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# Examination of Energy and Group Velocities in Positive and Negative Index Chiral Materials with and without Dispersion

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**Abstract:** Concepts of energy and group velocities, Poynting and propagation vectors are examined for both positive and negative index materials. Known definitions for these entities are explored in terms of the interplay of chirality and dispersion. It is shown that in isotropic materials, energy and group velocities are identical, and negative index may be realized via topological chirality and dispersion.

## 1. Background and Motivation

The phenomenon of negative refraction has drawn considerable attention in recent years. The effect is shown to manifest itself in a variety of materials (including those classified as metamaterials). However, the determination of negative index is often based on the mutual relationships between several known propagation characteristics associated with electromagnetic waves, such as energy velocity, group velocity, phase velocity, the wave vector and the Poynting vector. Due to the variability in such approaches, a consistent set of rules does not usually emerge upon perusal of the literature. Recently, in an attempt to establish the condition required to ensure negative refractive index in a chiral material, field solutions were obtained for (essentially) TEM propagation in unbounded chiral media [1]. The result, based on the requirement that the Poynting energy flow be contra-directional relative to the wave vector, yielded a condition for the magnitude of the chirality parameter ( $\kappa$ ) that agreed with that reported in the literature. In this work, the electromagnetic laws are revisited in order to examine more closely the definitions of energy and group velocities, with particular attention to chiral materials. Thereafter, the dispersive and chiral properties necessary to generate positive index behavior, and the more unusual negative index behavior, are scrutinized in some detail.

## 2. Mathematical Approach

In the simplest case, we assume an isotropic, non-chiral and unbounded material in which the energy and group velocities are defined as follows:

$$v_e = \frac{\langle S \rangle_{av}}{\langle w \rangle_{av}}, \quad \text{and} \quad v_g = \left( \frac{\partial \omega}{\partial \beta} \right)_{\omega_0}, \quad (1)$$

where  $\langle S \rangle_{av}$  and  $\langle w \rangle_{av}$  are the time-average Poynting vector and total energy density,  $\omega_0$  is the carrier, and  $v_e$  and  $v_g$  are energy and group velocities respectively. After showing straightforwardly that the above relations for a non-chiral material are actually equivalent, we

then proceed to re-examine the relations for a chiral material. In this case, the previously known field solutions [1] are used to derive the velocity relations. The results are interpreted in terms of dispersion conditions in order to demonstrate positive or negative index behavior. We surmise that combining chirality with topological dispersion may yield an environment where the high-magnitude chirality parameter needed for realizing negative index behavior is achieved extrinsically.

### **3. References**

[1] M.R. Chatterjee and P.P. Banerjee, "Investigation of negative refractive index in reciprocal chiral materials," in Proceedings of SPIE, Katherine Creath, ed., Vol. 6285 (SPIE, San Diego, CA, 2006), no. 628504.

[2] A. Bers, "Note on Group velocity and energy propagation," Am. J. Phys. **68**, 482-484 (2000).