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Enhancing Manufacturing Planning and Control Systems Through Artificial Intelligence Techniques

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Chapter 7

Enhancing manufacturing planning and control systems with artificial intelligence techniques

Ronald Dattero, John J. Kanet and Edna M. White

Abstract Manufacturing planning and control systems are currently dominated by systems based upon Material Requirements Planning (MRP). MRP systems have a number of fundamental flaws. A potential alternative to MRP systems is suggested after research into the economic batch scheduling problem. Based on the ideas of economic batch scheduling, and enhanced through artificial intelligence techniques, an alternative approach to manufacturing planning and control is developed. A framework for future research on this alternative to MRP is presented.

Introduction

American industry wastes billions of dollars each year because of inadequate procedures for controlling inventory and production. It could be argued that a good deal of this waste is attributable to the manner in which computers are used (or perhaps misused) in production and inventory control. Certainly the benefits of computers are not being fully realized; 'Most currently available software systems address only a portion of the overall control problem' (Maxwell *et al.*, 1983).

Over the past 20 years, large manufacturing firms have switched from traditional reorder point systems (usually based on the Economic Order Quantity) to computerized Material Requirements Planning (MRP) systems. The American Production and Inventory Control Society is the major force behind the MRP movement (Krajewski and Ritzman, 1987) with Orlicky (1975), Plossl (1973), and Wight (1974) spearheading it. In fact, Orlicky (1975) has gone as far as to call MRP 'the new way of life in manufacturing'.

Unfortunately, MRP has not succeeded in solving all of manufacturing's problems. It has been said that MRP systems provide 'necessary but incomplete planning information to managers' as 'the full benefits of computer-based systems for planning production are yet to be realized' (Maxwell *et al.*, 1983).

This chapter argues that the major reason MRP is not the 'way of life' is that MRP systems were developed to operate under the third generation computer environment of the late 1960s and early 1970s. Naturally, MRP systems, as developed, cannot take full advantage of the computer capabilities presently available. Today, computers operate at least 100 times faster than their third generation counterparts. Fifth generation computers (which are likely to be fully developed in the next few years) are expected to operate at speeds at least an additional 100 times faster. Fifth generation computers are also expected to incorporate parallel processing, supporting even more extensive and sophisticated systems.

MRP systems have a number of inherent weaknesses that reduce production performance, and will be described and assessed in the next section. Following this, the ideas of economic batch scheduling, which provide a basis for an alternative to MRP systems, will be presented. A framework for future research on this alternative to MRP is then presented.

MRP systems

A typical manufacturing planning and control (MPC) system consists of three parts: front end, engine, and back end (Vollmann *et al.*, 1984). The front end is the set of activities and systems for overall direction setting, such as demand planning, production planning, and the master production schedule (MPS). The engine is the set of systems for accomplishing the detailed material and capacity planning such as MRP, detailed capacity planning, and material and capacity plans. The back end is the set of execution systems such as shop-floor control systems and purchasing systems. These manufacturing planning and control systems are often simply referred to as MRP systems.

A typical MRP system is illustrated in Figure 7.1. As the figure shows, the system takes a schedule of marketing requirements as a major input and produces two major outputs: a schedule of planned manufacturing orders, and a set of order release prompts to the shop floor and to purchasing.

The MRP system divides the manufacturing task into subtasks such as master scheduling, shop floor control, and inventory planning. Subtasks that are fairly standard, in the installations we have seen, are denoted with solid boxes in Figure 1. Other subtasks (modules) such as maintenance planning, purchasing, and tool planning are often present as well. A rough-cut capacity planning module, used to aid the development of master schedules, is also available in many implementations. For example, IBM's software package MAPICS (1985) includes such a module.

Fundamental weaknesses of MRP systems

> 'Traditional MRP has offered little more than a computerized method of keeping voluminous records on material, and the resulting resource

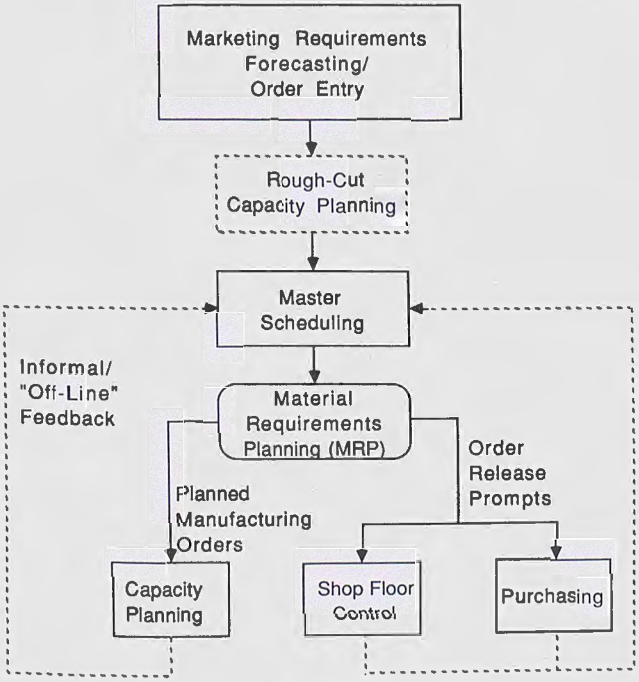

Figure 7.1 MRP system architecture

> requirements. There has never been an attempt, in any but the most superficial way, to account for the actual resource capacity in production planning and control. It has always been handled in an iterative, ad hoc, manual fashion. The manual approach is often a frustrating and impossible task'. Gershwin *et al.* (1984).

As the previous quotation states, MRP systems suffer from a number of inherent weaknesses. The focus here is on two major weaknesses. First, MRP systems frequently do not include capacity planning in their scheduling, and when capacity is considered, only rough-cut capacity planning or infinite capacity assumptions are used. Second, in MRP systems, a simplifying assumption is made that production lead time is constant. These weaknesses are discussed in more detail below.

In MRP systems, the lot size decision is made independently of machine capacity and order sequencing. Orders are sequenced into the shop scheduling system based upon planned, constant lead times. The sequence through the shop is controlled by the shop scheduling system. There exists little formal protocol governing the format of feedback whenever the material plan causes a capacity or sequencing problem. The type of feedback that does exist is informal. The MRP system first plans materials and then imposes this plan on capacity planning and sequencing modules. Capacity planning is done by projecting the load pattern that the material plan imposes on the factory. Consequently, resulting machine load reports can be quite misleading, and their value as a planning tool is significantly impaired.

In MRP systems, the effects of order sequencing are simply not considered in the material planning step; planned lead times are viewed as static parameters based upon historical average order flow times (or even guesses about flow times). This static view can lead to wasteful overplanning of material. For example, consider two manufactured parts that differ only slightly in their design, and thus have almost the same processing time and the same routing through production. Suppose 100 units of each part have the same due date. If both parts cannot be produced at exactly the same time (which is often the case), some sequencing decisions must be made.By not acknowledging the sequencing, the MRP system forces material to be available for both orders early enough to allow either order to be produced first. In other words, lead times are assumed to be constant at any point in time, whereas in reality they vary (sometimes dramatically) according to the current load on the plant. When many orders are involved, the problem is greatly compounded. The following section describes the ideas of economic batch scheduling, which may be an excellent starting point in overcoming these weaknesses in MRP systems.

The Economic Batch Scheduling problem

As early as 1957, researchers were reporting results on what has come to be called the 'Economic Batch Scheduling' (EBS) problem. Figure 7.2 provides a historical perspective of the evolution of research since the early work of Vazsonyi (1957) and others on this problem. We are concerned here with only a brief description of this problem; for a detailed review of this research the reader should consult Elmaghraby (1978).

The EBS problem can be briefly stated as follows: Given a set of products produced by a single machine and their forecasted demand, find a schedule of production that satisfies demand and minimizes total costs (holding costs plus setup costs). While the problem is easy to state, finding a solution to it is far from trivial. In fact, the computational difficulty of this problem can be shown to be in the NP class (Park, 1987). This may, at least partially, explain why — after a number of early research reports on this combination scheduling/lot sizing problem — the theme of most of the research which followed tended to fall into one of two major branches. In both branches, the problem was broken down into two subproblems, perhaps in an attempt to 'divide and conquer'. Unfortunately, the problem has yet to be put back together properly.

The first research direction assumed the problem to be entirely a matter of determining an economic lot size. No regard is given for the possible machine interference that might result when the economic order quantities

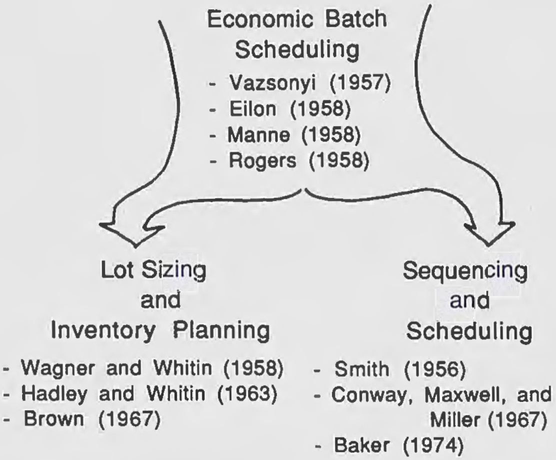

Figure 7.2 Historical division in economic batch scheduling

for each product are derived independently. This research direction is typified by the well-known paper by Wagner and Whitin (1958) and the large body of inventory literature that has since evolved (for example, Hadley and Whitin, 1963; and Brown, 1967).

The second research direction assumed that the batch sizes are given, and concentrated entirely upon the sequencing aspects of the problem. The early research of Smith (1956) is typical of the tremendous effort that has been extended on this half of the problem (for example, Conway *et al.*, 1967; and Baker, 1974).

There has been considerable success in solving the EBS problem for the single machine case (Park, 1987), but the multiple machine problem remains to be solved. Given the current productive rate of research in artificial intelligence (AI) and operations research (OR) and the nearness of fifth generation computers, it seems likely that good approaches to the EBS problem will be developed within the next few years. Due to the computational complexity of the multiple machine EBS problem, it is unlikely that optimal solutions will be possible for reasonable size problems, but good heuristic solutions seem quite likely.

The merging of ideas from AI and OR

The most promising remedy to the problems of MRP systems appears to be to return to the economic batch scheduling problem and solve it directly (Kanet and Dattero, 1986). In particular, an economic batch scheduler would be in the centre of the MPC engine rather than MRP. This economic batch scheduler would have the same capabilities as MRP in exploding the bill of materials, but the logic in scheduling and planning would be different.

Recently, there has been much optimism regarding the application of AI to issues such as manufacturing planning and control; 'joining hands with AI, management science and OR can aspire to tackle every kind of problem-solving and decision-making task the human mind confronts' (Simon, 1987).

One notable result of this collaboration between AI and OR, is the acceptance of sufficing rather than optimizing. 'Good' (sufficing) solutions to very difficult problems (even problems in the NP class) are possible through the use of heuristics and intelligent search methods. Ow and Smith (1987) have tackled difficult job-shop scheduling problems through domain-specific knowledge that supports opportunistic reasoning (that is, performing those actions which appear to be the most promising in terms of the current state) and hierarchical organization structures which control and coordinate the solution search activity.

Kusiak (1987) classifies these new scheduling ideas which originate from AI as follows: *hierarchical*, *non-hierarchical*, *script-based* (*skeleton*), *opportunistic*, and *constraint-directed*. In *hierarchical* scheduling, the overall scheduling problem is solved first at an aggregate data level and then detailed at lower (less aggregated) data levels. In *non-hierarchical* scheduling, the entire problem is solved with no problem decomposition. In *script-based* scheduling, schedule *skeletons* or templates are developed and stored in a database until needed. In *opportunistic* scheduling, the scheduling action that appears the most promising in terms of the current stage of the schedule is performed. In *constraint-directed* scheduling, constraints (such as number of machines, due dates, etc.) provide guidance and bounds in the search for 'good' schedules. An extensive survey of artificial intelligence based scheduling systems is given by Steffen (1986).

From EBS to MPC systems

Once a sufficing, if not exact, solution has been found to the EBS problem, it becomes possible to develop a computerized MPC system free of the weaknesses of MRP systems. In this section, an outline of such a system is given.

The proposed system addresses the multi-machine case where customer orders are for assembled products. The system first focuses on finding feasible solutions to the stated problem and then refining the solution. This will be achieved through a controlled computer search.

Figure 7.3 provides an overview of how the overall MPC problem might be approached, incorporating the basic ideas of economic batch scheduling. A central feature of this system is a search algorithm which takes as input a set of marketing requirements of finished products, and produces as output a 'good', feasible, 'low cost', detailed production timetable (Gantt chart) for every manufacturing resource. By feasible, we mean that all customer requirements are met without exceeding the stated capacity of any resource. By 'low cost' we mean that at least some effort is expended in determining

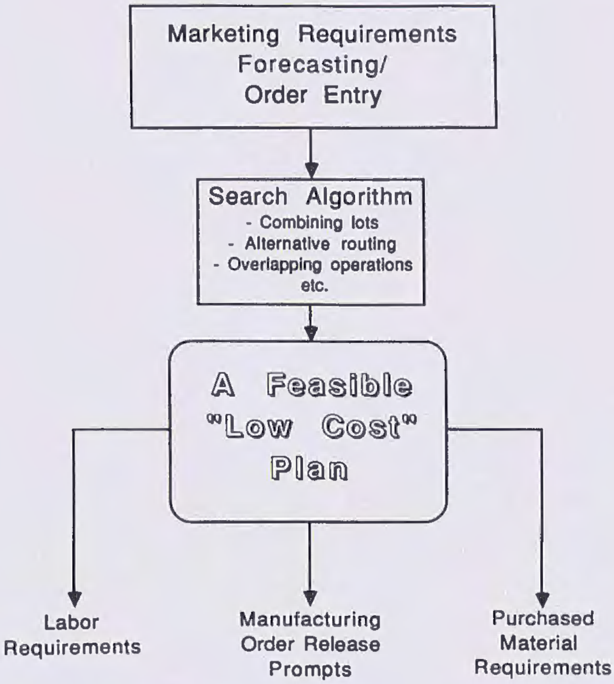

Figure 7.3 An alternative MPC system

a feasible schedule with a satisfactory cost level, though not necessarily the lowest possible cost. A 'good' timetable is measured against some user-defined objective(s); again with a satisfactory rather than an optimal solution sought. As Figure 7.3 shows, the resulting schedule would then be used to develop labour requirements, reports and manufacturing order release prompts. The production plan would also imply a schedule of purchased material requirements which would be input to a purchased materials inventory management subsystem.

Although the complete approach as described above is still on the drawing board, the basic spirit of this approach is already on the way to becoming reality. For example, the ISIS project of Westinghouse (Fox and Smith, 1984) and the PATRIARCH project at Carnegie-Mellon University (Morton, 1985) both appear to be headed in the general direction that we are suggesting here. Additionally, commercial software products which employ finite scheduling appear to be gaining acceptance, such as OPT by Creative Output, Inc. (Goldratt, 1980) and SCHEDULEX by Numetrix Ltd. (Schengili, 1986).

The use of search algorithms

A key feature of the approach proposed here is the use of a search component to arrive at a production plan. In the terminology of production and inventory control, this approach employs a finite capacity planning algorithm because it will not permit work centres to be scheduled at beyond their capacity. We envisage a two-stage search approach. The first stage of the search procedure would be deployed simply to find a feasible schedule (plan). Once a feasible plan is available, the second phase of the search procedure would be deployed to find a 'low cost' plan. Figure 7.4 illustrates our thoughts on how these search algorithms might be employed.

The proposed system draws from the expert systems model in certain aspects such as an explanation facility. For example, in searching for a feasible plan, the search algorithm might be employed for some user-defined maximum time period. Whenever the search algorithm fails to find a feasible schedule, it would report this, indicate the apparent reason, and suggest alternative courses of action. The search algorithm would take into account the possibility of combining lots, alternative routings, overtime, etc., in an attempt to find a good feasible schedule.

Similarly, the user might wish to know the consequences of a proposed change. For example, 'supposing customer A increases her order quantity from 100 to 140?' The system should respond with a set of feasible alternative strategies for accomplishing the change such as rerouting of other orders to provide the capacity required, splitting the batch size of this or a previous order that uses the same resources, and so on.

Another aspect of the expert system model incorporated into the proposed system concerns alternative choices. For example, suppose a marketing manager wishes to change the scheduled shipping date of a given order.

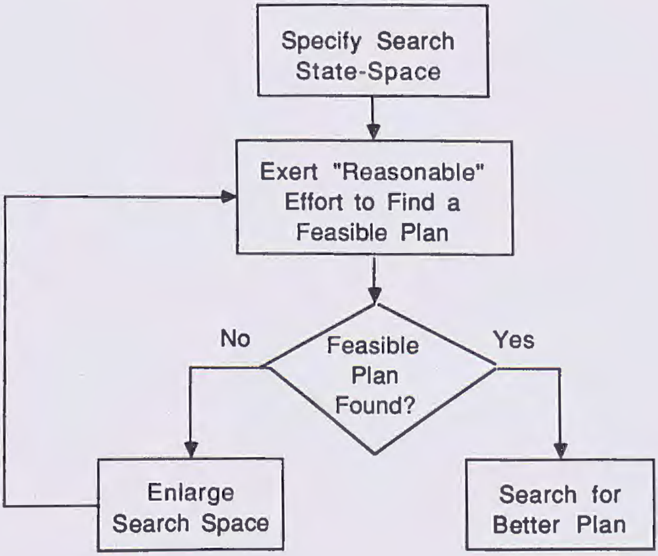

Figure 7.4 General search strategy

The search routine might first determine that there is no feasible way to accommodate this change, but the algorithm might also suggest relaxing the shipping date of some other product or scheduling overtime. The marketing and production managers would then decide how to reformulate the problem statement, and the search routine would again be deployed to find a feasible solution.

There would exist a hierarchy of ways that managers could choose to consider the problem statements. After a feasible solution is found, the second stage of the algorithm would be deployed to search for a 'good' possible solution to the current problem statement (according to some user-defined criterion). Like the previous component, this phase of the search would also be terminated after some predetermined time period. The user would then be briefed on the consequences of this proposed change, in terms of its effect on the predefined objective(s).

Overall system architecture

Figure 7.5 illustrates our thoughts on the overall architecture of the type of manufacturing planning and control system that we envisage. At the heart of the system is the current statement of the production plan. We can think of this as a database showing the detailed schedule of every manufacturing resource over the entire planning horizon. Personnel from marketing, production control, purchasing, etc., would have limited capability, through a supervisory algorithm, to query the current production plan; to make changes in the current status of the resources; to explore the ramifications of changing the production plan; and to change the production plan.

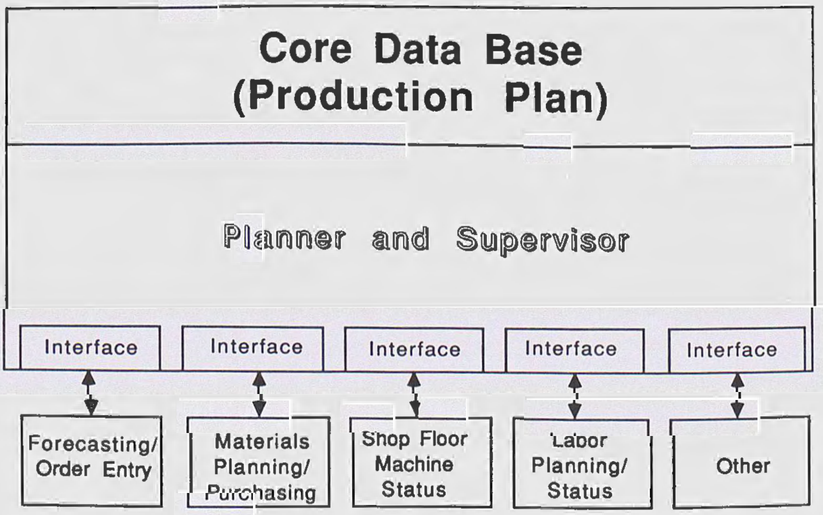

Figure 7.5 An alternative MPC system architecture

Each of the interfaces depicted in Figure 7.5 would have similar features, and would be designed with the same search methodology as described in Figure 7.4 and the discussion above. What would differ among the interface modules would be the set of alternatives available in the reformulation hierarchy. For example, the production control manager might have the option of exploring the use of alternative (possibly more costly) job routing through the factory. The marketing manager may not have such an alternative available to her; but might be the only one with the authority to decrease a marketing requirement. Nevertheless, the same reformulative two-stage search methodology would prevail at each planning interface. Other interfaces to maintenance planning, material handling, etc., would be facilitated in a similar fashion.

The contrast with MRP

Our alternative system differs conceptionally from the traditional MRP system in a number of important ways:

(1) Unlike an MRP system, it would simultaneously take into account both material and capacity, in attempting to find a feasible plan. Lot sizing and sequencing would be done concurrently.
(2) Unlike an MRP system, it would either find a feasible manufacturing plan or interact with the user to determine the next course of action.
(3) Unlike an MRP system, it would not only search for a feasible plan, but would also exert reasonable effort to find a 'good' manufacturing plan.
(4) Unlike an MRP system, it would provide a formal set of computer-aided feedback protocols that would always ensure that the firm was following an achievable production plan.

Future research directions

The outline of our MPC system suggests future research along a number of avenues. Figure 7.6 summarizes what we believe to be the most beneficial directions for future research in this area.

The three major avenues are:

(1) Development of the mathematical foundations of computer search and of the underlying theory of economic batch scheduling.
(2) Development and design of search algorithms.
(3) Development of the system architecture and overall mode of operation.

Efforts along any one of these three avenues could, and probably should, be run in parallel as results found along one avenue are likely to have an

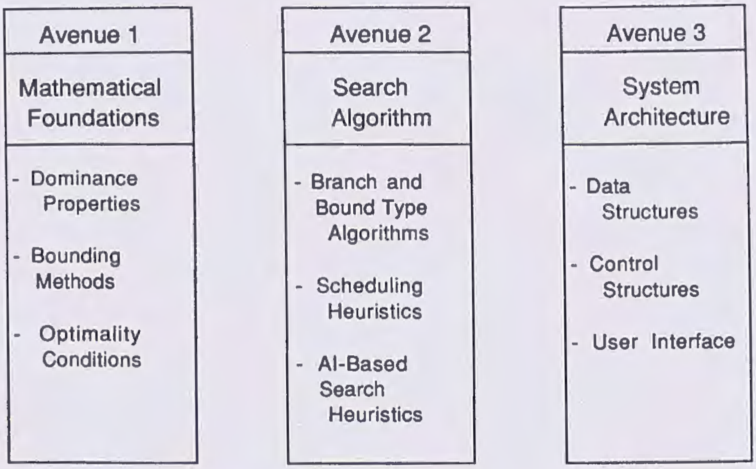

Figure 7.6 Research avenues

impact on the others. For example, progress in the development of new theoretical knowledge in (1) could certainly be exploited profitably in the design of improved search algorithms in (2). Likewise, developments in the search algorithm in (2) seem likely to facilitate certain types of improvement in the user interface design in (3).

Development of the mathematical foundations

In our opinion, developing a solid theoretical foundation is a major step toward the development of truly superior MPC systems. For the type of MPC systems we envisage, it will be necessary to draw on all the pertinent knowledge and theory available. There is a clear need to develop further the theoretical work of the Economic Batch Scheduling problem. Future research topics in this area would include; determining dominance properties among production plans, establishing necessary and sufficient conditions for the optimality and/or feasibility of proposed production plans, developing bounding methods for bounding the objective function values of production plans, etc.

A major research theme would be to specify the solution domain in which any search for a production plan would have to be conducted. An important goal would be to try to minimize this domain by determining and applying any dominance properties that might exist so that subsets of production plans might be eliminated from consideration. For example, in job shop scheduling, the set of active schedules is known to form a dominant set (Baker, 1974) for 'regular' measures of performance. An immediate question for research would be to determine if an analogous result exists for EBS problems. Considerable research has been conducted over the years on the mathematical aspects of inventory and scheduling. However, we now see the need to concentrate future efforts on the combined inventory/scheduling problem statement.

Development of search algorithms

In parallel with the continued development of mathematical bases would be the design and development of the basic search methods that form the core of the approach we suggest. To a degree, efforts along this avenue would be directed towards applying the types of mathematical results outlined above. However, because of the problem's complexity, there will always be the need to investigate heuristic solution methods. Heuristics can be used to limit or control the complexity of the search procedure, for example, by providing trial solutions for bounding partial solutions during the search. Interestingly, continued mathematical development of the type described above might have the added benefit of inspiring higher quality heuristics. For example, special case analysis might find necessary conditions for optimal solutions to a simplified problem version, and form a basis for a heuristic to the original and more complicated problem statement.

There already exists a solid foundation in the area of heuristic development, both from the literature of job shop scheduling and the literature of AI. For example, the use of a 'priority dispatching rule' might be thought of as a quick way to arrive at a completion of a partial solution. Considerable knowledge has already been accumulated on the properties of such rules. For a review of this line of research, see Blackstone *et al.* (1982).

The development of heuristics such as priority dispatching functions concentrates upon exploiting the peculiarities of scheduling-related problems. However, research results that provide general heuristic problem-solving tools might also be appropriate to the type of problem we address here. This is where research in the general field of AI might have some application. A currently prevailing theme in AI research is the development of intelligent search strategies, (see Pearl, 1984 for a thorough treatment of heuristic search strategies). The use of AI methods in manufacturing logistics is already underway. For example, in AI-based research at Carnegie-Mellon, Fox and Smith (1984) have used 'constrained-directed' search and Ow and Smith (1986, 1987) are using 'opportunistic reasoning and hierarchical organization structure' in the job shop scheduling domain. Additionally, Ow and Morton (1985) have reported using a 'beam search' in a simple scheduling problem. The development of search procedures for manufacturing problems will continue to benefit from the discovery of good heuristic techniques — both the kind that are more 'problem specific', such as with priority dispatching rules, as well as those which are useful in any search situation, such as the 'beam search' approach.

Development of system architecture

A research avenue of some importance to the development of manufacturing planning and control systems is what we call here the design of the 'system architecture'. This includes topics such as how planning systems function (that is, their control structure), how the user interfaces with the system (for example, issues in the ergonomics of screen design), as well as data representation issues such as file design and memory management.

As identified in a recent report (Abraham *et al.*, 1985), there are a number of important criteria that must be considered in evaluating a system architecture for manufacturing planning and control. They claim that such systems must be robust, flexible, and responsive. We foresee the need for research that finds design features which address these types of criteria. A possible method for research along this avenue might be to develop prototype systems using some of the tools available in expert systems development. For example, declarative programming languages like PROLOG, and AI programming shells like KEE, Knowledge Craft, and ART, could be very useful for quickly prototyping a particular systems architecture.

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