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ABSTRACT

This article suggests to the educator an example of combining conceptual notions that should be familiar to students of business. In addition the example, which combines the notions of economic lot size and the learning curve, serves as a means for dealing with those critical of the many assumptions that often precede model building. This exercise or ones similar to it should give critics and/or students some appreciation of the complexities which are unveiled as we attempt to model closer to "real-world" applications.

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A PEDAGOGICAL EXERCISE IN CONCEPT INTEGRATION:
Or Relaxing Assumptions and Moving Toward the Real World

Those academics who "do battle" in the "trenches of quantitative methods" are oftimes snared by the "damned if you do/damned if you don't" dilemma. This is particularly true for those of us who attempt to "model." Criticism is heaped upon the poor professor who "assumes away reality" in his models. How often have economics professors listened to that time worn criticism? On the other hand as the model-builder begins to relax his assumptions to move closer to "reality" (wherever that may be!) the complexity of the models greatly grieves the average student.

Two basic concepts which should be familiar to students of business are: (1) the economic lot size (ELS) model and (2) the learning curve. One might attempt to contribute to a student's concept integration by combining notions wherever possible. And these two basic concepts do lend themselves to integration. However, the aforementioned dilemma raises its menacing horns for those so inclined toward this educational foray.

The following is presented as an example of our dilemma:

The Basic Model

The basic problem of determining the most economic production schedule is one which most production-aligned industry encounters. Much has been written on the production scheduling problem in the literature of production/operations management, industrial engineering, operations research, quantitative decision-making and various other subject areas [1,5,9,13,17,18].

One of the simplest approaches to the economic production lot size (ELS) problem is a model suggested by Buffa [5]. A graphical illustration of this model appears in figure 1.

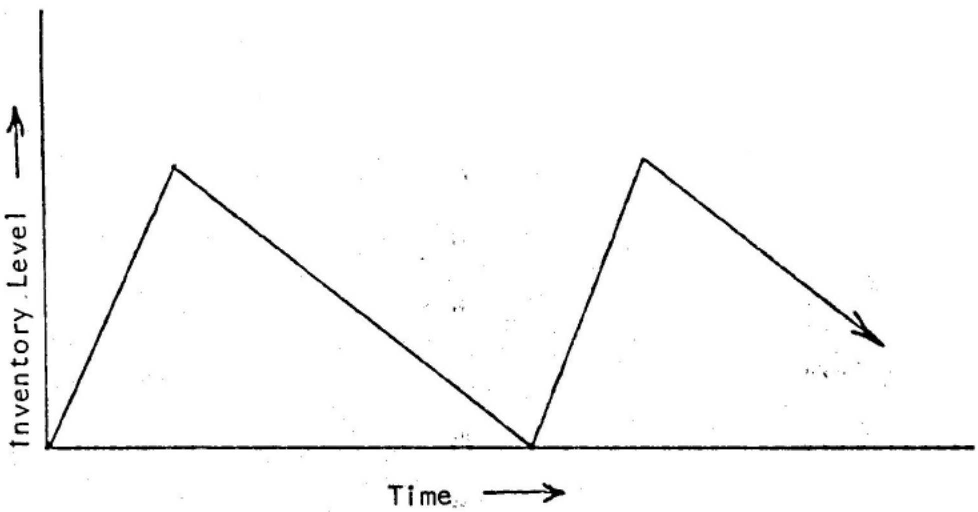

Figure 1

The assumptions of the basic model include: (a) a constant rate of production, (b) a constant rate of sales or usage, and (c) simultaneous production and usage.

In order to construct the basic mathematical model, let:

D = annual demand (in units)

U = usage (or sales) per day

P = production per day

H = holding cost as a percent of unit cost

S = setup cost per production run

C = cost per unit

TC = total cost per year

TMC = manufacturing cost per year

THC = holding cost per year

TSC = setup cost per year

X = optimal number of units per production run

X/P = time required to produce optimum run

Now:

$$TC = TSC + THC + TMC \tag{1}$$

Substituting we get:

$$TC = DS/X + XHC(1-U/P)/2 + DC \tag{2}$$

In order to minimize the total cost function, we take the first derivative with respect to X and set this equal to zero.

$$TC' = DS/X^2 + HC(1-U/P)/2 = 0 \tag{3}$$

Solving for X we get:

$$X = \sqrt{\frac{2DS}{HC(1-U/P)}} \tag{4}$$

That X is a minimum is verified by checking the second derivative.

<u>Learning</u>

Just as the basic model just presented, almost all mathematical ELS models and their variations that have appeared in the literature incorporate the underlying assumption of a constant production rate which, in effect, ignores the learning phenomenon. One notable exception is Keachie and Fontana [15] who have examined certain effects of learning on optimal lot sizes. The assumption of a constant production rate is rarely, if ever, satisfied. It is both logically and intuitively reasonable that as a worker repeats a certain task, he becomes more proficient in that particular task. Over a period of time, the rate at which the worker produces increases.

L.L. Thurstone was one of the first to publich the results of his investigation of the learning phenomena [20] . Thurstone found that "the learning curve in the speed-time form has an initial positive acceleration which changes to a negative acceleration when the attainment has reached one-third the limit of practice. His conclusion is contingent on the assumption that the learning curve in the speed-amount form is hyperbolic, an assumption which has been empirically shown to be safe for the majority of learning curves." [20]

Ettlinger [10] suggested that an exponential or growth curve is more representative of learning than the hyperbolic function suggested by Thurstone. Ettlinger carefully points out similarity of form between the two types of equations (hyperbolic and exponential) and suggests that the use of an exponential or growth curve is justified from the viewpoint of simplicity.

According to Hein [12] the learning curve takes on the form of an exponential curve, shown below modified for our particular use:

$$\bar{L} = TX^{-b}$$

where $\bar{L}$ = average unit time as a percent of the first unit

T = time to produce the first unit

X = unit number

b = a constant factor representing the rate of learning

Abernathy and Baloff [3] point out that the values of b found in practice have been in the range $0 \leq b \leq 1$. If $b=1$, then $\bar{L} = T/X$ and as X approaches infinity, $\bar{L}$ approaches zero. We know this is not possible. Any action takes time. However, if a constant "A" is included as an upper limit, it drops out in the first differentiation process, so we choose not to include it at this point.

Learning as a Function in the ELS Model

Including the learning notion in the basic ELS model is demonstrated in figure 2.

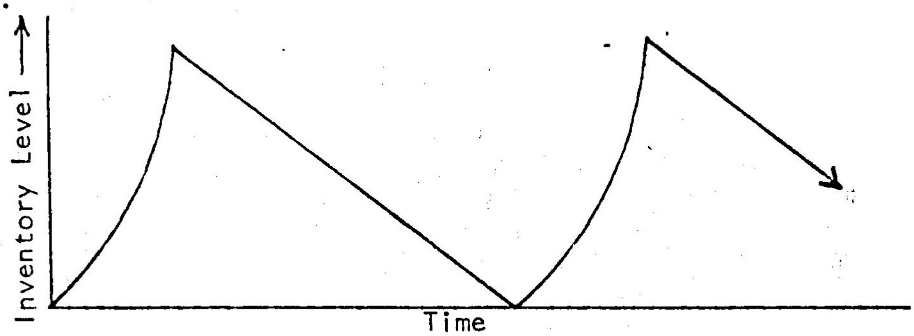

Figure 2

The underlying assumptions of the model are:

1) simultaneous production and usage
2) constant demand
3) production must exceed demand
4) exponential learning
5) complete dislearning between lots
6) units can be used immediately upon production
7) infinite horizon

Continuing to employ the above stated symbols, under normal conditions (steady state production) the average inventory for this model is

$$\bar{I}_n = X(1-U/P)/2 \tag{6}$$

where $\bar{I}_n$ = average inventory (normal conditions)

Considering the learning phenomena, the average inventory is

$$\bar{I}_L = X(1-U/R)/2 \tag{7}$$

where $\bar{I}_L$ = average inventory (with learning)

R = average daily production rate during learning cycle

From the basic model above,

$$THC = \bar{I}_n HC \text{ (steady state production)} \tag{8}$$

or $$THC = \bar{I}_L HC \text{ (with learning)} \tag{9}$$

Now $$P = N/L \tag{10}$$

where N = number of labor units available for production period

L = labor input per unit (steady state production during producing stage)

and assuming a constant number of labor units:

$$R = N/\bar{L} \tag{11}$$

where $\bar{L}$ = average labor input per unit (with increasing productivity during the producing stage due to learning)

Given $$\bar{L} = TX^{-b} \tag{12}$$

the ratio of the production rate (P) during normal conditions to the production rate (R) under learning is

$$P/R = \frac{N/L}{N/\bar{L}} = \bar{L}/L$$

or $$R = PL/\bar{L} \tag{13}$$

and substituting the learning function for $\bar{L}$

$$R = PL/(TX^{-b} \tag{14}$$

Since $\bar{I}_L = X(1-U/R)/2$

and we make the necessary substitutions,

$$\bar{I}_L = X(1-UTX^{-b}/PL)/2 \tag{15}$$

Considering the average labor cost per unit under learning:

$$\bar{C}_L = C_{ul}TX^{-b} = C_{ul}\bar{L} \tag{16}$$

where $\bar{C}_L$ = average labor cost per unit produced (under learning)

C_{ul} = cost per unit of labor

Cost per unit (C) under normal and learning conditions are as follows:

$$C_1 = C_L + C_F = C_{ul}L + C_F \tag{17}$$

where C_1 = cost per unit (normal conditions)

C_L = labor cost per unit

C_F = fixed cost (materials, etc.) per unit

and $$C_2 = \bar{C}_L + C_F = C_{ul}TX^{-b} + C_F \tag{18}$$

where C_2 = cost per unit (under learning)

Recalling the basic model

$$TC = TSC + THC + TMC \tag{1}$$

$$TC = DS/X + HC\bar{I} + DC \tag{2}$$

substituting (under learning)

$$TC = DS/X + HC_2\bar{I}_L + DC_2 \tag{19}$$

$$TC = DS/X + H(C_{ul}TX^{-b} + C_F)X(1-UTX^{-b}/PL)/2 + D(C_{ul}TX^{-b} + C_F) \tag{20}$$

To minimize the total cost function (under learning), we take the first derivative with respect to X and set this equal to zero. The formula, simplified, is as follows:

$$TC' = -(SD) + HC_FX^2/2 - (1-b)HC_F(UTX^{2-b})/2PL + (1-b)HC_{ul}TX^{2-b}/2 -$$

$$(1-2b)HC_{ul}T^2UX^{2-2b}/2 - (bC_{ul}TX^{1-b}D) = 0 \tag{21}$$

Thus the total cost function is minimized by optimizing the number of units produced in the run with learning as a factor.

Summary and Conclusions

Now that learning is included in the basic ELS model, this should logically produce a more realistic "optimal number of units per production run."

Whereas, with the basic ELS model we can solve a general equation for a solution, equation 21 does not lend itself to a general solution equation for X. Of course if the parameters of equation 21 can be determined, a "search" approach can be used for an optimal solution of X. However, gathering data on the so-called "constants" or "parameters" would be time consuming and expensive within an organizational setting. For example, "b", the constant factor representing the rote of learning is not easily determined and would differ with each task, production run, and worker. A "new" optimum lot size would have to be determined for each production run.

This exercise and similar demonstrations should convince even the most hardened critic that:

a) the relaxation of assumptions in models is not impossible.

b) integration of basic concepts that move us closer to "reality" is not impossible.

c) as we move closer to "reality" our models become significantly more complex.

d) as we move closer to "reality" our models do not lend themselves to general solutions due to mathematical limitations (see equation 21).

Other questions which such a demonstration should envoke from the inquisitive student would include: Is the increased complexity of the model worth the effort? Do the cost savings of the model with learning greatly surpass cost savings of the basic model (without the learning concept)?

It may be simple enough to integrate these two concepts (ELS and learning) in a theoretical manner in a classroom setting and obtain understanding on a "gut" level how they relate to one another. On a formal, mathematical level, however, is the average student interested in how the concept integration or assumption relaxation affects the basic model? Is the average student overwhelmed by this type of response to his "lack of reality" criticism? Excluding the sophisticated student, most educators would probably agree that a steady diet of demonstrations (as suggested herein) would quickly gorge our hungry (?) students. But, given an occasional snack of the above, even the most critical, self-appointed "practitioner" should gain some appreciation of solving "real-world" problems.

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