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A Probability Model for Strategic Bidding on The Price is Right

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Abstract

The TV game show The Price is Right features a bidding auction called "Contestants' Row" that rewards the player (out of 4) who bids closest to an item's value without overbidding. This paper considers ways in which the players can maximize their win probability based on their bidding order. We consider marginal strategies in which players assume their opponents are bidding their perceived value of the merchandise. Each player has available to them the information provided by the bids that preceded their own. We consider conditional strategies in which players adjust their bids knowing other players are using strategies. The last bidder has a large advantage in both scenarios due to receiving the most information from opposing players and being able to bid the minimal amount over an opponent's bid without incurring extra risk. Finally, we measure how a player's confidence can affect their winning probability.

Keywords: Auction; Marginal strategy; Normal distribution; Order statistics; Simulation study.

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1 Introduction

The Price Is Right (TPiR) is a well-known game show that has been running on American television since 1956. Over the course of 60 years, its hosts have included Bill Cullen, Bob Barker, and Drew Carey. The show features several games in which contestants from the audience compete to win prizes by guessing the retail price of some featured merchandise. In this paper, we concentrate on the "Items up for bid" segments, the most commonly played game in which four contestants from the audience guess the value of a piece of merchandise, and the bidder who is closest without going over the actual value wins the merchandise. This is a particular bidding scenario that will motivate players to consider underbidding in order to avoid having their bid invalidated, but also allows them to bid a value immediately above a previous bid if they feel that bid is under the value of the merchandise.

1.1 Background

Game theory has provided a helpful stochastic framework for auction bidding optimization [9] but studies do not consider settings such as this one where overbidding eliminates the player in that game. In general, most games are symmetric, where each player's chance of winning is determined by strategy. In this case, bid order imposes a unique asymmetry. Unlike first-price auctions or independent private value auctions, the information advantage increases from the first bidder to the last, while the overbid penalty affects all the players, but not equally.

While this article keeps the focus on the probability models that show us how to characterize and optimize player bids, Berk et al. [3] provided the first comprehensive study of bidding behavior on TPiR and considered simple rules for rational bidders. Based on basic principles of rational bidding, they found contestants' strategies appeared to be suboptimal. From bidding data recorded during the 1994 season of TPiR, their research shows unequivocally that it is advantageous to bid last, which may come at no surprise. Specifically, they found that all four players ended up overbidding in 13% of games, and in 42% of the games, all four players bid below the actual value. While the last bidder was the most frequent winner, they argue that all players failed to bid rationally nearly half the time.

Prior to this study, Bennett and Hickman [1] investigated data from Berk [2] and identified rational bids for the fourth player (bidding one dollar more than a previous bidder or bidding one dollar). They found suboptimal bidding occurred more frequently in earlier games and postulated that players learned how to bid optimally through game playing experience.

Estelami [4] expanded on the research of Berk et al. [3], and studied the impact of product-related factors on the players' understanding of different product categories. Healy and Noussair [5] conducted an experimental study that showed similar suboptimal bidding behavior to that found in Berk et al. [9]. Lee et al. [8] used the bids of the individual players to construct an aggregate bid that is superior to estimates of individual players. Holbrook [7] considers fundamental relationships between the bidding behavior of the players and the kinds of merchandise that is up for auction, and specifically the way television (TPiR in particular) affects that relationship.

Mendes and Morrison [10] present optimal strategies for symmetric games (each player has equal footing), including those where overbidding disqualifies the bidder. Although they consider a game that is similar to TPiR in one of their examples, it does not reflect the sequential bidding aspect of Contestants' Row, where more information is available to players who bid later. In this paper, our interest is solely on how the auction-winning probability is affected by different strategies of the four players, where the game is not symmetric.

1.2 Marginal versus Conditional Strategy

In Section 3, we introduce a marginal strategy for bidding. Marginal strategies assume each player maximizes their chance of winning without consideration of player bids that follow their own. While this strategy will likely be ineffectual (except for Player 4), it serves as a building block for more realistic strategies. We introduce conditional strategies in Section 4 for which players use information from previous player bids and also seek to maximize winning probability by considering possible bids by players that follow them. We outline the difference between the two strategies from Sections 3 and 4 in a simple example.

1.3 Example

Four players are bidding on Contestants' Row. The first player bids \$820, and the next player bids \$850. Suppose Player 3 believes the value of merchandise up for bid is \$800. What is the best bidding strategy for Player 3? If Player 3 uses a marginal strategy, they do not consider the potential bids by Player 4 and we will show the marginal strategy is to bid one dollar because it maximizes their chance of winning among the three bids, according to Player 3's belief. Of course, this bid is not optimal, because Player 4 can eliminate Player 3 from this auction by bidding two dollars.

The conditional strategy takes this into consideration. Player 3 can increase their chance of winning (according to their belief) by shrinking their bid below \$800. However, if they bid too far below that value, they increase the risk that Player 4 will bid a dollar more, which will greatly diminish their chance of winning. Player 3 must adopt a defensive strategy knowing Player 4 has this unique advantage. For this case (see Section 4), we use simulation to find that Player 3 maximizes their winning probability by bidding 4% below their perceived value. In this example, Player 3 would bid \$768.

1.4 Outline of Paper

Our primary interest is in the TPiR bidding model described above for the game called "Contestants' Row". However, in Section 2 we will first consider a simpler model with two bidders that will show the ramifications of the overbidding penalty. Section 3 describes the marginal strategies, and these serve as a foundation for the more-realistic conditional strategies described in Section 4. Some of the conditional strategies are determined through Monte Carlo simulation based on simple stochastic models that assume players generate independent merchandise assessments. In Section 5, we consider the effect of player uncertainty on bidding using a belief distribution.

We may treat each player's guess of the merchandise value as a random variable: let X_i be estimated value for the merchandise by Player i, where $i = 1, 2, 3, 4$. For all scenarios, we assume the actual unknown value of the merchandise is η and that

$V(X_i) = \sigma^2 < \infty$ for $i = 1, 2, 3, 4$. In scenarios in which Player i will bid some value that is possibly different from their belief X_i, we will denote the bid as b_i.

2 Two-Player Game

Suppose we have two players each with an independent guess at the true price η that is random, with $P(X_i \leq \eta) = 0.5$ for $i = 1, 2$. The guess that is closest to η wins the game (at this stage, we are not yet considering the TPiR penalty for over-bidding). We assume Player 1, with no extra information outside their own personal assessment, uses their guess X_1 for their bid ($b_1 = X_1$). Player 2 bids after Player 1.

Theorem 1. Let $\epsilon > 0$ be the closest Player 1 is allowed bid to X_1. Then Player 2 maximizes their chance of winning the game by bidding $b_2 = X_1 + \epsilon$ if $X_1 < X_2$, and will bid $b_2 = X_1 - \epsilon$ if $X_2 < X_1$. This strategy is consistent with "Proposition 1" for the behavior of a rational bidder in [3].

Proof. In the $3! = 6$ possible orderings of (X_1, X_2, η), Player 1 wins in just two of them: $X_2 < X_1 < \eta$ and $\eta < X_1 < X_2$. $P(X_1 > \eta, X_2 > \eta) = 1/4$ and given both variables are larger than η, $P(X_1 > X_2) = 1/2$. The same holds for $(X_1 < \eta, X_2 < \eta)$, so that $P(X_2 < X_1 < \eta) + P(\eta < X_1 < X_2) = 1/4$. □

The theorem does not hold without independence. For example, with $\eta = 1/2$ and joint density $f(x_1, x_2) = 4I(x_2 < x_1 < \eta) + 4I(\eta < x_1 < x_2 < 1)$, Player 1 wins every time. Consider the dependent case where X_1 and X_2 are distributed normally: $X_1 \sim N(\eta, \sigma^2)$, $X_2 \sim N(\eta, \sigma^2)$ and suppose that the bidders have (positively) correlated evaluations. If $Cov(X_1, X_2) = \rho\sigma^2$, we can use numeric integration to evaluate $P(X_2 < X_1 < \eta) + P(\eta < X_1 < X_2)$. If $\rho > 0$, the probability Player 1 wins goes from 1/4 up to 1/2 as ρ increases from zero to one. If $\rho < 0$, then by symmetry the probability that Player 1 wins decreases down to zero as ρ decreases to -1.

One may ask if Player 1 can increase the probability of winning by shifting their bid away from X_1 for the normal case. Figure 1 depicts the contours of the

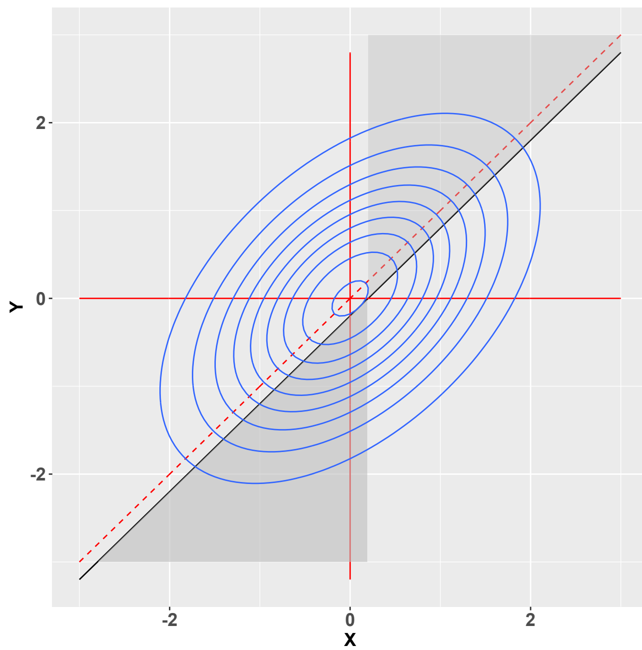

Figure 1: Two-bidder problem ($x = X_1, y = X_2$): what happens if Player 1 bids $X_1 + \delta$ (where $\eta = 0$ and $\delta = +0.2$ above) instead of X_1. Blue lines represent contours of joint density, and the shaded area shows the regions in which Player 1 wins the auction.

bivariate distribution for X_1, X_2 with correlation $\rho = 1/2$. The shaded area showing the regions in which Player 1 can win the auction are shifted away from the origin (0.2 units to the right in the figure), which necessarily decreases its probability for any bivariate normal distribution with positive (finite) variances and $|\rho| < 1$. That is, any shift in the bid from X_1 to the left or right will necessarily decrease the probability that is accumulated between the y-axis and the red-dotted line. This provides a simple graphical proof that there is no potential gain in bidding some amount either smaller or greater than X_1 (without knowledge of Player 2's bid).

Overbid Disqualification

If we incorporate the constraint that an overbid disqualifies the player, the optimal bid process is more interesting. We will first consider how to optimize the bid of Player 2, given that Player 1 bids at the assessed value. Theorem 2 below assumes players have an independent and identically distributed belief distributions which are not necessarily normal. In the theorem's condition, note that if X_1 and X_2 are

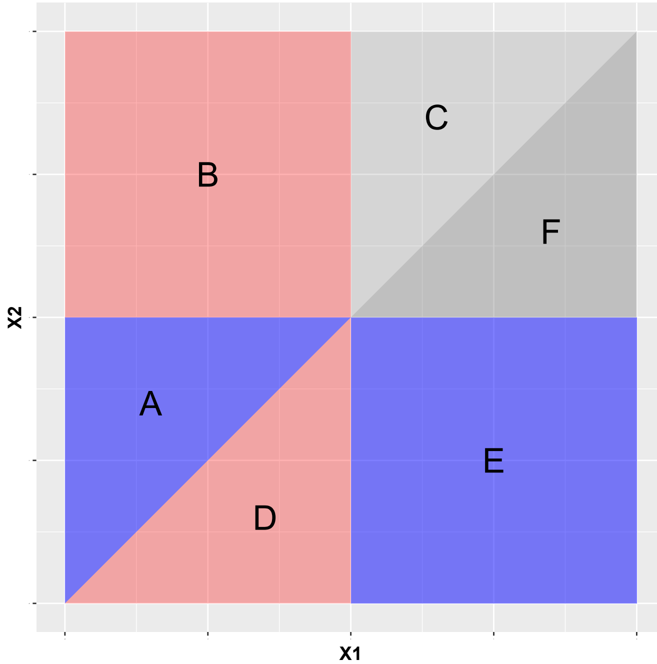

Figure 2: Joint bids at which Player 1 wins (B,D), Player 2 wins (A,E), or neither player wins (C,F)

independent, then $P(X_2 < X_1 < \eta) = 1/8$.

Theorem 2. Suppose η is the median value of the belief distribution for both players, and suppose $P(X_2 < X_1 < \eta)$ is bounded above by $1/4$. If Player 1 bids the assessed value of $b_1 = X_1$, then the optimal bid for Player 2 depends on whether X_1 is larger than X_2. If $X_1 < X_2$, then Player 2 should bid $b_2 = X_1 + \epsilon$. If $X_2 < X_1$, then Player 2 should bid $b_2 = \epsilon > 0$.

Proof. There are 3!=6 possible arrangements of X_1, X_2, and η. They are labeled on Figure 2 as

A. $X_1 < X_2 < \eta$	B. $X_1 < \eta < X_2$	C. $\eta < X_1 < X_2$
D. $X_2 < X_1 < \eta$	E. $X_2 < \eta < X_1$	F. $\eta < X_2 < X_1$

Let P_2 be the winning probability for Player 2. It can be shown that if $X_1 < X_2$ (cases A,B,C), then P_2 will increase (from $\mathrm{P}(A)$ to $\mathrm{P}(A \cup B)$) as Player 2's bid decreases from X_2 down to X_1. Once decreased below X_1, P_2 is zero, then increases to $\mathrm{P}(B)$ as Player 2's bid decreases to zero. In case $X_2 < X_1$ (cases D,E,F), P_2 increases from $\mathrm{P}(E)$ to $\mathrm{P}(E \cup F)$ as Player 2's bid decreases to zero. On the right, it is maximized at $X_1 + \epsilon$, where $P_2 = \mathrm{P}(D)$, which is strictly less than $\mathrm{P}(E \cup F)$ by assumption. $\square$

Normal Case: Suppose that Player 1 bids $b_1 = X_1$, where $X_i \sim N(\eta, \sigma^2)$, $i = 1, 2$ with $Corr(X_1, X_2) = \rho$. It is easy to show that if (X_1, X_2) have a bivariate normal distribution with zero mean, unit variance, and positive correlation coefficient ρ, then

$$\alpha = P(X_1 > 0, X_2 > 0) = \int_0^\infty \phi(t)\Phi(at)dt,$$

where $a = \rho/\sqrt{1-\rho^2}$, and (ϕ, Φ) are the density and cumulative distribution (respectively) for the standard normal distribution. If $\rho > 0$, this probability is bounded by $1/4 \leq \alpha \leq 1/2$ as ρ increases from zero to one. At $\rho = 1/2$, for example, $\alpha = 1/3$. If we consider the six possible orderings of X_1, X_2, and the actual value η, we see there is only one scenario (D) in where Player 1 can win if Player 2 uses the optimal strategy. If $\rho < 0$, then we replace α with $\tilde{\alpha} = 1/2 - \alpha$. For example, if $\rho = -1/2$, then $\tilde{\alpha} = 1/6$. The inequalities above still hold with $0 \leq \tilde{\alpha} \leq 1/4$.

3 Marginal Strategies for Four Bidders

The actual TPiR Contestants' row game involves four players, and in this section we will consider optimal strategies for each one. There is inherent advantage in bidding after the other players, but in this section, we focus only on optimizing each player's chance of winning under the assumption that the other players are bidding what they believe is the value of the item up for bid. That is, only one bidder exhibits strategy. Although this approach may not be directly applicable to most game show settings, where all four players typically use different strategies simultaneously, it will illuminate some important effects of strategy for each player. Players are allowed to bid within one dollar ($\epsilon = 1$) of a previous bid, and the smallest possible bid is \$1.00.

Theorem 3: Suppose that the player estimates (X_1, X_2, X_3, X_4) are independently generated from the same distribution with median equal to the merchandise value.

If players bid their perceived value, the most frequent winner of the contest will be the player with the second-smallest bid.

Proof. This result follows from binomial probabilities; for example, the probability that all four bids are disqualified is $(1/2)^4 = 1/16$. The probability that the smallest bid wins the auction is the probability that exactly three bids out of four are over the true value: $4(1/2)^4 = 4/16$. The probability that the second, third and fourth-smallest bids win, then, are $6/16, 4/16, 1/16$, respectively. $\square$

With only four players, we have $4! = 24$ possible orderings for (X_1, X_2, X_3, X_4). We will refer to any specific ordering using the short notation [1234], for example, to represent the scenario $X_1 < X_2 < X_3 < X_4$. With no other information besides the observed ordering, this particular scenario would imply Player 1 has a 0.25 chance to win, Player 2 has a 0.375 chance, and so on.

In this section we examine individual players exhibiting strategy (possibly bidding something other than their believed value based on the observed previous bids) in order to maximize win probability. Results for all the strategies in this section are based on changing one variable at a time, considering how the player's believed value compares to the other bids that have been observed by that time. For Player 2, there are two cases, three for Player 3, and four for Player 4. In some of the results that follow, the potential improvement gained by bidding a value other than X will be dependent on the distribution. In those cases, we treat (X_1, X_2, X_3, X_4) as multivariate normal with identical mean η, constant variance $V(X_i) = \sigma^2, i = 1, 2, 3, 4$ and $Cov(X_i, X_j) = 0$ for $i \neq j$. Later, we consider special models in which the correlation between X_i and X_j is $\rho \neq 0$, for $i \neq j$.

In most cases, we consider scale shifts for a player's bid because these allow the player to invoke both the mean and variance of their belief distribution. However, in some cases (e.g., X_i is below other players' observed bids) it is more helpful to consider a location shift that can be optimized in terms of variance alone (i.e., without the effect of the normal distribution location parameter).

Because only one player exhibits a strategy in each part of this section, we will denote a previous player's bid as x_i instead of b_i to emphasize this point. For example, when Player 3 uses strategy to optimize their probability of winning, that player observes the previous two bids as x_1 and x_2.

3.1 Marginal Strategy for Player 1

Player 1 has the least amount of information to use as strategy, so assuming the players that follow will bid at their assessed value (X_2, X_3, X_4), the only chance Player 1 has to modify their winning probability is to shift this first bid from their believed value of X_1. Without shifting, if the bids are uncorrelated, the winning probabilities for all four players are all $(\frac{15}{16})(\frac{1}{4}) \approx 0.234$ (because the chance they all overbid is $1/16 = 0.0625$). However, if Player 1 shrinks their bid to a fraction of what they believe the item is worth, it turns out they can improve their chances. Because the effect depends on the underlying distribution, we use the normal assumptions and consider a location shift.

Without loss of generality, suppose the other three bidders use belief distribution $X_i \sim N(0,1)$, $i = 2,3,4$, and Player 1 considers shrinking the bid by some $\delta > 0$, so the cumulative distribution function (CDF) of Player 1's bid is $\Phi_1(t) = \Phi(t + \delta)$. Given $X_1 < 0$, the conditional probability Player 1 wins by bidding $b_1 = X_1$ is equal to the probability $X_i \notin (b_1, 0)$, $i = 2,3,4$, i.e., $[1 - (\Phi(0) - \Phi(b_1))]^3$. By off-setting the bid by δ, Player 1 can achieve win probability

$$\int_{-\infty}^{0} [1 - (\Phi(0) - \Phi(y))]^3 \, \phi_1(y) dy = \int_{-\infty}^{0} \left[\Phi(y) + \frac{1}{2}\right]^3 \phi(y + \delta) dy.$$

It can be shown, numerically, that this win probability is maximized at $\delta = -0.712$, that is, by Player 1 bidding about 0.7 standard deviations under the believed value.

For Player 1 to implement such a strategy that depends on σ, they need to have knowledge about their uncertainty in evaluating the merchandise up for bid. Suppose the value of the merchandise is \$1000 and the belief distribution has standard deviation $\sigma = \$100$. In the optimal case in which the standard deviation is known, Player 1 will shrink their bid by \$71. Ascertaining σ^2 may not be straightforward, as it does not pertain to the player's uncertainty, but to the natural variability between independent bidders. Cause and effects regarding bidding uncertainty have been studied [11], but estimation of variability in a player's belief distribution is not investigated in this paper.

3.2 Marginal Strategy for Player 2

If the other players bid their believed value, it is not immediately clear what the optimal strategy should be for Player 2. If X_2 is just above x_1, it would make sense for Player 2 to bid $x_1 + 1$. If X_2 far exceeds x_1, this might not seem to be the best approach. On the other hand, if $X_2 < x_1$, it might be better to shrink the bid, especially if x_1 and X_2 are close in value. In this setting, compared to the realistic game show setting, there are fewer potential consequences for bidding one dollar because Player 3 does not have the option to bid one dollar more than Player 2. This will change for multi-player strategies in the next section. Here, we consider these two potential ways of improving the probability Player 2 wins: (a) shrink the bid toward 0 in the case $X_2 < x_1$, and (b) shrink back the bid toward $x_1 + 1$ if $X_2 > x_1$.

In the case $x_1 < X_2$, it can be shown that the optimal marginal strategy is to bid $x_1 + 1$. Given $x_1 < X_2$, suppose there exists $\theta \in (0, 1)$ that maximizes the win probability for the bid of $x_1 + \theta(X_2 - x_1)$. Then the conditional win probability is strictly decreasing as a function of θ for any belief distribution for which the median matches the true merchandise value. The proof is relegated to the appendix. Table 1 in the appendix shows how the probability of winning increases from $\frac{17}{96} = 0.177$ to $\frac{39}{96} = 0.406$ if Player 2 bids a dollar over Player 1 in this case.

In the case $X_2 < x_1$, the optimal shift depends on the belief distribution, which we assume is normal. It can be shown that Player 2 can optimize win probability by shifting their bid 0.439 standard deviations to the left of the perceived value. Win probability initially increases from $\frac{7}{48} = 0.146$, and then returns back to $\frac{3}{32} = 0.094$ as the bid approaches one dollar. The proof is detailed in the appendix, and relies on similar numeric integration used to prove the previous result.

3.3 Marginal Strategy for Player 3

Player 3 has more chances to optimize their bid given the available information from the first two bidders. If we denote those bids according to their order ($x_{1:2} < x_{2:2}$), then there are three cases in which we search for an optimal way to modify their believed value of X_3:

a. If $X_3 < x_{1:2}$, adjust the bid to $\theta_3 X_3$ for some value of $0 \leq \theta_3 \leq 1$.

b. If $x_{1:2} < X_3 < x_{2:2}$, find the optimal value in between the first two bids: $x_{1:2} + \lambda_3(X_3 - x_{1:2})$.

c. If $x_{2:2} < X_3$, find optimal bid value of the form $x_{2:2} + (X_3 - x_{2:2})\gamma_3$ for some $\gamma_3 > 0$.

In case (a), if X_3 is less than the previous bids, Player 3's winning probability is maximized by shrinking to the minimum (one dollar). Naturally, this only makes sense in the context of the marginal strategy, knowing Player 4 will not bid \$2 to maximize their own chance of winning. In the appendix, we show that leaving the bid as is, Player 3 wins in four equally likely scenarios (three in which X_3 is the smallest and one in which $X_4 < X_3$), so the probability of winning is $(\frac{3}{24})(\frac{4}{16}) + (\frac{1}{24})(\frac{6}{16}) = \frac{3}{32} = 0.047$. We show that if Player 3 bids $X_3 - \delta$, then winning probability increases with δ, up to $\frac{1}{8} = 0.125$, which is the probability the other three players overbid (and Player 3 will not overbid by bidding one dollar).

In case (b), when X_3 is in between the previous two bids, Player 2 maximizes the chance of winning by bidding $x_{1:2} + 1$. It is shown in the appendix that if Player 3 bids $x_{1:2} + \lambda_3(X_3 - x_{1:2})$, then the probability of winning is maximized at $\lambda_3 = 0$. At $\lambda_3 = 1$, the win probability is 0.104, but increases to 0.177 as λ_3 decreases to zero.

In case (c), we want to find the optimal amount to bid above the previous two bids. It turns out Player 3 will maximize winning probability in this case by bidding $x_{2:2} + 1$, which one might expect. With a bid of X_3, there are 2 out of 24 scenarios in which Player 3 will have the second highest bid, and 6 out of 24 with the highest bid, so the probability of winning would be $(\frac{6}{24})(\frac{1}{16}) + (\frac{2}{24})(\frac{4}{16}) = \frac{7}{192} = 0.036$. If Player 3 bids one dollar more than $x_{2:2}$, the second highest bid now has a 10/16 chance of winning. Depending on X_4, the probabilities of winning with the highest bid are either 6/16 or 1/4, and the probability of winning increases to $(\frac{2}{24})(\frac{10}{16}) + (\frac{4}{24})(\frac{4}{16}) + (\frac{2}{24})(\frac{6}{16}) = \frac{1}{8} = 0.125$. Using the same approach as case (b), it can be shown that the winning probability increases as Player 3's bid shrinks toward $x_{2:2}$.

3.4 Marginal Strategy for Player 4

In the fourth case, we assume the first three contestants bid their believed value (i.e., there is no strategy exhibited in the first three bids). There are no bids to follow, so this case is the easiest to directly optimize. Given the bidding behavior of the contestants, Player 4 might consider bidding just above the highest competitive bid that the player deems is not overbid (Proposition 1 from [3]). We will call this Strategy I. For example, if X_4 is the second-highest value of the four, then Player 4 will bid just above the third highest. Using this tactic, we can show Player 4 will win the game with probability 0.470.

Consider alternative naïve Strategy II where Player 4 simply bids just above the highest bid. Using Strategy II, the winning probability is reduced to 0.130. Note that Strategy II would be more successful in a setting in which the first three bidders use strategy. In that case they are likely to underbid more due to the severe penalty for overbidding. We can show, using the same approach as before, that Strategy I maximizes the probability of winning for Player 4. The proof follows directly from the proofs for the other marginal strategies in the appendix.

Figure 3 shows how the winning probabilities change as a function of (positive) correlation. At some point (near $\rho = 0.785$), if the contestants' guesses are correlated enough, the second strategy is actually better. However, as correlation becomes more negative, Strategy I winning probability increases (up to 0.5) and Strategy II probability further decreases.

3.5 Summary of Marginal Strategies

Four players generate independent random evaluations X_1, X_2, X_3, X_4 of merchandise valued at η, with $P(X_i < \eta) = 0.5$ and $V(X_i) = \sigma^2$. In each case below, one player (Player i) exhibits a strategy that allows them to bid a value other than X_i.

1. Assuming $X_i \sim N(\eta, \sigma^2)$, Player 1 maximizes winning probability by bidding $x_1 - 0.712\sigma$.

2. For Player 2, who observes the first bid as x_1, if $X_2 > x_1$, win probability is maximized by bidding $x_1 + 1$. If $X_2 < x_1$, Player 2 maximizes winning probability by bidding $x_1 - 0.439\sigma$ (based on normal assumptions).

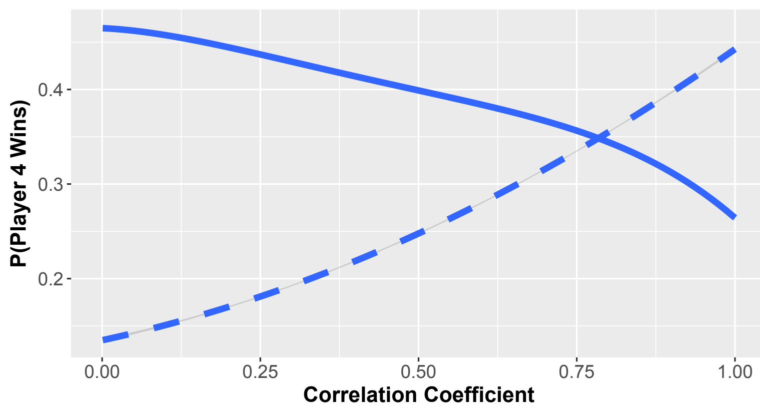

Figure 3: Probability Player 4 wins (as ρ goes from 0 to 1) for two strategies. The blue line is for Strategy I. The red line is for Strategy II. They have equal probability at ρ= 0.79.

3. For Player 3, who observes bids ordered $x_{1:2} < x_{2:2}$, probability of winning is maximized by bidding one dollar if $X_3 < x_{1:2}$. If $x_{1:2} < X_3 < x_{2:2}$, Player 3 should bid $x_{1:2} + 1$, and if $x_{2:2} < X_3$, the bid should be $x_{2:2} + 1$.

4. Player 4 maximizes win probability by bidding a dollar more than the largest bid under X_4, and by bidding a dollar if X_4 is smaller than the three previous bids.

4 Conditional Player Strategies

In this section we present a more practical assessment of the "Items up for bid" game on TPiR in which all players use strategies simultaneously. The marginal strategies presented in the previous section do not offer promising remedies for players to exploit, but they serve as a foundation to the more applicable bidding solutions presented here. In this case, we will use the empirical results from simulations to suggest the optimal bid for any player, when necessary. Previous bids are not necessarily the same as that player's belief, so they will be more correctly labeled b_i rather than x_i. For Player 4, the strategy will remain the same because there are no actions by the other players to follow the last bid. That is, the marginal strategy of Player 4 in the previous section is identical to the conditional strategy in this framework.

For the other players, we will consider different modifications of their believed value and search to find which bid turns out to be optimal *after* players that follow react to that bid. Player 3 will bid based on knowing the bids of Players 1 and 2, but we will determine their optimal bids based on how the auction turns out after all four players have bid. For example, we found that if $X_2 > b_1$, the best marginal strategy for Player 2 was to bid $b_1 + 1$. But if we base the bid choice on results in which all four players use strategy, Player 2 will bid more than $b_1 + 1$, as we might expect.

We consider the basic conditional strategy for all four players using the following approach:

(a) Player 4 uses the same marginal strategy that maximizes their winning probability based on observing the three previous bids.

(b) Player 3 will construct a bid based on observing the first two bids, but also based on which bid turns out to be most successful given the strategy exhibited by Player 4.

(c) Player 2 constructs a bid based on observing the first bid and optimizes it by considering various bids and finding which one maximizes winning probability given the strategies shown by Player 3 in (b) and Player 4 in (a).

(d) Player 1 adjusts their bid (away from their believed value X_1) according to simulation results that maximize their chance of winning, given the strategies of the three players that bid subsequently.

Each player's conditional strategy uses the actual bid of the previous players, so it does not matter if those other players are bidding their actual belief or if they are bidding a lesser value in order to avoid the risk of overbidding. We next consider the conditional strategies for the first three players, based on how modifying their bid changes winning probability in light of the player decisions that follow their bid.

4.1 Conditional Strategy for Player 3

We return to consider the optimal strategy for Player 3, now based on simulation results, given that Player 4 uses the optimal strategy described in Section 3. The set up is the same as before: there are three cases in which we can find a best way to modify their believed value of X_3 based on the other two ordered bids $(b_{1:2}, b_{2:2})$ observed by Player 3:

a. If $X_3 < b_{1:2}$, adjust the bid to $\theta_3 X_3$ for some value of $0 \leq \theta_3 \leq 1$.

b. If $b_{1:2} < X_3 < b_{2:2}$, find the optimal value in between the first two bids: $b_{1:2} + \lambda_3(X_3 - b_{1:2})$.

c. If $b_{2:2} < X_3$, find the optimal bid value of the form $b_{2:2} + (X_3 - b_{2:2})\gamma_3$ for some $\gamma_3 > 0$.

The previous (marginal) results were based on Player 4 bidding the believed value X_4. In this case, where Player 4 uses an optimal strategy, the results will be different.

The following results are based on 1,000,000 simulations. In case (a), if X_3 is less than the previous bids, then Player 3's winning probability is maximized at θ_3 = 0.963, so their best chance of winning will be to bid $b_3 = 0.963X_3$. In case (b), when X_2 is in between the previous two bids, Player 2 maximizes the chance of winning at $\lambda_3 = 0.249$, so the bid will be $b_{1:2} + 0.249(X_3 - b_{1:2})$. In case (c), Player 3 will still maximize winning probability in this case by bidding $b_{2:2} + 1$ (winning probability is a decreasing function of γ_3).

4.2 Conditional Strategy for Player 2

Next, we reconsider the optimal bids for Player 2, who will bid based on observing only the result of the first bidder. If we use the same framework as before, then we are looking for values of

a. If $X_2 < b_1$, change the bid to $\theta_2 X_2$ for some value of $0 \leq \theta_2 \leq 1$.

b. If $b_1 < X_2$, find the optimal value of γ_2 and bid $b_1 + \gamma_2(X_2 - b_1)$

Results are based on 1,000,000 simulations. For case (a), Player 2 increases the winning probability by shrinking the bid. In this case, simulation shows it is minimized at $\theta_2 = 0.975$, so Player 2 will bid 97.5% of their believed value if it is below the observed bid by Player 1. In the case $b_1 < X_2$, it is now more profitable to bid more than just over the previous bidder. Player 2 will maximize the overall winning probability by bidding $\gamma_2 = 0.345$ standard deviations over the first bidder.

4.3 Conditional Strategy for Player 1

Finally, when all the other players are using optimal strategies, Player 1 can maximize his or her chance of winning by bidding some value $b_1 = \theta_1 X_1$. Using the previous marginal strategy, we determined that the probability of winning was maximized by bidding about 0.71 standard deviations under the perceived value. In this scenario where the bidders following Player 1 will also adjust their bidding behavior, simulation shows Player 1 will maximize winning probability by bidding 97.5% of the believed value.

4.4 Simulation Results Based on Conditional Strategies

Given that the four players each produce independent random evaluations X_1, X_2, X_3, X_4 of merchandise valued at η, the best conditional strategies for each player are as follows:

1. Assuming $X_i \sim N(\eta, \sigma^2)$, Player 1 maximizes the win probability by bidding $0.975X_1$.

2. For Player 2, who observes the first bid as b_1, if $X_2 > b_1$, then the win probability is maximized by bidding $b_1 + 0.345(X_2 - b_1)$. If $X_2 < b_1$, then Player 2 maximizes winning probability by bidding $0.975X_2$.

3. For Player 3, who observes bids ordered $b_{1:2} < b_{2:2}$, the probability of winning is maximized by $0.963X_3$. If $b_{1:2} < X_3 < b_{2:2}$, then Player 3 should bid $b_{1:2} + 0.249(X_3 - b_{1:2})$, and if $b_{2:2} < X_3$, the bid should be $b_{2:2} + 1$.

4. Player 4 maximizes win probability by bidding a dollar more than the largest bid under X_4, and by bidding a dollar if X_4 is smaller than the three previous bids.

If all four players use conditional strategies, Player 4 still comes out with a large advantage, winning over half the time ($P_1 = 0.075$, $P_2 = 0.116$, $P_3 = 0.216$, $P_4 = 0.569$). These results are based on 10,000,000 simulations. We will compare and contrast the three different strategies discussed thus far in the final discussion section. In the next section, we consider how player uncertainty changes the strategy for an optimal bid.

5 Additional Player Strategies

The models from the last two sections illuminate how any player can aim to improve their probability of winning by modifying their bid according to the past bids they observe. In this section, we examine how the effect of player uncertainty (or confidence) affects the outcome of the auction. We first consider a group of four bidders, each with a distribution that describes their belief about the value of the item up for auction. If the true value is η, then the i^{th} bidder will value the item at $X_i \sim N(\eta, \sigma^2)$, so σ^2 represents the natural variability that reflects error in their personal assessment of the item up for sale. But the bidder also has a confidence in their bid which is characterized by another variance component δ^2; their belief is characterized by $Y_i \sim N(X_i, \delta^2)$.

For example, if the fourth bidder has observed three bids (b_1, b_2, b_3), they will attempt to optimize their bid by maximizing their chance of winning using their belief distribution. That is, they will choose b_4 so that they have the best chance of winning, assuming the true value of the item is X_4, but knowing they will not win the auction if they overbid, which is influenced by δ^2.

5.1 Example with Belief Distribution

Suppose there are four bidders, with $b_1 = 386$, $b_2 = 426$, and $b_3 = 502$. The fourth bidder has a belief distribution $Y_4 \sim N(420, 50^2)$. That is, $X_4 = 420$ and $\delta = 50$. Because they are the last bidder, their optimal bid will be 1, 387, 427, or 503. Recall with the naïve strategy, Player 4 would bid 503, and the optimal marginal or conditional strategy is a bid of 387.

According to Player 4's belief uncertainty, if they bid 1 dollar, then the probability they win is equal to the probability the true value of the item up for bids is below 418, which they believe is $P(Y_4 < 386) = 0.25$. If they bid 387, then they believe the probability they will win to be $P(386 < Y_3 < 426) = 0.30$. If they bid 427, $P(426 < Y_4 < 503) = 0.40$, and if they bid 503, $P(Y_3 > 502) = 0.05$. Using their uncertainty to optimize the bid, Player 4 believes they have the best chance of winning by bidding 427.

This example shows how the fourth player optimizes strategy according to their

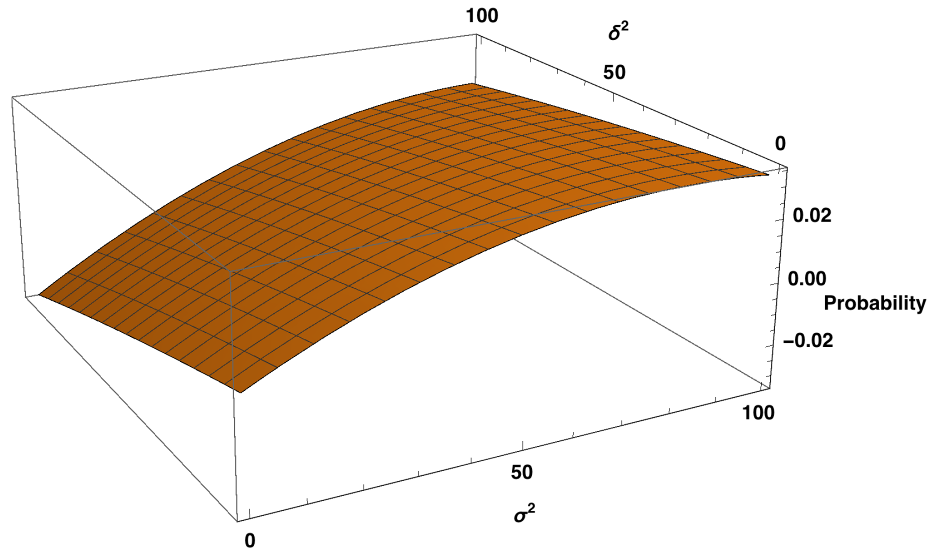

Figure 4: Win probability for Player 4 as a function of variability and uncertainty.

own belief system. But if that belief was also randomly generated (along with the beliefs of the other players), the true probability of winning the game might be different from 0.40 given $b_4 = 427$. Assume for now that the first three bids are relatively close to each bidder's evaluation. If $\eta = 510$ and $\sigma = 100$, then generating $(X_1, X_2, X_3, X_4) = (386, 426, 502, 420)$ is not an unreasonable Monte Carlo outcome, but despite Player 4's optimized game plan, Player 3 would be the auction winner.

Figure 4 shows how the information variance (σ^2) and belief confidence (with variance component δ^2) affects the probability that Player 4 wins. As σ^2 increases, Player 4's win probability increases. The less information garnered by the players allows Player 4 to have increasing leverage by bidding last. However, as the player's belief confidence decreases (δ^2 increases) the win probability starts to goes down, but is not strictly decreasing. In this example, Player 4 seems to do best when the variability between players is some fixed amount larger than the variability that characterizes player confidence.

The belief distribution could be used by the other players in order to increase their own win probability, including helping them in anticipating the next player's bid. As a simple example, suppose $X_3 = 420$, $Y_3 \sim N(X_3, 50^2)$, and the previous bids are $b_1 = 536$ and $b_2 = 551$. In this case, Player 3 need not be concerned about the previous bids, since they are deemed to grossly overvalue the merchandise ($P(Y_4 > 536) = 0.01$). Player 3 has the choice to bid one dollar, but at the obvious risk that Player 4 would bid \$2. Player 3 wants to find the optimal bid b_3 that anticipates the bid by Player 4, attempting to ensure Player 4 cannot achieve an overwhelming win probability by either bidding $b_4 = 1$ or $b_4 = b_3 + 1$.

Perhaps the best Player 3 can do is to assume Player 4 has the identical belief distribution, and therefore find a bid b_3 that splits $P(0 < Y_3 < b_3)$ and $P(b_3 < Y_3 < b_1)$. Assuming Player 4 is rational, they will choose based on which either one is maximized, so Player 3 optimizes them by making them close to equal, in this case around $b_3 = 420$. On the other hand, assumptions about your opponents' rationality may be unjustified! Perhaps entering a larger bid would avoid the asymmetric loss incurred if Player 4 chooses to bid $b_3 + 1$ despite having a higher expected gain bidding $b_4 = 1$. For example, Player 3 can still secure a 40% chance of winning by bidding $b_3 = 433$ and inducing Player 4 to bid one dollar.

6 Discussion

In this paper we have outlined optimal strategies for the bidding players on Contestants' Row during the game show "The Price is Right". The fundamental two-player strategies discussed in the introduction set the framework for constructing optimal marginal and conditional bids for the four-player game. We model player-to-player uncertainty using a random distribution with median equaling the merchandise value. Some of the optimal bidding strategies work for all possible distributions, but some results (e.g., when a player's bid is lower than a previous bid, but another player bids after that player) are distribution dependent, and we used the normal distribution to illustrate.

The asymmetry of the game leads to highly unequal outcome probabilities that greatly favor the last bidder. Figure 5 aggregates the outcomes for the three bidding schemes we considered: (a) the naïve strategy in which everyone bids their perceived value, (b) the marginal strategy, in which every player optimizes under the assumption that other players will bid their perceived value, and (c) the con-

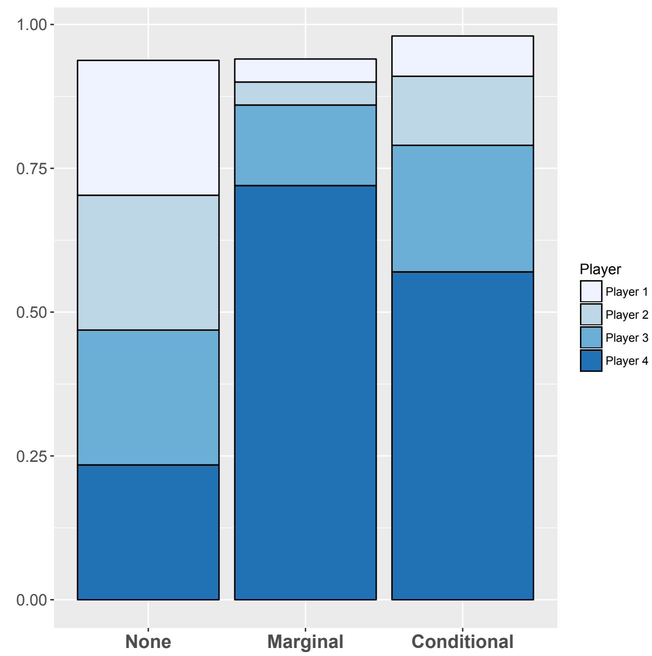

Figure 5: Player win probabilities for different strategies

ditional strategy, in which players try to optimize their bid based on simulation outcomes that rely on other bidders using strategy. The middle bar in Figure 5 shows what happens if every player uses a marginal strategy, ignoring the fact that other players are potentially bidding something other than their perceived value. Such a strategy is catastrophic for every player except the last bidder, as one might expect. While each player using conditional strategy works to maximize their own win probability, Figure 5 shows the probability of all players overbidding is also reduced 2.5 times (from 6.25% down to 2.5%).

In the previous section, we modeled player strategies based on their own acknowledged uncertainty in the value of the merchandise up for bid. It turns out that this additional uncertainty can significantly alter a player's bid strategy. This model framework with two variable components also allows other players to consider bids that reflect an opponent's uncertainty.

The asymmetric loss in this study does not represent typical auctions observed in business and industry, but it does provide further information about how rational bidders can adapt their bidding behavior to incorporate new auction constraints. While few studies have investigated the rational decision theory observed in TPiR since Bennett and Hickman [1] and Berk et al.[3], these results can provide a benchmark for future studies that study empirical results for asymmetric auctions, such as Estelami [4], Hendricks and Porter [6], and Perrigne and Vuong [11].

Although most of the results in this paper focus on players that generate independent assessments of the merchandise, the TPiR game set up is sure to induce correlations between bids. While this limits the dominion of our results, it also suggests an avenue for potential research. For example, data from TPiR episodes will not reveal the bidding intentions of the game-show players. A separate study would probably be needed to show bias in a player's perception of merchandise value. However, data may reveal effects of "anchoring" among player bids. Anchoring refers to the human tendency to rely too heavily on an initial piece of information, such as a previous bid, when making decisions. For example, we might conjecture that the second bidder is more likely to bid below the merchandise value if the first bidder grossly undervalues the item up for bid.

Overall, results show that significant gains can be won by players who take advantage of the unique rules of the "Items up for bid" segment on The Price is Right. It was relatively straightforward to show how the last player can optimize their chance of winning, but this paper showed that the other three players can also augment their bids to significantly increase their chance of winning.

Appendix

Player Two Marginal Strategy when $X_1 < X_2$

In the case $X_1 < X_2$, it can be shown the optimal strategy is to bid $X_1 + 1$. To prove this we consider the twelve equally likely orderings of the four bidders given $X_1 < X_2$. In half of those scenarios (e.g., [1234]) it is plain to see that the win probability for Player 2 is maximized if Player 2 bids the minimum amount larger than X_1. For example, with [1234], Player 2 already has a 6/16 chance of winning (the binomial probability that two of four bids are larger than the target value and two are less). By shrinking to $X_1 + 1$ (with no exhibited strategy from Player 3 and Player 4), that chance goes up 4/16 (the binomial probability that three bids are larger than the target and one is smaller), so that the probability of winning increases to 10/16.

Given $X_1 < X_2$, suppose there exists $\theta \in (0,1)$ that minimizes Player 2's win probability based on bidding $X_1 + \theta(X_2 - X_1)$. If we designate the order statistics for (X_1, X_2, X_3, X_4) as $X_{1:4} < X_{2:4} < X_{3:4} < X_{4:4}$, the twelve possible orderings are listed in the table below, along with the conditional win probabilities for Player 2 (P_0 = probability of winning with the original bid versus P_1 = probability of winning by bidding at $X_1 + 1$). The final column lists the conditional win probability for Player 2 as a function of θ, where Player 2's bid is $X_1 + \theta(X_2 - X_1)$. Aggregating the P_0 column (and dividing by 12) we find that the conditional probability of Player 2 winning, given $X_1 < X_2$, is 0.177. By bidding $X_1 + 1$ *instead* of X_2, that conditional probability increases to 0.406.

Table 1: Conditional win probabilities for Player 2 under the 12 equally likely orderings in which $X_1 < X_2$.

$X_{1:4}$	$X_{2:4}$	$X_{3:4}$	$X_{4:4}$	P_0	P_1	P(Win $\vert\theta$)
1	2	3	4	$\frac{6}{16}$	$\frac{10}{16}$	$4 + 6(1-\theta)$
1	2	4	3	$\frac{6}{16}$	$\frac{10}{16}$	$4 + 6(1-\theta)$
1	3	2	4	$\frac{4}{16}$	$\frac{4}{16}$	$4 + 4\theta\ln(\theta) - 6(1-\theta)\ln(1-\theta)$
1	3	4	2	$\frac{1}{16}$	$\frac{4}{16}$	$4 - 4\theta^2 + 8\theta\ln(\theta)$
1	4	2	3	$\frac{4}{16}$	$\frac{4}{16}$	$4 + 4\theta\ln(\theta) - 6(1-\theta)\ln(1-\theta)$
1	4	3	2	$\frac{1}{16}$	$\frac{4}{16}$	$4 - 4\theta^2 + 8\theta\ln(\theta)$
3	1	2	4	$\frac{4}{16}$	$\frac{10}{16}$	$4 + 6(1-\theta)$
3	1	4	2	$\frac{1}{16}$	$\frac{6}{16}$	$1 + 5(1-\theta) + 6\theta\ln(\theta) - 4(1-\theta)\ln(1-\theta)$
3	4	1	2	$\frac{1}{16}$	$\frac{5}{16}$	$1 + 4(1-\theta)$
4	1	2	3	$\frac{4}{16}$	$\frac{10}{16}$	$4 + 6(1-\theta)$
4	1	3	2	$\frac{1}{16}$	$\frac{5}{16}$	$1 + 5(1-\theta) + 6\theta\ln(\theta) - 4(1-\theta)\ln(1-\theta)$
4	3	1	2	$\frac{1}{16}$	$\frac{6}{16}$	$1 + 4(1-\theta)$

To compute conditional win probabilities for the bid $X_1 + \theta(X_2 - X_1)$ as a function of θ, we can assume $X_i \sim U(0,1)$ without loss of generality, so that the probability change is linear between X_1 and X_2. In the case X_3 and/or X_4 are between X_1 and X_2, we average over one uniform variable, or (in the case [1342], for example) we average over two order statistics $(U_{1:2}, U_{2:2})$. For example, in the case [1324], we first condition on $X_3 = t$ and find $P(Win|X_3 = t) = 4(1 - \theta/t)$ if $\theta < t$, and $P(Win|X_3 = t) = 4 + 6(1-\theta)/(1-t)$ if $t < \theta < 1$. Adding up the conditional probabilities in the last column (each weighted with 1/12 probability), we have

$$P(Win|\theta) = \frac{78 - 44\theta - 5\theta^2 - 2(1-\theta)(17-3\theta)\ln(1-\theta) + 6\theta(6+\theta)\ln(\theta)}{192}.$$

This probability is strictly less than 39/96 for $\theta > 0$ and converges to the original win probability (0.177) as θ increases to one.

Player Two Marginal Strategy when $X_2 < X_1$

Without loss of generality, suppose players use belief distribution $X_i \sim N(0,1)$, $i = 1,2,3,4$, and Player 2 considers shrinking the bid by some $\delta > 0$, so the CDF of Player 2's bid is $\Phi_2(t) = \Phi(t+\delta)$. Given $X_2 < X_1$, for Player 2 to win, we need $X_2 < \delta$, $X_1 > 0$, and both X_3 and X_4 not in the interval $(X_2 - \delta, 0)$. These last two conditional probabilities are denoted

$$Q_{21}(x_2) = 1 - [\Phi(0) - \Phi(x_2 - \delta)] = \Phi(x_2 - \delta) + \frac{1}{2}$$

when $x_2 < \delta$, otherwise $Q_{21}(x_2) = 0$. Including only the scenarios in which $X_2 < X_1$, the probability of Player 2 winning is

$$\int_{-\infty}^{\delta}\int_{\delta}^{\infty} Q_{21}(x_2)^2\phi(x_2)\phi(x_1)dx_2dx_1 + \int_0^{\delta}\int_{-\infty}^{x_1} Q_{21}(x_2)^2\phi(x_2)\phi(x_1)dx_2dx_1.$$

$$= \frac{1}{2}\int_{-\infty}^{\delta} Q_{21}(x_2)^2\phi(x_2)dx_2 - \int_0^{\delta}\left(\Phi(x_2) - \frac{1}{2}\right)Q_{21}(x_2)^2\phi(x_2)dx_2.$$

By numeric integration, it can be shown this function is maximized at 0.439.

Player Three Marginal Strategy when $X_3 < X_1 < X_2$

We may express $P(X_3 < X_1 < X_2) = \int_{\Omega_0(312)} f(\boldsymbol{x})d\boldsymbol{x}$, where $f(\boldsymbol{x}) = \prod_{i=1}^4 f(x_i)dx_i$ is the density function for $\boldsymbol{X} = (X_1, X_2, X_3, X_4)$ and $\Omega_0(312)$ is the set $-\infty < x_3 < x_1 < x_2 < \infty, -\infty < x_4 < \infty$.

Without loss of generality, let $X_i \sim N(0,1)$ for $i = 1,2,3,4$. If Player 3 offsets the bid by δ, then the probability Player 3 wins in this scenario is $P(\Omega_\delta(312)) =$

$\int_{\Omega_\delta(312)} \prod_{i=1}^4 f(x_i)dx_i$ where $\Omega_\delta(312)$ is the set

$$\{x_3 \in (-\infty, \delta)\} \cap \{x_2 \in \mathbb{R}^+\} \cap \{x_1 \in (0, x_2)\} \cap \{x_4 \in (-\infty, x_3) \cup (0, \infty)\}$$

With $\delta = 0$, the probability Player 3 wins with this particular order can be deduced using binomial probabilities (as before). There are 3 out of 24 equally likely orderings in which x_3 is the smallest bid out of four, and one out of 24 in which x_3 is the second smallest [4312], so the probability of winning would be $(\frac{3}{24})(\frac{4}{16}) + (\frac{1}{24})(\frac{6}{16}) = \frac{3}{32} = 0.047$. It is also easy to show that $P(\Omega_\delta(312))$ is increasing in δ, and levels off at $\frac{1}{8} = 0.125$, which is the probability the other three players overbid.

Player Three Marginal Strategy when $X_1 < X_3 < X_2$

If $\theta \in (0,1)$, we consider the bid $X_1 + \theta(X_3 - X_1)$. If $\theta = 1$, and Player 3 uses X_3 as the bid, then there are four out of 24 scenarios in which X_3 is the second smallest bid (win probability $= 6/16$) and four in which X_3 is third smallest (win probability $= 4/16$), so Player 3's win probability in this case is $(\frac{4}{24})(\frac{6}{16}) + (\frac{4}{24})(\frac{4}{16}) = \frac{5}{48} = 0.104$.

In this case, it can be shown that the win probability increases as θ decreases to zero. Using the same assumptions as in the previous proof, let $\Omega_\theta(132)$ be the set of bids in which Player 3 wins. Then $\Omega_\theta(132)$ is defined as

$$\{x_1 \in \mathbb{R}^-\} \cap \{x_2 \in \mathbb{R}^+\} \cap \{x_3 \in (x_1, \frac{-(1-\theta)x_1}{\theta})\} \cap \{x_4 \in (-\infty, \frac{-(1-\theta)x_1}{\theta}) \cup (0, \infty)\}.$$

Because $g(t) = (1-t)/t$ is strictly decreasing in $0 < t < 1$, this area is strictly increasing as θ decreases from one to zero.

Specifically, Figure 6 shows how we integrate over more area (for X_1, X_3) as θ decreases from one to zero, where the entire quadrant $x_1 < 0, x_3 > 0$ is included. As θ approaches 0, and Player 3 bids the minimum amount over X_1, the probability of winning increases to $(\frac{6}{24})(\frac{10}{16}) + (\frac{2}{24})(\frac{4}{16}) = \frac{17}{968} = 0.177$.

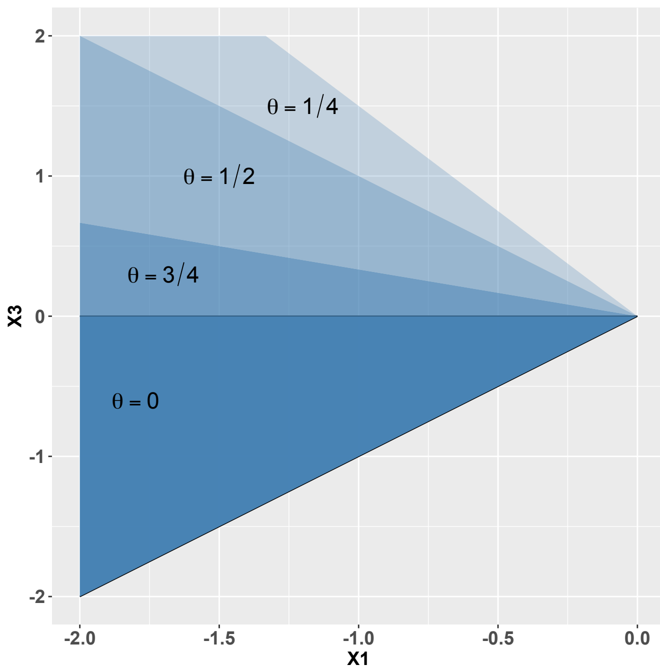

Figure 6: Increasing domain in (X_1, X_3) as θ goes from 1 (dark blue) to 3/4, 1/2, and 1/4 (lightening shades of blue)

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