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University of Dayton. Department of Mathematics

Abstracts of the Colloquium Talks: Fall 2006

Department of Mathematics

Convex Independence in Multipartite Tournaments

Dr. Darren Parker

Abstract: During the past 5 years, R. Westhoff, M. Wolf, and I have been developing tools to help study two-path convexity in multipartite tournaments. In particular, we have studied various convex invariants, such as rank, Helly number, and Radon number. While our initial results merely gave upper bounds for these parameters, out later work has determined the relationships between these invariants. In particular, we determine when the Helly number, Radon number, and rank are all equal. To an extent, we have done this for general multipartite tournaments. This talk will discuss our results for clone-free multipartite tournaments.

A Survey of Results on Transitive Subtournaments

Dr. Arthur Busch

Abstract: A tournament is an oriented complete graph. The score sequence of a tournament is the sequence of out-degrees of the vertices, usually in non-decreasing order. A tournament is transitive if the arc set induces a total ordering on the vertices. Erdos and Moser showed that every tournament of order n contains a transitive subtournament of order $log(n) + 1$, and using a nice probabilistic argument, that for each n there is some tournament of order n which does not contain any transitive subtournament of order $2log(n) + 1$. We will generalize this problem by asking similar questions about the set of all tournaments with a given score sequence. We also will discuss problems relating to tournaments that admit a partition of the vertex set into 2 subsets, each of which induces a transitive subtournament.

Stochastic Valuation of Mortgage Servicing Rights

Dr. Vikas Gupta

Abstract: Mortgage servicing rights (MSR) have historically been valued with a static interest rate environment assumption. Whereas, IO (Interest only strips, similar in nature to MSR) trade under a stochastic based OAS approach. Recently, some large market participants have gravitated towards a stochastic valuation approach in valuation of their MSR portfolios.

National City Mortgage, with a $160 Billion servicing portfolio, is evaluating the merits of stochastic based valuation of this portfolio. This valuation framework requires a robust term structure model (which forecast future interest rates based on current market conditions: yield curve, volatility, etc). The LIBOR market Model (Rebanato, 2002) provides a state of the art framework for a term structure model and is being evaluated by National City Mortgage.

This talk describes the mechanics of stochastic valuation of MSRs, and discusses the pros and cons of the approach. Implications of this choice to the risk management (hedging) of the MSR asset are explained. Challenges in calibrating, validating, and implementing the LMM model are also discussed.

Vikas Gupta is Senior Vice President for National City Mortgage responsible for MSR research and performance. In his position Vikas overseas prepayment analytics of National City's retained servicing portfolio, prepayment modeling, and valuation methodology development.

Before joining National City Mortgage, Vikas held positions in Secondary Marketing and Servicing Valuation for M&T Bank. Vikas has a Ph.D. in Materials Science from Vanderbilt University and an MBA from the University of Michigan's Ross School of Business.

Option Pricing for An Inhomogeneous Stochastic Differential Equation

Dr. Ruihua Liu

Abstract: In this work we studied an option pricing problem where the underlying asset follows an inhomogeneous stochastic differential equation that is different from the usual geometric Brownian motion (GBM) model for Black-Scholes formula. For the inhomogeneous model, exact analytical formula for option price does not exist. So we studied approximate methods instead. We compared several approaches including finite difference for PDE, approximate Fourier Transform and Edgeworth expansion. We developed a new method based on Taylor series. It has been demonstrated that the new method outperforms the other approaches.

Fractional q-Calculus on a Time Scale

Dr. Paul Eloe

The study of fractional q-calculus in this paper serves as a bridge between the fractional q-calculus in the literature and the fractional q-calculus on a time scale

$$T = q^{N0} \cup \{0\} with\ 0 < q < 1$$

By use of time scale calculus notation, we find the proof of many results more straight forward. We develop some properties of q-laplace transform. Then we use these properties to solve fractional q −difference equations.

Experience Studies and Applications at Union Central Life Insurance Company

David Harrison

Abstract: Annuities are a financial instrument wherein the owner pays a fixed stream of cash flows over a fixed amount of time into an account that pays a fixed interest rate. We are able to compute the value of annuities since the value of any financial instrument is the future value of all of its cash flows and this calculation is taught in many introduction level finance courses.

Life insurance is another financial instrument similar to an annuity in that a fixed amount – called a premium – is paid at regular intervals over the life of the insurance contract. However, unlike an annuity, the amount paid to the beneficiary is paid at the time of death, which is unpredictable. Then, certain assumptions are made based on how long a person is expected to live. These assumptions are called mortality assumptions which are broken down by age group and help insurance companies determine fair premiums to charge for policies. The Society of Actuaries compiles tables that give the mortality probability for every age group and the Union Central Life Insurance company uses tables compiled from data collected between 1975 and 1980 for the majority of its policies. This table is called the 1975-1980 Basic Table, or the 75/80 Table.

These tables are expectations and do not reflect exactly how long the American population will live due to the probabilistic nature of ones life expectancy as well as, in most part, to the improvement of

mortality over the years. Companies therefore check these mortality assumptions to make sure insurance policies are priced correctly. This check is a study done over all policies called the experience study. The results of the experience study are used in many facets of the insurance industry including pricing policies, financial analysis, dividend payment and other simulations.

Adaptations of "Smart Money & Naïve Speculators"
Bridget Hilgeford
Abstract: In his paper, "Smart Money and Naïve Speculators," F. Albert Wang explores a model of speculative trading involving both noise traders and informed investors with emphasis on an informed investor's strategies to exploit typical noise trading. By adjusting one of the initial assumptions within this model and recalculating the mathematical results, one is able to thoroughly investigate the structure of the model. The implications of the new results over time can then be seen graphically and analyzed for their effect on the trading parties involved. As an extension of the scope of the paper, I have developed a computer program within VBA to supply a more immediate means of analyzing the model and its graphical results with regard to changing initial conditions. The program and its mechanics will also be discussed.

Pricing American options with Monte Carlo Simulation
Fatima Bousso
Abstract: American option has been an active area in financial mathematics. Valuing American options is an extremely challenging problem. The difficulty in pricing American options stems from the possibility of early exercise, and the early exercise decision must be determined as part of the solution. It has been shown that in general a closed-form formula for American option prices does not exist. Therefore, much efforts have been concentrated on approximate methods. In this project, I implemented two numerical methods based on Monte Carlo simulation. The first one combines simulation with a tree and the second one searches for the optimal exercise prices using the Newton's method. I used the two methods to value American Put options on non-dividend paying stocks and compared the methods with the Implicit and Explicit Finite difference methods for solving the PDE, and with an analytical approximation method proposed by Ju. The programming was done using Matlab and VBA.

Information Content in the Disparity of Call and Put Options Implied Volatilities
Brook Bisrat
Abstract: Implied volatility, based upon an option pricing model, generates a theoretical value for an option that is equal to its current market price. The factors that affect an option's implied volatility are exercise price, rate of return, maturity date and the price of the option. Using the Black-Scholes option pricing model and Chicago Board of Trade commodity options data, the implied volatility can be derived.

The implied volatility of call options derived based upon Black Scholes option pricing model is usually different than that of put options because of the non-lognormal distribution of underlying assets' prices. If the implied volatility difference between these call and put options is due to the market's outlook on the distribution of the future underlying asset prices, it should contain information about future price movements.

The results of this study provide proof of the relationship between the implied volatility difference and future price changes. These results suggest that implied volatiles are not only relevant to future realized volatilities, but also related to the first moment of the underlying asset price distribution. Academia will find the results useful to better their understanding of the relation between derivatives and underlying

assets' markets. On the other hand traders can take advantage of the information content in the implied volatility differences to improve upon or create new trading strategies.