{Equation (15)}$$

Thus, for ideal gases, the change in entropy between two states depends on both temperature and pressure and is represented by Equation (15).

Thermomechanical Exergy Content of a Flow Stream

Figure 3 shows a control volume with a mass flow with enthalpy $\dot{H}_i$ entering, a mass flow with enthalpy $\dot{H}_e$ leaving, work $\dot{W}$ added, and heat $\dot{Q}$ rejected across a boundary at temperature T.

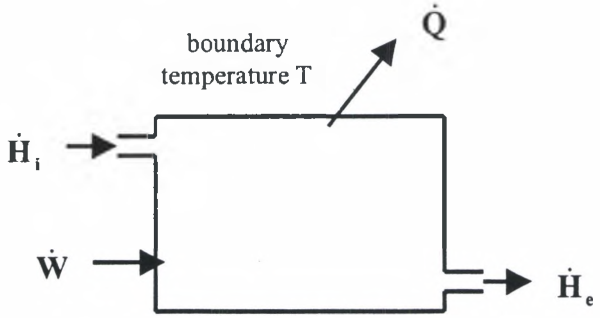

Figure 3. Control volume with energy flows.

Assuming steady state conditions, there is no storage of mass, energy, entropy or exergy in the control volume. The First Law for this system states:

$$\begin{aligned} &\dot{W} + \dot{H}_i - \dot{Q} - \dot{H}_e = 0 \\ &\dot{Q} = \dot{W} + \dot{H}_i - \dot{H}_e \end{aligned} \qquad \text{Equation (16)}$$

The Second Law written in entropy form states:

$$\dot{S}_i - \dot{S}_e - \frac{\dot{Q}}{T} + \dot{S}_{gen} = 0 \qquad \text{Equation (17)}$$

where $\dot{S}_i$ and $\dot{S}_e$ are the entropies of the mass flows entering and exiting the system, and $\dot{S}_{gen}$ is the entropy generated by the process.

The maximum work $\dot{W}_{max}$ that could be done by a system corresponds to the theoretical condition in which no irreversibilities exist and no entropy is generated by the process. Thus, setting $\dot{S}_{gen}$ to zero and solving for $\dot{Q}$ results in:

$$\dot{Q} = T(\dot{S}_i - \dot{S}_e) \qquad \text{Equation (18)}$$

Combining the Equations (16) and (18) by eliminating $\dot{Q}$ results in the following equations:

$$\begin{aligned}\dot{W}_{max} + \dot{H}_i - \dot{H}_e &= T(\dot{S}_i - \dot{S}_e)\\ \dot{W}_{max} &= (\dot{H}_e - \dot{H}_i) - T(\dot{S}_e - \dot{S}_i)\\ \dot{W}_{max} &= (\dot{H}_e - T\dot{S}_e) - (\dot{H}_i - T\dot{S}_i)\end{aligned} \qquad \text{Equation (19)}$$

$\dot{W}_{max}$ becomes exergy $\dot{X}$ when either the inlet or exit state is the state of the environment. In evaluating a stream of exergy, where the inlet state is that of the surrounding environment denoted by subscript "o", the exit state by "$_1$", and the mass flow rate by $\dot{m}$, the exergy content of the flow stream is:

$$\dot{X}_1 = \dot{m}(h_1 - T_o s_1) - \dot{m}(h_o - T_o s_o) \qquad \text{Equation (20)}$$

Thermochemical Exergy Content of a Flow Stream

Equation (20) is used to determine the exergy content of a flow stream when no chemical reactions take place. With chemical reactions, Equation (20) must be modified by replacing the thermal enthalpy and entropy values with ones that include both the chemical and thermal contributions.

When a substance exists in equilibrium with the environment in elemental form, that substance is said to be "stable"[3]. For example, N_2, O_2, and H_2 are the stable forms of nitrogen, oxygen, and hydrogen. Chemical compounds that do not naturally exist in the environment have non-zero chemical potential at environmental conditions; they are not in equilibrium with the environment. The chemical enthalpy for such a substance is the work needed to form the compound from stable, environmental elements. At standard conditions of 25°C and 1 atmosphere (indicated by a superscript "o"), this value is known as the "enthalpy of formation" for the substance and is denoted by $\overline{h}_f^o$. The total thermochemical enthalpy of a compound includes both the chemical and sensible enthalpies. If the compound is at a state other than standard conditions, the change in sensible enthalpy can be determined by subtracting the sensible enthalpy value at standard conditions $\overline{h}^o$ from the enthalpy at the state of interest, $\overline{h}$. Thus, the total thermochemical enthalpy is:

$$\text{total enthalpy} = \overline{h}_f^o + \overline{h} - \overline{h}^o \qquad \text{Equation (21)}$$

Determining entropy values is somewhat different. Unlike enthalpy, entropy depends on pressure as well as temperature, even for ideal gases. The Third Law of Thermodynamics defines the absolute entropy of substances. The Third Law states that a pure crystalline substance at absolute zero temperature has zero entropy. Tables of absolute entropy $\bar{s}^o$ at various temperatures are published[2,4] for many gases that can be considered ideal at one atmosphere of pressure. When a gas exists at a partial pressure other than one atmosphere, as is the case for each component in a mixture of gases at one atmosphere, an adjustment Δs must be made to account for the pressure difference. Thus, thermochemical entropy values for ideal gases are represented as:

$$\text{total entropy} = \bar{s}^o + \Delta s \qquad \text{Equation (22)}$$

Equation (15) describes the change in entropy of an ideal gas going through a process from state 1 to state 2. If the process is isothermal, the first term becomes zero, and the change in enthalpy is:

$$\Delta s = -R \ln\left(\frac{P_2}{P_1}\right) \qquad \text{Equation (23)}$$

Thus, if P_i is the partial pressure of component i of a mixture of gases, and P_o is the mixture pressure of one atmosphere, the total entropy of the component is made up of both the absolute chemical entropy and the change in entropy due to the pressure difference, determined as:

$$\text{total entropy} = \bar{s}^o - R \ln\frac{P_i}{P_o} \qquad \text{Equation (24)}$$

Chemical analyses frequently use molar quantities rather than mass quantities. The superscript bar on the entropy and enthalpy values indicates that these are per-mole quantities. Replacing the sensible enthalpy and entropy values in Equation (20) with these "total" values and indicating the molar quantity of component i as $\dot{N}$, the thermochemical exergy content of a flow stream is:

$$\dot{X}_i = \dot{N}_i\left[\bar{h}_f^o + \bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_i}{P_o}\right)\right] \qquad \text{Equation (25)}$$

Work of Compression

"Useful exergy" is the exergy that is used by a process to achieve some desired purpose. For an air compressor, useful exergy is the work potential, or quantity of exergy, that is available in the compressed air as well as the quantity of exergy contained in any reclaimed heat stream. Figure 4 shows a control volume with a single flow with enthalpy $\dot{H}_1$ entering, a single flow with enthalpy $\dot{H}_2$ leaving, work $\dot{W}$ produced, and heat $\dot{Q}$ added across a boundary at temperature T.

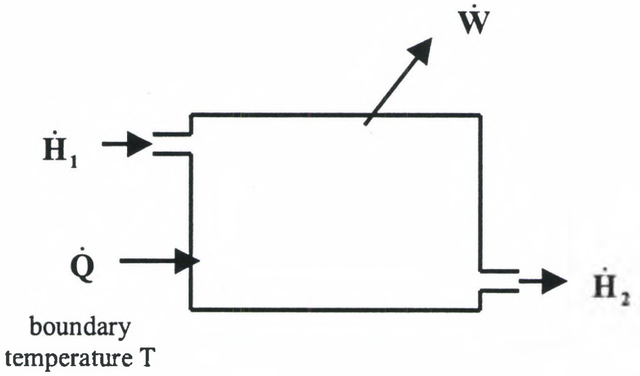

Figure 4. Control volume with energy flows.

In a steady state scenario, there is no storage of mass, energy, entropy or exergy in the control volume. The First Law for this system states:

$$\begin{aligned} &\dot{Q} + \dot{H}_1 - \dot{W} - \dot{H}_2 = 0 \\ &\dot{Q} = \dot{H}_2 - \dot{H}_1 + \dot{W} \end{aligned} \qquad \text{Equation (26)}$$

The Gibbs relation of Equation (12) is restated here:

$$TdS = dH - VdP \qquad \text{Equation (12)}$$

Replacing TdS with its equivalent δQ, integrating between inlet and exit states 1 and 2, and expressing the values in terms of rates per unit mass, we obtain:

$$\int \delta Q = \int dH - \int VdP$$

$$Q = H_2 - H_1 - m\int_1^2 vdP$$ Equation (27)

$$\dot{Q} = \dot{H}_2 - \dot{H}_1 - \dot{m}\int_1^2 vdP$$

Comparing Equation (27) with Equation (26), the work is found to be:

$$\dot{W} = -\dot{m}\int_1^2 vdP$$ Equation (28)

For a polytropic compression process, $Pv^k = C$, where P is pressure, v is specific volume, and C is a constant. For isothermal compression, k = 1. At low pressure, air acts as an ideal gas. Thus,

$$Pv = RT$$
$$v = \frac{RT}{P}$$ Equation (29)

On a flow basis, the work associated with raising the pressure of air from state 1 to state 2 in an isothermal compression process with no losses, the *reversible work*, is:

$$\dot{W} = -\dot{m}\int_1^2 vdP = -\dot{m}RT\int_1^2 \frac{dP}{P} = -\dot{m}RT\ln\left(\frac{P_2}{P_1}\right)$$ Equation (30)

Ideal Gas Considerations

The general relationship between pressure P, specific volume v and temperature T for a gas is:

$$Pv = zRT$$ Equation (31)

where R is the universal gas constant. For an ideal gas, the compressibility factor z equals 1. At low pressures and high temperatures, gases approach ideal gas behavior, and the ideal gas assumption is a good one[2]. As temperature and pressure approach the critical values for the gas, behavior departs from ideal gas behavior. For the air compression process in the case study of Chapter III, the pressure of air is raised from 0 psig (14.7 psia) to 100 psig (114.7 psia) and temperature is held constant at 50°F.

To determine if the ideal gas assumption is valid at 100 psig, we consult the generalized compressibility charts for gases to determine the value for z. If z is very close to 1, the gas closely emulates ideal gas behavior. First, the relationship between critical point values and test conditions is determined, by calculating "reduced pressure" and "reduced temperature". Table 1 lists critical point values for the primary components of air: nitrogen and oxygen.

Table 1. Critical point values for air[2].

Air Component	Critical Point Values
Temperature, N_2	227.1 R
Pressure, N_2	492 psia
Temperature, O_2	278.6 R
Pressure, O_2	736 psia

Assuming that air is a mixture of 79% nitrogen and 21% oxygen by volume, and with total pressure of 114.7 psia, partial pressures, "reduced pressures", and "reduced temperatures" for N_2 and O_2 are calculated as follows:

Nitrogen, N_2

P_{N_2} = partial pressure of N_2

$P_{N_2} = P_{tot} \times 0.79 = 114.7\,\text{psia} \times 0.79 = 90.6\,\text{psia}$

Pr_{N_2} = "reduced pressure" of N_2

$Pr_{N_2} = P_{N_2}/Pcr = 90.6\,\text{psia} / 492\,\text{psia} = 0.18$

Tr_{N_2} = "reduced temperature" of N_2

$Tr_{N_2} = T_{N_2}/Tcr = 510R / 227.1\,R = 2.25$

Oxygen, O_2

P_{O_2} = partial pressure of O_2

$P_{O_2} = 114.7\,\text{psia} \times 0.21 = 24.1\,\text{psia}$

Pr_{O_2} = "reduced pressure" of O_2

$Pr_{O_2} = 24.1\,\text{psia} / 736\,\text{psia} = 0.03$

Tr_{O_2} = "reduced temperature" of O_2

$Tr_{O_2} = 510R / 278.6\,R = 1.83$

Entering the generalized compressibility charts using these reduced values, the compressibility factors for nitrogen and oxygen in air, at standard pressure and 50°F are found to be:

$$z_{N_2} = 0.99$$

$$z_{O_2} = 0.99$$

For the pressure range of the case study, the compressibility factors of the primary components of air are very close to 1. Furthermore, in an isothermal compression process, the gas is maintained at constant temperature throughout the process. In a real, polytropic compression process, the temperature of the gas increases during compression. Therefore, the gas departs further from the critical point, and more closely approaches ideal gas behavior. Thus, the ideal gas assumption appears valid for the industrial air compression example investigated in the next chapter.

CHAPTER III

THERMOMECHANICAL EXERGY: AIR COMPRESSION PROCESS

Compressed air is used in virtually every industrial plant as a source of exergy for tools, actuators, and a myriad of manufacturing processes. For this analysis, a typical scenario is considered with a compressor installed indoors. Conditions for the indoor surroundings are temperature T_1 and pressure P_1, while the outdoor conditions (the environment) are T_o and P_o. For the following analysis, the compressed air system is defined as the compressor, the aftercooler and distribution system, and the end use for the compressed air. It is assumed that all heat of compression is removed and compressed air exits the process at the indoor temperature $T_2 = T_1$. Figure 5 illustrates these conditions.

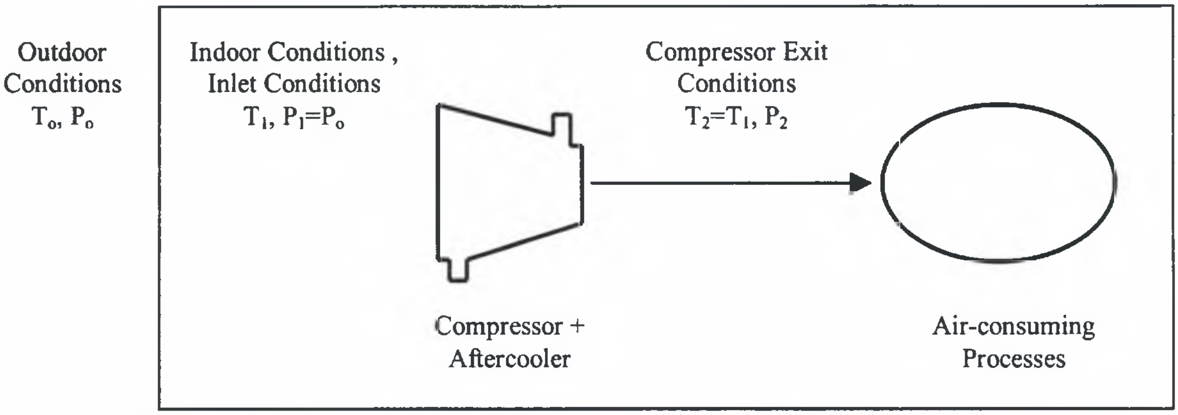

Figure 5. Industrial setting for air compressor, with environmental conditions.

Energy (First Law) Analysis

For a First Law analysis of the air compression process, we apply the laws of Conservation of Mass and Conservation of Energy. Figure 6 shows the physical condition with energy flows. The control volume boundary is indicated as a solid line. Air enters with enthalpy h_1 and exits with enthalpy h_2. Work input $\dot{W}$ is needed to achieve compression. Heat of compression $\dot{Q}$ is rejected to the environment.

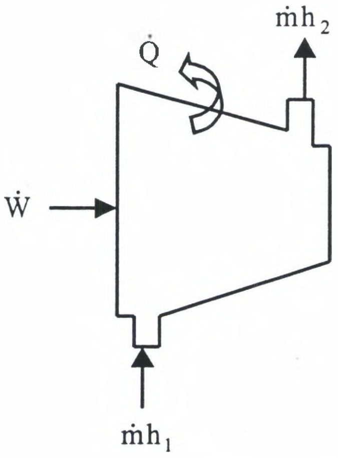

Figure 6. Schematic diagram of air compression process with energy flows.

An air compressor is a steady-flow device. No mass is stored in the control volume over time. Thus, the Conservation of Mass equation,

$$\dot{m}_1 - \dot{m}_2 = \frac{dm}{dt} \qquad \text{Equation (32)}$$

reduces to:

$$\begin{aligned} \dot{m}_1 - \dot{m}_2 &= 0 \\ \dot{m}_1 = \dot{m}_2 &\equiv \dot{m} \end{aligned} \qquad \text{Equation (33)}$$

Similarly, assuming potential and kinetic energy are negligible, the Conservation of Energy equation,

$$\Sigma\dot{E}_{in} - \Sigma\dot{E}_{out} = \frac{dE}{dt} \qquad \text{Equation (34)}$$

reduces to

$$\begin{aligned} \Sigma\dot{E}_{in} &= \Sigma\dot{E}_{out} \\ \dot{W} + \dot{m}h_1 &= \dot{Q} + \dot{m}h_2 \end{aligned} \qquad \text{Equation (35)}$$

Solving for $\dot{Q}$ gives:

$$\dot{Q} = \dot{W} - \dot{m}(h_2 - h_1) \qquad \text{Equation (36)}$$

As is apparent from this formulation, heat, work, and enthalpy are all treated as equivalent forms of energy. However, their abilities to do work, e.g. their exergies, are not equivalent.

An ideal compressor would increase the pressure of the gas without increasing the temperature: the process would be isothermal. In this case, it is assumed that there would be enough heat transferred from the compressed gas to maintain constant temperature. For an ideal gas, enthalpy is a function of temperature only. In the isothermal process, there would be no change in temperature from state 1 to state 2. Thus, $h_1 = h_2$ and Equation (36) would reduce to $\dot{Q} = \dot{W}$. This indicates that from a First Law perspective all work entering the control volume would be converted to heat and the energy content of the compressed air would be the same as the energy content of the uncompressed, input air.

In a real compression process, the temperature of the compressed gas rises during compression, because of irreversibilities and because there is not enough cooling to keep the temperature constant. However, aftercoolers are used to remove the remaining heat of compression and moisture from the compressed air. Thus, the compressed air reaches end use locations at approximately the temperature of the indoor surroundings and the assumption of an isothermal process for the overall system is valid.

The First Law efficiency of any process is defined by Equation (6) as the ratio of useful energy output to valued energy input. For a compression process, the useful energy output includes the enthalpy change of the compressed gas and any heat $\dot{Q}_r$ reclaimed for a useful purpose. If the electricity supplied is $\dot{W}_{ele}$, the energy efficiency of the compression process is:

$$\eta_1 = \frac{\dot{m}(h_2 - h_1) + \dot{Q}_r}{\dot{W}_{ele}}$$

Equation (37)

$$\eta_1 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{Q}_r}{\dot{W}_{ele}}$$

For an isothermal process with no heat reclamation, $\dot{Q}_r = 0$ and $T_1 = T_2$. The energy efficiency of the process is zero. All energy (electricity) supplied to the process is given off as waste heat. If this seems counter-intuitive, it is because it is obvious that compressed air is useful. But the First Law analysis seems to indicate that compressing air is a waste of energy. Apparently, First Law analysis does not tell the whole story.

Exergy (Second Law) Analysis

An exergy formulation of the Second Law for any process, where $\dot{X}$ represents exergy flow, is as follows:

$$\Sigma\dot{X}_{in} - \Sigma\dot{X}_{out} - \dot{X}_{destroyed} = \frac{dX_{cv}}{dt} \qquad \text{Equation (38)}$$

For steady-state processes, no exergy storage occurs in the control volume, and the general equation becomes:

$$\Sigma\dot{X}_{in} - \Sigma\dot{X}_{out} = \dot{X}_{destroyed} \qquad \text{Equation (39)}$$

These equations make clear one difference between the First and Second Laws: unlike energy, which is conserved, exergy is not conserved. Some exergy is destroyed or lost in every real process.

Since the air does not undergo any chemical reactions and assuming potential and kinetic energy may be neglected, exergy values of the air at states 1 and 2 are evaluated using Equation (20):

$$\begin{aligned} \dot{X}_1 &= \dot{m}(h_1 - T_o s_1) - \dot{m}(h_o - T_o s_o) \\ \dot{X}_2 &= \dot{m}(h_2 - T_o s_2) - \dot{m}(h_o - T_o s_o) \end{aligned} \qquad \text{Equation (40)}$$

The exergy content of $\dot{Q}$, as defined in Equation (5), is:

$$\dot{X}_{h.t.} = \dot{Q}\left(1 - \frac{T_o}{T_s}\right) \qquad \text{Equation (5)}$$

where T_s is the temperature at which heat leaves the system. This is the temperature of the cooling air or water at the immediate location where it leaves the compressor. Figure 7 shows the flows of exergy to and from the compressor control volume.

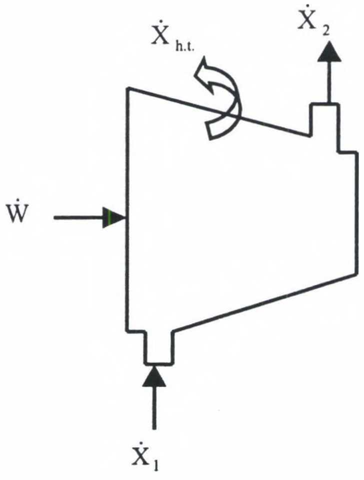

Figure 7. Schematic diagram of compressor with exergy flows.

Inserting the above relations into Equation (39) and rearranging, an expression is obtained for the exergy destroyed in the air compressor:

$$\dot{X}_{destroyed} = \Sigma\dot{X}_{in} - \Sigma\dot{X}_{out}$$

$$\dot{X}_{destroyed} = \dot{W} + [\dot{m}(h_1 - T_o s_1) - \dot{m}(h_o - T_o s_o)] - [\dot{m}(h_2 - T_o s_2) - \dot{m}(h_o - T_o s_o)] - \dot{Q}\left(1 - \frac{T_o}{T_s}\right)$$

$$\dot{X}_{destroyed} = \dot{W} + \dot{m}(h_1 - T_o s_1) - \dot{m}(h_2 - T_o s_2) - \dot{Q}\left(1 - \frac{T_o}{T_s}\right)$$

$$\dot{X}_{destroyed} = \dot{W} + \dot{m}(h_1 - h_2) + \dot{m}T_o(s_2 - s_1) - \dot{Q}\left(1 - \frac{T_o}{T_s}\right)$$

$$\dot{X}_{destroyed} = \dot{W} + \dot{m}C_p(T_1 - T_2) + \dot{m}T_o(s_2 - s_1) - \dot{Q}\left(1 - \frac{T_o}{T_s}\right)$$

Equation (41)

where state 2 is the state of compressed gas exiting the process, state 1 is the inlet state, and state 0 is the ambient state of the surroundings. The change in entropy, s_2-s_1, is derived in Chapter II, Equation (15):

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$ Equation (15)

Thus, the exergy destroyed in compressing the air is:

$$\dot{X}_{destroyed} = \dot{W} + \dot{m}C_p(T_1 - T_2) - \dot{m}RT_o\ln\left(\frac{P_2}{P_1}\right) - \dot{Q}\left(1 - \frac{T_o}{T_s}\right) \qquad \text{Equation (42)}$$

When considering the entire compressed air system, additional exergy is lost in the distribution system and at the end uses. Compressed air leaks result in exergy losses due to unrestrained expansion when pressure drops from the supply pressure P_2 to atmospheric pressure P_o. Pipe flow friction also results in pressure drops. In addition, when compressed air is supplied to end uses at higher pressures than the equipment requires, the excess pressure is discharged to the environment. These pressure drops further contribute to system exergy losses. The exergy loss due to air leaks, where $\dot{m}_{leak}$ represents the mass flow rate of air leaks, is:

$$\dot{X}_{leak} = -\dot{m}_{leak}RT_o\ln\left(\frac{P_o}{P_2}\right) \qquad \text{Equation (43)}$$

The exergy loss incurred by providing compressed air at unnecessarily high pressure P_2 greater than the required process pressure P_c is:

$$\dot{X}_p = -\dot{m}RT_o\ln\left(\frac{P_c}{P_2}\right) \qquad \text{Equation (44)}$$

Thus, the useful exergy of the overall process is the exergy supplied to the system $\dot{W}_{ele}$ minus exergy destroyed and exergy losses:

$$\dot{X}_{useful} = \dot{W}_{ele} - \left\{\dot{W}_{ele} + \dot{m}C_p(T_1 - T_2) + \dot{m}T_o\left[C_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)\right] - \dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_o}{P_2}\right) - \dot{m}RT_o\ln\left(\frac{P_c}{P_2}\right)\right\}$$

$$\dot{X}_{useful} = \dot{m}C_p(T_2 - T_1) + \dot{m}T_o(R\ln\left(\frac{P_2}{P_1}\right) - C_p\ln\left(\frac{T_2}{T_1}\right)) + \dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o\ln\left(\frac{P_2}{P_c}\right)$$

Equation (45)

The *exergetic efficiency* of the compression process using Equation (7) is:

$$\eta_2 = \frac{\dot{X}_{useful}}{\dot{X}_{supplied}} = \frac{\dot{W}_{ele} - (\dot{X}_{destroyed} + \dot{X}_{leak} + \dot{X}_p)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{m}T_o(R\ln\left(\frac{P_2}{P_1}\right) - C_p\ln\left(\frac{T_2}{T_1}\right)) + \dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o\ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

Equation (46)

Equation (46) is a general formulation of the exergetic efficiency of an air compression process in terms of the inlet, outlet, and environment temperatures and pressures. The terms in the numerator represent the usable exergy in the process. The first term is the enthalpy increase of the air during compression from state 1 to state 2. The second term represents the exergy change due to raising the pressure, and the third is the exergy change due to raising the temperature. The fourth term is the available work in the stream of rejected heat. The fifth term is the exergy loss due to leaks, and the last term is exergy loss due to pressure drops in the distribution system and discharge of excessively high-pressure compressed air.

Case Study

To illustrate and compare the use of energy and exergy concepts in the industrial setting, an actual set of air compressor recommendations from the UD Industrial Assessment Center is presented. By measuring current draw and loading cycle times, the compressor was found to be loaded about 33% of the time. Air for compression comes from inside the compressor room. No heat of compression is reclaimed. The continuously running compressor loads when plant air pressure drops to 110 psi, and unloads when pressure reaches 120 psi. Some leaks were apparent in distribution lines and at end-uses. Table 2 lists system parameters.

Table 2. Compressed air system parameters.

Compressor type	Rotary screw
Compressor motor horsepower	30 hp
Supply voltage	480 VAC, 3-phase
Measured current draw, loaded	33 Amps per phase
Measured current draw, unloaded	17 Amps per phase
Typical fraction of cycle time loaded	33%
Average outdoor air temperature	50°F
Average indoor air temperature	80°F
Ambient air pressure	14.7 psia
Compressed air pressure	115 psig = 129.7 psia
Minimum required plant air pressure	95 psig
Estimated system leaks	10% of system capacity[5]

To improve the air compression process, several recommendations were made. Plant equipment required 95 psig supply pressure for correct operation. Allowing for a 10 psi pressure drop in the aftercooler, filtration and distribution system components, the target compressor supply pressure P_c was established at

105 psig. Thus, the first improvement recommendation was to lower the activation setpoint pressure to 100 psi and standby setpoint pressure to 110 psi. This would reduce average plant air pressure from 115 to 105 psi., cutting energy costs and reducing wear-and-tear on the pneumatic equipment. The second recommendation was to revise the intake air ductwork to bring in cooler outside air for compression. The third was to install dampers to direct cooling air inside during the winter for space heating, and outside during the summer. The fourth recommendation was to implement a regular leak repair program. The final recommendation was to replace the compressor with a smaller, more appropriately-sized unit, to more closely match system capacity with actual plant requirements.

In the sections below, the improvement provided by each of these measures in isolation is evaluated, as compared to the present system, Case 0. For the final case, the improvement achieved by applying all the recommended measures at once is evaluated. This case represents a "best practices" scenario.

Operating Parameters

To evaluate the system, knowledge of the following parameters is required: loaded and unloaded power draw $\dot{W}_{ele}$, volume flow rate $\dot{V}$ of air, the gas constant R for air, density ρ of air, mass flow rate $\dot{m}$ of air, and the specific heat C_p of air. Assuming an average power factor for the motor of 86%, power draw was measured to be:

$$\dot{W}_{ele} = \frac{33\ \text{Amps} \times 480\ \text{V} \times \sqrt{3}}{1000\ \text{VA/kVA}} \times 0.86\ \text{kW/kVA} = 23.6\ \text{kW loaded}$$

$$\dot{W}_{ele} = \frac{17\ \text{Amps} \times 480\ \text{V} \times \sqrt{3}}{1000\ \text{VA/kVA}} \times 0.86\ \text{kW/kVA} = 12.2\ \text{kW unloaded} \qquad \text{Equation (47)}$$

To estimate the volume flow rate of air, an industry-standard estimate that this type of compressor generates about 4 scfm per brake horsepower is used[5]. For this compressor, the volume flow rate is about:

$$\dot{V} = 4\ \text{scfm/hp} \times 30\ \text{hp} = 120\ \text{scfm} \qquad \text{Equation (48)}$$

The gas constant for air is calculated from the universal gas constant and the molar mass of air. Values of R for air, with two different unit combinations, are:

$$R = \frac{R_u}{M} = \frac{1.986\frac{\text{Btu}}{\text{lbmol}\cdot\text{R}}}{28.97\frac{\text{lbm}}{\text{lbmol}}} = 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}$$

Equation (49)

$$R = \frac{R_u}{M} = \frac{10.73\frac{\text{psia}\cdot\text{ft}^3}{\text{lbmol}\cdot\text{R}}}{28.97\frac{\text{lbm}}{\text{lbmol}}} = 0.370\frac{\text{psia}\cdot\text{ft}^3}{\text{lbm}\cdot\text{R}}$$

The density of air depends on temperature and pressure, but compressed air calculations are carried out at standard conditions of $P_o = 14.7$ psia and $T_o = 50°F$.

$$\rho = \frac{P}{RT} = \frac{14.7\,\text{psia}}{0.370\frac{\text{psia}\cdot\text{ft}^3}{\text{lbm}\cdot\text{R}}\times(460+50)\text{R}} = 0.078\frac{\text{lbm}}{\text{ft}^3}$$ Equation (50)

The mass flow rate of air in the compression process is determined from the density and volume flow rate. The density of air depends on temperature and pressure, but compressed air calculations are carried out at standard pressure. The units for volume flow rate are scfm - cubic feet per minute at standard conditions. The mass flow rate is determined as:

$$\dot{m} = \rho\dot{V} = 0.078\frac{\text{lbm}}{\text{ft}^3}\times 120\frac{\text{ft}^3}{\text{min}}\times\frac{60\text{ min}}{\text{hr}} = 560.9\frac{\text{lbm}}{\text{hr}}$$ Equation (51)

Leaks are assumed to account for 10% of total system capacity[5]. With this assumption, the mass flow rate of compressed air leaks to the atmosphere is:

$$\dot{m}_{\text{leak}} = 10\%\times\dot{m}$$

$$\dot{m}_{\text{leak}} = 10\%\times 560.9\frac{\text{lbm}}{\text{hr}} = 56.1\frac{\text{lbm}}{\text{hr}}$$ Equation (52)

Finally, the specific heat of air at standard conditions is about[2]:

$$C_p = 0.239 \frac{\text{Btu}}{\text{lbm} \cdot \text{R}}$$

Case 0: Base System Efficiency

For the process as presently configured, indoor air enters the compressor at temperature $T_1 = 80°F = 540R$ and pressure $P_1 = P_o = 14.7$ psia, and exits at temperature $T_2 = T_1$ and pressure $P_2 = 115$ psig = 129.7 psia. No heat is reclaimed. The energy efficiency of the process is calculated according to Equation (37):

$$\eta_1 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{Q}_r}{\dot{W}_{ele}} \qquad \text{Equation (37)}$$

Because $T_2 = T_1$, the enthalpy change is zero. In addition, because no heat is reclaimed, $\dot{Q}_r$ is zero. Thus, the entire numerator is zero and the process has 0% energy efficiency.

The exergetic efficiency for a loaded air compressor is calculated according to Equation (46).

$$\eta_2 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{m}T_o(R \ln\left(\frac{P_2}{P_1}\right) - C_p \ln\left(\frac{T_2}{T_1}\right)) + \dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak}RT_o \ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o \ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

Equation (46)

All useful exergy obtained from the process is due to the pressure increase in the air from P_o to P_c, while leaks and pressure losses reduce useful exergy. Thus, the first, third, and fourth terms in the numerator of Equation (46) are zero and the exergetic efficiency for the compression process when the compressor is loaded is defined as:

$$\eta_2 = \frac{\dot{m}RT_o \ln\left(\frac{P_2}{P_1}\right) - \dot{m}_{leak}RT_0\ln\left(\frac{P_2}{P_0}\right) - \dot{m}RT_0\ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left\{\begin{array}{l}\left[\left[560.9\frac{\text{lbm}}{\text{hr}}\times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\times 510\text{R}\times\ln\left(\frac{129.7\text{psia}}{14.7\text{psia}}\right)\right]-\left[56.1\frac{\text{lbm}}{\text{hr}}\times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\times 510\text{R}\times\ln\left(\frac{129.7\text{psia}}{14.7\text{psia}}\right)\right]\right] \\ -\left[560.9\frac{\text{lbm}}{\text{hr}}\times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\times 510\text{R}\times\ln\left(\frac{129.7\text{psia}}{119.7\text{psia}}\right)\right]\end{array}\right\}\times\frac{1\,\text{kW}}{3412\,\text{Btu/hr}}}{23.6\,\text{kW}}$$

$$\eta_2 = \frac{10.9\,\text{kW}}{23.6\,\text{kW}} = 46\%$$

Equation (53)

The overall process includes unloaded compressor run time as well as loaded run time. Measurements from the plant assessment and information about plant operations revealed that, although it runs full-time, the compressor is loaded for only about 33% of the time. Thus, the average power draw including unloaded running time is about:

$$\dot{W}_{avg} = (23.6\,\text{kW x } 33\%) + (12.2\,\text{kW x } 67\%) = 16\,\text{kW} \qquad \text{Equation (54)}$$

When compressing air, the useful exergy of the process is 10.9 kW, as shown in the numerator of Equation (53). Constant pressure is maintained in the supply header. Thus, leaks result in exergy loss even when the compressor is unloaded. When running in an unloaded mode, the useful exergy of the process is -1.26 kW, an exergy loss due to leaks. The average overall process efficiency is:

$$\eta_2 = \frac{(10.9\,\text{kW x } 33\%) + (-1.26\,\text{kW x } 67\%)}{16\,\text{kW}} = 17\% \qquad \text{Equation (55)}$$

Case 1: System Efficiency with Lower Discharge Pressure

In this case, the compression process efficiency due to lowering the discharge pressure is discussed. Like the base case, indoor air enters the compressor at temperature $T_1 = 80°F = 540R$ and pressure $P_1 = P_o = 14.7$ psia and exits at temperature $T_2 = T_1$. In this case, the average discharge pressure is reduced to $P_c = 105$ psig to more closely match equipment needs while allowing for a 10 psi pressure drop in the distribution system. No heat is reclaimed.

The energy efficiency of the process is calculated according to Equation (37):

$$\eta_1 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{Q}_r}{\dot{W}_{ele}} \qquad \text{Equation (37)}$$

The energy efficiency of the process does not depend on pressure. Because $T_2 = T_1$, the enthalpy change is zero. In addition, because no heat is reclaimed, $\dot{Q}_r$ is zero. Thus, the entire numerator is zero, and the process again has 0% energy efficiency.

As discussed in Chapter II, the work of compression depends on temperature and pressure of the inlet and discharge air streams. Reducing the required discharge pressure would result in a reduction of work input in the amount $\Delta\dot{W}$:

$$\Delta\dot{W} = \dot{m}RT_o\left[\ln\left(\frac{P_2}{P_1}\right) - \ln\left(\frac{P_c}{P_1}\right)\right]$$

$$\Delta\dot{W} = 560.9\frac{\text{lbm}}{\text{hr}} \times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}} \times 510\text{R} \times \left[\ln\left(\frac{129.7\text{psia}}{14.7\text{psia}}\right) - \ln\left(\frac{119.7\text{psia}}{14.7\text{psia}}\right)\right] \times \frac{1\,\text{kW}}{3412\,\text{Btu/hr}}$$

$$\Delta\dot{W} = 0.5\,\text{kW}$$

Equation (56)

The work input is thus:

$$\dot{W}_{ele} = \dot{W}_{ele} - \Delta\dot{W}$$
$$\dot{W}_{ele} = 23.6\,\text{kW} - 0.5\,\text{kW} = 23.1\,\text{kW} \qquad \text{Equation (57)}$$

The exergetic efficiency for the loaded air compressor is calculated as in the base case according to Equation (46) with the modified value $\dot{W}_{ele}$. In this case, the temperature-dependent terms are again zero. Since the compressor discharge pressure is reduced, all P_2 terms are changed to P_c, and the exergy loss due to excessive pressure is zero.

$$\eta_2 = \frac{\dot{m}RT_o \ln\left(\frac{P_c}{P_1}\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_c}{P_o}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left\{\begin{array}{l} 560.9\frac{\text{lbm}}{\text{hr}} \times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}} \times 510\text{R} \times \ln\left(\frac{119.7\text{psia}}{14.7\text{psia}}\right) \\ -56.1\frac{\text{lbm}}{\text{hr}} \times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}} \times 510\text{R} \times \ln\left(\frac{119.7\text{psia}}{14.7\text{psia}}\right) \end{array}\right\} \times \frac{1\,\text{kW}}{3412\,\text{Btu/hr}}}{23.1\,\text{kW}} \qquad \text{Equation (58)}$$

$$\eta_2 = \frac{10.9\,\text{kW}}{23.1\,\text{kW}} = 47\%$$

Including the effect of unloaded running time, the average power draw is about:

$$\dot{W}_{avg} = (23.1\,\text{kW x } 33\%) + (12.2\,\text{kW x } 67\%) = 15.8\,\text{kW} \qquad \text{Equation (59)}$$

When compressing air, the useful exergy of the process is 10.9 kW, as shown in the numerator of Equation (58). When running in unloaded mode, the useful exergy of the process is -1.26 kW as shown in Case 0. The average overall process efficiency is:

$$\eta_2 = \frac{(10.9\,\text{kW x } 33\%) + (-1.26\,\text{kW x } 67\%)}{15.8\,\text{kW}} = 17.4\% \qquad \text{Equation (60)}$$

This analysis shows that the overall process is slightly more efficient when delivering air at a lower discharge pressure. This is true because less work is required to compress the air to the lowered discharge pressure. The loss associated with discharging the air supplied at excessively high pressure is eliminated. Since the system power draw is reduced, efficiency is increased and operating cost is lowered.

Case 2: System Efficiency with Reduced Inlet Air Temperature

In the base case, air for compression is drawn from the indoor surroundings at 80°F, while the average outdoor temperature is 50°F. By drawing air from outdoors, the temperature-dependent terms in the efficiency equation would have finite values. Thus, the first- and second-law efficiencies would be improved.

Using Equation (37), the energy efficiency for the loaded compressor is:

$$\eta_1 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{Q}_r}{\dot{W}_{ele}}$$

$$\eta_1 = \frac{560.9\,\frac{\text{lbm}}{\text{hr}} \times 0.239\,\frac{\text{Btu}}{\text{lbm}\cdot\text{R}} \times (80-50)\text{R} \times \frac{1\ \text{kW}}{3{,}412\ \text{Btu/hr}} + 0}{23.6\,\text{kW}} \qquad \text{Equation (61)}$$

$$\eta_1 = \frac{1.2\,\text{kW}}{23.6\,\text{kW}} = 5\%$$

Since the work of compression depends on temperature and pressure of the inlet and discharge air streams, reducing the inlet temperature would reduce the actual work input $\dot{W}_{ele}$. The reduction in work input $\Delta\dot{W}$ is:

$$\Delta\dot{W} = \left[\dot{m}C_p(T_2 - T_{1\,new}) - \dot{m}T_oC_p\ln\left(\frac{T_2}{T_{1\,new}}\right)\right] - \left[\dot{m}C_p(T_2 - T_1) - \dot{m}T_oC_p\ln\left(\frac{T_2}{T_1}\right)\right] \qquad \text{Equation (62)}$$

Because $T_2 = T_1$ in the base scenario, the second bracketed term is zero. Thus, the reduction in work would be:

$$\Delta\dot{W} = \left[\dot{m}C_p(T_2 - T_{1\,new}) - \dot{m}T_oC_p\ln\left(\frac{T_2}{T_{1\,new}}\right)\right]$$

$$\Delta\dot{W} = \left(560.9 \times 0.239 \times (80-50) - 560.9 \times 510 \times 0.239 \times \ln\frac{(460+80)}{(460+50)}\right) \div 3412 \qquad \text{Equation (63)}$$

$$\Delta\dot{W} = 0.03\,\text{kW}$$

The process work input is thus:

$$\dot{W}_{ele} = \dot{W}_{ele} - \Delta\dot{W}$$
$$\dot{W}_{ele} = 23.6\,\text{kW} - 0.03\,\text{kW} = 23.57\,\text{kW} \qquad \text{Equation (64)}$$

The exergetic efficiency for the loaded air compressor is calculated according to Equation (46), with modified values for T_1 and $\dot{W}_{ele}$. Thus, the exergetic efficiency becomes:

$$\eta_2 = \frac{\dot{m}C_p(T_2 - T_{1\,new}) + \dot{m}T_o\left(R\ln\left(\frac{P_2}{P_1}\right) - C_p\ln\left(\frac{T_2}{T_{1\,new}}\right)\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o\ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left[\begin{array}{l} 560.9\times 0.239\times(80-50) + 560.9\times 510\times\left(0.069\times\ln\left(\frac{129.7}{14.7}\right) - 0.239\times\ln\left(\frac{540}{510}\right)\right) \\ -56.1\times 510\times\left(0.069\times\ln\left(\frac{129.7}{14.7}\right)\right) - 560.9\times 510\times\left(0.069\times\ln\left(\frac{129.7}{119.7}\right)\right) \end{array}\right] \div 3412}{23.57\,\text{kW}}$$

$$\eta_2 = \frac{10.9\,\text{kW}}{23.57\,\text{kW}} = 46\%$$

Equation (65)

Including the effect of unloaded running time, the average power draw is:

$$\dot{W}_{avg} = (23.57\,\text{kW x } 33\%) + (12.2\,\text{kW x } 67\%) = 15.9\,\text{kW} \qquad \text{Equation (66)}$$

When compressing air, the useful exergy of the process is 10.9 kW. In unloaded mode, the useful exergy of the process is -1.26 kW. The average overall process efficiency is:

$$\eta_2 = \frac{(10.9\,\text{kW x } 33\%) + (-1.26\,\text{kW x } 67\%)}{15.9\,\text{kW}} = 17\% \qquad \text{Equation (67)}$$

This analysis suggests that the efficiency of the compression process is affected very little by taking in cooler air. This is true because the exergy content of compressed air is dominated by the effect of raising pressure, while the temperature effect is very small. However, a small cost savings would result because the compressor power input is reduced.

Case 3: System Efficiency when Reclaiming Heat of Compression

The process efficiency can be improved by reclaiming the heat of compression. One way to use this heat is to use the heated air or water to provide space heating in the facility during winter months. Under this scenario, the quantity of energy purchased for space heating is reduced by the quantity of heat reclaimed. As discussed earlier, for an isothermal process like the base case, the quantity of heat is equal to the electrical energy supplied to the compressor. For this analysis, all of the heat is assumed to be used for 6 months per year. From Equation (37) with $T_1 = T_2$, the energy efficiency for the loaded compressor during the winter months, and that for the summer months, would be about:

$$\eta_1 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{Q}_r}{\dot{W}_{ele}}$$

$$\eta_1 = \frac{\dot{Q}_r}{\dot{W}_{ele}} = \frac{23.6\,\mathrm{kW}}{23.6\,\mathrm{kW}} = 100\%\ \text{in winter} \qquad \text{Equation (68)}$$

$$\eta_1 = \frac{\dot{Q}_r}{\dot{W}_{ele}} = \frac{0\,\mathrm{kW}}{23.6\,\mathrm{kW}} = 0\%\ \text{in summer}$$

When the compressor is unloaded and running, heat is removed by cooling air or water. The energy efficiency when all heat is reclaimed is 100%. During the summer months, the compressor is operated exactly as in the base case; the loaded exergetic efficiency is 46%. From Equation (46), the exergetic efficiency for the loaded compressor during heating season when heat is reclaimed at 100°F becomes:

$$\eta_2 = \frac{\dot{m}RT_o \ln\left(\frac{P_2}{P_1}\right) + \dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak}RT_o \ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o \ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left\{\begin{array}{l} 560.9 \times 0.069 \times 510 \times \ln\left(\frac{129.7}{14.7}\right) - 56.1 \times 0.069 \times 510 \times \ln\left(\frac{129.7}{14.7}\right) \\ - 560.9 \times 0.069 \times 510 \times \ln\left(\frac{129.7}{119.7}\right) \end{array}\right\} \div 3412 + 23.6\,\mathrm{kW} \times \left(1 - \frac{460+50}{460+100}\right)}{23.6\,\mathrm{kW}}$$

$$\eta_2 = \frac{13\,\mathrm{kW}}{23.6\,\mathrm{kW}} = 55\%$$

Equation (69)

When the compressor is unloaded and running, it draws 12.2 kW and the heat removed by cooling air or water can be reclaimed. The exergetic efficiency in this case, when all heat is reclaimed but no other usable exergy is obtained, is:

$$\eta_2 = \frac{\dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak} R T_o \ln\left(\frac{P_2}{P_o}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{12.2\,\text{kW} \times \left(1 - \frac{510}{460+100}\right) - 1.26\,\text{kW}}{12.2\,\text{kW}} \qquad \text{Equation (70)}$$

$$\eta_2 = \frac{1.1\,\text{kW} - 1.26\,\text{kW}}{12.2\,\text{kW}} = \frac{-0.16\,\text{kW}}{12.2\,\text{kW}} = -0.01\% \cong 0\%$$

The average power draw including loaded and unloaded running time, is:

$$\dot{W}_{avg} = (23.6\,\text{kW x } 33\%) + (12.2\,\text{kW x } 67\%) = 16\,\text{kW} \qquad \text{Equation (71)}$$

When compressing air, the useful exergy of the process is 13 kW; when running in an unloaded mode, the useful exergy of the process is about 0 kW. The average overall process efficiency is:

$$\eta_2 = \frac{(13\,\text{kW x } 33\%) + (0\,\text{kW x } 67\%)}{16\,\text{kW}} = 27\% \qquad \text{Equation (72)}$$

During the summer months, the conditions are exactly like the base case. Thus, for half the year, the useful exergy is 10.9 kW when loaded and -1.26 kW when unloaded. To compare the process efficiency to the other cases, the annual average efficiency for the compression process with wintertime heat reclamation is determined to be:

$$\eta_2 = \frac{[(13\,\text{kW} \times 33\%) + (0\,\text{kW} \times 67\%)] \times 50\% + [(10.9\,\text{kW} \times 33\%) + (-1.26\,\text{kW} \times 67\%)] \times 50\%}{16\,\text{kW}} = 22\%$$

Equation (73)

This analysis shows that the process efficiency is increased as a result of reclaiming the heat of compression, which would otherwise be discarded. Exergy recovered by reclaiming the heat of compression is the difference between work input for this case and the base case:

$$\dot{X}_{\text{loaded}} = 13\,\text{kW} - 10.9\,\text{kW} = 2.1\,\text{kW}$$
$$\dot{X}_{\text{unloaded}} = 0\,\text{kW} - 0\,\text{kW} = 0\,\text{kW}$$
Equation (74)

Thus, the average annual exergy draw for the process, including load cycling, heat reclamation for 6 winter months, and no heat reclamation for 6 summer months would be:

$$\dot{W}_{\text{avg}_{\text{ann}}} = [\text{average power draw, winter}] + [\text{average power draw, summer}]$$
$$\dot{W}_{\text{avg}_{\text{ann}}} = [(23.6\,\text{kW} - 2.1\,\text{kW}) \times 33\% + (12.2\,\text{kW} - 0\,\text{kW}) \times 67\%] \times 50\% + [(23.6\,\text{kW} \times 33\%) + (12.2\,\text{kW} \times 67\%)] \times 50\%$$
$$\dot{W}_{\text{avg}_{\text{ann}}} = 15.6\,\text{kW}$$
Equation (75)

Case 4: System Efficiency with Reduced Air Demand

Due to the many difficulties encountered in determining the total quantity of compressed air leakage in an industrial plant, industry personnel often assume that 10% to 20% of total capacity is lost to leaks[5]. For this analysis we assume a leakage rate of 10% of total system capacity and that all leaks are repairable. Under current conditions, the compressor is loaded for 33% of run time, and the compressor capacity is about 120 scfm. Thus, the average flow rate of air is about:

$$120\ \text{scfm} \times 33\% = 40\ \text{scfm}$$
Equation (76)

Repairing leaks would increase the time the compressor runs unloaded. If leaks comprise 10% of total capacity, the plant requirement after leaks were repaired would be about:

$$40\ \text{scfm} - (120\ \text{scfm} \times 10\%) = 28\ \text{scfm}$$
Equation (77)

The new fraction of run time that the compressor would be loaded would be about:

$$28\ \text{scfm} / 120\ \text{scfm} = 23\%$$
Equation (78)

In this case, the temperature dependent terms and the heat reclamation term are zero. In addition, all leaks are repaired, so the exergy loss due to leaks is zero. The loaded exergetic efficiency is calculated as:

$$\eta_2 = \frac{\dot{m}RT_o \ln\left(\frac{P_2}{P_1}\right) - \dot{m}RT_o \ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left\{560.9 \times 0.069 \times 510 \times \ln\left(\frac{129.7}{14.7}\right) - 560.9 \times 0.069 \times 510 \times \ln\left(\frac{129.7}{119.7}\right)\right\} \times \frac{1}{3412}}{\dot{W}_{ele}} \qquad \text{Equation (79)}$$

$$\eta_2 = \frac{12.1\,\text{kW}}{23.6\,\text{kW}} = 51\%$$

The difference between this case and the base case is that the fraction of loaded run time would be reduced from 33% to 23%. Thus, the average power draw would be:

$$\dot{W}_{avg} = (23.6\,\text{kW x } 23\%) + (12.2\,\text{kW x } 77\%) = 14.8\,\text{kW} \qquad \text{Equation (80)}$$

The average process efficiency including load cycling effects and elimination of leaks would be:

$$\eta_2 = \frac{(12.1\,\text{kW x } 23\%) + (0\,\text{kW x } 77\%)}{14.8\,\text{kW}} = 19\% \qquad \text{Equation (81)}$$

This analysis shows that the power draw would decrease and some cost savings would result. Simply repairing system leaks would slightly increase the exergetic efficiency of the process, but repairing system leaks is best undertaken along with correct re-sizing of the compressor. This combination would further decrease costs and improve exergetic efficiency, as shown in the following sections.

Case 5: System Efficiency with Correctly-sized Compressor

Under current conditions, the compressor is loaded for 33% of run time and the system capacity is about 120 scfm. Thus, the plant requirement for compressed air is about:

$$120\text{ scfm x } 33\% = 40\text{ scfm} \qquad \text{Equation (82)}$$

A compressor that could provide 40 scfm output would be loaded for 100% of the time. The mass flow rate for the new compressor would be:

$$\dot{m} = \rho\dot{\forall} = 0.078\frac{\text{lbm}}{\text{ft}^3} \times 40\frac{\text{ft}^3}{\text{min}} \times \frac{60\text{ min}}{\text{hr}} = 187.2\frac{\text{lbm}}{\text{hr}} \qquad \text{Equation (83)}$$

Using the industry standard that rotary screw compressors generate about 4 scfm per brake horsepower[6], the correct size for current compressed air requirements would be:

$$40 \text{ scfm} / 4 \text{ scfm/hp} = 10 \text{ hp} \qquad \text{Equation (84)}$$

Assuming the 10-hp motor is 88% efficient and fully loaded, the power draw for the correctly-sized compressor would be about:

$$\frac{10 \text{ hp x } 0.746 \text{ kW/hp}}{88\%} = 8.5 \text{ kW} \qquad \text{Equation (85)}$$

Energy efficiency for the system is determined by Equation (37).

$$\eta_1 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{Q}_r}{\dot{W}_{ele}} \qquad \text{Equation (37)}$$

As in the base case, $T_2 = T_1$ and the enthalpy change is zero. In addition, because no heat is reclaimed, $\dot{Q}_r$ is zero. Thus, the process has 0% energy efficiency.

The exergetic efficiency for the loaded air compressor is determined according to Equation (46).

$$\eta_2 = \frac{\dot{m}C_p(T_2 - T_1) + \dot{m}T_o\left(R\ln\left(\frac{P_2}{P_1}\right) - C_p\ln\left(\frac{T_2}{T_1}\right)\right) + \dot{Q}\left(1 - \frac{T_o}{T_s}\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o\ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

Equation (46)

Just as in the base case, all useful exergy obtained from the process is due to the pressure increase of the compressed air. Thus, the temperature-dependent and heat reclamation terms in the numerator are all zero. However, with a correctly-sized compressor, the power input $\dot{W}_{ele}$ is significantly lower and the mass flow rate is reduced as shown by Equation (83). Thus, exergetic efficiency for the compression process when the compressor is loaded is:

$$\eta_2 = \frac{\dot{m}RT_o\ln\left(\frac{P_2}{P_1}\right) - \dot{m}_{leak}RT_o\ln\left(\frac{P_2}{P_o}\right) - \dot{m}RT_o\ln\left(\frac{P_2}{P_c}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left\{\begin{array}{l} 187.2\times0.069\times510\times\ln\left(\frac{129.7\text{psia}}{14.7\text{psia}}\right) - 56.1\times0.069\times510\times\ln\left(\frac{129.7}{14.7}\right) \\ -187.2\times0.069\times510\times\ln\left(\frac{129.7}{119.7}\right) \end{array}\right\}\times\frac{1}{3412}}{8.5\,\text{kW}} \qquad \text{Equation (86)}$$

$$\eta_2 = \frac{2.8\,\text{kW}}{8.5\,\text{kW}} = 33\%$$

Because the compressor is sized to run fully loaded for 100% of the time, the average power draw and average overall process efficiency are the same as the loaded values: 8.5 kW and 33%. The exergetic efficiency for this case is lower than that of the base case because leaks were not repaired and are thus a much larger proportion of the compressor output capacity.

Case 6: "Best Practices": Combined Effects of Cases 1 through 5

The best installations simultaneously apply all improvement measures. In this case, the energy efficiency becomes 100% during heating season, because heat reclamation is provided, and 0% when no heat is reclaimed. To properly evaluate the combined improvements, the measures must be implemented in the proper sequence. The exergy method starts with by determining the actual requirements at the end use, and works back toward the compressor. It thus begins by minimizing the consumption of compressed air. In Case 4, the plant's actual compressed air requirement $\dot{V}$ could be reduced to 28 scfm by repairing leaks. The mass flow rate at this reduced output would be:

$$\dot{m} = \rho\dot{V} = 0.078\frac{\text{lbm}}{\text{ft}^3}\times28\frac{\text{ft}^3}{\text{min}}\times\frac{60\,\text{min}}{\text{hr}} = 131\frac{\text{lbm}}{\text{hr}} \qquad \text{Equation (87)}$$

The correct compressor size for the new compressed air requirements would be about:

$$28 \text{ scfm} / 4 \text{ scfm/hp} = 7 \text{ hp} \qquad \text{Equation (88)}$$

Assuming the 7-hp motor is 85% efficient, the power draw for the correctly-sized compressor would be about:

$$\dot{W}_{ele} = \frac{7 \text{ hp x } 0.746 \text{ kW/hp}}{85\%} = 6.1 \text{ kW} \qquad \text{Equation (89)}$$

By the method in Case 1, the reduction in electrical work input due to discharge pressure reduction would be:

$$\Delta\dot{W}_P = \dot{m}RT_o \ln\left(\frac{P_2}{P_1}\right) - \dot{m}RT_o \ln\left(\frac{P_c}{P_1}\right)$$
$$\Delta\dot{W}_P = 131\frac{\text{lbm}}{\text{hr}} \times 0.069\frac{\text{Btu}}{\text{lbm}\cdot\text{R}} \times 510\text{R} \times \left[\ln\left(\frac{129.7\text{psia}}{14.7\text{psia}}\right) - \ln\left(\frac{119.7\text{psia}}{14.7\text{psia}}\right)\right] \times \frac{1\,\text{kW}}{3412\,\text{Btu/hr}}$$
$$\Delta\dot{W}_P = 0.1\,\text{kW}$$

Equation (90)

As for Case 2, the reduction in work input due to lowering the intake air temperature is calculated as:

$$\Delta\dot{W}_T = \dot{m}C_p(T_2 - T_o) - \dot{m}T_oC_p\ln\left(\frac{T_2}{T_o}\right)$$
$$\Delta\dot{W}_T = \left(131 \times 0.239 \times (80-50) - 131 \times 510 \times 0.239 \times \ln\frac{(460+80)}{(460+50)}\right) \div 3412 \qquad \text{Equation (91)}$$
$$\Delta\dot{W}_T = 0.01\,\text{kW}$$

Thus, we find that the actual electrical work input would be:

$$\dot{W}_{ele} = \dot{W}_{ele,old} - \left(\Delta\dot{W}_P + \Delta\dot{W}_T\right) = 6.1\,\text{kW} - (0.1\,\text{kW} + 0.01\,\text{kW}) = 6\,\text{kW} \qquad \text{Equation (92)}$$

By correctly sizing the compressor, the system would run loaded 100% of the time. Thus, the average overall exergetic efficiency would be the same as the loaded efficiency, and would be calculated using Equation (46) with the leak term eliminated:

$$\eta_2 = \frac{\dot{m}C_p(T_2 - T_{1o}) + \dot{m}T_o\left(R\ln\left(\frac{P_c}{P_o}\right) - C_p\ln\left(\frac{T_2}{T_o}\right)\right) + \dot{Q}\left(1 - \frac{T_o}{T_s}\right)}{\dot{W}_{ele}}$$

$$\eta_2 = \frac{\left[131 \times 0.239 \times (80 - 50) + 131 \times 510 \times \left(0.069 \times \ln\left(\frac{119.7}{14.7}\right) - 0.239 \times \ln\left(\frac{540}{510}\right)\right)\right] \div 3412 + 6\,\text{kW} \times \left(1 - \frac{510}{560}\right)}{6\,\text{kW}}$$

$$\eta_2 = \frac{3.4\,\text{kW}}{6\,\text{kW}} = 56\%$$

Equation (93)

Conclusions

Table 3 summarizes the process efficiency values for each case. Table 3 clearly demonstrates that energy efficiency analysis does not provide enough information to make good decisions on how to improve the process. The energy efficiency information suggests that the only useful improvement is heat reclamation.

Table 3. Comparison of compressed air processes.

Condition	Description	Loaded η_1	unloaded η_1	average η_1	loaded η_2	unloaded η_2	Average η_2
Case 0, base case	indoor air, no heat reclaim, oversized compressor, excessively high pressure setting	0%	0%	0%	46%	0%	17%
Case 1	Case 0 with lowered pressure setting	0%	0%	0%	47%	0%	17.4%
Case 2	Case 0 with lower-temperature inlet air	5%	0%	1.6%	46%	0%	17%
Case 3	Case 0 with heat reclamation	100%*	100%*	100%*	55%*	0%*	22%**
Case 4	Case 0 with reduced air demand	0%	0%	0%	51%	0%	19%
Case 5	Case 0 with properly-sized compressor	0%	-	0%	33%	-	33%
Case 6	"Best Practice" case: improvements 1-5 simultaneously implemented	100%	-	100%	56%	-	56%

* Process efficiencies when heat is reclaimed. Does not include the effect of 6 months with no heat reclamation.

** This value calculated for overall process, including load cycling and seasonal heat reclamation (see Case 3).

In contrast, exergy analysis is able to quantify the improvements from each recommendation. Exergy analysis also indicates that the single improvement which leads to the most significant process efficiency gain is matching the compressor capacity to the task. This example visibly demonstrates the usefulness of exergy analysis in support of industrial resource minimization.

Table 3 reveals how process efficiency is affected by improving various aspects of the process. But does the financial picture provide the same results? A good measure of expected cost savings is the power draw. Electrical costs for industrial facilities are determined primarily on the basis of two values: *peak demand* and *total consumption*. Peak demand is the facility's peak power draw during a billing period and is usually measured in kilowatts (kW). Energy consumption is the total electricity use measured in kilowatt-hours (kWh) during the billing period. Reducing the electrical power draw of continuously-operating equipment such as air compressors reduces both peak demand and total electricity consumption.

Table 4. Comparison of power draw for air compression processes.

Condition	Description	Power draw $\dot{W}_{ele}$, (kW)	Avg. process power draw $\dot{W}_{avg}$, (kW)	Avg. process power reduction from base case, (kW)	Avg. process power reduction, (% of base case)
Case 0, base case	Indoor air, no heat reclaim, oversized compressor, excessively high pressure setting	23.6	16	0	0%
Case 1	Case 0 with lowered pressure setting	23.1	15.8	0.2	1.3%
Case 2	Case 0 with lower-temperature inlet air	23.57	15.9	0.1	0.6%
Case 3	Case 0 with heat reclamation	21.5*	15.6**	0.4**	2.5%**
Case 4	Case 0 with reduced air demand	23.6	14.8	1.2	7.5%
Case 5	Case 0 with properly-sized compressor	8.5	8.5	7.5	46.9%
Case 6	"Best Practice" case: improvements 1-5 simultaneously implemented	6.0	6.0	10	62.5%

* This value reported as power draw during compressor loading, with all heat reclaimed.

** These values calculated for overall process, including load cycling and seasonal heat reclamation (see Case 3).

Table 4 lists the power draw of the compressor for Cases 0 through 6. Comparing the reductions in required power from with the efficiency improvements of Table 3, it is apparent that by both measures, the top priority for improving the system is matching the compressor size with the plant compressed air demand. Repairing leaks and eliminating inappropriate uses of compressed air would reduce the required compressor capacity and should be done in conjunction with properly sizing the compressor. After these are accomplished, the efficiency gain and cost savings from heat reclamation would provide the next most significant efficiency increase and cost savings. Reducing pressure is next and bringing in outside air would be last priority. These last two measures are nearly insignificant from a process efficiency standpoint, but provide marginal cost savings because they reduce the process power draw.

CHAPTER IV

THERMOCHEMICAL EXERGY: ALUMINUM MELT FURNACE

Metal casting is a highly energy-intensive industry. In audits conducted by the University of Dayton Industrial Assessment Center, melting furnace efficiencies less than 40% have been routinely observed. In this section, each step in the melting process is analyzed. Energy and exergy losses are quantified, and improvement concepts prioritized. For this analysis, a typical natural gas-fired "reverb" furnace for melting aluminum is considered. Conditions for the indoor surroundings are temperature T_1 and pressure P_1, while the outdoor conditions, the *environment*, are T_o and P_o. The melting process consists of the following operations: natural gas combustion, heat exchange between hot products of combustion and aluminum material, mixing of exhaust gases with ambient air, and heat reclamation (Figure 8).

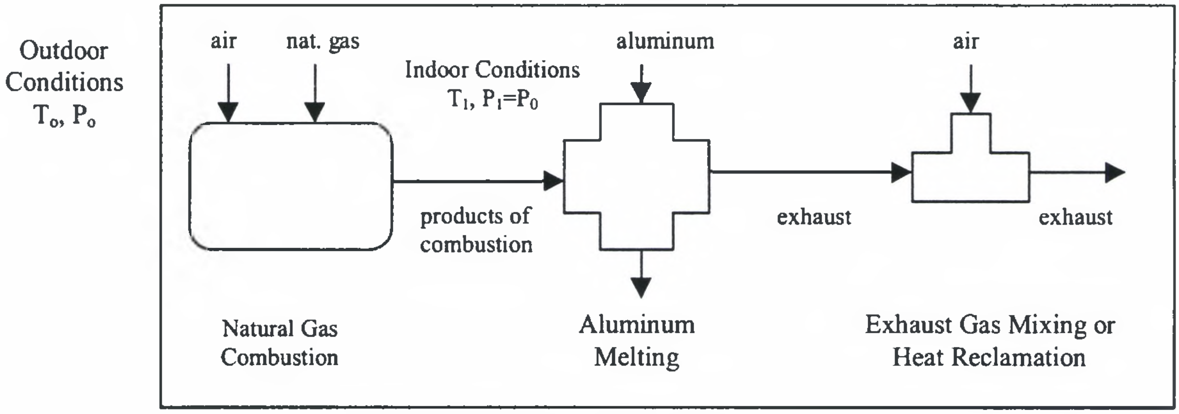

Figure 8. Aluminum melting process.

Combustion Process

The following assumptions apply to this analysis: natural gas is 100% methane, the N_2 is inert during combustion, the combustion chamber is adiabatic, and complete combustion occurs. All C is oxidized to CO_2 and all H is oxidized to H_2O. The environmental atmosphere (or *surroundings*) is defined as follows: air is composed of 79% N_2 and 21% O_2 on a molal basis, average outdoor temperature is 50°F, and atmospheric pressure is 14.7 psia. The chemical equation for natural gas combustion under these conditions, where a_{th} represents the stoichiometric quantity of air and F the multiple of stoichiometric, such that (a_{th} x F) equals total combustion air supplied, is:

$$CH_4 + (a_{th} \times F)(O_2 + 3.76\,N_2) \rightarrow CO_2 + 2\,H_2O + (a_{th} \times F - 2)\,O_2 + (a_{th} \times F \times 3.76)\,N_2$$

Equation (94)

In the case of stoichiometric combustion, F = 1 because the exact quantity of air is supplied that would be needed for perfect, complete combustion. Solving for a_{th} results in a value of 2 moles of O_2.

Energy Analysis

For a First Law analysis of the combustion process, the laws of Conservation of Mass and Conservation of Energy are applied. Figure 9 shows the physical condition with mass and energy flows. The control volume boundary is indicated as a solid line. Air and fuel enter with enthalpies h_a and h_{ng} respectively, while products of combustion exit with enthalpy h_p. The process is modeled as adiabatic: there is no energy loss due to heat transfer.

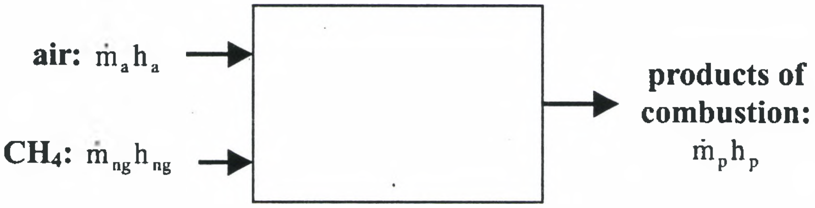

Figure 9. Schematic diagram of combustion chamber with energy flows.

Since the combustion chamber is a steady-state device, no mass is stored in the control volume over time. Thus, the Conservation of Mass equation

$$\dot{m}_a + \dot{m}_{ng} - \dot{m}_p = \frac{dm}{dt}$$

Equation (95)

reduces to:

$$\dot{m}_a + \dot{m}_{ng} = \dot{m}_p$$

Equation (96)

Similarly, the Conservation of Energy equation,

$$\Sigma\dot{E}_{in} - \Sigma\dot{E}_{out} = \frac{dE}{dt} \qquad \text{Equation (97)}$$

reduces to

$$\begin{aligned} \Sigma\dot{E}_{in} &= \Sigma\dot{E}_{out} \\ \dot{E}_a + \dot{E}_{ng} &= \dot{E}_p \\ \dot{m}_a h_a + \dot{m}_{ng} h_{ng} &= \dot{m}_p h_p \end{aligned} \qquad \text{Equation (98)}$$

assuming that changes in kinetic and potential energy are negligible. When chemical reactions take place, the enthalpy of each component includes a chemical term in addition to the sensible terms. The enthalpy of formation, $\bar{h}_f^o$, represents the energy required to form the component from substances that exist in stable form in the environment. Because the constituents of air are chemically stable at environmental conditions, the enthalpy of formation for air is zero. Further, the constituents of air can be treated as ideal gases, so their sensible enthalpy is a function of temperature. Thus, using the total enthalpy for the fuel and combustion products, the First Law equation is rewritten:

$$\dot{m}_a C_{pa}(T - T_o) + \frac{\dot{m}_{ng}}{M_{ng}}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_{ng} = \sum \frac{\dot{m}_p}{M_p}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_p \qquad \text{Equation (99)}$$

where M represents molar mass of fuel or combustion products.

The First Law efficiency of any process is defined by Equation (6) as the ratio of useful energy output to valued energy input. For a combustion process, the useful energy output is the enthalpy of the products. For this analysis, the combustion process is assumed to be adiabatic. Thus, all thermal energy exiting the process is accounted for in the combustion products. The energy efficiency of the combustion process is:

$$\eta_I = \frac{\dot{H}_{\text{useful out}}}{\dot{H}_{in}} = \frac{\dot{H}_{\text{products}}}{\dot{H}_{\text{reactants}}}$$

$$\eta_I = \frac{\sum \frac{\dot{m}_p}{M_p}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_p}{\dot{m}_a C_{pa}(T - T_o) + \frac{\dot{m}_{ng}}{M_{ng}}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_{ng}}$$ Equation (100)

Exergy Analysis

Referring to Figure 10, an exergy balance for the steady-state combustion chamber is:

$$\dot{X}_a + \dot{X}_{ng} - \dot{X}_{\text{destroyed}} - \Sigma\dot{X}_p = 0$$ Equation (101)

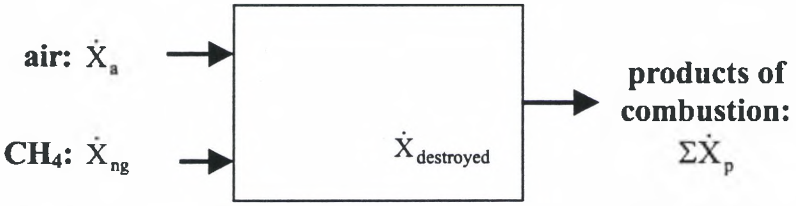

Figure 10. Schematic diagram of combustion chamber with exergy flows.

In a generalized steady-state combustion reaction, reactants enter the control volume with enthalpy H_r and entropy S_r. Products leave with enthalpy H_p and entropy S_p. Heat Q may enter the control volume at boundary temperature T, and work W may leave the control volume. Entropy is added to the control volume with heat, and some entropy is generated within the control volume. Referring to Figure 11, the energy and entropy balances are:

$$Q - W + H_r - H_p = 0$$

$$S_r - S_p + \frac{Q}{T} + S_{gen} = 0$$ Equation (102)

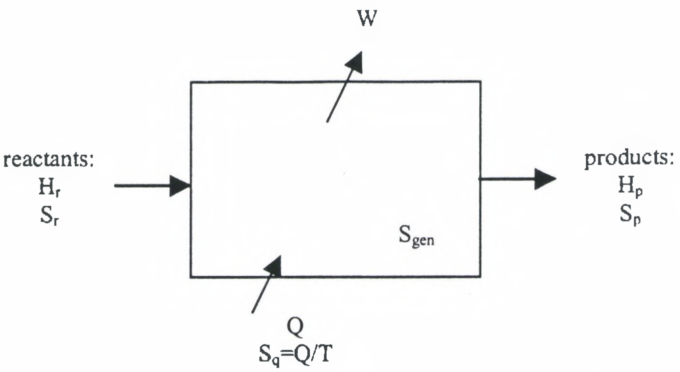

Figure 11. Control volume with energy and entropy flows.

If the reaction were reversible, the work out would be maximized. This corresponds to the case in which no entropy is generated in the control volume; $S_{gen} = 0$. After solving Equations (102) for Q and combining the energy and entropy balances, the maximum work W_{max} is found to be:

$$\begin{aligned} Q &= W + H_p - H_r \\ Q &= T(S_p - S_r) \\ W_{max} &= (H_r - TS_r) - (H_p - TS_p) \end{aligned} \qquad \text{Equation (103)}$$

In combustion with hydrocarbon fuels, some of the heat may be converted into useful work. In the process, the system also becomes more disordered because the energy in the molecular bonds is released when the bonds are broken. This relationship between enthalpy and entropy is captured in a property called Gibbs free energy, G. Gibbs free energy is defined as[7]:

$$G = H - TS \qquad \text{Equation (104)}$$

The combustion reaction can be thought of as occurring in two primary steps. First, the molecular bonds making up the fuel are broken. Doing so releases energy. Second, new compounds are formed from environmental elements and the components of the fuel. New molecular bonds are created, forming the products of combustion (POC). The fuel and air, the reactants, enter the reaction with Gibbs function G_r, and the products exit with Gibbs function G_p. The exergy released from a chemical reaction is related to the change in the Gibbs function of the reaction[8]. This change in Gibbs function is:

$$\Delta G = G_p - G_r \qquad \text{Equation (105)}$$

Comparing Equation (105) with Equations (103) and (104), the reversible work obtained from the chemical reaction is opposite in sign to the change in Gibbs function. This is because in the combustion reaction with a hydrocarbon fuel, there is a release of useful energy, and G is negative. Therefore, the reversible work available from the reaction is the opposite of the change in Gibbs function[8].

$$W_{rev} = -\Delta G = G_r - G_p = (H_r - TS_r) - (H_p - TS_p) \qquad \text{Equation (106)}$$

If the temperature at the boundary where heat is exchanged with the environment is that of the environment T_o, the reversible work is the exergy of the reaction.

$$W_{rev} = X_{reaction} = (H_r - T_oS_r) - (H_p - T_oS_p) \qquad \text{Equation (107)}$$

The chemical analysis at the beginning of this chapter reveals that the components comprising the products of combustion (POC) are CO_2, H_2O, O_2, and N_2. The POC is a mixture of these components discharged at atmospheric pressure. Thus, each component has a partial pressure P_i, determined as the component mole fraction y_i times the atmospheric pressure P_o. The exergy value of each component CO_2 and H_2O in the POC is:

$$\dot{X}_i = -\dot{N}_i\left[\bar{h}_f^o + \bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_i}{P_o}\right)\right]_{CO_2 \text{ or } H_2O} \qquad \text{Equation (108)}$$

Because the components of air are stable at environmental conditions, the enthalpy of formation for O_2 and N_2 is zero. Thus, the exergy values for these components are:

$$\dot{X}_i = -\dot{N}_i\left[\bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_i}{P_o}\right)\right]_{O_2 \text{ or } N_2} \qquad \text{Equation (109)}$$

The Second Law of Thermodynamics states that for any real process, entropy or disorder increases; therefore, exergy decreases. The decrease in exergy is determined by inserting these thermochemical exergy values into Equation (101). $\dot{X}_{reactants}$ represents the exergy supplied to the process in the fuel and air. The exergetic efficiency of the combustion process using Equation (7) is:

$$\eta_2 = \frac{\dot{X}_{\text{useful out}}}{\dot{X}_{\text{in}}} = \frac{\dot{X}_{\text{reactants}} - \dot{X}_{\text{destroyed}}}{\dot{X}_{\text{reactants}}}$$ Equation (110)

Case Study

This analysis evaluates the combustion process in a natural gas fired aluminum melt furnace, and is taken from an actual case study by the University of Dayton Industrial Assessment Center. Operating conditions for each burner are summarized in Table 5.

Table 5. Combustion operating parameters for aluminum melt furnace[9].

Fuel	Natural Gas, idealized as CH_4
Volumetric Air/Fuel Ratio	9.85
Combustion Air, Fuel Temperature	77°F
Fuel flow rate	80.11 scfm
Combustion air flow rate	790 scfm
Burner output	5 million Btu/hr
Furnace melt capacity per burner	2,000 lbm/hr aluminum

Equation (94) and the air/fuel ratio are used to determine the stoichiometric fraction F.

$$CH_4 + (2\times F)(O_2 + 3.76\,N_2) \rightarrow CO_2 + 2\,H_2O + (2\times F - 2)\,O_2 + (2\times F\times 3.76)\,N_2$$

$$9.85\frac{\text{moles air}}{\text{mole}\,CH_4}\times 1\,\text{mole}\,CH_4 = 2\times F\times(1+3.76)\,\text{moles air}$$ Equation (111)

$$F = 1.035$$

The molar quantities and mole fractions of the products are found by evaluating the molar coefficients in Equation (111) with F=1.035 (Table 6).

Table 6. Makeup of the products of combustion.

Component	Mole quantity	Mole fraction (mf)
CO_2	1	0.092
H_2O	2	0.184
O_2	0.069	0.006
N_2	7.781	0.717
Total Mixture	10.850	0.999

Energy Analysis. Since the constituents of air are stable at environmental conditions, their enthalpy of formation is zero. Using ideal gas properties, inlet conditions of Table 5, and the enthalpy of formation values[2], Equation (99) is used to determine the thermochemical enthalpies of the reactants as follows:

$$\dot{E}_a = \dot{m}_a C_{pa}(T - T_o)_a = 0.071\frac{\text{lbm}}{\text{ft}^3}\times 790\frac{\text{ft}^3}{\text{min}}\times 60\frac{\text{min}}{\text{hr}}\times 0.24\frac{\text{Btu}}{\text{lbm}\cdot\text{R}}\times(77 - 77)\text{R} = 0\frac{\text{Btu}}{\text{hr}}$$

$$\dot{E}_{ng} = \frac{\dot{m}_{ng}}{M_{ng}}\left(\bar{h}_f^o + C_p(T - T_o)\right)_{ng}$$

$$= \frac{\left(0.039\frac{\text{lbm}}{\text{ft}^3}\times 80.11\frac{\text{ft}^3}{\text{min}}\times 60\frac{\text{min}}{\text{hr}}\right)}{16.043\frac{\text{lbm}}{\text{lbmol}}}\times\left(-32{,}210\frac{\text{Btu}}{\text{lbmol}} + 8.697\frac{\text{Btu}}{\text{lbmol}\cdot\text{R}}\times(77 - 77)\text{R}\right) = -376{,}364\frac{\text{Btu}}{\text{hr}}$$

Equation (112)

The thermochemical enthalpy of the product gas mixture is determined as follows:

$$\dot{E}_p = \sum\frac{\dot{m}_p}{M_p}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_p = \left[\frac{\dot{m}}{M}(\bar{h}_f^o + \bar{h} - \bar{h}^o)\right]_{CO_2} + \left[\frac{\dot{m}}{M}(\bar{h}_f^o + \bar{h} - \bar{h}^o)\right]_{H_2O} + \left[\frac{\dot{m}}{M}(\bar{h} - \bar{h}^o)\right]_{O_2} + \left[\frac{\dot{m}}{M}(\bar{h} - \bar{h}^o)\right]_{N_2}$$

$$\dot{E}_p = \left[\frac{\dot{m}}{M}(-169{,}300 + \bar{h} - 4{,}027.5)\frac{\text{Btu}}{\text{lbmol}}\right]_{CO_2} + \left[\frac{\dot{m}}{M}(-104{,}040 + \bar{h} - 4{,}258.0)\frac{\text{Btu}}{\text{lbmol}}\right]_{H_2O} + \left[\frac{\dot{m}}{M}(\bar{h} - 3{,}725.1)\frac{\text{Btu}}{\text{lbmol}}\right]_{O_2}$$
$$+\left[\frac{\dot{m}}{M}(\bar{h} - 3{,}729.5)\frac{\text{Btu}}{\text{lbmol}}\right]_{N_2}$$

Equation (113)

To solve for the outlet condition, the mass flow rate for each product term is required. The mass balance equation, gas properties, the volume flow rates of Table 5, and the preceding characterization of the are all used to determine component mass flow rates. Values are determined below and summarized in Table 7. For the combustion air:

$$\dot{m}_{air} = \rho_{air} \times \dot{V}_{air} = 0.071\frac{lbm}{ft^3} \times 790\frac{ft^3}{min} \times 60\frac{min}{hr} = 3365\frac{lbm}{hr}$$

$$\rho_{O_2} = \frac{P_{O_2}}{RT} = \frac{14.7psia}{0.3353\frac{psia \cdot ft^3}{lbm \cdot R} \times (460+100)R} = 0.0782\frac{lbm}{ft^3}$$

$$\rho_{N_2} = \frac{P_{N_2}}{RT} = \frac{14.7psia}{0.383\frac{psia \cdot ft^3}{lbm \cdot R} \times (460+100)R} = 0.0685\frac{lbm}{ft^3}$$

$$\dot{m}_{N_2} = \rho_{N_2} \times 79\% \times \dot{V}_{air} = 2565\frac{lbm}{hr}$$

$$\dot{m}_{O_2} = \dot{m}_{air} - \dot{m}_{N_2} = 3365 - 2565 = 800\frac{lbm}{hr} \qquad \text{Equation (114)}$$

For the natural gas fuel:

$$\dot{m}_{CH_4} = \rho_{CH_4} \times \dot{V}_{CH_4} = 0.0393\frac{lbm}{ft^3} \times 80.11\frac{ft^3}{min} \times 60\frac{min}{hr} = 189\frac{lbm}{hr}$$

$$\dot{m}_C = \dot{m}_{CH_4} \times \frac{M_C}{M_{CH_4}} = 189\frac{lbm}{hr} \times \frac{12}{16} = 142\frac{lbm}{hr} \qquad \text{Equation (115)}$$

$$\dot{m}_{H_2} = \dot{m}_{CH_4} \times \frac{M_{H_2}}{M_{CH_4}} = 189\frac{lbm}{hr} \times \frac{4}{16} = 47\frac{lbm}{hr}$$

Table 7. Property values for reactants.

Component of Reactants	$\dot{m}$ (lbm/hr)	M (lbm/lbmol)	$\bar{h}_f^o$ (Btu/lbmol)	$\bar{h} - \bar{h}^o$ (Btu/lbmol)	$\dot{H}$ (Btu/hr)
air	3,365	28.97	0	0	0
CH_4	189	16	-32,210	0	-380,400
O_2	800	32	-	-	-
N_2	2,565	28	-	-	-
C	142	12	-	-	-
H_2	47	2	-	-	-
Total	3,554				-380,400

Knowledge of the reactant properties leads to evaluation of the properties for the products of combustion. Because mass is conserved, the mass flow rate of the products must equal that of the reactants. In addition, because no nuclear transformations take place during combustion, the mass of each element in the products equals that of the reactants. The three unknown mass flow rates are determined as follows, and listed in Table 8.

$$\dot{m}_{O_2}\Big|_{free} = (2\times F-2)\times\rho_{O_2}\times(mf_{O_2}\times\dot{V}_{air}) = 7\%\times\rho_{O_2}\times\left(21\%\times\dot{V}_{air}\right)$$

$$\dot{m}_{O_2}\Big|_{free} = 0.07\times0.0782\frac{lbm}{ft^3}\times\left(0.21\times790\frac{ft^3}{min}\times60\frac{min}{hr}\right) = 54.5\frac{lbm}{hr}$$

$$\dot{m}_{H_2O} = \dot{m}_{H_2}\times\frac{M_{H_2O}}{M_{H_2}} = 47\frac{lbm}{hr}\times\frac{18}{2} = 423\frac{lbm}{hr}$$

$$\dot{m}_{CO_2} = \dot{m}_{POC} - \left(\dot{m}_{H_2O} + \dot{m}_{O_2}\Big|_{free} + \dot{m}_{N_2}\right) = 3{,}554 - (423 + 54.5 + 2{,}565) = 512\frac{lbm}{hr}$$

Equation (116)

Because the combustion chamber is assumed to be adiabatic, the First Law of Thermodynamics requires that the enthalpy entering the process with the reactants equals the enthalpy leaving with the products. The enthalpy at the outlet state is determined from enthalpy of formation and thermal enthalpy values[2], with a reference state of 77°F.

Table 8. Property values for products.

Component of Products	$\dot{m}$ (lbm/hr)	M (lbm/lbmol)	$\bar{h}_f^o$ (Btu/lbmol)	$\bar{h}$ (Btu/lbmol)	$\bar{h}^o$ (Btu/lbmol)	$\dot{H}$ (Btu/hr)
CO_2	512	44	-169,300	*50,138.3*	4,027.5	-
H_2O	423	18	-104,040	*41,267.2*	4,258.0	-
$O_{2,\,free}$	54.5	32	0	*33,373.4*	3,725.1	-
N_2	2,565	28	0	*31,867.4*	3,729.5	-
Total	3,554					-380,400

Solving Equation (99) iteratively for $\bar{h}$ yields a product mixture temperature of 3,602°F, and the $\bar{h}$ values of Table 8.

This analysis assumes that the combustion process is adiabatic: no heat is lost from the combustion chamber, so all enthalpy in the reactants is accounted for in the products. As Equation (117) shows, the energy efficiency of a combustion process equals the ratio of enthalpies of products to reactants. Thus, the energy efficiency of the adiabatic combustion process is 100%:

$$\eta_I = \frac{\dot{H}_{\text{useful out}}}{\dot{H}_{in}} = \frac{\dot{H}_{\text{products}}}{\dot{H}_{\text{reactants}}}$$

$$\eta_I = \frac{-380{,}400\,\frac{\text{Btu}}{\text{hr}}}{-380{,}400\,\frac{\text{Btu}}{\text{hr}}} = 100\%$$

Equation (117)

From an energy perspective, it appears that adiabatic combustion is perfect. All energy entering the combustion chamber with the reactants is recovered in the products. The exergy method, however, paints a different picture.

Exergy Analysis. The exergy analysis for this combustion process is similar to the energy analysis, except the exergy content of each component is used, and some exergy is lost because the available work of chemical fuels, highly ordered sources of energy, is degraded to heat, a highly disordered form[8]. Equation (101), the process exergy balance, is restated below for convenience:

$$\dot{X}_{\text{destroyed}} = \dot{X}_a + \dot{X}_{ng} - \Sigma\dot{X}_p$$

Equation (101)

Recognizing that $\dot{N} = \frac{\dot{m}}{M}$ and inserting the component exergies of Equations (108) and (109) into Equation (101), the exergy balance is written:

$$\begin{aligned}
\dot{X}_{\text{destroyed}} = &-\frac{\dot{m}_{O_2}}{M}\left[\bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{O_2}}{P_o}\right)\right]_{O_2} - \frac{\dot{m}_{N_2}}{M}\left[\bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{N_2}}{P_o}\right)\right]_{N_2} \\
&- \frac{\dot{m}_{CH_4}}{M}\left[\bar{h}_f^o + \bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{CH_4}}{P_o}\right)\right]_{CH_4} + \frac{\dot{m}_{CO_2}}{M}\left[\bar{h}_f^o + \bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{CO_2}}{P_o}\right)\right]_{CO_2} \\
&+ \frac{\dot{m}_{H_2O}}{M}\left[\bar{h}_f^o + \bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{H_2O}}{P_o}\right)\right]_{H_2O} + \frac{\dot{m}_{O_2}}{M}\left[\bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{O_2}}{P_o}\right)\right]_{O_2} \\
&+ \frac{\dot{m}_{N_2}}{M}\left[\bar{h} - \bar{h}^o - T_o\left(\bar{s}^o - R\ln\frac{P_{N_2}}{P_o}\right)\right]_{N_2}
\end{aligned}$$

Equation (118)

From the preceding energy analysis, it is known that the sum of enthalpies entering with the reactants is equal to the sum of enthalpies leaving with the products of combustion. Thus, the enthalpy terms of Equation (118) cancel, and the equation is simplified to:

$$\dot{X}_{destroyed} = -\frac{\dot{m}_{O_2}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{O_2}}{P_o}\right)\right]_{O_2} - \frac{\dot{m}_{N_2}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{N_2}}{P_o}\right)\right]_{N_2} - \frac{\dot{m}_{CH_4}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{CH_4}}{P_o}\right)\right]_{CH_4}$$

$$+\frac{\dot{m}_{CO_2}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{CO_2}}{P_o}\right)\right]_{CO_2} + \frac{\dot{m}_{H_2O}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{H_2O}}{P_o}\right)\right]_{H_2O} + \frac{\dot{m}_{O_2}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{O_2}}{P_o}\right)\right]_{O_2} +$$

$$\frac{\dot{m}_{N_2}}{M}\left[-T_o\left(\bar{s}^o - R_u \ln\frac{P_{N_2}}{P_o}\right)\right]_{N_2}$$

Equation (119)

In Equation (119), each term is the component of exergy that is associated with the entropy at that state. The Second Law of Thermodynamics states that for any real process, entropy increases. Thus for this case, the entropy of the products of combustion is greater than that of the reactants. This indicates that during combustion, the reactants have been degraded to a more disordered state, and therefore, their capacity to do work is less. This lowered capacity to do work is the exergy destroyed, and results from the increase in entropy. With reference environmental conditions of $T_o = 77°F$ and $P_o = 1$ atmosphere, the properties of each component^2 are summarized in Table 9.

Table 9. Property values for calculating component exergies.

Reactants	$\frac{\dot{m}}{M}$ (lbmol/hr)	$\bar{s}^o$, 77°F (Btu/lbmol-R)	R_u (Btu/lbmol-R)	$-R_u \ln\frac{P_i}{P_o}$ (Btu/lbmol-R)	$T_o\dot{S}$ of reactants (Btu/hr)
O_2	25	48.982	1.986	3.0994	699,193
N_2	91.61	45.743	1.986	0.4681	2,273,337
CH_4	11.81	44.49	1.986	0	282,154
Total					3,254,685

Products	$\frac{\dot{m}}{M}$ (lbmol/hr)	$\bar{s}^o$, 3605°F (Btu/lbmol-R)	R_u (Btu/lbmol-R)	$-R_u \ln\frac{P_i}{P_o}$ (Btu/lbmol-R)	$T_o\dot{S}$ of products (Btu/hr)
CO_2	12.25	75.98	1.986	4.7385	502,263
H_2O	23.5	65.02	1.986	3.3619	859,293
O_2	0.85	64.98	1.986	10.1604	68,982
N_2	91.61	61.44	1.986	0.6607	3,044,555
Total					4,474,907

Recognizing that the enthalpy terms cancel because the combustion process is adiabatic, the exergy destroyed in the reaction is given by Equation (101) as:

$$\dot{X}_{destroyed} = \Sigma\dot{X}_{reactants} - \Sigma\dot{X}_{products} = -T_oS_r - (-T_oS_p)$$
$$\dot{X}_{destroyed} = -3{,}254{,}685\frac{Btu}{hr} - (-4{,}474{,}907\frac{Btu}{hr}) = 1{,}220{,}222\frac{Btu}{hr} \quad \text{Equation (120)}$$

As Equation (120) shows, combustion destroys a significant portion of the available work in the reactants; the exergy in the products is less than that of the reactants by the amount $\dot{X}_{destroyed}$.

The energy obtained from the combustion reaction is the heat released: the enthalpy of combustion $\dot{H}_c$. Enthalpy of combustion is defined as[2] the heat leaving a combustion reaction if the products were returned to the state of the reactants. In this case, the reactants enter at a temperature of 77°F. Table 10 lists the thermodynamic properties needed for calculating enthalpy of combustion[2].

Table 10. Thermodynamic properties of reactants and products at 77F.

Reactants	mdot (lbm/hr)	M (lbm/lbmol)	$\bar{h}_f^o$ (Btu/lbmol)	$\bar{h}^o$, 77°F (Btu/lbmol)	H_r (Btu/hr)
O2	800	32	0	3,725.1	0
N2	2,565	28	0	3,729.5	0
CH4	189	16	-32,210		-380,400
Total	3,554				-380,400

Products	mdot (lbm/hr)	M (lbm/lbmol)	hfo (Btu/lbmol)	h, 77°F (Btu/lbmol)	H_p (Btu/hr)
CO2	539	44	-169,300	4,027.5	-1,970,036
H2O	423	18	-104,040	4,258.0	-2,444,940
O2	27	32	0	3,725.1	0
N2	2,565	28	0	3,729.5	0
Total	3,554				-4,414,976

Thus, the heat of combustion is:

$$\dot{H}_c = \dot{H}_{products} - \dot{H}_{reactants} = \Sigma\frac{\dot{m}}{M}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_{products} - \Sigma\frac{\dot{m}}{M}(\bar{h}_f^o + \bar{h} - \bar{h}^o)_{reactants}$$
$$\dot{H}_c = -4{,}414{,}976\frac{Btu}{hr} - (-380{,}480\frac{Btu}{hr}) = -4{,}034{,}576\frac{Btu}{hr} \quad \text{Equation (121)}$$

The lower heating value LHV of the combustion reaction is defined as the quantity of heat available from a combustion reaction if the products were returned to the state of the reactants, and the water in the products is in the vapor state. Thus, LHV=-$\dot{H}_c$. The negative of the Gibbs function of reaction -ΔG is approximately equal to the lower heating value of the fuel[12]. Thus, the chemical exergy available from the fuel is approximated as the LHV.

$$\dot{X}_{\text{reactants}} = -\Delta G \cong \text{LHV} = -\text{Hc} = 4{,}034{,}576\frac{\text{Btu}}{\text{hr}} \qquad \text{Equation (122)}$$

The exergetic efficiency of the process as defined by Equation (110) is thus:

$$\eta_2 = \frac{\dot{X}_{\text{useful out}}}{\dot{X}_{\text{in}}} = \frac{\dot{X}_{\text{reactants}} - \dot{X}_{\text{destroyed}}}{\dot{X}_{\text{reactants}}}$$

$$\eta_2 = \frac{(4{,}034{,}576 - 1{,}220{,}222)\frac{\text{Btu}}{\text{hr}}}{4{,}034{,}576\frac{\text{Btu}}{\text{hr}}} = 0.698 \qquad \text{Equation (123)}$$

Thus the exergetic efficiency of the combustion process is about 69.8%. Over 30% of the exergy in the fuel is destroyed by combustion, even before the thermal energy is put to any useful purpose.

Conclusions

This analysis reveals that although the energy efficiency of the adiabatic combustion process is 100%, the exergetic efficiency is only 69.8%; nearly one third of the exergy in the fuel is lost even before the heated products are put to any useful purpose. Clearly, this process is irreversible, and a significant proportion of the work capacity of the fuel is lost. First Law analysis gives no indication of this lost potential.

The component of exergy destroyed that is associated with an increase in entropy is far larger than the enthalpy component. This is because the highly ordered state of the chemical fuel is degraded significantly by combustion. The entropy component values $T_0\dot{S}$ of Table 9 reveal that a significant portion of exergy is lost in heating the inert nitrogen to combustion temperature. This suggests three approaches for improving the combustion process. First, keeping the quantity of excess air to a minimum would result in less nitrogen being introduced to the combustion process. Second, if an inexpensive source of pure oxygen were available, the combustion air could be enriched with oxygen[8], decreasing the proportion of nitrogen provided. Third, preheating the combustion air would reduce the quantity of fuel needed to heat the reactants. All of these methods would decrease the exergy loss to the inert nitrogen. Thus, in addition to evaluating the magnitude of exergy destruction due to combustion, exergy analysis also points out ways to improve the process.

Melting Process

The aluminum melting process is modeled as a heat exchanger in which the material stream to be heated is aluminum, and the heating stream is the products of combustion (POC). In some furnace designs, the POC are in direct contact with the aluminum; however, we assume no mixing because the POC are in gaseous phase while the aluminum is in solid/liquid phase. Figure 12 illustrates the concept, and identifies energy and exergy flows.

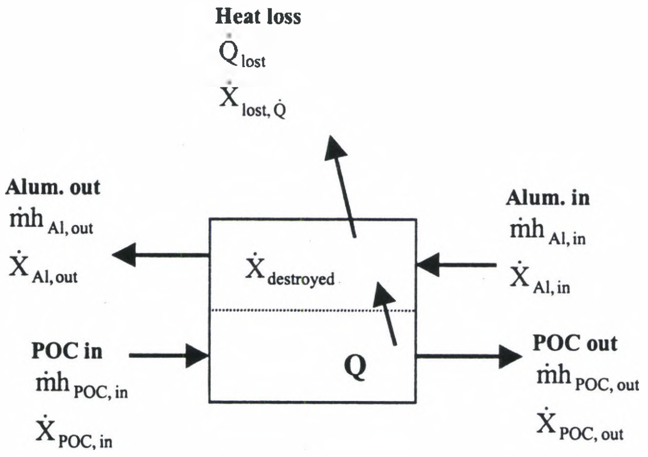

Figure 12. Aluminum melting process modeled as heat exchanger.

Energy Analysis

Referring to Figure 12 and assuming that heat losses from the steady-state heat exchanger are negligible so that $\dot{Q}_{lost} = 0$, the First Law energy balance is written as follows:

$$\begin{aligned} \Sigma\dot{E}_{in} &= \Sigma\dot{E}_{out} \\ \dot{m}h_{Al,in} + \dot{m}h_{POC,in} &= \dot{m}h_{Al,out} + \dot{m}h_{POC,out} \\ \dot{m}_{POC}(h_{POC,in} - h_{POC,out}) &= \dot{m}_{Al}(h_{Al,out} - h_{Al,in}) \end{aligned} \qquad \text{Equation (124)}$$

The energy change in the hot stream involves only sensible energy transfer since no chemical changes or phase changes occur. For the aluminum, the process is complicated by its change in phase from solid to liquid. This change is comprised of three steps. First, solid aluminum at the temperature of the surroundings enters the HX and is heated to melting temperature. Next, a constant-temperature addition of heat overcomes the cohesive forces of the crystal lattice, and the solid melts to liquid. Finally, the liquid aluminum is heated sensibly from melting temperature to an elevated temperature. An energy balance for this process is:

$$\sum \dot{m}_i C_{pi}(T_{in} - T_{out})_{POC} = \dot{m}_{Al}[C_p(T_{melt} - T_{in})_{solid} + h_{melt} + C_p(T_{out} - T_{melt})_{liquid}]_{Al} \qquad \text{Equation (125)}$$

As described in the combustion section of this chapter, the energy available from the combustion reaction is equal to the lower heating value LHV of the fuel. With this consideration, the energy efficiency of the melting process is:

$$\begin{aligned} \eta_I &= \frac{\dot{H}_{useful\,out}}{\dot{H}_{in}} = \frac{\Delta\dot{H}_{Aluminum}}{-\dot{H}_c} = \frac{\Delta\dot{H}_{Aluminum}}{LHV} \\ \eta_I &= \frac{\dot{m}_{Al}[C_p(T_{melt} - T_{in})_{solid} + h_{melt} + C_p(T_{out} - T_{melt})_{liquid}]_{Al}}{LHV} \end{aligned} \qquad \text{Equation (126)}$$

Exergy Analysis

An exergy formulation of the Second Law for the steady-state melting process, where $\dot{X}$ represents exergy flow, is as follows:

$$\Sigma\dot{X}_{in} - \Sigma\dot{X}_{out} - \dot{X}_{lost,\dot{Q}} - \dot{X}_{destroyed} = 0 \qquad \text{Equation (127)}$$

Since the heat exchanger is assumed to be adiabatic (there is no heat exchange with the environment), $\dot{X}_{\text{lost},\dot{Q}} = 0$. The POC transfer exergy to the aluminum, and exit the process with whatever exergy is left. Thus, exergy discharged in the exhaust is lost, since no further use is made of it. In addition, exergy is destroyed in the heat transfer process, due to the irreversibilities in heat transfer through a finite temperature difference. This exergy destroyed is calculated from the exergy balance:

$$\begin{aligned}
\dot{X}_{\text{destroyed}} &= \Sigma\dot{X}_{\text{in}} - \Sigma\dot{X}_{\text{out}} \\
\dot{X}_{\text{destroyed}} &= (\dot{X}_{\text{POC, in}} + \dot{X}_{\text{Al, in}}) - (\dot{X}_{\text{POC, out}} + \dot{X}_{\text{Al, out}}) \\
\dot{X}_{\text{destroyed}} &= (\dot{X}_{\text{POC, in}} - \dot{X}_{\text{POC, out}}) - (\dot{X}_{\text{Al, out}} - \dot{X}_{\text{Al, in}})
\end{aligned} \qquad \text{Equation (128)}$$

The exergetic efficiency of a heat exchanger is defined as the ratio of the exergy gain of the heated stream (aluminum) to the exergy loss of the heating stream (hot POC)[12]. Thus, the exergetic efficiency of the heat exchanger is defined as:

$$\eta_{2,\text{HX}} = \frac{\dot{X}_{\text{Al, out}} - \dot{X}_{\text{Al, in}}}{\dot{X}_{\text{POC, in}} - \dot{X}_{\text{POC, out}}} \qquad \text{Equation (129)}$$

When considering the efficiency of the entire melting process, the valued exergy supplied is not the change in exergy of the POC, but the total exergy entering the process with the POC. Thus, the exergetic efficiency of the entire melting process is formulated as follows:

$$\eta_2 = \frac{\dot{X}_{\text{useful out}}}{\dot{X}_{\text{valued in}}} = \frac{\dot{X}_{\text{Al, out}} - \dot{X}_{\text{Al, in}}}{\dot{X}_{\text{POC, in}}} \qquad \text{Equation (130)}$$

Case Study

Continuing with the case study from the combustion process, each burner in the melt furnace provides the capability to melt 2,000 pounds of aluminum per hour. The material to be melted is idealized as pure aluminum. The molten aluminum is heated to a temperature of 1,400°F, while the temperature of outgoing exhaust, a mixture of POC and ambient air, was measured to be about 1,400°F. Table 11 lists the thermodynamic properties of aluminum and the products of combustion (POC) needed for this analysis.

Table 11. Thermodynamic properties of aluminum and POC[4,2].

Melting point, Al	933.61 K	1,221°F
Molar mass Al	26.982 g/mol	0.059 lbm/mol
Heat of melting Al, h_{melt}	2.580 kcal/mol	172.1 Btu/lbm
Specific heat, $C_{p,\ solid\ Al}$	6.9 cal/mol K	0.256 Btu/lbm°F
Specific heat, $C_{p,\ liquid\ Al}$	7.590 cal/mol K	0.281 Btu/lbm°F
Entropy, $s^o_{melt,solid\ Al}$	14.303 cal/mol K	0.610 Btu/lbm°F
Entropy, $s^o_{melt,liquid\ Al}$	17.067 cal/mol K	0.632 Btu/lbm°F
C_p, CO_2		0.325 Btu/lbm°R
C_p, H_2O		0.648 Btu/lbm°R
C_p, O_2		0.278 Btu/lbm°R
C_p, N_2		0.303 Btu/lbm°R

Energy Analysis. Equation (131) is the energy balance for the process. All values are known except the exit temperature T_{out} of the POC. The aluminum is assumed to enter the furnace at a temperature of 77°F, and is heated to 1,400°F; the POC enter the furnace at the temperature of combustion, 3,602°F. An energy balance on the melting operation gives:

$$\sum \dot{m}_i C_{pi}(T_{in} - T_{out})_{POC} = \dot{m}_{Al}[C_p(T_{melt} - T_{in})_{solid} + h_{melt} + C_p(T_{out} - T_{melt})_{liquid}]_{Al} \qquad \text{Equation (131)}$$

Solving Equation (131), the temperature of the POC exiting the melting process is found to be 2,767°F. Because the HX is assumed to be adiabatic, all the energy transferred from the hot POC is accounted for in the aluminum. Thus, the HX energy efficiency is 100%.

$$\eta_1 = \frac{\Delta\dot{H}_{Aluminum}}{\Delta\dot{H}_{POC}} = \frac{\dot{m}_{Al}[C_p(T_{melt} - T_{in})_{solid} + h_{melt} + C_p(T_{out} - T_{melt})_{liquid}]_{Al}}{\sum \dot{m}_i C_{pi}(T_{in} - T_{out})_{POC}}$$

$$\eta_1 = \frac{2000\frac{lbm}{hr}[0.256\frac{Btu}{lbmF}(1221-77)F + 172.1\frac{Btu}{lbm} + 0.281\frac{Btu}{lbmF}(1400-1221)F]}{3554\frac{lbm}{hr} \times 0.347\frac{Btu}{lbmF}(3700-2767)F} \qquad \text{Equation (132)}$$

$$\eta_1 = \frac{1{,}029{,}918\frac{Btu}{hr}}{1{,}029{,}918\frac{Btu}{hr}} = 100\%$$

As previously described, the energy efficiency of melting is the ratio of the enthalpy change of aluminum to the enthalpy available from the combustion reaction. Therefore, the energy efficiency of the melting process is actually about:

$$\eta_I = \frac{\dot{H}_{\text{useful out}}}{\dot{H}_{\text{in}}} = \frac{\Delta\dot{H}_{\text{Aluminum}}}{-\dot{H}_c} = \frac{\Delta\dot{H}_{\text{Aluminum}}}{\text{LHV}}$$

$$\eta_I = \frac{\dot{m}_{\text{Al}}[C_p(T_{\text{melt}} - T_{\text{in}})_{\text{solid}} + h_{\text{melt}} + C_p(T_{\text{out}} - T_{\text{melt}})_{\text{liquid}}]_{\text{Al}}}{-\dot{H}_c}$$

$$\eta_I = \frac{2000\frac{\text{lbm}}{\text{hr}}[0.256\frac{\text{Btu}}{\text{lbmF}}(1221 - 77)\text{F} + 172.1\frac{\text{Btu}}{\text{lbm}} + 0.281\frac{\text{Btu}}{\text{lbmF}}(1400 - 1221)\text{F}]}{4{,}034{,}576\frac{\text{Btu}}{\text{hr}}}$$

$$\eta_I = \frac{1{,}029{,}918\frac{\text{Btu}}{\text{hr}}}{4{,}034{,}576\frac{\text{Btu}}{\text{hr}}} = 26\%$$

Equation (133)

Exergy Analysis. Exergy of the aluminum at inlet and exit conditions is determined starting with the assumption that the aluminum enters the melting chamber at a temperature of 77°F. To determine the exergy of the aluminum at outlet conditions, three components are added to the inlet exergy: 1) the change in exergy due to sensible heating from 77°F to the melt temperature of 1,221°F, 2) the change in exergy of melting and 3) the change in exergy from sensible heating of the liquid aluminum from melt temperature to the final temperature of 1,400°F. The change in exergy due to melting is calculated from:

$$\Delta\dot{X}_{\text{melt}} = \dot{m}[h_{\text{melt}} - T_o(s_{\text{out}} - s_{\text{in}})] \qquad \text{Equation (134)}$$

using h_{melt} = 172.1 Btu per lbm[4]. Table 12 lists the values resulting from this analysis.

Table 12. Exergy for aluminum.

Aluminum	$\dot{m}$ (lbm/hr)	C_p (Btu/lbm-R)	T_{in} (R)	T_{melt} (R)	T_{out} (R)	$s_{o,\,in}$ (Btu/lbmR)	$s_{o,\,out}$ (Btu/lbmR)	$s_{o,\,77\,F}$ (Btu/lbmR)	$\dot{X}_{in}$ (Btu/hr)	$\Delta\dot{X}_{added}$ (Btu/hr)	$\dot{X}_{out}$ (Btu/hr)
Solid	2000	0.256	537	1681		0.251	0.530	0.251	0	285,458	
Melting	2000		1681	1681		0.530	0.632	0.251		234,195	
Liquid	2000	0.281		1681	1860	0.632	0.661	0.251		70,421	
Total	2000								0	590,074	590,074

The POC enters the melting process at the exit temperature of the combustion chamber. Assuming the heat exchanger is adiabatic, the enthalpy rise of the aluminum must equal the enthalpy loss of the POC. This analysis results in a POC exhaust temperature of 2,767°F. With knowledge of the exhaust temperature, the exergy values and change in exergy of the POC in the melting process can be evaluated. Table 13 lists the property and exergy values for the POC.

Table 13. Exergies for constituents of the products of combustion (POC).

POC in	$\dot{m}$ (lbm/hr)	C_p (Btu/lbm-R)	T_{in} (R)	T_{melt} (R)	M (lbm/lbmol)	$s_{o,\,in}$ (Btu/lbmR)	$s_{o,\,out}$ (Btu/lbmR)	$s_{o,\,77\,F}$ (Btu/lbmR)	$\dot{X}_{in}$ (Btu/hr)	$\dot{X}_{out}$ (Btu/hr)
CO2	512	0.326	4062	3227	44	1.719	1.643	1.160	434,011	315,628
H2O	423	0.649	4062	3227	18	3.596	3.494	2.504	719,644	513,443
O2	54.5	0.278	4062	3227	32	2.040	1.975	1.531	38,575	27,809
N2	2565	0.303	4062	3227	28	2.187	2.116	1.634	1,977,030	1,425,592
Total	3554.5								3,169,260	2,282,472

Equation (128) is used to calculate the quantity of exergy destroyed due to heat exchange in the melting process. Using the values from Tables 12 and 13, the exergy destroyed in the melting process is:

$$\dot{X}_{destroyed} = (\dot{X}_{POC,\,in} - \dot{X}_{POC,\,out}) - (\dot{X}_{Al,\,out} - \dot{X}_{Al,\,in})$$

$$\dot{X}_{destroyed} = (3{,}169{,}260 - 2{,}282{,}472)\frac{\text{Btu}}{\text{hr}} - (590{,}074 - 0)\frac{\text{Btu}}{\text{hr}} = 296{,}715\frac{\text{Btu}}{\text{hr}} \qquad \text{Equation (135)}$$

The exergetic efficiency of the heat exchange is:

$$\eta_{2,\,HX} = \frac{\dot{X}_{Al,\,out} - \dot{X}_{Al,\,in}}{\dot{X}_{POC,\,in} - \dot{X}_{POC,\,out}} = \frac{590{,}074\frac{\text{Btu}}{\text{hr}}}{886{,}778\frac{\text{Btu}}{\text{hr}}} = 66.5\% \qquad \text{Equation (136)}$$

The exergy available from the chemical reaction is given by Equation (122):

$$\dot{X}_{reactants} = 4{,}034{,}576\frac{\text{Btu}}{\text{hr}} \qquad \text{Equation (122)}$$

Thus, the exergetic efficiency of the overall process, considering both exergy lost with the exhaust POC and exergy destroyed, is calculated as follows:

$$\eta_2 = \frac{\dot{X}_{Al,\,out} - \dot{X}_{Al,\,in}}{\dot{X}_{reactants}} = \frac{590{,}074\frac{\text{Btu}}{\text{hr}}}{4{,}034{,}576\frac{\text{Btu}}{\text{hr}}} = 14.6\% \qquad \text{Equation (137)}$$

Conclusions

Melting Heat Exchange. The energy efficiency of heat transfer in an adiabatic heat exchanger is 100%. However, exergy analysis reveals that the useful work provided to the HX is only used at a rate of 68.5%. Thus, there is room for improvement in the heat exchanger itself.

Exergy analysis reveals that nearly half of the exergy added was used to raise the temperature of the solid aluminum to melting temperature. Using the hot exhaust products to preheat the solid aluminum prior to melting would recover some of the enthalpy and exergy of the exhaust, and increase the process efficiency.

Overall Melting Process. The overall melting process consists of combustion and melting of solid aluminum. First Law analysis resulted in a process efficiency of 100% for the adiabatic combustion process, and 26% for melting. Thus, the overall process energy efficiency is:

$$\begin{aligned} \eta_{I\,\text{process}} &= \eta_{I\,\text{combustion}} \times \eta_{I\,\text{melting}} \\ \eta_{I\,\text{process}} &= 100\% \times 26\% = 26\% \end{aligned} \qquad \text{Equation (138)}$$

Equation (137) shows that the exergetic efficiency of the overall melting process is just 14.6%. The fuel purchased to melt aluminum contains a large quantity of exergy, but less than 15% of it is actually used to melt aluminum. Nearly one third of the fuel's exergy is destroyed in combustion and more is destroyed in the melting of the aluminum. The rest is discharged to the environment from the exhaust stack. Table 14 and Figure 13 shows how exergy is used, destroyed, and lost throughout the process. Figure 13 graphically illustrates a very important point: nearly half of the exergy supplied to the process is lost, because it is discharged to the environment with the exhaust. At a discharge temperature of over 2,700 degrees Fahrenheit, this is a valuable exergy stream.

Table 14. Accounting of exergy in the overall aluminum melting process.

Exergy	Quantity	
	(Btu/hr)	(%)
Fuel exergy supplied	4,034,576	100.0%
Xdestroyed, combustion	1,220,222	30.2%
Xexiting combustion, supplied to melt	2,814,354	69.8%
Xdestroyed, melt	296,715	7.4%
Xused, Aluminum melting	590,074	14.6%
Xlost, discharged in exhaust.	1,927,566	47.8%

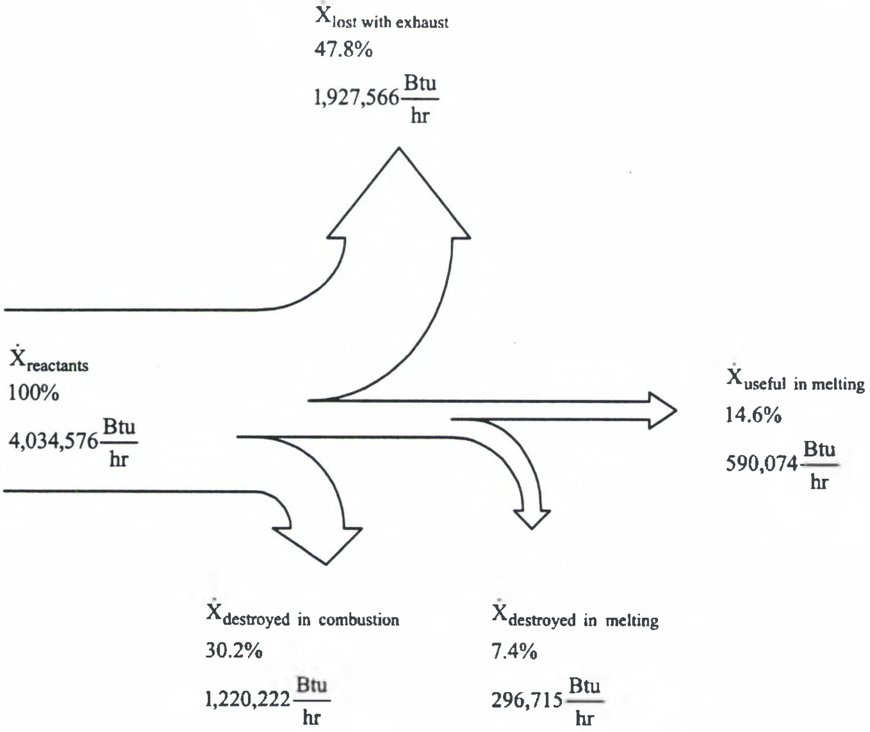

Figure 13. Exergy flow diagram for aluminum melt furnace.

Both η_1 and η_2 indicate that the aluminum melting process is highly inefficient. However, exergy analysis showed that a significant portion of the fuel's value is destroyed in the combustion process, while energy analysis did not. In addition, exergy analysis points out that heat transfer is an irreversible process, and provides a measure of the exergy destroyed. Exergy is destroyed in the heat exchange process because heat transfer through a finite temperature difference increases the system's entropy. First Law analysis does not capture this detail. Thus, exergy analysis reveals where losses occur in a process and leads to determination of improvement measures.

Exhaust Heat Reclamation

The exergy analysis of the combustion process revealed that a large quantity of exergy is expended simply to heat the inert nitrogen in the combustion air. In addition, the analysis of the melting process indicates that a significant quantity of energy and exergy is available in the exhaust products. This suggests that heat reclamation would improve both the energy and exergy efficiencies of the overall fuel use. In this section, a conceptual heat exchanger (HX) design for preheating the combustion air is presented.

The current exhaust stack design is shown in Figure 14. The POC is exhausted from the melting process at a temperature of 2,767°F. Ambient air is mixed with these hot exhaust products to lower the temperature in the exhaust stack. This is done to protect the exhaust stack and downstream components from the elevated temperature.

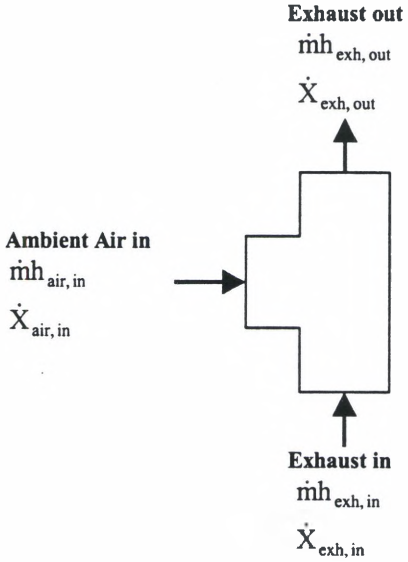

Figure 14. Existing exhaust / air mixing process.

Measured values of the mixture temperature of the exhaust POC + ambient air are about 1,400°F. Mixing reduces temperature, enthalpy, and exergy in the exhaust. With this design, the reduction in energy and exergy of the exhaust is generally not considered waste, because the exhaust is simply discharged to the atmosphere as a process output. However, it would be considered a loss if the heat in the exhaust stream were to be reclaimed for a useful purpose.

Reclaiming this heat would require a HX. The design presented here for preheating combustion air involves installing a concentric shell around the vertical, cylindrical exhaust stack. Exhaust gas would be discharged through the existing inner exhaust pipe, while the incoming combustion air passes through the annular space between the exhaust pipe and the shell. The exhaust pipe wall keeps flows separate, and provides heat transfer surface. Figure 15 illustrates the concept.

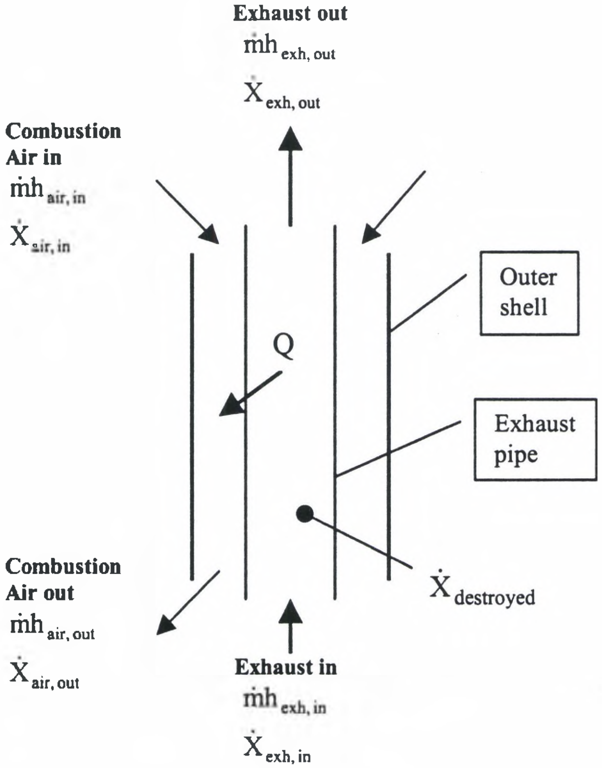

Figure 15. Concentric shell HX (in cross section) for combustion air preheating.

Adding this type of heat exchanger to the exhaust stack could be done in two ways: 1) installing a shell around the existing exhaust pipe above the mixing chamber, or 2) closing off the mixer intake, adding the shell, and creating a new mixing chamber above the HX if necessary. This is an important difference because the inlet temperature of the hot stream would be significantly different. Downstream of the mixing chamber, the temperature is about 1,400°F. If the mixing air intake were closed, the POC temperature at the inlet of the HX would be over 2,700°F. This would retain the exergetic value of the 2,700°F exhaust stream, allowing it to be put to good use.

Heat Transfer Coefficient

To evaluate the usefulness of the concentric shell HX described above, the overall heat transfer coefficient U is needed. To calculate U, it is assumed that heat is transferred through the exhaust pipe wall by convection and conduction, and no heat is transferred through the outer shell wall. Thus, the overall heat transfer coefficient U is derived from an inner annulus surface convection coefficient h_1, the conduction coefficient k_w for the wall material, and an inner pipe surface convection coefficient h_2. Figure 16 shows the physical condition, along with the electrical analogy used to develop U.

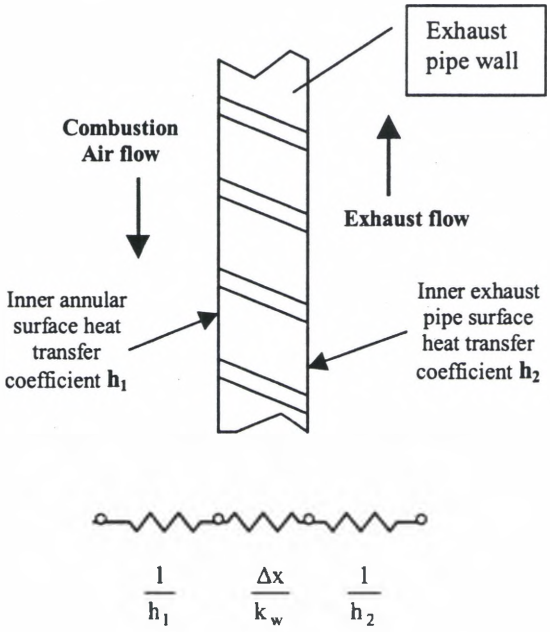

Figure 16. Exhaust pipe wall and electrical analogy for heat flow resistances.

The overall resistance to heat flow is the sum of the three resistances in series. Thus, the overall heat transfer coefficient for the concentric tube heat exchanger is:

$$U = \frac{1}{\frac{1}{h_1} + \frac{\Delta x}{k_w} + \frac{1}{h_2}} \qquad \text{Equation (139)}$$

An empirical correlation for the Nusselt Number for the inner surface of an annulus, where D_h represents the hydraulic diameter and k_f the thermal conductivity of the fluid, is[10]:

$$Nu = 0.023 \times Re^{0.8} \times Pr^{0.4} = \frac{h_1 \times D_h}{k_f} \quad \text{Equation (140)}$$

For fully developed, turbulent flow inside a circular tube of diameter D, the correlation is[10]:

$$Nu = 0.023 \times Re^{0.8} \times Pr^{0.3} = \frac{h_2 \times D}{k_f} \quad \text{Equation (141)}$$

Table 15 lists physical dimensions from an actual "reverb" melt furnace, and fluid properties for calculation of heat transfer coefficients. We assume the exhaust products have the same properties as air.

Table 15. Exhaust stack dimensions and fluid properties for air.

D, Inner exhaust pipe diameter	32 inches
Exhaust pipe material	Plain steel
Δx, Exhaust pipe wall thickness	0.125 inch
k_w, Pipe thermal conductivity	$35 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot {}^\circ\text{F}}$
Outer shell diameter	36 inches
ρ, density of fluid	$\rho = \frac{P}{RT}\left(\frac{\text{lbm}}{\text{ft}^3}\right)$
$\mu_{exhaust}$, viscosity of exhaust products	$0.094 \frac{\text{lbm}}{\text{ft} \cdot \text{hr}}$
$k_{f\ exhaust}$, thermal conductivity of exhaust products	$0.035 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot {}^\circ\text{F}}$
$C_{p,exhaust}$, specific heat of exhaust products	$0.266 \frac{\text{Btu}}{\text{lbm} \cdot {}^\circ\text{F}}$
μ_{air}, viscosity of combustion air	$0.055 \frac{\text{lbm}}{\text{ft} \cdot \text{hr}}$
$k_{f\ air}$, thermal conductivity of combustion air	$0.0195 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot {}^\circ\text{F}}$
$C_{p,air}$, specific heat of combustion air	$0.24 \frac{\text{Btu}}{\text{lbm} \cdot {}^\circ\text{F}}$

The expressions defining the Reynolds Number Re and the Prandtl Numbers Pr are[10]:

$$Re = \frac{\dot{m} D_h}{\mu A}$$
$$Pr = \frac{C_p \mu}{k_f} \quad \text{Equation (142)}$$

Using the values of Table 15 and Equations (139) through (142), the heat transfer coefficients for the inner surface of the annulus and the inner surface of the exhaust pipe, and the conduction coefficient for the pipe wall are, respectively:

$$h_1 = 3.2\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}}$$
$$h_2 = 0.95\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}} \qquad \text{Equation (143)}$$
$$\frac{\Delta x}{k_w} = 0.0003\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}}$$

The overall heat transfer coefficient for the HX is estimated to be about:

$$U = \frac{1}{\left(\frac{1}{3.2}+0.0003+\frac{1}{0.95}\right)}\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}} = 0.732\frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}} \qquad \text{Equation (144)}$$

Heat Exchanger Analysis

The Effectiveness-NTU method[11] is used for heat transfer analysis of the heat exchanger. Referring to Figures 15 and 17, under steady-state conditions the heat transferred from the hot exhaust stream and to the cold air stream is:

$$\dot{Q} = \dot{m}_h C_{ph}(T_{hi} - T_{ho}) = \dot{m}_c C_{pc}(T_{co} - T_{ci}) \qquad \text{Equation (145)}$$

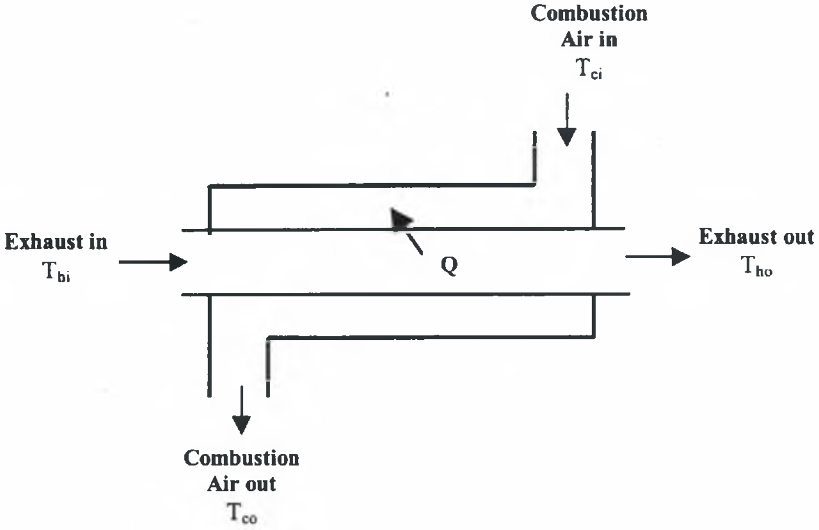

Figure 17. Schematic diagram of concentric tube HX with inlet and outlet temperatures.

The maximum heat transfer $\dot{Q}max$ would occur in an infinitely long HX in which the temperature of the exhaust out reached T_{ci} and the temperature of the combustion air out reached T_{hi}. For this condition, $\dot{Q}max$ is limited by the fluid with the minimum heat capacity, so that:

$$\dot{Q}_{max} = \left(\dot{m}C_p\right)_{min}(T_{hi} - T_{ci}) \qquad \text{Equation (146)}$$

The effectivenessε of the heat exchanger is defined as the ratio of actual heat transfer to maximum possible heat transfer:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\dot{m}_h C_{ph}(T_{hi} - T_{ho})}{\left(\dot{m}C_p\right)_{min}(T_{hi} - T_{ci})} = \frac{\dot{m}_c C_{pc}(T_{co} - T_{ci})}{\left(\dot{m}C_p\right)_{min}(T_{hi} - T_{ci})} \qquad \text{Equation (147)}$$

Thus, the actual heat transferred would be:

$$\dot{Q} = \varepsilon\left(\dot{m}C_p\right)_{min}(T_{hi} - T_{ci}) \qquad \text{Equation (148)}$$

For the Effectiveness-NTU method, effectiveness is a function of the ratio of fluid heat capacities C_r and the number of transfer units NTU. These values are defined as[11]:

$$\begin{gathered} NTU = \frac{UA}{\left(\dot{m}C_p\right)_{min}} \\ Cr = \frac{\left(\dot{m}C_p\right)_{min}}{\left(\dot{m}C_p\right)_{max}} \\ \varepsilon = \frac{1 - e^{[-NTU(1-Cr)]}}{1 - Cre^{[-NTU(1-Cr)]}} \end{gathered} \qquad \text{Equation (149)}$$

The overall heat transfer for the HX is determined by solving the following equation:

$$\dot{Q} = UA(T_{hbulk} - T_{cbulk}) \qquad \text{Equation (150)}$$

Fluid parameters and flow conditions are determined using ideal gas properties, the values of Table 13, and Equations (145) through (150). The result of this approach is a set of equations which can be solved simultaneously for heat transfer information.

Results of Analysis

The desired outcome from using the concentric-tube HX is to preheat combustion air to as high a temperature as possible, so that energy and exergy is reclaimed from the process waste stream. A simple computer program was developed that incorporates the analytical method outlined in the previous section. The program was used as a design tool and run for several cases to estimate the heat transfer under different conditions. Table 16 lists the variables and results for a concentric shell HX with inner pipe diameter of 32 inches, outer shell diameter of 36 inches. A listing of the program is given in the Appendix.

Table 16. Heat transfer analysis results.

	Input Variables				Model Output				
Run	T_{hi} (°F)	T_{ci} (°F)	Length (ft)	HX area	U (Btu/hr-ft^2°F)	T_{ho} (°F)	T_{co} (°F)	Q (Btu/hr)	ε
1	1,400	77	20	Single	0.7747	1296.8	199.6	157,080	0.0927
2	1,400	77	20	Doubled	0.6440	1202.3	311.9	238,820	0.1776
3	1,400	77	30	Single	0.7027	1248.3	257.2	204,350	0.1362
4	1,400	77	30	Doubled	0.5553	1118.0	412.1	283,100	0.2530
5	2,700	77	20	Single	0.6240	2485.0	332.4	249,650	0.0974
6	2,700	77	20	Doubled	0.4465	2277.1	579.5	323,160	0.1916
7	2,700	77	30	Single	0.5211	2379.1	458.4	297,490	0.1454
8	2,700	77	30	Doubled	0.3512	2090.3	801.5	345,200	0.2762

In the fifth column of Table 16, the notation "doubled" means that the HX surface area was doubled by dimpling or folding the inner stack to increase its surface area. Preliminary analysis of the data indicates that the HX could likely preheat combustion air to between 200°F and 800°F. Lower temperatures correspond to single heat transfer area and exhaust mixed with ambient air. Higher temperatures correspond to increased heat transfer surface area and unmixed exhaust air.

Conclusions

The facility investigated in the case study melts an average of 1.9 million pounds of aluminum per month. Thus, the annual quantity of melted aluminum is about:

$$1.9\times10^{6}\,\frac{\text{lbm}}{\text{mo}}\times12\,\frac{\text{mo}}{\text{yr}}=22.8\times10^{6}\,\frac{\text{lbm}}{\text{yr}} \qquad \text{Equation (151)}$$

The melt furnace under consideration has a melt capacity of 4,000 pounds per hour and facility total melt capacity is 10,000 pounds per hour. The melt furnaces operate in melting mode for about:

$$22.8\times 10^{6}\,\frac{\text{lbm}}{\text{yr}}\times\frac{1\,\text{hr}}{10{,}000\,\text{lbm}}=2{,}280\,\frac{\text{hr}}{\text{yr}} \qquad \text{Equation (152)}$$

We assume that 40% of the aluminum is melted using the furnace of our case study. If all combustion air for the 4,000 pound-per-hour furnace could be preheated to 300°F, the Table 14 shows that the quantity of reclaimed heat reclaimed would be over 200,000 Btu/hr. Under current operation, this heat is obtained from combustion of natural gas. Thus, using reclaimed heat would reduce the consumption of natural gas. Considering that natural gas has a heat capacity of about 1,000 Btu per hundred cubic feet (ccf) and the facility pays about $0.43 per ccf, the fuel and cost savings would be about:

$$\begin{aligned} &200{,}000\,\frac{\text{Btu}}{\text{hr}}\times 2{,}280\,\frac{\text{hr}}{\text{yr}}=456\,\frac{\text{MMBtu}}{\text{yr}}\\ &456\,\frac{\text{MMBtu}}{\text{yr}}\times\frac{1\times 10^{6}\,\text{Btu}}{\text{MMBtu}}\times\frac{1\,\text{ccf}}{1{,}000{,}000\,\text{Btu}}=4{,}560\,\frac{\text{ccf}}{\text{yr}} \qquad \text{Equation (153)}\\ &4{,}560\,\frac{\text{ccf}}{\text{yr}}\times\frac{\$0.43}{\text{ccf}}=\$1{,}961\,\text{per year} \end{aligned}$$

This analysis indicates that significant savings could be achieved with the concentric tube HX. If such a heat recovery unit could be installed for $1,961 or less, the simple payback would be less than one year.

The energy and exergetic efficiencies of the process would both be improved with the use of the concentric tube HX. The energy efficiency would be increased from 26% because the quantity of useful energy would be increased. From Equation (133), the energy efficiency with heat reclamation would be about:

$$\eta_{I}=\frac{1{,}029{,}918\,\frac{\text{Btu}}{\text{hr}}+200{,}000\,\frac{\text{Btu}}{\text{hr}}}{4{,}034{,}576\,\frac{\text{Btu}}{\text{hr}}}=30\% \qquad \text{Equation (154)}$$

Using the annual average outdoor temperature of 50°F as T_o and assuming the heat is reclaimed at an average stack temperature T_r of about 1,200°F, the useful exergy contained in the reclaimed heat stream is about:

$$\dot{X}_{\text{reclaimed}} = \dot{Q}_r\left(1-\frac{T_o}{T_r}\right)$$
$$\dot{X}_{\text{reclaimed}} = 200{,}000\frac{\text{Btu}}{\text{hr}}\times\left(1-\frac{460+50}{460+1200}\right) = 138{,}554\frac{\text{Btu}}{\text{hr}} \qquad \text{Equation (155)}$$

Using Equation (137), the exergetic efficiency of the process would be increased from 14.6% to about:

$$\eta_2 = \frac{590{,}074\frac{\text{Btu}}{\text{hr}}+138{,}554\frac{\text{Btu}}{\text{hr}}}{4{,}034{,}576\frac{\text{Btu}}{\text{hr}}} = 18\% \qquad \text{Equation (156)}$$

Refinement of the HX, possibly by modifying the surface to enhance heat transfer, is likely to increase the temperature to which the combustion air can be preheated. This would improve the energy efficiency because the quantity of heat reclaimed would be increased. In addition Equation (155) shows that as the temperature increases, the exergetic efficiency is improved. On the other hand, the cost increase of a refined HX over the simple concept presented here would likely be minimal. Even with the simple concept, it appears that heat reclamation with the concentric tube HX may be cost-effective. As the concept is refined, the cost effectiveness is likely to be improved.

CHAPTER V

ENERGY INFORMATION: ELECTRICAL LOAD FACTOR

For industrial plant assessments and building energy audits, "Load Factor" is a useful indicator of energy utilization efficiency. Load Factor depends on three parameters of electricity use: consumption, demand, and length of the billing period.

Actual energy consumption is determined from the facility's electric meter. It is the total quantity of electrical energy consumed during the billing period, measured in kilowatt-hours, kWh. Peak electrical demand is a measure of the peak power draw for a facility and is measured in kilowatts, kW. Typically, demand is defined as the maximum energy consumption over a 30-minute window during a billing period. The 30-minute window with the highest energy consumption over the billing period is the *peak period.* The consumption during the peak period is stored by the meter. Dividing this peak quantity of energy by the length of the period results in the peak electrical demand. For a facility whose highest 30-minute consumption measured 400 kWh, the peak electrical demand would be:

$$\text{Peak Demand} = \frac{400\,\text{kWh}}{30\,\text{min} \times \dfrac{1\,\text{hr}}{60\,\text{min}}} = 800\,\text{kW} \qquad \text{Equation (157)}$$

If the facility continuously operated at 800 kW for 24 hours per day and 7 days per week over a 30-day billing period, the quantity of energy consumed would be the maximum possible. This maximum possible energy consumption would be:

$$\text{Maximum consumption} = 800\,\text{kW} \times 24\,\text{hr/dy} \times 30\,\text{dy} = 576{,}000\,\text{kWh} \qquad \text{Equation (158)}$$

We define Load Factor as:

$$\text{Load Factor} = \frac{\text{actual exergy consumption (kWh)}}{\text{maximum possible exergy consumption (kWh)}}$$

$$\text{Load Factor} = \frac{\text{kWh/period}}{\text{kW} \times \text{(hr/period)}} = \frac{\text{kWh/period}}{\text{peak demand (kW)} \times 24\ \text{hr/dy} \times \text{days in billing period}}$$

Equation (159)

Higher load factors are desirable because they result in lower electrical costs. This is true because the purchased electricity at the peak demand is utilized more effectively when the load factor is high.

Load Factor and Exergy Analysis

Because a 100%-efficient device could convert all supplied electrical energy to work, the available work in a quantity of electricity is equal to the quantity of electrical energy. Thus, for electricity, exergy content equals energy content.

Equation (159) suggests two general ways to improve the load factor. The first is to increase the numerator, the actual exergy consumed, while holding the denominator constant. If a company plans to increase production output while minimizing electricity costs, this first method of increasing load factor should be the goal. Holding the denominator constant means not increasing peak demand. This can be done by carefully scheduling the operation of electrical equipment. Rather than increasing output by adding more equipment and operating it during the peak part of the day, output may be increased by operating existing equipment during off-peak times, or replacing oversized, inefficient machinery with better equipment. In this way, the total electrical consumption increases but demand is not increased. The result is better use of the exergy supplied at the demand value, and lower average cost of electricity. In addition, the total cost of electricity is significantly lower than it would be by simply adding additional equipment and running it during on-peak periods.

The second way to increase the load factor is to decrease the denominator of Equation (159). Often, a facility's peak demand is needed for only a few minutes per day. By replacing or eliminating oversized equipment and rescheduling power-intensive operations, short-term power spikes can be eliminated.

Replacing continuously-operating equipment with more efficient alternatives also lowers demand. If no expansion is planned, the goal should be to improve load factor by decreasing both numerator and denominator: the facility demand and electricity consumption. These measures reduce facility peak demand, improve the load factor, and reduce electricity costs.

Equation (159) establishes the maximum possible exergy consumption as the basis of comparison, and defines Load Factor as the ratio of actual exergy consumption to this maximum possible. This is a form of exergetic efficiency: a variation of Equation (7). A high load factor indicates better utilization of electric exergy. In the following case study, a demonstration is presented of how load factor analysis, a form of exergy analysis, can be used to improve the efficiency of industry.

Case Study

As an example, consider a facility with peak demand of 800 kW, and electrical consumption of 250,000 kWh over a 30-day period. According to Equation (159), the load factor for this period was:

$$\text{Load Factor} = \frac{250{,}000\,\text{kWh}}{800\,\text{kW} \times 30\,\text{dy} \times 24\,\text{hr/dy}} = 43\% \qquad \text{Equation (160)}$$

The value of 43% indicates that, on average, the facility operates at 43% of peak demand.

Analysis of Facility Operation

As demonstrated by the preceding equations, the load factor is a measure of how effectively the peak demand is used. This information can be used in several ways.

Facility Operating Schedule. First, it can help the analyst determine how many shifts the facility is operated. Table 17 lists the maximum theoretical load factor for 1, 2 and 3-shift operations according to Equation (159) and with the assumption that during down shifts, electrical consumption is zero.

Table 17. Maximum load factor for multi-shift operations.

Facility Operation	Hours of Operation	Maximum Load Factor
1 shift	8 hr/dy	33%
2 shifts	16 hr/dy	66%
3 shifts	24 hr/dy	100%

For the example facility, a load factor of 43% was calculated. The analyst may conclude that the facility operates more than one shift.

Reduction in Electrical Consumption. The second way load factor can be used is to determine the potential for reductions in exergy consumption. If the facility manager reports that the facility is operated for only one shift, the analyst would conclude that the load factor is too high at 43%, and might investigate whether a significant portion of the equipment is left running during down shifts. In this case, simply turning off equipment during downtime would decrease the load factor, but result in energy cost savings. In the same way, a two-shift facility with a load factor greater than 66% might be a good candidate for electricity consumption reductions.

Billing Error Analysis. A third way to use Load Factor is to identify billing errors. A load factor exceeding 100%, implies that the facility consumes more power than theoretically possible. For facilities with standard electrical billing rates, this indicates errors in meter readings, defective meters, or billing errors. Energy managers can use this information to track down these types of problems.

Verification of Electricity Bills. A fourth way to use load factor is to verify whether time-of-use billing is worthwhile. In 24-hour facilities with time-of-use billing structures, load factors greater than 100% can be encountered when the actual peak demand occurs during off-peak hours and is not recorded on the utility bill. With these rate structures, facility managers can achieve electrical cost savings by scheduling electrical loads with high power draw for time periods that the utility deems "off-peak". For example, billing demand is determined as the larger of: 1) a fraction (perhaps 50%) of the highest demand that occurs during off-peak periods, or 2) 100% of the highest demand that occurs during on-peak periods. Thus, the billing demand for a period may be lower than the measured facility peak demand, and load factor could be higher than 100%. In this case, the load factor serves to show that the time-of-use rate is providing lower bills than standard rates.

Reduction in Electrical Demand. A fifth way load factor can be used is to investigate the opportunity for demand reduction. If the example facility actually operates for 2 shifts per day, the best possible load factor would be 66%. Thus, an actual load factor of 43% indicates that there is room for improvement: peak demand may be unnecessarily high. If power-intensive processes can be rescheduled or eliminated, or if the power draw of continuously-operating equipment can be reduced, the peak demand would be reduced. Thus, the denominator of Equation (159) would be reduced and load factor increased. This would indicate that the exergy supplied at the demand value was used more effectively. Average and total electricity costs would thus be reduced.

Continuous Monitoring of Facility or Process. A sixth way to use the load factor is to alert energy managers to changes in the process. For example, if production output remained constant but load factor decreased, energy managers might look for equipment problems or unexpected changes in production processes. By regularly calculating and tracking the load factor for a whole facility or an individual process, trends in production exergy use can be determined, and process troubleshooting can be facilitated.

Conclusions

Load factor analysis is a useful tool for whole-facility and individual process assessment. By calculating the facility load factor from utility bills or measured data, an analyst can evaluate six areas prior to even beginning an assessment:

1. Expected facility operating schedule
2. Potential for reduction of electricity consumption
3. Whether billing errors occurred
4. Whether time-of-use rates provide savings
5. Opportunity for electrical demand reduction
6. Process changes or problems

Load factor can be viewed as a measure of how much of the peak purchased electrical exergy is actually used. In the ideal case, a production facility would operate continuously at its peak demand, and no short-term demand spikes would drive costs up. In this ideal case, the load factor would be 100%. This ideal load factor indicates that the exergy available from the purchased electricity, at the facility's peak demand, was completely put to use. The load factors in Table 15 are ideal values with which to compare the effectiveness of actual facility electricity use.

One formulation for the exergetic efficiency of an energy consuming process is:

$$\eta_2 = \frac{\dot{X}_{\text{actual}}}{\dot{X}_{\text{ideal}}} \qquad \text{Equation (160)}$$

Like the exergy analyses of Chapter II and III, Load Factor also compares actual and ideal performance, and is formulated in the same way as Equation (160). For these reasons, we consider Load Factor analysis to be a type of exergy analysis. Load Factor analysis also points out the sources of inefficiency, and suggests ways to reduce losses. Thus, Load Factor analysis, a form of exergy analysis, promotes effective use of electricity.

CHAPTER VI

INDUSTRIAL ASSESSMENT METHODOLOGY

Energy is a fundamental economic input. Without significant energy input, materials cannot be mined; goods cannot be produced, transported, refrigerated and distributed; information cannot be disseminated and research cannot be conducted in sufficient quantity to maintain modern desires. It seems logical, therefore, that this important economic commodity should be used effectively.

From a thermodynamic standpoint, the most rigorous evaluation of energy use considers not only energy quantity, but also quality. The value of energy is in its capacity to produce useful work, its exergy. This thesis describes how exergy analysis can be used to evaluate, design and improve industrial processes. It demonstrates that exergy analysis is a practical tool for informed product and system design and improvement.

The exergy analysis techniques developed here compare the actual and minimum exergies needed to perform a task. This indicates that the analysis should begin by evaluating the task and its desired output, and the minimum exergy needed to achieve this task. Thus, it suggests that a particular procedure should be followed.

First, the desired output of a process is determined. The types of questions asked at this stage should be:

- What is the desired product of this process?
- What is the desired service of this system?
- Are these products or services necessary?

If the answer to the third question is, "Yes," the second step is to establish an ideal with which to compare the existing or proposed process or system. The kinds of questions to investigate are:

- What are the boundary conditions: how does the product or system interact with the environment?
- What is the theoretical ideal, reversible process for generating this product or service?
- What are the options for accomplishing this service?
- What are the options for generating this product?

These questions address the idea of how the product or service is to be provided. Each option is evaluated with the goal of determining the quantity and quality of the required inputs and outputs. The energy and exergy requirements, material inputs, and labor demands are assessed at this stage. In addition, a fundamental task at this stage is to evaluate the outputs. Thus, an additional question to be answered at this stage is:

- For each option, what is the quantity of energy and material that is discharged from the process?

The third stage of the process involves evaluating the optional processes and services in terms of their inputs and outputs. The following types of questions are answered at this stage.

- What is the quality of the effluents of the process?
- Can the discharged material or energy streams be used as input to other processes?
- Does this option minimize destruction of the quality of energy and materials?
- Which options are the best based on these criteria?

Applying this procedure to existing processes points out the sources of waste, and provides information about how to improve the process. For the case studies of Chapters III, IV and V, exergy analysis suggested several ways the process could be improved. The exergy method is even more effective, however, during the initial design process. While it is nearly always more expensive to retrofit an existing system with improvements, the incremental cost of designing exergetic efficiency into the process or system at the outset is often minimal.

To design and install a concentric tube heat exchanger as part of the initial installation of an aluminum melt furnace would likely require some additional labor and capital expense. However, the retrofit of an existing furnace would be significantly more difficult. Shutting down a melt furnace is not an attractive option because of the subsequent solidification of residual material in the furnace. In addition, these melt furnaces often provide material for 24-hour, 7 day per week, year-round operations, and finding time to shut one down is difficult from a production standpoint. To install such a unit as a retrofit, the project would probably need to be undertaken while the furnace is hot. This example demonstrates that although the exergy method is useful in improving existing processes, it is even more valuable as a design tool.

Used as a design tool, this method often leads to designs with high resource efficiencies and lower first costs. In the air compression example, if the "best practices" system had been designed from the outset, the initial purchase price of the system would have been significantly lower, and operating costs would have been less all along.

Another example was observed at a client facility of The University of Dayton Industrial Assessment Center. In response to potential EPA regulations, this company built a multi-million dollar facility to treat wastewater from a plating operation. Unfortunately, this decision was fully implemented before an assessment was conducted. As part of the assessment, the need for the process was questioned. This initiated a spirited debate among management as to whether it was cheaper to outsource the entire plating process. Thus, the entire waste stream could have been eliminated through a policy decision. Next, by looking at the plating line, a few simple measures were found that would reduce the quantity of waste water generation from 100,000 gallons per day to about 10,000 gallons per day. These measures included delaying the dip rate and employing counter-current rinse tanks. If this system analysis had been undertaken initially, the size and initial and operating costs of the wastewater treatment plant could have been dramatically reduced. In addition, the increased concentration of chemicals in the low-flow design would have enabled the company to recycle plating chemicals from the effluent. This would have saved the company hundreds of thousands of dollars per year in reduced chemical costs. In this example, the success of the assessment method is shown as a way to improve an existing system. On the other hand, failure to use this method in the design phase has cost the company millions of dollars in initial and operating costs.

Conclusion

Exergy analysis provides a framework for evaluating processes and services. Using this method, the quality of process inputs and outputs, as well as their quantities, are determined. With knowledge of the quality and quantity of inputs and outputs, the most resource-efficient system can be selected. Knowledge of quality and quantity of outputs helps the analyst determine how to handle the effluents of a process. For example, if a given material makes up a necessary waste stream from a process, knowledge of its value suggests how and where it could be used as input to another process.

The approach to industrial systems assessment as described above points to sources of waste and inefficiency, and suggests ways to improve an existing system or a new design. Using this methodology, labor, material, energy, exergy and capital resources are used in the most effective way.

CHAPTER VII

CONCLUSIONS AND FUTURE DIRECTIONS

Energy efficiency analysis is an outgrowth of the First Law of Thermodynamics while **exergy analysis** derives from both the First and Second Laws of Thermodynamics. As the preceding case studies show, exergy analyses can lead to quite different conclusions than energy analyses. Exergy analysis points to areas of inefficiency or losses that are invisible to First Law analyses. For example, the First Law analysis of Chapter IV indicated that the combustion process is 100% efficient. However, the exergy analysis showed that nearly one-third of the exergy in the fuel is destroyed during combustion.

Exergy analysis leads in two potential directions for improving system efficiency. The first direction is improvement of an existing system. With this direction, questions that arise include:

- How might the existing system be improved?
- What would be the value by which it is improved?
- What would the economic considerations be?

The second direction is complete redesign of a system. By designing a system with exergetic resource efficiency in mind, the first cost can be nearly the same or even less than for a workable, but inefficient design. As an example, consider the compressed air plant of Chapter III. Proper sizing of the compressor during initial design would have resulted in a smaller compressor and significantly lower first cost. The exergetic efficiency of the system would have been more than twice that of the existing, typical system. In addition, the operating cost would have been lower from the outset. Over time, a considerable sum of money would have been saved.

In the case of the air compressor, the simple payback was so short that retrofitting with a smaller compressor was cost-effective. Often, however, the cost of refitting a system with exergy-efficient features makes it hard to justify. In these cases, not employing resource-efficient design at the outset can be an irreversible decision. These examples serve to illustrate that, by using appropriate design principles at the outset, a resource-efficient process or system need not be prohibitively expensive.

The value in exergy analysis lies in its ability to point out sources of and solutions to resource inefficiency. Rather than simply quantifying the energy consumption and losses in a process, exergy analysis reveals the work potential of the energy uses and points to the sources of losses. Quality as well as quantity is considered. With this information, the analyst can develop the proper priorities for designing or improving a system or process. A process with high exergetic efficiency uses energy from a source that is matched in quality with the needs of the process. Exergy, or work potential, is not wasted and energy is used more effectively.

The case studies in this thesis serve to show that the exergy method can be used to analyze not only energy systems, but also information and materials processing systems. It is thus a powerful analytical tool for designing and improving systems so that they utilize resources most effectively.

Suggested Future Efforts

In the industrial environment, compressed air is used for a multitude of applications. Many of these make extremely poor use of the high-quality electricity needed to produce the compressed air. It would be useful to determine common uses of compressed air and their exergetic efficiencies. As part of the effort, alternative methods of accomplishing the same tasks should be identified, and their exergetic efficiencies determined. This information could be packaged as a sourcebook of industrial improvement opportunities, and used as a resource for industrial plant assessments.

The concentric tube heat exchanger of Chapter IV shows promise as a device for heat reclamation on high-temperature furnaces. This design should be further developed and tested. One useful refinement would be design of the heat transfer surface to increase the quantity of heat recovered. This would improve its cost-effectiveness. In energy-intensive industries such as metal casting, a large proportion of the fuel exergy is discharged to the environment. Clever design can improve such processes so that the utilization of energy resources optimized. Such improvements often manifest themselves in decreased operating costs, and improved environmental stewardship. Use of the concentric tube HX may prove to be such an enhancement.

In addition to these two specific tasks, the relationships between economics and process efficiencies should be further investigated. Under what conditions does process improvement and retrofit make sense? What conditions render a decision irreversible, such that exergetic efficiency must be designed into the process at the outset, and retrofitting is not justifiable? Do systems with high exergetic efficiencies necessarily require higher first costs?

APPENDIX

The following pages are listings of the computer program used to analyze heat transfer in the concentric-tube HX design.

```
% Matlab m-file DTS6.M        routine for solving systems of equations.
% User must have put new equations into EQNS.M prior to running this.
% Also requires initial values from INITVALS.M as input.
%
initvals;         % Establish # of unknowns
f=(zeros(1,nvar))';      % Initialize all arrays
x=(zeros(1,nvar))';
pd=zeros(nvar,nvar)';    % pd is a square matrix.  Others col. vectors.
fold=(zeros(1,nvar))';
xold=(zeros(1,nvar))';

initvals;         % Establish initial values (first guesses) for
unknowns.

% -Begin iteration loop.  Allow max of 15 iterations.  If no
convergence, then quit.
iter=1;
while iter<15

  % -Subprograms-
  eqns;     % Evaluate the equations at initial values: form f matrix.
  pardif;   % Calculate partial differentials of f's: form pd matrix.

  % -Calculate deltax matrix by [deltax]=inverse[pd]*[f]
  deltax=inv(pd)*f;      % Matrix of changes in value of the unknowns
  x=x-deltax;            % Matrix of new values of independent variables

  % -Convergence check-
  if max(abs(deltax)) < (0.001*max(abs(x)))
    iter=15;
  end

  % -finishing the "while iter" loop-
  iter
  deltax
  iter=iter+1;
end; % -end of "while iter" loop

% -Continuing with program after convergence-
exitvals;
% -The x matrix is the matrix of current values of the unknowns.

end; % -END OF PROGRAM-
```

```
% Matlab m-file INITVALS.M
% Wayne Bader
%   Feb 29, 2000 [concentric tube, counterflow HX]
% This subprogram is for use with the Newton-Raphson routine NEWTON.M
% for solving systems of equations.  User must enter the appropriate
% initial values and number of unknowns/equations here...
% this becomes input for NEWTON.M.
% NEWTON.M also requires system equations (see EQNS.M).
%
nvar=6;           % Number of equations/unknowns

%-system constants
Pc=14.7;    % inlet air pressure (psia)
Rc=0.37;    % gas constant for air (ft3*psia/lbm-R)
Png=14.8;   % inlet nat. gas pressure (psia)
Rng=0.671;  % gas constant for NG (ft3*psia/lbm-R)

Cpc=0.24;   % specific heat for air (Btu/lbm-F)
kc=0.0195;  % thermal conductivity, air (Btu/hr-ft-F)
Prc=0.677;  % Prandtl No, air
muc=0.055;  % fluid viscosity, air (lbm/ft-hr)

Cph=0.27;   % specific heat for p.o.c. (Btu/lbm-F)
kh=0.035;   % thermal conductivity, p.o.c. (Btu/hr-ft-F)
Prh=0.714;  % Prandtl No, p.o.c.
muh=0.094;  % fluid viscosity, p.o.c. (lbm/ft-hr)

      Thi=2700;   % inlet p.o.c. temp (F)
      Tci=77;           % inlet air temp (F)
      L=20;       % length of HX (ft)
      D=32/12;    % inner dia, main exhaust stack (ft)
      Ds=36/12;   % dia, outer shell (ft)

Dh=Ds-D;    % hydraulic diameter, annulus (ft)
Ah=pi*(D/2)^2;    % cross-sect. area, main stack (ft2)
Ac=pi*((Ds/2)^2-(D/2)^2);     % cross-sect. area, annulus (ft2)
A=pi*D*L;   % HX area for heat transfer (ft2)
thk=0.125/12;     % stack wall thickness (ft)
kw=35;            % thermal conductivity, stack wall (Btu/hr-ft-F)

% -Initial values for system unknown variables-
Tco=Tci+100;      % cold air outlet T (F)
mdotc=5000; % flow rate cold air (scfh)
mdoth=5300; % flow rate p.o.c. (scfh)
Tho=Thi-100;      % p.o.c. outlet T (F)
U=0.7;            % overall HX heat transfer coefficient
Q=590000;   % heat transfer to cold fluid (Btu/hr)

x(1)=Tco;
x(2)=mdotc;
x(3)=mdoth;
x(4)=Tho;
x(5)=U;
x(6)=Q;

% -END OF SUBPROGRAM--
```

```
% Matlab m-file PARDIF.M
% This subprogram is for use with the Newton-Raphson routine NEWTON.M
% for solving systems of equations.  PARDIF.M computes a nvar x nvar
% matrix of partial differentials of the system functions.
%

dx=0.00001; % incremental change in independent variable

% -save current vals of "f" matrix as "fold"
  fold=f;
  xold=x;

%-Wayne's method-
for col=1:nvar
  x(col)=x(col)*(1+dx); % new value of independent variable x(col)
  eqns;                 % new values of "f" matrix with new x(col),
holding other x's constant.
  pd(:,col)=(f-fold)./(x(col)*dx);% partial derivatives for current
column of "pd" matrix. (nvar cols)
  x=xold;         % set indep. variables back to original vals to hold
constant for next pd
end;

% -END OF SUBPROGRAM--
```

```
% Matlab m-file EQNS.M
%   Feb 29, 2000 [concentric tube, counterflow HX]
% This subprogram is for use with the Newton-Raphson routine NEWTON.M
% for solving systems of equations.  User must enter the appropriate
% system equations here ... this becomes input for NEWTON.M.
% NEWTON.M also requires initial values for unknowns (see INITVALS.M).
%
% -Eqn #1-
f(1)=-x(2)+2*Pc/(Rc*(x(1)+460))*(0.014*x(1)^2-35.262*x(1)+50787);

% -Eqn #2-
f(2)=-x(3)+x(2)+2*Png/(Rng*(x(1)+460))*(0.0014*x(1)^2-
3.5629*x(1)+5149.2);

% -Eqn #3-
f(3)=-x(6)+x(2)*Cpc*(x(1)-Tci);

% -Eqn #4-
f(4)=-x(6)+x(3)*Cph*(Thi-x(4));

% -Eqn #5-
numer=1-exp((-x(5)*A)/(x(2)*Cpc)*(1-(x(2)*Cpc)/(x(3)*Cph)));
denom=1-(x(2)*Cpc)/(x(3)*Cph)*exp((-x(5)*A)/(x(2)*Cpc)*(1-
(x(2)*Cpc)/(x(3)*Cph)));
f(5)=-x(6)+(numer/denom)*x(2)*Cpc*(Thi-Tci);
%...in above eqns, make sure that Cpmin and Cpmax are correctly set...

% -Eqn #6-
Reh=x(3)*D/(muh*Ah);
Rec=x(2)*Dh/(muc*Ac);
f(6)=-
x(5)+1/(1/(kh/D*(0.023*Reh^0.8*Prh^0.3))+(thk/kw)+1/(kc/Dh*(0.023*Rec^0
.8*Prc^0.4)));

% -END OF SUBPROGRAM--
```

```
% Matlab m-file EXITVALS.M
% Wayne Bader
%   Feb 29, 2000 [concentric tube, counterflow HX]
% This subprogram is for use with the Newton-Raphson routine NEWTON.M
% for solving systems of equations.  User must enter the appropriate
% initial values and number of unknowns/equations here... this becomes
input for NEWTON.M.
% NEWTON.M also requires system equations (see EQNS.M).

Thi
Tho=x(4)
Tci
Tco=x(1)
mdotc=x(2)
mdoth=x(3)
U=x(5)
Q=x(6)
eff=Q/(mdotc*Cpc*(Thi-Tci))

NG=4000*Q/100000;
junk=['Annual NG use (ccf) is'];
disp(junk)
NG
% -END OF SUBPROGRAM--
```

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