

**A GUIDE FOR INTEGRATING THE GRAPHING CALCULATOR
IN THE ADVANCED PLACEMENT CALCULUS CURRICULUM**

MASTER'S PROJECT

**Submitted to the School of Education
University of Dayton, in Partial Fulfillment
of the Requirements for the Degree
Master of Science in Education**

by

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June 1994**

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A handwritten signature in black ink, appearing to be 'JW', is written over the horizontal line and extends slightly below it.

ACKNOWLEDGEMENTS

This work could not have been accomplished without the support, patience, understanding, and love of my wife, Jodi, and our daughters, Hannah and Holly.

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CHAPTER I

INTRODUCTION

The development of the graphing calculator is presenting mathematics teachers with new ideas and teaching techniques that could supplement the current Advanced Placement mathematics curriculum. In addition, the College Board has revised the existing testing program so that graphing calculators are required for the advanced placement test during the 1994-95 school year. However, many mathematics teachers are not familiar with graphing calculators and how they can be utilized in the classroom. Therefore, a handbook that provides ideas, teaching suggestions, and lesson plans for specific topics in the advanced placement curriculum would be useful for mathematics teachers that desire to incorporate graphing calculators in their own classroom situation.

The purpose, then, of this handbook is to fill a need that exists in the field of mathematics instruction. In order to prepare students for the advanced placement tests and the other changes that have been mandated by the state of Ohio and the National Council of Teacher of Mathematics (NCTM), changes must also be made in instructional techniques and activities that students are involved with in the classroom. This handbook provides the classroom teacher ideas, activities, and lesson plans to follow that incorporate the use of the graphing calculator within the calculus

I have been involved in the teaching of advanced placement calculus for the past 6 years. During this period of time, I have used various types of calculators in teaching my advanced placement classes. Recently I began using a TI-81 graphing calculator in my advanced placement classes, and have realized what an excellent tool this calculator is for teaching concepts in calculus, and for expanding some of the ideas I currently discuss in class.

ASSUMPTIONS AND LIMITATIONS

There are many opportunities to incorporate the use of the graphing calculator throughout the high school mathematics curriculum. However, the range of this handbook is restricted to the utilization of the the graphing calculator in the advanced placement calculus curriculum. It is probable that the graphing calculator will be incorporated throughout the high school mathematics curriculum through a "trickle down" process. By including this type of calculator at the highest level of the curriculum, it is inevitable that they will be used in the lower areas as teachers and students become familiar with their function and application in the mathematics classroom. However, the scope of this handbook remains at the calculus level.

Although different types of graphing calculators are currently available, this project will be limited to utilizing the Texas Instruments

calculator, the TI-82. It will be possible to apply many of the ideas, procedures, and activities to other types of graphing calculators.

It is also assumed that all students in the classroom will have access to a graphing calculator. Not all school systems have graphing calculators available to their students, and it is possible that all students would not be able to purchase one of their own. It is suggested that mathematics departments request a classroom set of graphing calculators for the students to use in the mathematics classrooms, or if possible, require students to purchase this type of calculator at the beginning of the school year.

DEFINITIONS

Graphing Calculator. This is considered to be any calculator that has the capability of generating graphs of equations that are built into the calculator or that are defined by the user. Currently, Texas Instruments, Casio, Hewlett-Packard, and Sharp all manufacture graphing calculators.

Advanced Placement Calculus. A calculus course that is designed for high school students that follows curriculum that is set forth by The College Board. Upon completing the course, students may take the advanced placement test that is administered by The College Board, and possibly acquire college credits depending upon the score they receive on the Advanced Placement test and the college they attend.

CHAPTER II

REVIEW OF LITERATURE

A commonly held perception among many educators is that calculators in the classroom, at all levels, have a negative effect on learning. This view is expressed in statements such as “students who use calculators do not learn basic skills” or “students will not learn to graph functions if they have graphics calculators.” In the 1800’s when lead pencils and paper were available for all school children, there was also great concern among the school teachers of mathematics. What will happen, they worried, when their students have this new technology? What about their ability to do mental arithmetic? Won’t this ability decline and therefore cause major disaster for all mathematics education? Those in mathematics education that feel that the use of calculators, and more specifically, graphing calculators, should look back into history to see that these fears are unfounded. The development of the graphing calculator should be viewed upon as a tool that will improve and enhance the current mathematics curriculum instead of a deterrent to instruction.

Many other educators share the same belief that the graphing calculator can be a powerful teaching tool. It is thought that precalculus students benefit from an intuitive understanding of functions gained through the use of graphics calculators, and calculus students gain a deeper understanding of functions and their graphs by interactively using

graphics and algebraic capabilities of calculators (Dion, 1990). It has also been suggested that graphing calculators empower students by giving them the ability to use the power of visualization to do mathematics, and that through this power students can add a geometric or visual representation to a problem in order to investigate traditional algebraic topics (Demana & Waits, 1992a). This is a valid point as it is difficult in teaching math to find ways to give students a concrete example for mathematics concepts that basically abstract. Students seem to grasp difficult concepts much easier when the concept can be presented visually, and they can actually “see” results. It is much the same in teaching the game of golf, the instructor can tell the student how to hold a club and how to swing it, but unless they get visual input and a hands on experience they will not understand the concept being taught. They must “see” and “feel” what is being presented to them.

The National Council of Teachers of Mathematics (NCTM) has also supported the use of technology in the classroom. According to the NCTM, “the teacher of mathematics, in order to enhance discourse should encourage and accept the use of computers, calculators, and other technology” , and also state that “the most fundamental consequence of changes in patterns of instruction in response to technology rich classroom environments is the emergence of a new classroom dynamic in which teachers and students become natural partners in developing mathematical ideas and solving mathematical problems” (National Council

of Teachers of Mathematics, 1989).

Even though there are many proponents of graphing calculators and the use of calculators in the classroom, there are many concerns that arise as well. For instance, a major concern of many educators is “what is there left to teach?” It is important to realize that the graphing calculator is being proposed as a tool to supplement instruction, not replace it. As Dion suggests, “the effective use of scientific and graphics calculators requires a solid understanding of the mathematical concepts involved” (Dion, 1990, p. 567). Joseph Mercer concurs in this ideas as he surmises that the time that is saved in calculations can be put to use by “teaching about graphs, how to model real-world problems with graphs, how to glean information from graphs, and how to model graphs with equations” (Mercer, 1992, p. 416).

It is important to note that although there are many teachers, administrators, and other professionals in the field of mathematics education that have determined that using graphing calculators improves mathematics instruction and the learning of mathematics, but the research to this point is not conclusive in supporting this claim. However, Anthony Ralston suggests that current research does in suggest that reasoned use of calculators in any grade enhances the learning of mathematics generally. He explains that the benefits of the use of calculators will be significant; that student interest will be enhanced, the drudgery of pencil and paper worksheets will be replaced by calculator

explorations for patterns and rules, mental math will actually be emphasized more in order to facilitate estimation and checking answers for reasonableness, and students will learn that mathematics is an experimental as well as an exact science (Ralston, 1991).

In summary, current research and trends indicate that the uses of the graphing calculator in the classroom are powerful and could change the shape of mathematics instruction in the future. If the opportunity arises, the mathematics teacher should try to utilize the graphing calculator to supplement current methods of instruction in the classroom.

CHAPTER III

A GUIDE FOR INTEGRATING THE GRAPHING CALCULATOR IN THE ADVANCED PLACEMENT CALCULUS CURRICULUM

AP Calculus Lesson Plan

Topic: Graphing $c \cdot f(x)$, $f(c \cdot x)$, $-f(x)$, $f(-x)$, $|f(x)|$

Overview:

The use of the graphing calculator can make it quite easy for students to see how the period and amplitude of functions can be affected by constants or absolute value. In this lesson, students will graph a function, then graph variations of that function and interpret the resulting graphs.

Objective(s):

-the student will describe the amplitude and period changes in the graphs of the functions $c \cdot f(x)$, $f(c \cdot x)$, $-f(x)$, $f(-x)$, $|f(x)|$.

Procedure(s):

I. Have students enter the equation $y = \sin x$ in Y1, and sketch its graph. Use a window size so that $-\pi \leq x \leq 2\pi$ with a scale of $\pi/2$ and $-2.5 \leq y \leq 2.5$ with a scale of 1.

II. Define amplitude and period using the function in I (period 2π , and the amplitude = 2).

III. Have students then enter the following equations:

$$Y2 = 1.5 * \sin x$$

$$Y3 = 2 * \sin x$$

$$Y4 = 2.5 * \sin x$$

Questions: What does the constant do to the graph of the equation?
Does the constant change the amplitude or period?

IV. Have students enter these equations for Y2, Y3, and Y4:

$$Y2 = -\sin x$$

$$Y3 = -2 * \sin x$$

$$Y4 = -2.5 * \sin x$$

Question: What does the negative constant do to the graph of the equation?
Does it change the amplitude or the period?

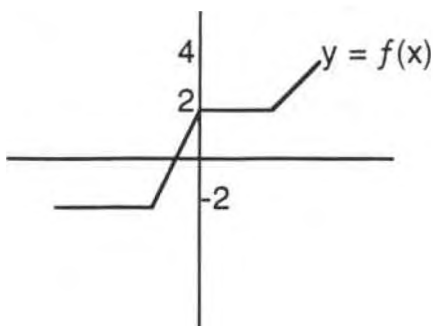
V. Ask students to give a “rule of thumb” for the graph of $c * f(x)$ and graph the following equations using this rule (no calculator!):

$$y = 10 \sin x$$

$$y = -23 \sin x$$

$$y = 100 \sin x$$

VI. Show students the following graph of a function:



Discuss how $y = f(x)$ would be changed by $2f(x)$ and $-4f(x)$.

VII. Enter the following equations and analyze their graphs:

$$Y2 = \sin 2x$$

$$Y3 = \sin 4x$$

$$Y4 = \sin .5x$$

Questions: What does the constant do to the graph of the equation?
Does the constant change the amplitude or period?

VIII. Enter the following equations and analyze their graphs:

$$Y2 = |\sin x|$$

$$Y3 = -|\sin x|$$

Questions: What change does the absolute value make in the graph?
Is there a change in amplitude or period?

IX. Have students graph the equations below, but first have students guess or visualize what the graph should look like. Give the amplitude and period for each graph.

1. $y = 4 \cos x$
2. $y = -4 \cos x$
3. $y = |4 \cos x|$
4. $y = -5 \cos 2x$
5. $y = 3 \cos .5x$
6. $y = |.5 \cos 3x|$

X. Reverse thinking. Have the students determine what values for k will give the function the given amplitude or period.

1. $y = \sin kx$, period 4π .
2. $y = k \cos kx$, amplitude $.25$.
3. $y = \tan kx$, period 2 .
4. $y = (-1/k)\sin kx$, amplitude 3 .

XI. Give the students a graph of a function as in part VI, and have them graph variations of that graph using the skills they learned in this lesson.

XII. The "friendly" window. Set the window of the TI-82 so that the X_{\min} and the X_{\max} are multiples of 94, and the Y_{\min} and Y_{\max} are multiples of 62. This will give the user integer values when using the trace function of the TI-82.

For instance, a "standard" window is as follows:

$$\begin{aligned} X_{\min} &= -9.4, & X_{\max} &= 9.4, & X_{\text{scl}} &= 1 \\ Y_{\min} &= -6.2, & Y_{\max} &= 6.2, & Y_{\text{scl}} &= 1 \end{aligned}$$

AP Calculus Lesson Plan

Topic: Graphing $f(x) + c$, $f(x + c)$

Overview:

This is an extension of the previous lesson, as the student will see how a constant can effect the graph of an equation. In this case, the constant is added or subtracted “inside” or “outside” of a function.

Objective(s):

-the student will describe the amplitude and period changes in the graphs of the functions $f(x) + c$ and $f(x + c)$

Procedure(s):

I. Have students enter the equation $y = x^2$. Window size $-9.4 \leq x \leq 9.4$ with a scale of 1, and $-6.2 \leq y \leq 6.2$ with a scale of 1.

II. After viewing the graph of $y = x^2$, have students enter the following equations, and graph all three:

$$Y2 = x^2 - 2$$

$$Y3 = x^2 + 3$$

Question: How does the constant added or subtracted to the original graph change its picture?

III. Have students theorize how the graphs below will appear:

Original Equation

New Equation

$$y = \sin x$$

$$y = (\sin x) + 1$$

$$y = \sin x$$

$$y = (\sin x) - 2$$

$$y = x^3$$

$$y = x^3 + 4$$

$$y = x^3$$

$$y = x^3 - 5$$

Have students verify their theory by graphing each with their calculator.

IV. Have students clear their current graphs, except for $Y1 = x^2$, and enter the following graphs:

$$Y2 = (x + 1)^2$$

$$Y3 = (x - 3)^2$$

Question: How does the constant “inside” the function alter the graph?

V. Have students theorize how the graphs below will appear:

Original Equation

New Equation

$$y = \sin x$$

$$y = \sin(x + 1)$$

$$y = \sin x$$

$$y = \sin(x + \pi)$$

$$y = x^3$$

$$y = (x + 2)^3$$

$$y = x^3$$

$$y = (x - 4)^3$$

Have students verify their theory by graphing.

VI. Combinations of both types of equations. Have students first try to determine what the graph of the equation will look like based on the graph of the original equation and what they have learned so far in this lesson. Then graph the equation using their calculator:

Original Equation

New Equation

$$y = x^2$$

$$y = (x - 2)^2 + 3$$

$$y = x^3$$

$$y = (x + 5)^2 + 7$$

$$y = \cos x$$

$$y = 3 + \cos(x - \pi)$$

VII. Putting it all together. These equations summarize the graphing concepts discussed in the last 2 lessons. Have students get a “mental” picture first, then graph the equation using their calculator.

1. $y = 2\sin(4x)$

2. $y = -|\cos(x/2)|$

3. $y = -(x - 2)^2 + 4$

4. $y = (x + 7)^3 - 2$

AP Calculus Lesson Plan

Topic: Vertical Asymptotes

Overview:

This is an activity where students can discover vertical asymptotes of rational functions. This activity was prepared for the TI-81 mini course at AMATYC in Indianapolis in 1992.

Objective(s):

-the student will find the vertical asymptotes of a rational function

Procedure(s):

Students should complete the following activity.

Directions: Use your calculator to sketch the graph of the following functions and record your sketch on paper. Window size $-9.4 \leq x \leq 9.4$ and $-6.2 \leq y \leq 6.2$.

1. For the equation:

$$y = \frac{4}{x - 1}$$

- What happens to the graph at $x = 1$?
- What is the domain of the function?
- Complete the following table of values for this function.

x	1.1	1.05	1.01	1.005	1.001	1.0001
y	?	?	?	?	?	?

x	.9	.95	.99	.995	.999	.9999
y	?	?	?	?	?	?

- What is happening to y as x approaches 1 but is bigger than 1?
- What is happening to y as x approaches 1 but is smaller than 1?

2. For the equation:

$$y = \frac{20x + 10}{(5x - 6)(x + 4)}$$

- What happens to the graph at $x = 6/5$ and $x = -4$?
- What is the domain of the function?
- Complete the following table of values for this function.

x	-3.9	-3.95	-3.99	-3.995	-3.999	-3.9999
y	?	?	?	?	?	?

x	-4.1	-4.05	-4.01	-4.005	-4.001	-4.0001
y	?	?	?	?	?	?

- What is happening to y as x approaches -4 but is bigger than -4 ?
- What is happening to y as x approaches -4 but is smaller than -4 ?
- Complete the following table of values for this function:

x	1.3	1.25	1.21	1.205	1.201	1.2001
y	?	?	?	?	?	?

x	1.1	1.15	1.19	1.195	1.199	1.1999
y	?	?	?	?	?	?

- What is happening to y as x approaches 1.2 but is bigger than 1.2 ?
- What is happening to y as x approaches 1.2 but is smaller than 1.2 ?

3. For the equation:

$$y = \frac{5 - x}{x - 2}$$

- Where is there a break in this graph?
- What is the domain of the function?
- What is happening to y as x approaches 2 but is bigger than 2 ?
- What is happening to y as x approaches 2 but is smaller than 2 ?

4. For the equation:

$$y = \frac{7}{x^2 + x - 6}$$

- Where is there a break in this graph?
- What is the domain of the function?
- What is happening to y as x approaches 2 but is bigger than 2?
- What is happening to y as x approaches 2 but is smaller than 2?
- What is happening to y as x approaches -3 but is bigger than -3?
- What is happening to y as x approaches -3 but is smaller than -3?

5. Guess the values where the graphs of these functions break. Check by graphing.

a. $y = \frac{5}{2x - 6}$

b. $y = \frac{4x + 15}{(x - 5)(x + 1)}$

c. $y = \frac{3}{x^2 + 1}$

d. $y = \frac{x^2 - 25}{x - 5}$

e. $y = \frac{4x + 4}{x^2 - 4x - 5}$

6. Answer the following questions:

- How did your predictions work in the problems 5c, 5d, and 5e?
- Explain what happened in 5c.
- Use your trace to explain what happened in 5d and 5e.

7. Conjecture.

- A vertical line that a graph approaches and never touches is called a vertical asymptote. Using what you learned above, summarize how you would find the vertical asymptote of a rational function without graphing.
- Describe what a graph does on either side of a vertical asymptote.

AP Calculus Lesson Plan

Topic: Graphing Piecewise Defined Functions

Overview:

It is possible to graph a piecewise defined function with the TI-82 calculator, and this function becomes very handy when discussing continuity, and right and left hand limits. This lesson simply discusses how to use the calculator to graph this type of function.

Objective(s):

-the student will graph a piecewise defined function

Procedure(s):

I. Consider the function defined as follows:

$$f(x) = \begin{cases} 2x - 1, & \text{if } x \leq 1 \\ x^2 - 1, & \text{if } x > 1 \end{cases}$$

This piecewise defined function is the combination of a line and a parabola. A likely discussion in a calculus class would involve the continuity of this function at the point $x = 1$. A graph of this function would be of help in this area.

II. Follow these steps in order to graph this function:

1. Choose the Y= key.
2. Enter the following equation for Y1:

$$(2x - 1)(x \leq 1) + (x^2 - 1)(x > 1)$$

3. Choose the GRAPH key to see the graph (Remember to check the WINDOW before graphing)

III. Another method for graphing a piecewise defined function is as follows:

1. Using the same function in II, select the Y= key.
2. Enter the following equations:

$$Y1 = (2x - 1) / (x \leq 1)$$

$$Y2 = (x^2 - 1) / (x > 1)$$

****Notice the division symbol****

3. Choose the graph key to see the graph.

IV. Try to graph these piecewise functions using both methods:

$$1. f(x) = \begin{cases} 4, & x > 3 \\ x + 1, & 0 \leq x \leq 3 \\ -x^2, & x < 0 \end{cases}$$

$$2. f(x) = \begin{cases} \sin x, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

$$3. f(x) = \begin{cases} x^3 - 1, & x > 2 \\ 2 - 3x, & x \leq 2 \end{cases}$$

$$4. f(x) = \begin{cases} -2x, & x > 2 \\ x^2, & 0 \leq x < 2 \\ -x^2, & x < 0 \end{cases}$$

AP Calculus Lesson Plan

Topic: Limits

Overview:

This lesson enables the student to understand the meaning of the idea that “as x approaches a the limit is L ” as they examine the behavior of functions at particular points. The table function of the TI-82 as well as the graphing capabilities of the calculator help the student “see” the concept that is being presented.

Objective(s):

- the student will state the definition of a limit
- the student will apply the definition of a limit to different types of functions

Procedure(s):

I. Define limit as follows:

$$L = \lim_{x \rightarrow a} f(x)$$

which says “the number L is the limit of the function f as x approaches a .”

II. Examine the function $(\sin x) / x$ as the values of x approach zero by creating a table as follows:

1. Choose the $Y=$ key and enter the function $(\sin x) / x$.
2. Choose the TblSet key, and set the Indpnt option to Ask .
3. Choose the TABLE key and enter the values 1, .9, .7, .5, .3, and .1, and 0.

III. Questions to ask students.

What is happening to the Y_1 values as the X values get closer to 0?
Why does the value of 0 for X yield “ERROR” in the Y_1 column?

III. Change the X table values as follows:

.1, .05, .025, .01, .009, and .001

IV. Questions to ask students.

As the X values get closer to 1, what is happening to the Y_1 values? Why does the X value of .001 yield a value of 1? (Have students compute the value of $(\sin .001) / .001$ with their calculator to get an accurate response)

V. Other things to investigate.

1. Examine the behavior of the function as x approaches 0 from the left by changing the chart values to negatives.
2. Have students graph the function and use the TRACE function to see how the function behaves as x approaches 0.

VI. Examine the behavior of the function $(2x^2 - x - 1) / (x^2 - 1)$ as x approaches 1 in the following manner:

1. Select Y= and enter the function $(2x^2 - x - 1) / (x^2 - 1)$.
2. Select WINDOW and enter values: $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, scale of 1 each axis.
3. Select GRAPH.
4. After viewing the first graph, select ZOOM and then Zoom In.
5. Move the cursor to a point on the graph so that x is as close to 1 as possible, and then press ENTER.
6. Repeat step 5, and note that that values for x are getting closer to 1.
7. Repeating step 5 a few times will get an x value extremely close to 1, and a y value very close to 1.5, which is, in fact, the limit that can be computed by hand.

VII. Have students repeat the process in VI for the following functions:

1. $\cos(\pi x) / (2x - 1)$ as x approaches $.5$.
2. $(2x^3 + 3x^2 - 4x - 6) / (x - \sqrt{2})$ as x approaches radical 2.

VIII. Questions for Discussion.

1. Does the limit as x approaches a for a function f always equal $f(a)$?
2. Are there functions that exist so that at a point a , the values of the function do not approach any number? (This can serve as a lead in for the next lesson)
3. Can the students relate the definition of a limit to the problems?

AP Calculus Lesson Plan

Topic: Limits-nonexistent & the “Pinching” Theorem

Overview:

This lesson demonstrates limits that are nonexistent, and also uses the Pinching Theorem to determine the limit of a function. It would be a good idea to review how to graph a compound function using the TI-82. The functions the students graph in this lesson would also be good for discussing right and left hand limits.

Objective(s):

- the student will give reasons why a function does not have a limit at a particular point
- the student will use the Pinching Theorem to evaluate a limit

Procedure(s):

I. Discuss the definition of a limit

- review that L is the limit as x approaches a .
- $f(a)$ does not always equal the limit as x approaches a .

II. Have students graph the function $y = ((x + 1)^2 - 2) / x$. Then, have students use the TRACE to see how the graph behaves around $x = 0$

III. Questions to ask students.

1. According to the definition of a limit, there must be a value L that the function is or is near when x approaches a . Is there such a value when x approaches 0 for this function?
2. What value does y go to when x approaches 0 from the right? the left?

IV. Have students graph the following functions and determine for what values would the limit not exist.

$$1. y = \sin (1 / x)$$

$$2. y = |x| / x$$

$$3. y = (x - 4) / (x - 1)$$

$$4. y = x^2 / (x + 2)$$

V. Define the Pinching Theorem.

Assume that the limit of $g(x)$ and $h(x)$ both exist as x approaches a , and that these limits are equal to L . If the function f satisfies the inequality

$$g(x) \leq f(x) \leq h(x)$$

for all x in an open interval containing a then the limit as x approaches a of $f(x)$ is also equal to L .

VI. To demonstrate the Pinching Theorem, have the students enter the following equations and graph:

$$Y_1 = 1 / (\cos x)$$

$$Y_2 = \cos x$$

$$Y_3 = (\sin x) / x$$

**Note that the limit as x approaches 0 of both Y_1 and Y_2 is 1

VII. Questions to ask students.

1. Is $1 / (\cos x) \leq (\sin x) / x \leq \cos x$? On what interval(s)?
2. If #1 is true, and the limit of $1 / \cos x$ and $\cos x$ as x approaches 0 is 1, then what must be the limit of $(\sin x) / x$?

VIII. Have students determine the limits of these functions using their calculator and the Pinching Theorem.

1. Find the limit as x approaches 0 for the function f if

$$1 - x^2 \leq f(x) \leq 1 + x^2 \quad \text{for all } x.$$

2. Find the limit as x approaches 3 for the function f if

$$6x - x^2 \leq f(x) \leq x^2 - 6x + 18 \quad \text{for all } x.$$

3. Find the limit as x approaches 0 for the function f if

$$1 - x^4 \leq f(x) \leq \sec x \quad \text{for } -\pi/2 \leq x \leq \pi/2.$$

AP Calculus Lesson Plan

Topic: The Derivative

Overview:

These graphing calculator activities are designed to supplement the teaching of the definition of the derivative. The graphing capabilities of the TI-82 are combined with the use of the definition of the derivative in order for the student to get a visual idea of how the slope of the tangent is directly related to the derivative. In the final activity, the students will

Objective(s):

- the student will state the mathematical definition of the derivative
- the student will relate the slope of a tangent line to the definition of the derivative.
- the student will be able to determine if a function is differentiable at a given point.

Procedure(s):

I. Review the definition of the derivative (this concept should have been introduced previously).

The derivative of the function f on the interval I , denoted by f' , is the function with values:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists for all x that are in the interval I .

II. Introduce the following function:

$$m(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

and have the students graph $y = m(x)$ for $h = 1.0, 0.1, 0.01,$ and 0.001 using the functions listed below:

$$f(x) = 2x + 1$$

$$f(x) = x^2$$

$$f(x) = \sin x$$

Students should be answering the question:

How do these graphs relate to the graph of the derivative of f ?

III. This activity helps the student decide if a function is differentiable at a given point. A function f and a number a are given. Using the function:

$$m = \frac{f(a+h) - f(a)}{h}$$

and $h = \pm 2.0, \pm 1.5, \pm 1.0, \pm 0.1, \pm 0.01,$ and $\pm 0.001,$ graph the secant lines $y = m(x - a) + f(a)$. Determine if f is differentiable at a .

1. $f(x) = x^2 - 2x + 1, a = 1$

2. $f(x) = \begin{cases} 2x - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases} \quad a = 1$

III. Formula discovery. Have the students complete the following task:

For the equations listed below, given the function f , graph:

$$y = m(x) = \frac{f(x+h) - f(x)}{h}$$

where $h = .001$. Compare the graph of $m(x)$ with the graphs of other functions you know, and conjecture a formula for $f'(x)$.

1. $f(x) = 2x + 3$

3. $f(x) = x^2 + 2x + 3$

5. $f(x) = x^3 - 5x^2$

2. $f(x) = x^2$

4. $f(x) = x^3$

AP Calculus Lesson Plan

Topic: The Derivative

Overview:

One of the most powerful capabilities of the TI-82 and other types of graphing calculators is the ability to graph and compute derivatives, maximum and minimum values of functions, and to graph tangent lines to graphs at particular points. In this lesson the student is exposed to the built-in functions of the TI-82 that deal with differentiation.

Objective(s):

- the student will use a graphing calculator to find the relative maximum and minimum values of a function
- the student will use a graphing calculator to find the value of the derivative of a function at a given point
- the student will use a graphing calculator to draw the tangent line to a graph at a given point

Procedure(s):

I. Have students enter the equation $y = x^3 - x^2 - 8x$, and graph this equation on their calculator. Manipulate the range so the "turns" of the graph can be seen.

II. Demonstrate how to find the minimum value of the graph in the following manner:

1. Select the CALC menu by pressing 2nd then TRACE.
2. To find the minimum, select 3 from the CALC menu.
3. The graph will be displayed, and will be asked to enter a lower bound.
4. Using the arrow keys, move the cursor so the lower bound is approximately $x=-3$, $y=-12$, and press ENTER.
5. Using the arrow keys again, move the cursor so the upper bound is approximately $x=4$, $y=16$, and press ENTER.

6. The calculator then asks for a guess. This helps the calculator find the correct root, and to find it more quickly. Use the arrow keys again and move the cursor to a point that is near the relative minimum, and press ENTER.
7. The result cursor is on the solution and the coordinate values are displayed.

III. Demonstrate finding the maximum following the same steps in II, only this time choosing maximum instead of minimum.

IV. Verify the validity of these calculations by finding the relative maximum and minimum by hand. (Relative minimum of -12 at $x=2$, relative maximum of 6.5185 at $x=-4/3$)

V. Another method to demonstrate the relationship between the derivative and the tangent line is to graph the tangent at a point on the graph. Use the same equation in I, and demonstrate the following:

1. Graph the equation again, making sure the "turns" are visible.
2. Select the DRAW menu by pressing 2nd, then PRGM.
3. Choose 5, then ENTER to draw a tangent line.
4. Using the arrow keys, move the cursor so the x-coordinate is 2, and press ENTER.
5. This result shows a horizontal tangent line at $x=2$. This would be a good time to reinforce the idea of why the tangent should be horizontal at this point.
6. Use this method to show the tangent line at other points on the graph.

VI. Here is an alternative method to drawing the tangent line to a graph as well:

1. Make sure that an equation is entered in Y_1 .
2. Select the DRAW menu, and 5 for Tangent line.
3. To draw a tangent line to the equation stored in Y_1 , select the Y-VARS menu, then 1 for function, and then 1 and ENTER to select Y_1 .
4. Type a comma, and then an x-coordinate to draw the tangent line at that point.
5. Type a $)$, then press ENTER.

6. The graph of the equation in Y_1 and the tangent line at the x -coordinate that was entered will be displayed.

VII. Demonstrate how to find the value of the derivative at a given point by the following process:

1. Enter an equation for Y_1 (The equation from the previous examples is fine).
2. Select the CALC menu, and 6 for dy/dx .
3. The current graph is displayed. Move the cursor with the arrow keys to the point to find the value of the derivative, and press ENTER.
4. The value of the derivative at that point is displayed.

VII. Equations/Problems to work on:

Find the relative maximum and minimum values of the given functions using your graphing calculator. Verify your answers by computing the values by hand.

1. $y = x^2 - 5x - 2$

2. $y = 3x^4 - 4x^3$

3. $y = x^4 - 6x^2 - 3$

4. $y = x \sin x, 0 \leq x \leq 2\pi$

5. $y = \cos(3x)$

Write an equation of the line tangent to the graph at the given point. Use your calculator to find the value of the derivative at the point (the slope), and graph the equation of the line to check your results.

6. $y = x^2 - 4, (2,0)$

7. $y = x^3 - 3x + 1, (-1,3)$

8. $y = \cos x + 1, (\pi,0)$

9. $y = \tan x + x, (0,0)$

10. $y = x^4 - 4x^2 + 4, (1,1)$

AP Calculus Lesson Plan

Topic: Applied Max/Min

Overview:

This lesson focuses on the application of determining the maximum and minimum values of a function. A “standard” calculus problem is to find the maximum volume of a rectangular solid, given the dimensions or material that will be used to construct the box. This problem can be solved *without* calculus methods by using a graphing calculator, and then can be verified by using calculus.

Objective(s):

-the student will solve a maximization problem using a graphing calculator

Procedure(s):

I. Present the problem.

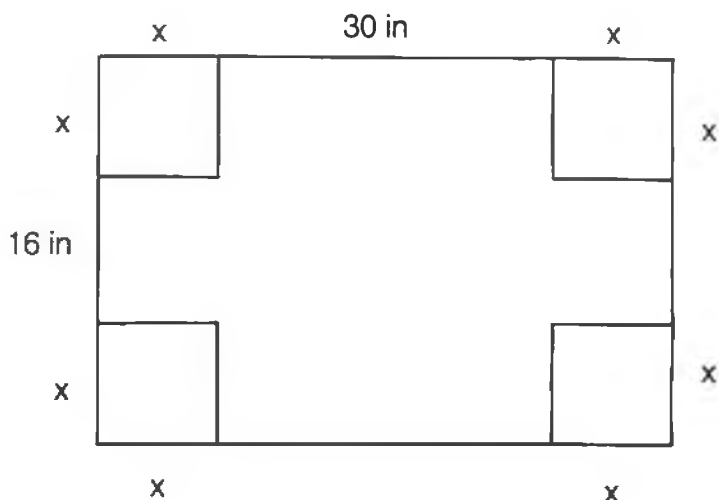
An open-topped box with a rectangular base is to be constructed from a sheet of cardboard that is 16 inches by 30 inches long by cutting squares of equal sides and folding the sides upward. What size squares should be cut from the corners in order to maximize the volume of the box?

II. Establish parameters for the problem, and determine an equation.

What (theoretically) would be the largest and smallest squares that could be cut from the corners?

What would the equation be for the volume of the box, considering that rectangular volume is the product of the length, width, and height?

(The following page shows a diagram of this problem)



Equation: Volume = $x(30 - 2x)(16 - 2x)$

Bounds: $0 \leq x \leq 8$

IV. Set up calculator to graph the equation:

Enter $Y_1 = x(30 - 2x)(16 - 2x)$

Window: $0 \leq x \leq 9.4$ with scale of 1, $0 \leq y \leq 940$ with a scale of 100
(this is a "user friendly" window)

V. Graph the equation and then use the TRACE to find the highest point on the graph. This point represents the highest or maximum volume (y coordinate), the x -coordinate represents the size of the square that must be cut from the corners of the box. It might also help to use the ZOOM to get a more precise answer.

VI. After finding a "calculator" answer, verify by finding the derivative, finding the critical points, and determining the maximum value. The calculus answer is:

$$x = 10/3, \quad \text{volume} = 19600/27$$

How close does the calculator come to this answer?

VI. Here are some more problems that can be solved using the same method. It is important that students verify their answer using calculus.

1. A sheet of cardboard 12 inches square is used to make an open box by cutting squares of equal size from the four corners and folding up the sides. What size squares should be cut to obtain a box with largest possible volume? (2 in. sq.)
2. An open box is to be made from a 3 ft. by 8 ft. rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume the box can have.
3. A rectangular field is to be bounded by a fence on three sides and a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fence.
4. A rectangular plot of land is to be fenced in using 2 kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining sides will use fencing selling for \$2 a foot. What are the dimensions of the plot of greatest area that can be fenced in at a cost of \$6000?

AP Calculus Lesson Plan

Topic: Newton's Method

Overview:

In the process of solving a problem or setting up a mathematical model, an equation in the form of $f(x) = 0$, and the solutions or zeros of the function are needed in order to solve the problem. Sometimes this process is easier said than done, and it is necessary to approximate solutions instead. Newton's Method is used to approximate the solution or solutions to an equation in the form $f(x) = 0$. The definition or approximation scheme for Newton's Method is as follows:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Objective(s):

-the student will use Newton's Method to find the zero(s) of a function

Procedure(s):

I. Demonstrate how to use Newton's Method without using the program

- Approximate the cube root of 5 by solving $x^3 - 5 = 0$.
- Approximate the solutions of $x^3 - x + 1 = 0$.

II. Give students the following program to enter:

Program Name: NEWTON

Disp "X0"

Input X

Lbl 1

X - Y₁ / Y₂ → X

Disp "X"

Disp X

Pause

```

If abs Y1 < 1E - 11
Goto 2
Goto 1
Lbl 2
Disp "X="
Disp X
Disp "Y="
Disp Y1
Y2 → D

```

III. Demonstrate how to use the program in the following manner:

Solve the equation $x^3 - x + 1 = 0$

1. Enter the equation $x^3 - x + 1$ in Y1.
2. Enter the derivative of the equation, $3x^2 - 1$ in Y2.
3. Select NEWTON from the program menu to execute the program.
4. The user must then choose a value for x_0 . This value should be a close to the solution as possible (Use -1 in this case).
5. As the user presses ENTER again and again, the approximations (x) get more and more exact, until a difference of .00000000001 is achieved.

In order to use this program for different functions, enter the equation in Y1 and its derivative in Y2.

IV. Have the students try these examples as well:

Find the roots of the following equations using Newton's Method:

1. $f(x) = x^4 - 5$
2. $f(x) = x^3 + x^2 + 3$

Notes:

It is known that Newton's Method will not always converge to a zero of the function. If this occurs in the program, stop and restart with a different initial guess. It would be good practice to get the initial guess from the graph of the function.

AP Calculus Lesson Plan

Topic: Riemann Sums

Overview:

The following is a program that can be utilized to draw a graph of a function and a Riemann sum approximation of its definite integral. The graph is the set of rectangles that make up the Riemann sum. The points t_j in the interval $[x_{j-1}, x_j]$ can be chosen to be the left endpoints (type 1), the right endpoints (type 2), the midpoints (type 3), or random points (type 4).

Objective(s):

-the student will evaluate a definite integral using Riemann sums

Procedure(s):

Program Name: RSUMS	If W=3	Lbl 4
ClrDraw	Goto 3	A+(J-1)*H→X
Disp "A"	If W=4	X→P
Input A	Lbl 1	X+H→Q
Disp "B"	A + (J-1)*H→X	H*Rand + X→X
Input B	X→P	Goto 5
Disp "N"	X+H→Q	Lbl 5
Input N	Goto 5	Y ₁ →F
Disp "POINT TYPE"	Lbl 2	F+R→R
Input W	A+J*H→X	Line(P,0,P,F)
(B-A)/N→H	X-H→P	Line(P,F,Q,F)
0→R	X→Q	Line(Q,F,Q,0)
1→J	Goto 5	IS > (J,N)
Lbl 0	Lbl 3	Goto 0
If W=1	A+J*H-H/2→X	Pause
Goto 1	X-H/2→P	H*R→R
If W=2	X+H/2→Q	Disp "RS="
Goto 2	Goto 5	Disp R

I. How to use the program.

Example: Evaluate the definite integral of $f(x) = \sin x$ on the interval $[0, \pi/2]$.

1. The function to be evaluated must be placed in Y1, in this case, the expression $\sin x$.
2. Set the Window size. Since this graph is a trigonometric graph, set the window so x goes from 0 to $\pi/2$, and the y goes from 0 to 1.
3. When executing the program you will be asked to enter the endpoints A and B (0 and $\pi/2$ for this example), and the number of points N. This will be followed by a prompt to enter the point type: 1 for left endpoints, 2 for right endpoints, 3 for midpoints, and 4 for random points. The graph is held on the screen until ENTER is pressed and the value of the sum is then displayed.

Notes:

- To change functions, the user must change the equation in Y1.
- Set the window size before executing the program.

III. Other examples to use:

Use midpoints with $n=6$ to approximate the integral:

$$1. \int_1^2 1/x \, dx$$

$$2. \int_0^{\pi} \cos x \, dx$$

Approximate the integral to five decimal places using Riemann sums:

$$1. \int_0^{\pi} x \sin 2x \, dx$$

$$2. \int_{-1}^3 (x^2 + 1)^{2/3} \, dx$$

AP Calculus Lesson Plan

Topic: Finding Areas by Integration

Overview:

In this lesson, the student will be shown how to find the area of a region that is bounded by 2 curves. In this activity, the graph of the 2 curves will be shown, and the region between the curves will be shaded. This helps give the student a good visual idea of what is being computed.

Objective(s):

-the student will find the area of a region bounded by 2 curves.

Procedure(s):

I. The problem:

Find the area of the region R bounded by the graphs of the equations $y = -x^2 + 5x - 6$ and $y = -x - 4$ between $x = 0$ and $x = 6$.

II. Activity:

1. Enter the equations in Y1 and Y2 and select graph. Students should determine if $Y1 > Y2$ or if $Y2 > Y1$.
2. Have the students find the points of intersection of the 2 curves by selecting intersection from the Calc menu (0 and 6 in the problem).
3. Have students shade the area to be calculated by selecting Shade from the Draw menu. Shade uses one of these formats:

Shade(lower function,upper function)

Shade(lower function, upper function, resolution)

Shade(lower function, upper function, resolution, X left)

Shade(lower function, upper function, resolution, X left, X right)

For this problem `Shade(Y2,Y1,1,0,6)` will show the graph needed.

4. Now, to calculate the integral, select fnInt from the Math menu.

$\text{fnInt}(Y1-Y2, X, 0, 6)$ calculates this integral.

5. To change the limits on the integral, or other parts of the problem, follow the format for fnInt as shown below:

$\text{fnInt}(\text{expression}, \text{variable}, \text{lower limit}, \text{upper limit})$

III. Other examples to demonstrate or have students compute:

Sketch the region bounded by the graphs of the given functions and then calculate the area of the region:

1. $y = 4 - x^2$, $y = x - 2$

2. $y = x^2$, $y = x^3$

3. $y = x^3$, $y = x$

4. $y = \sin x$, $y = \cos x$, $0 \leq x \leq \pi/4$

CHAPTER IV

SUMMARY

The graphing calculator can be a powerful tool for learning mathematics at all levels, not just at the highest level of the high school mathematics curriculum. The future of teaching mathematics seems to be heading towards utilizing this technology and other types of technology that are quickly becoming available. These tools not only help the teacher in terms of using less time, but also give students a visual sense of numbers, graphing, and other mathematical concepts. The author strongly suggests that schools examine the methods in which they provide mathematics instruction for their students and attempt to adopt the use of the graphing calculator in all levels of mathematics instruction.

It has been the intent of the author to provide meaningful and interesting ways for the classroom teacher to integrate the graphing calculator in the calculus classroom, and hopefully furnish teachers with thoughts and ideas to help improve their teaching and the education of their students.

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