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BULLETIN

APPLICATIONS FROM AERIAL NAVIGATION IN THE TEACHING OF MATHEMATICS AND GEOGRAPHY

—ROWLAND ANDERSON

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Floyd E. Perkins, Director

FOREWORD

This bulletin will probably be of most interest to junior and senior high school teachers of mathematics and geography and to instructors in mathematics in teacher training institutions. It is made up of parts of a more complete study which is on file at Teachers College, Columbia University, New York. It is an attempt to give a sample of the way in which mathematics ties up with air navigation but it makes no claim to completeness.

Dr. Rowland C. Anderson, the author, interrupted his teaching career at the St. Cloud State Teachers College in 1943 to join the U. S. Navy. He returned to the college in September 1948. During the period of absence from the classroom he served the navy as an air navigator and as an instructor of aerial navigation. After the shooting war was over he was engaged as a navigator in a squadron doing research in aviation. A part of this research was a study of hurricanes and typhoons about which the Navy wants more information.

A report of the navigator's job in connection with these studies — its mathematics and its thrills, is reported as "Families of Hyperbolas in Navigation" THE MATHEMATICS TEACHER National Council of Teachers of Mathematics Volume XLII No. 2 February 1949.

We believe the contents of this bulletin will give material for classroom mathematics and tie it up with a vital subject—air navigation.

Floyd E. Perkins

INTRODUCTION

Mathematics teachers, curriculum makers, and textbook writers have recognized in various ways the relationship between their subject and aerial navigation. However, little real effort has been made to show just how extensive this relationship can be.

The purpose of this project is to show the great variety of topics in which applications from aerial navigation may well be used to enrich a modern course of study in junior and senior high school mathematics. It aims to provide a body of professionalized subject matter for teachers who wish to make the subject interesting and practical for the air-minded students of our present day schools. It will show the type of mathematics actually used in the science of air navigation, and the mathematical principles that are the bases of the many devices and short cuts that make modern high speed navigation as accurate and safe as it now is. For both teacher and student, it will provide a background of information necessary for intelligent understanding and discussion of a topic of intense current and future interest. For an air age such as we find ourselves in today, the teacher who cannot tie up many school experiences with their counterparts in aviation is lacking in a most valuable professional tool.

An examination of the topics recommended for a modern course of study in secondary mathematics will show that a great many of the topics at all levels will profit from the enrichment of illustrative material drawn for aerial navigation. Frequent references will be made to the course suggested by the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, one of the best available studies in the field of modern mathematics curriculum making[1]. The following brief discussion of the relationships of aerial navigation to each of the major topics of this proposed curriculum will be amplified throughout the project to demonstrate the many opportunities for an alert teacher to link mathematics to this phase of modern life problems.

1. Arithmetic. Aerial navigation demands facility and accuracy in the understanding and use of numbers and the fundamental operations of arithmetic. It furnishes powerful demonstration of the approximate nature of numbers derived from measurement and the methods of computation and checking necessary for handling such numbers in a real life situation. The use of the slide rule and logarithms in various situations forms the foundation of much of the speed that characterizes aerial navigation for modern high speed planes.

2. Geometry. Navigation is a science based on the elementary meaning of geometry or earth measurement. The three dimensional problems of air travel and the applications of the spherical properties of the earth furnish an excellent departure point for combining plane and solid geometry in a modern setting. There are many opportunities for drawing and constructing basic figures, and for direct and indirect measurement of lengths and

1. The National Council of Teachers of Mathematics, Fifteenth Yearbook, THE PLACE OF MATHEMATICS IN SECONDARY EDUCATION, Bureau of Publications, Teachers College, Columbia University, New York, 1940.

angles. Many of the basic facts and propositions in such topics as similar figures, parallel lines, ratio and proportion find practical application in the field of navigation. Locus is an extremely important problem for the navigator. The use of coordinate systems in the geometry of position can well be demonstrated by their use in navigation.

3. Graphic representation of data. All forms of graphic representation are employed to show the relationships among the many variables with which a navigator must work. Many tables and graphs as well as numerous formulas are used to illustrate the ways in which functional relationships are encountered in practical situations.

4. Algebra. The use of the functional core of algebra with its tables, graphs, formulas and equations has already been referred to in the preceeding section. Navigation provides a surprising number of applications of signed numbers.

5. Trigonometry. From the suggested foundation in the seventh grade where such topics as scale drawings, measurement, and ratios are recommended, throughout the student's experience with trigonometry, numerous applications from navigation and map making can be used for supplementary informational and problem work. Trigonometry teachers have made use of many of these opportunities, but many of the applications have been artificial and not truly representative of the actual use in practical navigation.

6. Mathematical modes of thinking, habits, attitudes, types of appreciation. Many of the desirable habits that mathematics teachers hope to develop through training in mathematics are vitally important to the safety and life of a navigator and those depending on him. Correctness of computation, measurement, and drawing, the habits of estimating and checking, neatness in work and logging procedures are an integral part of good navigational practice.

The mathematical mode of thinking required in navigation is also worth pointing out to students as a model for their own thinking procedure. The navigator makes numerous observations for the purpose of accumulating data. He carefully records and evaluates these data. He draws conclusions, takes action on the information derived, and then constantly checks his results. He is working in a real situation involving many types of interdependent variables which he must understand and use.

7. History of mathematics. Because of the recent development of the science of aerial navigation, it has had little effect on the history of mathematics up to this point. It is highly probable that because of the importance of the field today and the need for its development to keep pace with developments in other phases of aviation, navigation may have some effects on the history of mathematics now in the making.

8. Correlated mathematical projects and activities. The very nature of the work and the interest of many students in aviation activities makes navigation an ideal field for related projects and activities. Many schools are now including various types of aviation activities. Mathematics clubs will find the topic stimulating and profitable. For exhibits and posters, many

types of related drawings will be a source of interest. The uses of some of the instruments, such as shooting the sun or stars with a sextant, make exciting and worth while mathematical projects.

No attempt has been made to include a complete course in navigation, but most phases are touched in their relation to the correlated mathematics. The organization is based strictly on the viewpoint of the mathematics teacher, but sufficient information is included to make the applications meaningful to the average secondary teacher.

The discussion is limited to aerial navigation. Many of the problems of surface navigation are closely related; but the present interest in aviation activities, the three-dimensional aspect of air navigation, and the speed at which planes operate with the attendant need for faster navigational procedures make the emphasis on air navigation desirable.

The fundamental job of the navigator is fairly easy to define. Basically, it is to bring his plane from the point of departure to his destination in the safest, most economical way. Actually the work involved is of considerably greater scope. The navigator must know with reasonable accuracy at all points along the way, where the plane is and under what conditions it is operating. He must be able to give, at any time, the information which will enable the pilot to fly to his point of departure, to the nearest point available for a safe landing, or to a satisfactory alternate destination. He must keep informed of weather conditions along the way, and whenever the conditions warrant, must alter his flight plan to provide for the safety of the plane, crew, and passengers or cargo. He must keep an accurate log of the flight, a complete diary of everything that happened in and about the plane during the flight. At the end of the flight he must have a complete chart showing the path of the plane actually traveled. The work is exacting, requires accuracy and concentration, and involves great responsibility. When operating for long hours, particularly at high altitudes, the navigator is subject to considerable strain and fatigue which makes the necessary accuracy even more difficult.

To ease the work of the navigator, to make it possible to keep up the high standard of performance required, and to make navigation a more nearly exact science, many devices and aids have been developed. Calculation has been cut to a minimum with much of the work being done with the aid of mechanical means or with tables. This has given some the impression that only a minimum of mathematics is actually used by the navigator. Particularly during the war, when large numbers of navigators were needed with only limited time available for training, the mathematical foundations of the processes were pretty largely neglected, and the emphasis placed on the use of materials and methods without regard to understanding. Nevertheless, a knowledge of the mathematics involved makes the work of the navigator more meaningful, more interesting, and provides a safety factor of no little value. Furthermore, the short cuts and devices now in use are the work of those who understood the mathematical foundations. Developments in the future will come from those who understand and have an insight into the processes involved rather than from those who merely perform computations mechanically.

SPHERICAL PROPERTIES OF THE EARTH AFFECTING NAVIGATION

A great many applications of mathematics may be drawn from the spherical properties of the earth and the coordinate system of reference for the earth's surface that form the background of all navigational procedures.

Although the earth is actually a spheroid with the polar diameter slightly less than the equatorial diameter, for all practical purposes in navigation it is treated as a sphere. It revolves about the sun in an elliptical orbit. It rotates about an axis inclined 23½° from the perpendicular to the plane of its orbit. The north and south poles are the ends of this axis of rotation. The equator is the great circle formed by the intersection of the surface of the earth with a plane perpendicular to the axis at its midpoint. Great circles passing through the poles form meridians.

The equator and the meridian through the Royal Observatory in Greenwich, England form the axis for the coordinate system of position on the earth known as latitude and longitude. The latitude of a point is its angular distance north or south of the equator measured along the meridian through the point. The longitude of a point is its angular distance east of west from the Greenwich or prime meridian. Both latitude and longitude may be defined in other ways to illustrate certain properties of spherical geometry.

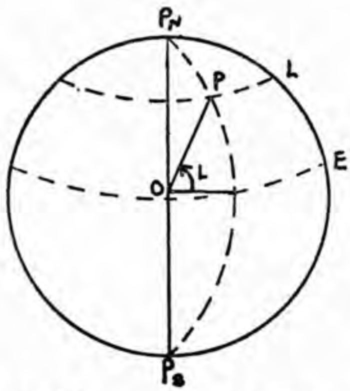


Fig. 1. Measurement of Latitude.

The parallels of latitude are small circles parallel to the equator or formed by the intersection of the sphere with planes perpendicular to the axis of the earth. The latitude of a point is equal to the angle subtended at the center of the earth by the arc of the meridian measured from the equator to the point.

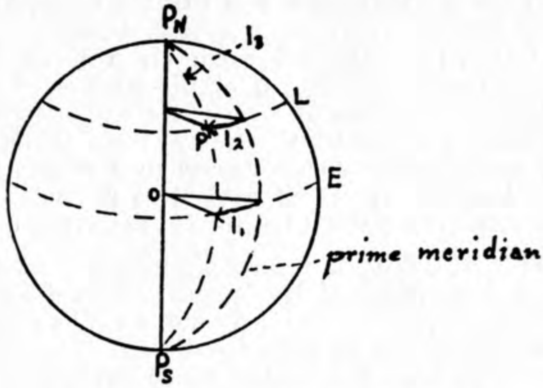


Fig. II. Measurement of Longitude.

The longitude of a point may be measured:

(1) l_1 as the degrees of arc of a great circle, the equator;

(2) l_2 as the arc of a small circle, the parallel of latitude through the point, or;

(3) l_3 as the angle at the pole between the prime meridian and the meridian of the point.

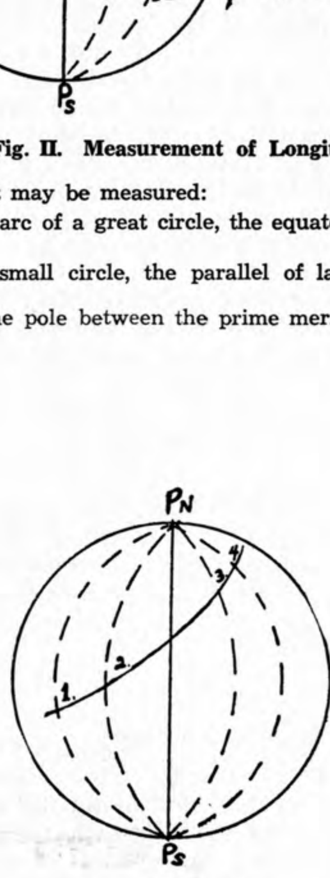


Fig. III. Rhumb Line

One other line on the sphere of great importance in navigation is the rhumb line—a line which cuts all meridians at the same angle. Such a line is also known as a loxadromic spiral as it spirals about the earth approaching the pole as its end point.

Angle 1 = angle 2 = angle 3 . . .

PROJECTION OF A SPHERE ON A PLANE SURFACE. From the time of recognition that the earth was a sphere, a major problem of cartography has been to put the spherical earth on a plane chart for navigation purposes. The exact translation of the surface of a 3-dimensional sphere to the 2-dimensional plane of a map is a mathematical impossibility comparable to squaring the circle. There have been attempts to avoid this problem by using global navigation. However such attempts have met with no general recognition, and as yet all navigation depends on the use of charts and maps presenting a distorted picture of the earth.

DEVELOPABLE SURFACES. Surfaces that may be placed flat without stretching or tearing are called developable. Both the cone and the cylinder are such developable surfaces. If a cone is cut in a straight line from apex to base, the surface may be spread out flat. Similarly, a cylindrical surface may be cut from base to base and rolled out with no distortion. In no way can any considerable part of a sphere be spread out on a plane without the distortion of tearing or stretching. Since the sphere is not developable, cartographers have made their maps by projecting the reference lines of the sphere on to some developable surface such as the cone, the cylinder, or the plane. A great many systems of projection have been devised, each intended to serve some particular purpose or to preserve some property of the sphere. Some of the properties which it might be desirable to preserve on the chart are: (1) True shape of the physical features, including correct angular relationships. (2) Equal areas, or the representation of the areas in their correct relative proportions. (3) True scale values for measuring distances. For navigation purposes the representation of great circles as straight lines and the representation of rhumb lines as straight lines are also desirable.

Since it is impossible to keep all these properties on a flat projection of a sphere, the choice of a projection is determined by the purpose for which it is to be used. Usually a compromise of several values is effected so that the chart maintains several of the desirable features by compromising some from each.

MERCATOR PROJECTION. The charts used for most navigation purposes are of the Mercator type. The Mercator projection is mathematically derived from a cylinder tangent to and along the equator. It is really a patchwork of tiny units of area made by minute squares of latitude and longitude, each of which is correct in its radius of small circle of latitude to radius of great circle of longitude relationship. The actual geodetic measurements given by the Smithsonian geographical tables show the following relationships between a minute of longitude and a minute of latitude at selected latitudes:[2]

2. William Chamberlin. "THE ROUND EARTH ON FLAT PAPER", National Geographic Society, Washington, D. C., 1947, p. 83. 50c.

Lat.	longitude	latitude	Secant	Mercator value of one minute of latitude-6087 x 1 min. lat. / 1 min. long.
20	5,722 ft.	6,053	1.064	6,439
40	4,670 "	6,071	1.305	7,913
60	3,051 "	6,092	2.000	12,154
79	1,165 "	6,106	5.241	31,903

An approximation of this projection may be made by assuming a cylinder to be placed around the earth tangent to it at the equator.

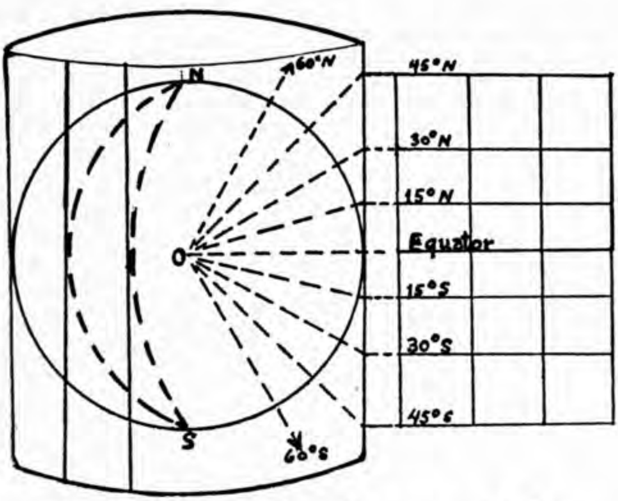


Fig. IV. Projection of a sphere on a cylinder.

If the planes of the meridians are projected onto the cylinder, they appear as parallel, equidistant, vertical lines. If the parallels of latitude are then projected from the center of the earth to the cylinder, they appear as parallel, horizontal lines. The interval between them increases as the latitude increases.

If the cylinder is then cut down one of the vertical lines and flattened out, the projection appears similar to the Mercator. Advantages of this type of chart for navigation are: (1) The meridians are equally spaced, parallel, vertical, straight lines. (2) The equator and parallels of latitude are parallel horizontal straight lines not equally spaced. (3) Rhumb lines are straight lines. (4) Local distortion of scale is comparatively small and within a limited area one minute of latitude may be used as a nautical mile to measure distances in any direction. The main disadvantages of this type of projection are: (1) The continuous expansion of scale. In polar areas an area appears many times as large as an area of the same size at the equator. Mercator charts are not used in latitudes above 75° (2) Great circles except the equator and meridians are curved lines. This means that to fly the shortest distance between two points on a conventional Mercator chart, a curved or series of rhumb lines must be used rather than a single rhumb line.

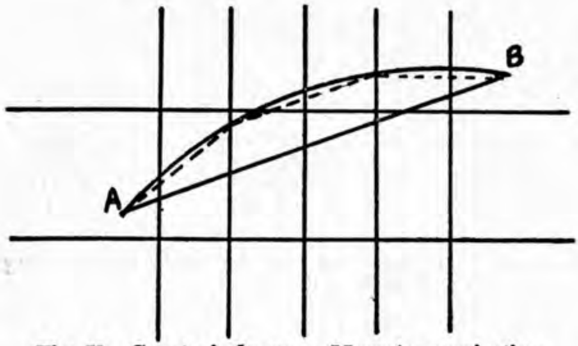


Fig. V. Great circle on a Mercator projection

A Mercator plotting chart for a small area sufficiently accurate for navigation purposes may be constructed with simple drawing instruments. The construction is based on the relationship that a minute of latitude at any part of a Mercator chart is equal to a minute of longitude at that point times the secant of the latitude. To demonstrate this point the following argument is presented.

In the following figure the notation is used: N = North Pole, C = center of earth, EE' is 1 minute of arc of the equator, NE and NE' are semi-meridians, PP' is 1 minute of arc of the small circle parallel of latitude at L degrees north. R'=R=radius of earth, r = radius of small circle PP'. A = arc of great circle EE', a = arc of small circle PP'.

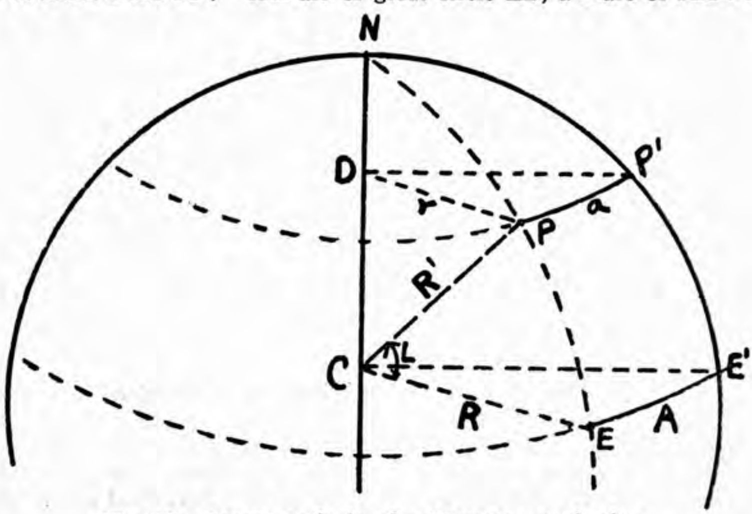


Fig. VI. Secant ratio in the Mercator projection.

On a Mercator projection PP' is equal to EE'.

Angle ECP = L (definition of latitude)

Angle DPC = angle ECP = L (alternate interior angles of parallel lines cut by a transversal)

Sector ECE'$\sim$PDP' (Corresponding parts are parallel)

$\frac{A}{a} = \frac{R}{r}$ (Corresponding parts of similar figures are proportional)

$\frac{R}{r} = \frac{R^1}{r} = \text{Sec } L$ (definition of secant)

A = a sec L (substitution)

= 1 minute of arc of a great circle = 1 nautical mile

= 1 minute of longitude x secant of latitude.

Below is the construction of a small area Mercator plotting chart with 40° N. mid-latitude. The longitude selected (74° W-76° W) has no bearing on the method of construction.

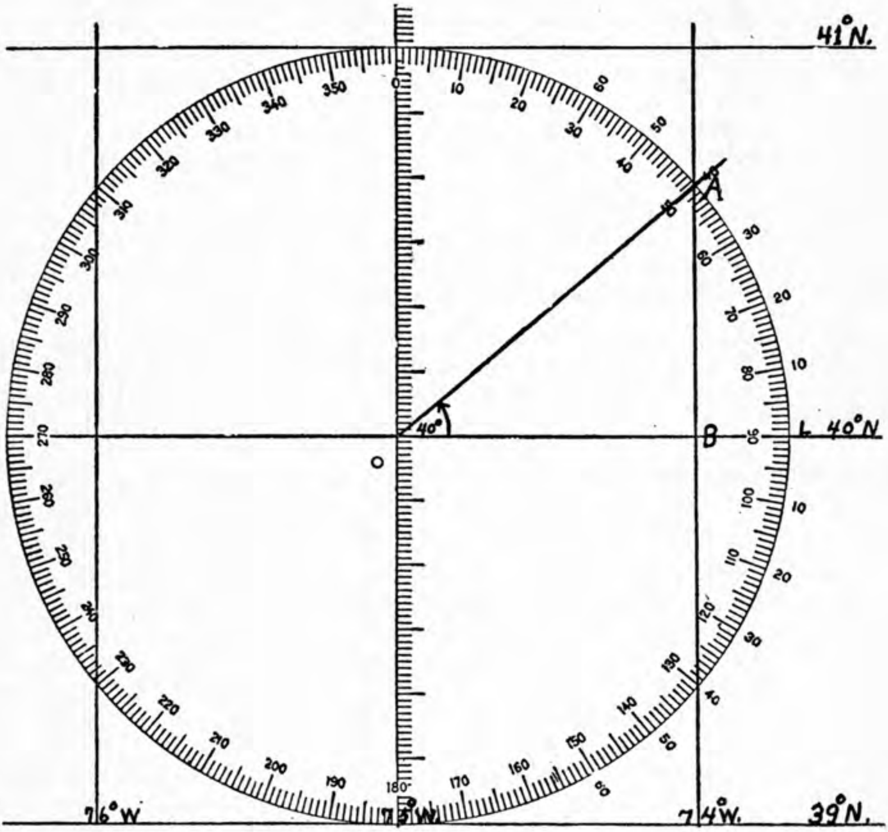


Fig. VII. Small area Mercator chart.

1. On mid-latitude line L at 0. construct angle = 40° (mid-latitude) cutting circle at A.
2. From A drop a perpendicular to L as AB.
   AB is a segment of the 74° W meridian.
3. Locate 76° W meridian in a similar manner.

In this type chart the scale of miles along the meridian is constant and the distance between the meridians is adapted so that the mercatorial secant ratio between latitude and longitude still obtains.

$$\frac{OA}{OB} = \frac{\text{1 degree of latitude}}{\text{1 degree of longitude}} = \sec 40^\circ$$

TRANSVERSE MERCATOR PROJECTION. For long flights, the difference between the Mercator course and the great circle is sufficient to become a matter of considerable importance. For example, the great circle route from Chicago, Illinois, to Gander, Newfoundland, lies on the north of the rhumb line between two points. On frequently traveled routes, oblique Mercator projections are now available. This projection is mathematically derived from a cylinder tangent to and along a great circle between the two terminal points. The distortion of scale and azimuth within 10 degrees either side of the line of tangency is negligible. For all practical purposes, all straight lines approximate arcs of great circles. By so changing the relation between the cylinder and the spheres, the meridians and parallels of latitude will change from their normal vertical and horizontal position to perpendicular oblique lines on the chart. To find the location of the intersections of the meridians and parallels with respect to the new line of tangency, spherical trigonometry is used to solve the triangle formed by the true pole, the transverse pole, and the point in question.

The principal disadvantages of this projecton is that a straight line is not a rhumb line. A course or bearing must be measured at the midmeridian between the two terminal points. Or the course may be divided into several sections with the direction of each section measured near its midpoint.

CONIC PROJECTION. The projection of the earth on a cone is the basis of the Lambert Conformal Maps used extensively for overland flight. The U. S. Sectional and Aeronautic charts are examples of such charts.

Again, althouth the projection is mathematically derived, the idea of the projection is illustrated by assuming the projection of the reference lines on a sphere from the center of the sphere on a right circular one passed through the surface of the earth intersecting the surface along two standard parallels of latitude. The standard parallels for the U. S. are 33 and 45 degrees. For northern regions 55° and 65° are used. At the standard parallels the scale is exact. Between them the earth is projected inward on the cone and the scale on the cone is slightly less than true. Outside the standard parallels, the earth is projected outward and the scale on the cone is too large.

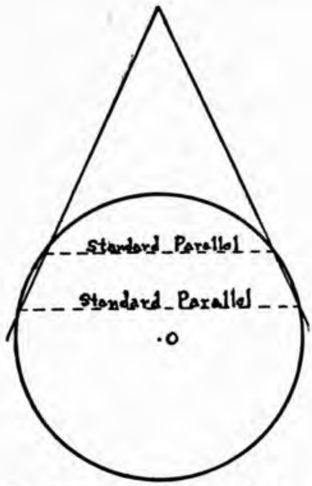


Fig. VIII. Conic projection.

Features of the Lambert Conformal charts and maps are:

(1) Meridians are straight lines converging toward a common point outside the chart (the apex of the cone).

(2) Parallels are arcs of concentric circles. Meridians are radii of these circles, center is at intersection of the meridians.

(3) Meridians and parallels intersect at right angles.

(4) Scale properties are very good—for most of U.S. the scale error is less than ½ of 1%.

(5) A straight line approximates the arc of a great circle.

(6) A straight line is not a rhumb line. Measurements of bearings and distances must be made as described for the oblique Mercator projection.

GNOMONIC—PROJECTION OF THE EARTH ON A PLANE. A third type of projection, the gnomonic, is made by projecting the sphere from the center on a plane tangent to the surface of the sphere at any desired point. The point of tangency most commonly used for such a map is the pole. The tangent plane is at right angles to the axis of the sphere. The area near the point of tangency is accurate but this accuracy decreases geometrically away from the center. If the North Pole is used as the center, only the Northern hemisphere is projected with the projection approaching infinity as it nears the equator.

In the gnomonic projection at the pole, the meridians are straight radial lines. The parallels are concentric circles at equal intervals. All straight lines through the pole are great circles that can be measured on the latitude scale on a meridian. Rhumb lines are curved.

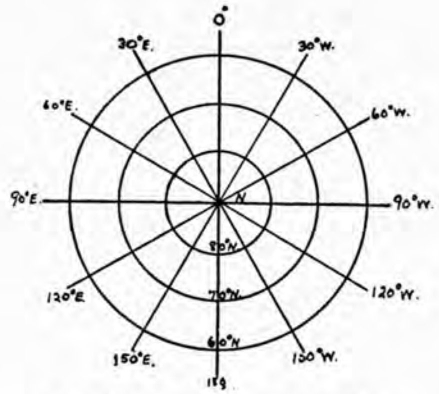


Fig. IX. Gnomonic projection.

This type of chart is used for polar navigation. It is also used in planning great circle flights for transfer to a Mercator chart later. The points at which the great circles cross desired meridians can be transferred to a Mercator chart, and these points joined with straight rhumb lines.

The plane of projection may be placed at any point on the sphere. For instance, for planning long distance flights from New York, a projection could be made on a plane tangent to the earth at New York. Such a chart would show at a glance the area crossed on a flight to any other city. A polar projection is satisfactory for a general map of the Northern hemisphere. This is the type of chart most popular for showing the relationship between countries in an air travel earth.

Other types of projections of the earth on a plane. There are two other relatively unimportant projections used for some purposes. These projections are: (1) the stereographic, from a pole of the earth and (2) the orthographic, from a point outside the sphere infinitely far away. Both of these are used for hemispheric rather than world representation. There is a distortion of distances that makes these types useless for navigation purposes but they are useful for showing general geographical relationships.

A great deal more could be said on the relation between cartography and mathematics; but since further discussion lies primarily outside the field of navigation, it is beyond the scope of this discussion. For a discussion of the trigonometry used in locating points on various types of projections, see page 95, "Round Earth on Flat Paper."[3]

3. Chamberlin, op. cit.

EDUCATIONAL IMPLICATIONS. The material in this chapter will be of interest primarily to the teacher and students of plane and solid geometry in grades 10 to 12 and above. It will aid in the development of spatial insight and the habit of noticing geometric relations in three dimensions and comparing such relations with those in a plane. It can be used to demonstrate the significance of geometry in human affairs.

In the informal geometry of grades 7 and 8, many of the elements of this topic may be introduced. The basic vocabulary of plane and solid geometry should be developed at this level. The relations of sphere and planes, great circles, and small circles, measurement of angles may be developed on an elementary level in the setting already familiar to the students of these grades through their knowledge of latitude and longitude.

The use of the secant function in the construction of the Mercator chart should be left until the eleventh grade trigonometry class when the student has sufficient maturity and geometric background to appreciate this trigonometric foundation of map construction.

MEASUREMENTS AND COMPUTATION WITH APPROXIMATE NUMBERS IN NAVIGATION

Few other fields of applied mathematics offer so wide an opportunity to demonstrate the use of approximate numbers and the computation with numbers derived from approximate measurement as does aerial navigation. A great many variables present many difficult problems of accurate measurement. The navigator must make these measurements, realize their limitations, and use them with judgment and discretion. He must constantly use averages to make his information more nearly representative of the varying conditions under which he may be operating. He must be able to estimate. He must constantly check new information against his estimates and previous information. He must be able to evaluate the degree of accuracy of any information, and must know the types of errors likely to occur.

The distortions of the various types of charts were discussed in detail in an earlier chapter. These must be recognized and accounted for in any plotting. Furthermore, in many areas there are errors in the information on the charts due to lack of adequate or accurate information for the chart makers. Since the scales used are fairly small, a small error on a chart may actually represent a rather large error in position.

MEASURING AND PLOTTING INSTRUMENTS

The instruments used in measuring and plotting on the charts are few and simple. Distances are measured or plotted by the use of dividers, with the spread of the points compared to the scale of miles on the latitude scale of the chart. Measurements are read to the nearest mile. Particular care must be taken to use the latitude scale near the mid-latitude of the area in which the measurement is made. Failure to observe this caution might easily result in an error of several miles, as on the same Mercator projection, 30 miles at latitude 45° is equal to 37 miles at latitude 30°.

Direction is most frequently measured or plotted using an aircraft plotter consisting of a transparent protractor and straight edge. It is measured to the nearest degree on a scale from 0° to 360° from true north as origin.

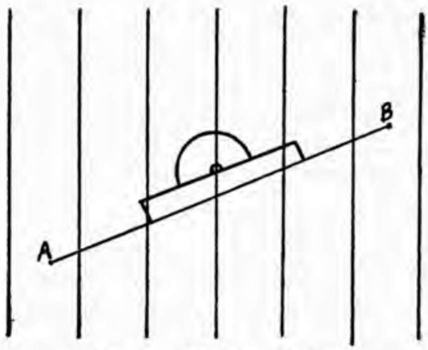


Fig. X Aircraft plotter.

To measure the direction of a line, lay the straight edge along the line so that the center hole of protractor lies over a longitude line of the chart. This meridian cuts the outer scale of the protractor circle at the angle the line makes with North: Caution: The angle and its reciprocal both appear on the scale and the reader must choose the proper reading.

Other means of measuring direction on charts are parallel rulers, celluloid triangles, and drafting machines. These instruments have not proved so satisfactory as the aircraft plotter and are not extensively used. A complete discussion of plotting instruments is contained in a Doctor of Education project report.[4]

VARIABLES AFFECTING NAVIGATION IN FLIGHT

1. Heading. Heading is the direction which the aircraft is moving through the air mass supporting it. This direction is measured to the nearest degree from zero to 360° of angular rotation from North by some type of compass.

Magnetic compasses use the North-South magnetic lines of force of the earth to show the direction the aircraft is headed. Such compasses are subject to two types of errors for which corrections must be made.

Variation is an error due to the displacement of the magnetic pole from the geographic pole. The amount of error for a given area is relatively con-

4. R. S. Gardner, INSTRUMENTS FOR THE ENRICHMENT OF SECONDARY SCHOOL MATHEMATICS, Type C Project, Teachers College, Columbia University, 1947.

stant and is shown on all navigation charts by isogonic lines joining places having the same variation. The amount of variation ranges from zero to ±22° in the United States. Particularly in the polar regions, variation is not constant and causes difficulty in navigation. True heading corrected for variation is called magnetic heading. In an area having 12° westerly variation, to fly a true heading of 030°, a magnetic heading of 042° would be required. Applying variation correctly is an extremely important part of navigation as applying 20° variation in the wrong direction would cause a total error of 40°. An error in addition of signed numbers here could cause serious difficulty.

The second type of magnetic compass error is deviation, caused by errors in the compass itself or from magnetic disturbances in the aircraft that affect the magnetic field about the compass. Such disturbances might be caused by electrical circuits or iron in the vicinity of the compass. Deviation must be determined for each indivdual compass by checkng the compass readings against known magnetic headings, at 15 or 30 degree intervals. Such a compass check is called "swinging ship." From a graph of the errors plotted against the magnetic headings, it is possible to compensate the compass, that is to remove part or all of the error. If it is not possible to remove all the error a correction card is made so the pilot or navigator knows what the compass should read to give any desired magnetic heading. This compass reading is called the compass heading. While deviation is usually constant in a given plane, changing electrical equipment or introducton of new metal into the structure might cause new errors. Planes should be swung at regular intervals and whenever any change is introduced that might cause any change in the magnetic fields.

An astrocompass uses sights on celestial bodies to give headings correct within one degree. Such a compass is used to accurately check magnetic compasses in flight. It is used extensively in polar regions and in emergencies when magnetic compasses fail to operate properly. Its disadvantages are the need for constant resetting for each reading and the need for visibility of the celestial body.

Below is a copy of the results of a typical compass swing. It illustrates the type of errors encountered and the use of positive and negative numbers in this type of work. In the preliminary or correcting swing, the deviation is checked on the four major headings. Coefficients C and B are the average deviations on North-South and East-West headings respectively. These factors are corrected by adjustment screws or bars in the compass. Then the compass is checked on headings of 45° intervals, the deviations recorded and averaged to give coefficient A. This error is removed by turning the lubber or index line of the compass. The residual deviations are graphed and a correction card is placed by the compass.

Date: 5/29/47

Aircraft Type and No: Stirling Z999 Adjusted by: A.C. WELLS

Compass Type and No.: P4-500D

CORRECTING SWING

(i) Approx. Course (M)	(ii) Astro Compass Course* (T)	(iii) Variation*	(iv) Actual Course (M)	(v) Aircraft Compass reads	(vi) Deviation (iv)−(v)	Coefficients
NORTH	349	10W	359	003	−4	$C = \frac{\text{Dev. on N} - \text{Dev. on S.}}{2}$
EAST	081	"	091	085	+6	$= \frac{-4-8}{2} = -6$
SOUTH	168	"	178	170	+8	
Coefficient C (with sign changed)				+6		
Make Compass read				176		
WEST	257	10W	267	265	+2	$B = \frac{\text{Dev on E} - \text{Dev on W.}}{2}$
Coefficient B (with sign changed)				−2		$= \frac{+6-2}{2} = +2$
Make Compass read				263		

CHECK SWING

(i) Approx. Course (M)	(ii) Astro Compass Course* (T)	(iii) Variation*	(iv) Actual Course (M)	(v) Aircraft Compass reads	(vi) Deviation (iv)−(v)	(vii) Coefficient A	(viii) Residual Deviations (vi)−(vii)
N.W.	307	10W	317	316	+1	+3	−2
NORTH	350	"	000	358	+2	+3	−1
N.E.	037	"	047	042	+5	+3	+2
EAST	083	"	093	089	+4	+3	+1
S.E.	125	"	135	134	+1	+3	−2
SOUTH	169	"	179	177	+2	+3	−1
S.W.	215	"	225	220	+5	+3	+2
WEST	263	"	273	269	+4	+3	+1
				Total:	+24 ÷ 8 = +3		

To remove +A, turn Lubber Line clockwise.
To remove −A, turn Lubber Line anti-clockwise.

* For use when swinging by Astro Compass.

RESIDUAL DEVIATIONS

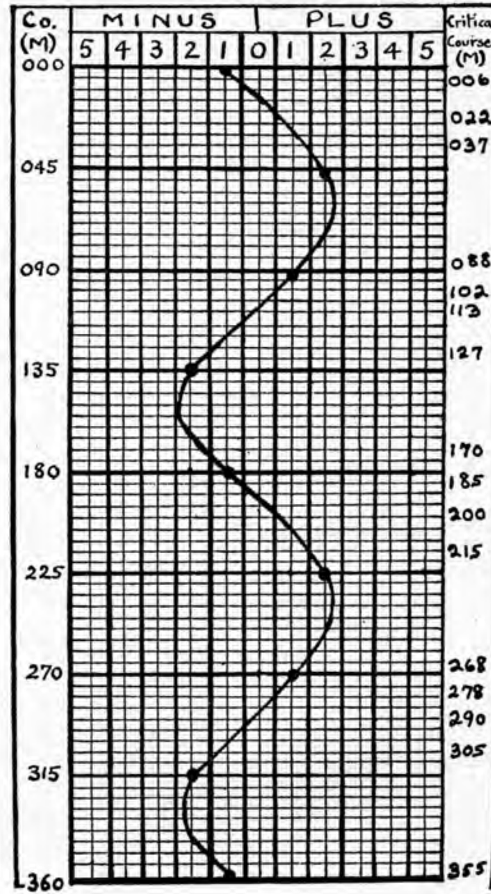


Fig. XI

2. Air speed. The second major variable is air speed, the rate at which the plane travels through the mass of air supporting it. This rate is registered on an air speed indicator in units of knots or miles per hour. A knot is one nautical mile per hour (1.15 land miles per hour). In the recent moves toward standardization between the U. S. Army and Navy, the knot was adopted as the standard unit. The accuracy of the air speed meter is checked by timing the flight of the plane at constant rates of 5 or 10 knot intervals back and forth over a measured course. Such a check would be made on a calm day but to eliminate any error that might be caused by wind the average of the rate over the course and back is used $R = \frac{R^1 + R^2}{2}$

A table of calibration is made and placed by the indicator. A sample table follows:

I.A.S.	120	130	140	150	160	170	180	190	200	210
C.A.S.	120	133	145	152	160	169	177	186	195	204

Tabble 1. Air Speed Calibration

The air speed indicator operates from a pilot tube which measures the pressure of air against the forward motion of the plane. Since the density of air is affected by temperature and altitude, a correction for these two factors must be applied to the calibrated air speed to give the true air speed of the plane. This is done on the slide rule part of a navigation computer. On a special slot for that purpose, altitude is placed opposite air temperature. Then on the outer dial true air speed is opposite calibrated air speed on the inner dial. For example, with an air temperature of -35° at 30,000 feet, a calibrated air speed of 120 knots would give a true air speed of 200 knots.

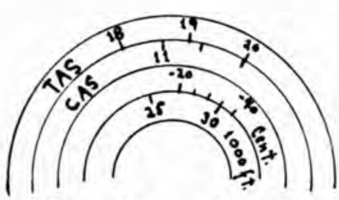


Fig. XII Air speed correction on computer

3. Altitude. Altitude is not so significant a variable in navigation as heading and air speed. However it must always be considered by the navigator as a safety factor in flight with all caution taken to be well above any obstructions on or near the path of flight. The height of a plane above the surface below is called absolute altitude. Safety altitudes for all areas are shown on navigation charts. Pressure altitude is an important factor in making the correction from calibrated to true air speed as illustrated above. Pressure altitude is the height of the plane in feet above sea level indicated by an altimeter. The most common type of altimeter consists of an aneroid barometer which can be set to the correct sea level atmospheric pressure. All major air fields transmit this information to planes in their areas. In long distance flights, a plane may go from a high pressure area to a low pressure area and cause an altimeter error of as much as several hundred feet. In mountainous areas or in areas of heavy traffic, such an error might be disastrous. Radio and radar altimeters of great accuracy are now used to show absolute altitude.

Another instrument concerned with altitude is the rate of climb indicator which shows the rate in feet per minute the plane is climbing or dropping. It measures change of altitude in terms of positive and negative numbers.

In flight the control of these first three variables is the responsibility of the pilot. The navigator knows the heading, air speed and altitude the pilot intends to fly. A good pilot with average air conditions should hold his heading within ± 2 degrees, air speed ± 5 knots and altitude ± 20 feet. Under adverse conditions these limits may be exceeded but a good pilot will try to average out any errors in heading and air speed. That is, if he finds he has been exceeding the desired heading by 5 degrees for a few minutes, he will fly the same amount on the other side for an approximately equal time. It would be an impossible task for the navigator to record and use all the variations that occur; so he uses the best average he can determine. Many planes are equipped with an automatic pilot which uses a gyroscopically stabilized set of controls to keep the plane within the above limits of the desired heading, air speed, and altitude.

4. Temperature. The temperature of the air outside the plane is an important factor in determining the true air speed of the plane. This is measured in degrees Centigrade by a metallic thermometer. Normally the temperature drops 2°C for each 1000 feet increase in altitude. Changes of temperature must be expected when crossing fronts between warm and cold air masses, or when passing from land to water areas and vice versa. Planes traveling at high speeds build up enough air compression heat to give an error in temperature which must be corrected. A table for this correction is usually included in the navigation kit.

5. Wind direction and velocity. The most difficult variable with which the navigator must contend is wind direction and velocity. Wind direction is measured in degrees from zero to 360 as the direction from which the wind blows. Wind velocity is measured in knots or miles per hour. The navigator gets his information on these variables in several different ways and is constantly alert to any signs of change. Before a flight and in flight if possible he gets the wind forecast from meteorological agencies. If flying

at 1500 feet or below, he can get a fairly accurate estimate of the wind up to that altitude by the effect of the wind on surface conditions. Above 1500 feet, surface wind conditions cannot be assumed. Normally direction and velocity increase with altitude. A table of wind velocities and their land and sea surface effects is the Beaufort scale. On weather maps the winds are given the Beaufort scale values with arrows showing the direction and one tail feather to show two units of force. Some of the ways used to determine wind direction are double drifts, wind stars, computation from vector triangles, between fixes, etc. In connection with the topic of variability, it is sufficient to point out here that wind determination is a real problem in navigation and subject to many errors. Wind direction within ±10 degrees and velocity within ±5 knots are desirable for satisfactory Dead Reckoning navigation.

6. Drift angle. Closely related to wind direction and velocity is the drift angle. Drift angle is the angle between the heading of the lane and the direction it actually travels in reference to the ground. It is measured in degrees left or right, or plus or minus. It is usually measured by some type of drift sight. Basically all drift sights consist of some kind of parallel grid lines on a transparent base. When these grid lines are parallel to the fore and aft axis of the plane, the index reads zero. By sighting through the transparent part and rotating the grid until objects on the ground move parallel to the grid lines, the grid lines are moved through an angle equal to the drift angle of the plane. A perfect reading requires absolute straight level flight. Since such a condition is rarely possible, several readings are taken and averaged. A gyro stabilized drift meter eliminates much of the possible error but even with such an instrument an average reading is preferred. Conditions of rough air, poor surface visibility, and surface glare are special problems in getting accurate drift readings. Because conditions change and the drift factor is so important in navigation, drift readings are taken on turning to any heading to be flown for any length of time and every 15 minutes on any maintained heading. Drift is also calculated from the heading and track made good between fixes and from a rather complicated formula involving changes in atmospheric pressure.

7. Track. Track is the path of the plane over the surface of the earth. It is measured in degrees from zero to 360. It is found by adding algebraically the true heading and the drift.

For example: TH 305° drift 7° left Track = 298°
TH 055° drift 14° right Track = 069°

Track may be found by measuring the direction between two fixes if there has been no change of heading between them. Track may be computed by the use of a wind vector triangle if T.H., TAS, and W/V are known.

8. Ground speed. Ground speed is the rate which the plane is traveling in reference to the surface of the earth below. It is measured in knots or miles per hour. It may be computed from wind vector triangle or from the time to travel between two known points if there has been no heading change. It may be computed from a formula by timing the passage of an object on the ground between two cross lines on the grid of certain types

of drift sights. Again several readings must be taken and averaged to get an accurate ground speed.

ACCURACY OF POSITIONS IN NAVIGATION

1. Visual positions. With proper identification, it is possible to get an exact position visually. This is one of the few cases where exactness is possible in navigation. Even visual methods present some difficulties. The difficulty of identification, of land features particularly from high altitude or at high speed is a real problem. At 40,000 feet, only the most prominent features are readily identifiable. In one of our present day high speed jet planes, with visibility of five miles, the pilot has slightly more than half a minute from the time a landmark comes into view until it is past. As the speeds increase this time will be even further reduced.

2. Dead reckoning positions. Since D. R. positions are computed by using the eight variables listed in the previous section, and since each of these is subject to errors, such positions could be considerably in error. It is easily possible to imagine a situation where a plane is flying in clouds and out of radio and radar contact of any known position for eight or ten hours with all possible errors accumulating instead of compensating each other. Under such adverse conditions it would be possible for a D. R. position to be several hundred miles in error.

3. Radio positions. Radio beams are usually very accurate. There are areas and conditions where beams split and bend and give false positions. The cone of silence over a radio transmitter is an accurate position if properly identified. False cones may cause the pilot some trouble. The accuracy of radio bearings depends on the distance from transmitter, signal strength, weather conditions, skill of the radio operator, and efficiency of the receiver. Bearings must be corrected for their deviation from great circle courses. Bearings taken in plane are relative bearings and any compass error would cause the same amount of error in the radio bearings. Radio bearings are subject to refraction or bending when passing over coast lines, islands, mountains or magnetic deposits.

4. Radar lines and positions. Radar lines and positions depend upon the efficiency of the gear, and the skill of the operators. They are subject to errors of identification of landmarks. Again the accuracy of the bearings are relative and their accuracy is no better than the plane's compass.

5. Celestial lines of position. The accuracy of celestial lines of position is conditional upon accuracy of the sextant, accuracy of the time, skill of the operator making the sights, straight and level flight of the plane, accurate computation and plotting. Under good conditions, a celestial line of position is expected to be consistently within ± 10 miles of actual position.

6. Loran lines of position. Loran provides the most consistently accurate lines of position with the maximum errors being less than 1 per cent of the distance of the plane from the transmitters. Near the transmitters this error will be a matter of yards. A thousand miles out from the transmitter, the maximum error will be less than 10 miles.

METHODS OF COMPENSATING FOR ERRORS

Since there are so many possibilities of errors in the various phases of navigation, precautions must be taken to use all information in the most intelligent manner. The constant use of estimating, averaging, and checking has been emphasized throughout the preceding discussion. If lines of position are available from only one source, it is good practice to take two such lines within a short time interval and use a mid-position and the mid-time. For example, if on a course of 090° sun lines a and b were obtained at times 1022 and 1026, average line c at time 1024 would be used as the best position.

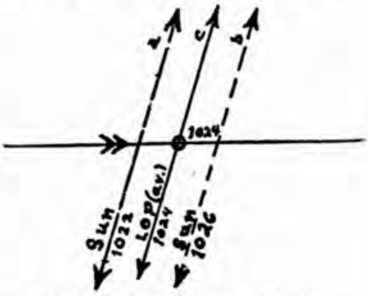


Fig. XIII. Average LOP.

When a single line of position is used and it disagrees with the DR position of the navigator, an average of the two is used at half the distance from the DR position to the line on a perpendicular from the DR position to the line. Such a position is called an EP (estimated position). The DR navigation is continued from this point.

Fig. XIV. Estimated position.

When only single lines are available, checks are made frequently (every 15 or 20 minutes) to insure the best possible positions.

To insure the finding of a destination after flying a considerable time with only single line of position available, a single line approach is made as follows: A plane has been flying on course 090 for target A for several hours with sun lines as the only aid to DR navigation. The navigator carefully estimates that the maximum error of his position may be as great as forty miles. While still some distance from A he shoots a sun line at 1340. A line parallel to the sun line is drawn through A as 1. On 1 he marks off 40 miles on one side of A as P. At 1350 he alters his course for P. When reaching P, he alters course again along 1. Since the probable error of the sun line is only ±5 miles, target A now lies within 5 miles of the course along 1 and under ordinary conditions should be visible. If the navigator under-estimates his maximum error, it is possible to turn away from the target on such an approach so adequate allowance must be made for accumulated errors when planning such an approach.

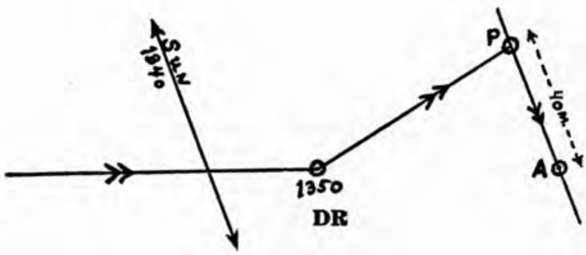
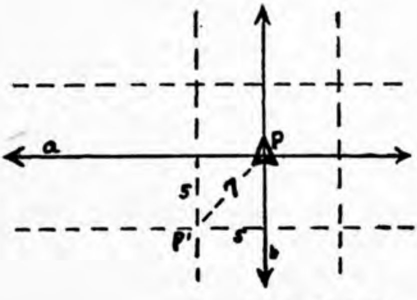


Fig. XV. Single line approach.

Lines of position from two different sources give more information than single lines. Such lines are chosen to intersect as near right angles as possible as the greater the deviation from a 90° angle of intersection, the greater the error in true position may be. For example, if lines of position, a and b, are each in error 5 miles, the determined position with 90° angle of intersection would not be more than 7 miles in error.



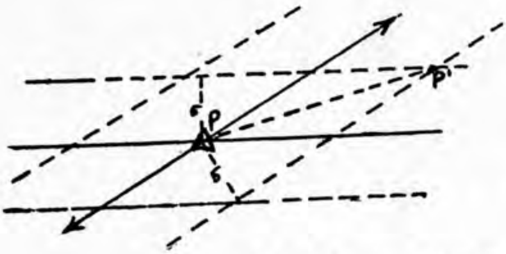
P-position determined

P'-actual position

PP'-error

Fig. XVII. Two LOP fix (30° intersection)

If angle of intersection were 30° instead of 90°, the maximum error would be considerably increased.



P-position determined

P'-actual position

PP'-error

Fig. XVII. Two LOP fix (30° intersection)

A three line of position fix is generally more reliable than a single or two line position.

If each line were accurate, all three would pass through the actual position and give a pinpoint fix.

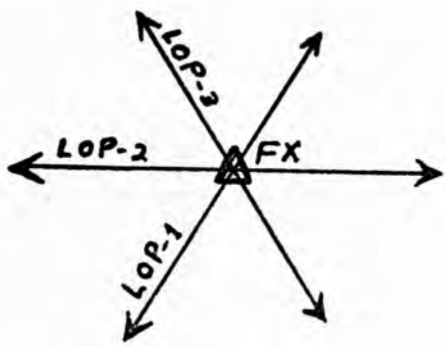


Fig. XVIII. Pinpoint Fix

However it is possible for all three lines to be in error and still pass through the same point so a pinpoint fix cannot safely be assumed to be an exact position.

Since any of the three lines may be in error, the navigator must assume them all equally good and the average position is taken as the fix position. The three lines will form a triangle and the incenter of the triangle equally distant from all three sides is used. This is normally found by bisecting the three angles. Since the information giving thes three lines originally is subject to error, actual construction of bisectors is a meaningless maneuver and visual selection of the center of the triangle is satisfactory. For a three line of position fix, the ideal condition is to have lines intersecting at angles of approximately 60°. For celestial lines, selection of stars at intervals of 120° will give such a triangle and tend to correct any constant error of instrument or observer.

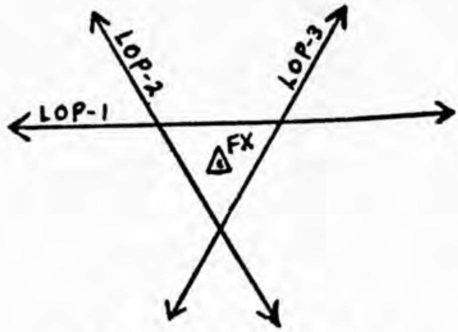


Fig. XIX. Three LOP fix.

Using more than three lines of position for a fix is possible but is not generally regarded as worth the extra time and trouble.

On long flights, a three line of position fix from Loran is usually plotted every 20 or 30 minutes. If the three lines are celestial lines, one such fix is usually plotted each hour. The errors of such fixes are not cumulative from one to the other.

Another safety measure for finding a target if it is not visible at the navigator's ETA (estimated time of arrival) is the square search. For a moving target, a relative square search is used with the pattern expanding relative to the movement of the target.

The navigator must consider all possible errors, try to compensate for them, and then finally make adequate provisions for safe arrival if his judgment in flight has been in error.

Educational implications. The approximate nature of measurement and the problems of computation with numbers derived from measurement should be stressed at all levels of mathematical training. Repetition of this important topic is essential to properly indoctrinate students with the idea that measurements are approximate and the accuracy of computation with such numbers is limited. The variability of such things as airspeed, drift, wind, and direction for a plane in flight and the difficulty of accurate measurements of these variables is easy to present. The need for estimating, for averaging, for constant checking can be seen with such variables more easily than with most other types of data.

The problems of aerial navigation can be used as illustrative material for this topic for all grades from seven through twelve and in college.

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