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Revelation of Nano Topology in Cech Rough Closure Spaces

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Revelation of Nano Topology in Čech Rough Closure Space

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Preliminaries

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Čech Rough Closure Space

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Continuity in Rough Closure Space

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Nano Topology in Čech Rough Closure Space

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References

Definition 1. A function $\mathcal{C} : P(X) \longrightarrow P(X)$, where $P(X)$ is a power set of a set X, is called a Čech closure operator for X provided the following conditions are satisfied:

(i) $\mathcal{C}(\emptyset) = \emptyset$

(ii) $A \subset \mathcal{C}(A)$ for each $A \subset X$

(iii) $\mathcal{C}(A \cup B) = \mathcal{C}(A) \cup \mathcal{C}(B)$ for each $A, B \subset X$.

Then the pair $(X, \mathcal{C})$, where X is a non-empty set and $\mathcal{C}$ is a Čech closure operator for X, is called a Čech closure space. If $(X, \mathcal{C})$ is a Čech closure space and $A \subset X$, then $\mathcal{C}(A)$ is called the closure of A in $(X, \mathcal{C})$. The Čech closure space $(X, \mathcal{C})$ is said to be Kuratowski(topological) space, if $\mathcal{C}(\mathcal{C}(A)) = \mathcal{C}(A)$ for each $A \subset X$.

Definition 2. Let U be a non empty set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements that belong to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respects to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 3. If (U, R) is an approximation space and $X \subseteq U$ is said to be a rough set (inexact) with respect to R if $B_R(X) = U_R(X) - L_R(X) \neq \emptyset$, that is, $U_R(X) \neq L_R(X)$. Otherwise the set X is said to be crisp (exact) with respect to R.

Definition 4. If (U, R) is an approximation space and $X, Y \subseteq U$. Then the set X is said to be rough subset of Y with respect to R if $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$. Note that every subset X of a rough set Y is a rough subset of Y.

Definition 5. Let $P(X)$ be the power set of a rough set X in the approximation space (U, R). A function $C : P(X) \to P(X)$ is called a Čech rough closure (simply, rough closure) operator for X if it satisfy the following conditions:

(i) $C(\emptyset) = \emptyset$

(ii) $A \subset C(A)$ for each $A \subset X$

(iii) $C(A \cup B) = C(A) \cup C(B)$ for each $A, B \subset X$.

Definition 6. A rough closure operator C is said to be finer than a rough closure operator C_1 on the same rough set X (or C_1 is coarser than C) if $C(A) \subset C_1(A)$ for each $A \subset X$ and it is denoted as $C > C_1$.

Remark 7. Let $P(X)$ be the power set of a rough set X in the approximation space (U, R). If a function $C : P(X) \to P(X)$ defined by $C(A) = A$ for all $A \subseteq X$, then clearly C is a finest rough closure operator and a rough topological closure operator on X. It gives rough discrete topology τ_C on X. Also if we define a function $C_1 : P(X) \to P(X)$ by

$$C_1(A) = \begin{cases} \emptyset & \text{if A=}\emptyset \\ X & \text{otherwise} \end{cases}$$

Then clearly C_1 is a coarsest rough closure operator and rough topological closure operator on X. Also it gives rough indiscrete topology τ_{C_1} on X.

Remark 8. Let (X, C) be a rough closure space and $A, B \subset X$. Then the following statements are true:

(i) If $A \subset B$, then $C(A) \subset C(B)$.

(ii) $C(A \cap B) \subset C(A) \cap C(B)$.

Proof

(i) Clearly we have, $C(A) \subset C(A) \cup C(B) = C(B)$.

(ii) Since $A \cap B \subset A$ and $A \cap B \subset B$, then $C(A \cap B) \subset C(A) \cap C(B)$.

Theorem 9. Let (X, C) be a rough closure space and $A, B \subset X$. Then the collection of all rough closed sets of a rough closure space (X, C) is closed under finite unions and arbitrary intersections.

Proof By the definition of C and for finite number n, we have $C(\bigcup_1^n A_i) = C(A_1) \cup C(A_2) \cup ... \cup C(A_n) = \bigcup_1^n A_i$. Since $\bigcap_1^\infty A_i \subset A_i$ for each $i = 1, 2, ...$, then $C(\bigcap_1^\infty A_i) \subset C(A_i) = A_i$ for each $i = 1, 2, ...$, and implies that $C(\bigcap_1^\infty A_i) \subset \bigcap_1^\infty A_i$. By the definition of C, we have $C(\bigcap_1^\infty A_i) = \bigcap_1^\infty A_i$.

Remark 10. In a rough closure space (X, C), $C(A)$ need not be a rough closed.

Definition 11. Let (X, C) be a rough closure space on X. A function $Int : P(X) \to P(X)$ is defined by $Int(A) = X - C(X - A)$ and is called a Čech rough interior (simply, rough interior) operator of A in (X, C).

Definition 12. A collection $\mathcal{B}$ of subsets of (X, C_R) is called a local base of the rough neighbourhood system of a set $A \subset X$ (resp. a point x)if and only if each $B \in \mathcal{B}$ is a rough neighbourhood of A (resp. of x) and every rough neighborhood of A (resp. of x) contains a $B' \in \mathcal{B}$.

Theorem 13. Let (X, C) be a rough closure space and $A, B \subset X$. Then the collection of all rough open sets of a rough closure space (X, C) is closed under arbitrary unions and finite intersections.

Proof: Proof is trivial by using de Morgan formula and $Int(A) = X - C(X - A)$.

Remark 14. In a rough closure space (X, C), $\emptyset$ and X are rough closed as well as rough open sets.

Theorem 15. Let (X, C) be a rough closure space. Then the following are true:

(i) $Int(\emptyset) = \emptyset$ and $Int(X) = X$.

(ii) $Int(A) \subset A$ for each $A \subset X$.

(iii) $Int(A) \subset Int(B)$ for each $A \subset B \subset X$.

(iv) $Int(A \cap B) = Int(A) \cap Int(B)$ for each $A, B \subset X$.

(v) $Int(A \cup B) \supset Int(A) \cup Int(B)$ for each $A, B \subset X$.

(vi) $Int(A) = A$ if and only if A is a rough open set.

Definition 16. Let (X, C_1) and (Y, C_2) be two rough closure spaces on rough sets X and Y respectively and a function $f : X \to Y$ is said to be rough continuous at a point $x \in X$ if $f(x) \in C_2(f(A))$ for $A \subseteq X$ and $x \in C_1(A)$.

Let (X, C_1) and (Y, C_2) be two rough closure spaces on rough sets X and Y respectively and a function $f : X \to Y$ is said to be rough continuous on X if it is rough continuous at each point of X, or equivalently, if $f(C_1(A)) \subseteq C_2(f(A))$ for each $A \subseteq X$.

Theorem 17. Let (X, C_1) and (Y, C_2) be two rough closure spaces on rough sets X and Y respectively and a function $f : X \to Y$ is said to be rough continuous on X if and only if $C_1(f^{-1}(B)) \subseteq f^{-1}(C_2(B))$ for every $B \subseteq Y$.

Proof. Assume $f : X \to Y$ is a rough continuous on X and take $A = f^{-1}(B)$ for every $B \subseteq Y$. Then $f(C_1(A)) \subseteq C_2(B)$ and implies $C_1(A) \subseteq f^{-1}(C_2(B))$. Therefore, $C_1(f^{-1}(B)) \subseteq f^{-1}(C_2(B))$ for every $B \subseteq Y$. Conversely, assume $C_1(f^{-1}(B)) \subseteq f^{-1}(C_2(B))$, for every $B \subseteq Y$. If we take $B = f(A)$ and $A_1 = f^{-1}(B) \supseteq A$ for $A \subseteq X$. Then we have $C_1(A_1) \subseteq f^{-1}(C_2(B))$ and implies that $f(C_1(A_1)) \subseteq C_2(B) = C_2(f(A))$. Since $A \subseteq A_1$, $f(C_1(A)) \subseteq f(C_1(A_1)) \subseteq C_2(f(A))$ for every $A \subseteq X$.

Theorem 18. Let (X, C_1) and (Y, C_2) be two rough closure spaces on rough sets X and Y respectively. If a function $f : X \to Y$ is rough continuous on X then the inverse image of every rough open(resp. closed) set in Y is rough open (resp. closed) set in X.

Theorem 19. Let (X, C_1), (Y, C_2) and (Z, C_3) be three rough closure spaces on rough sets X, Y and Z respectively and if functions $f : X \to Y$ and $g : Y \to Z$ are rough continuous on X and Y respectively, then $g \circ f : X \to Z$ is a rough continuous function.

Definition 20. Let (X, C) be a rough closure space and $A \subseteq X$. Then $int(A)$ is defined as the union of all rough open sets of (X, C) contained in A, that is $int(A) = \cup\{G \subseteq X : G \in \tau_C$ and $G \subseteq A\}$ and $cl(A)$ is defined as the intersection of all rough closed sets of (X, C) containing A in (X, C), that is $cl(A) = \cap\{F \subseteq X : F \in \tau_C'$ and $A \subseteq F\}$ and $Bd(A) = cl(A) - int(A)$ is called the boundary of A in (X, C).

Definition 21. Let (X, C) be a rough closure space, $A \subseteq X$, then the collection $\tau_{NC}(A) = \{X, \emptyset, int(A), cl(A), Bd(A)\}$ satisfies the following axioms:

(i) X and $\emptyset$ are in $\tau_{NC}(A)$.

(ii) The union of the elements of any sub-collection of $\tau_{NC}(A)$ is in $\tau_{NC}(A)$.

(iii) The intersection of the elements of any sub-collection of $\tau_{NC}(A)$ is in $\tau_{NC}(A)$.

That is, $\tau_{NC}(A)$ forms a topology on rough closure space (X, C) called the nano Čech topology on X w.r.t A. We call $(X, \tau_{NC}(A))$ as the nano Čech topological space. The elements of $\tau_{NC}(A)$ are called nano Čech-open sets and a set is said to be nano Čech-closed, if its complement is nano Čech-open.

Remark 22. Let (X, C) be a rough closure space and $A \subseteq X$.

(i) If $int(A) = \emptyset$ and $cl(A) = X$, then $\tau_{NC}(A) = \{X, \emptyset\}$, the indiscrete nano Čech topology on (X, C).

(ii) If $int(A) = cl(A) = A$, then the nano Čech topology, $\tau_{NC}(A) = \{X, \emptyset, int(A)\}$.

(iii) If $int(A) = \emptyset$ and $cl(A) \neq X$, then $\tau_{NC}(A) = \{X, \emptyset, cl(A)\}$.

(iv) If $int(A) \neq \emptyset$ and $cl(A) = X$, then $\tau_{NC}(A) = \{X, \emptyset, int(A), Bd(A)\}$.

(v) If $int(A) \neq cl(A)$ where $int(A) \neq \emptyset$ and $cl(A) \neq X$, then $\tau_{NC}(A) = \{X, \emptyset, int(A), cl(A), Bd(A)\}$ is the discrete nano Čech topology on (X, C).

Theorem 23. If $(X, \tau_{NC}(A))$ is a nano Čech topological space, $x \in \tau_{NC}\text{-}cl(S)$ if and only if $G \cap S \neq \emptyset$ for every nano Čech-open set G containing x, where $S \subseteq X$.

Theorem 24. If $(X, \tau_{NC}(A))$ is a nano Čech topological space and $S \subseteq X$, then

(i) $\tau_{NC}\text{-}cl(S) = S$ if and only if S is nano Čech-closed set.

(ii) $\tau_{NC}\text{-}int(S) = S$ if and only if S is nano Čech-open set.

Theorem 25. If $(X, \tau_{NC}(A))$ is a nano Čech topological space, then

(i) τ_{NC}-$cl(int(A)) = [Bd(A)]^c$.

(ii) τ_{NC}-$cl(cl(A)) = X$.

(iii) τ_{NC}-$cl(Bd(A)) = [int(A)]^c$.

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THANK YOU