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Abstract

To estimate power plant reliability, a probabilistic safety assessment might combine failure data from various sites. Because dependent failures are a critical concern in the nuclear industry, combining failure data from component groups of di®erent sizes is a challenging problem. One procedure, called data mapping, translates failure data across component group sizes. This includes common cause failures, which are simultaneous failure events of two or more components in a group. In this paper, we present methods for predicting future plant reliability using mapped common cause failure data. The prediction technique is motivated by discrete failure data from emergency diesel generators at U.S. plants. The underlying failure distributions are based on homogeneous Poisson processes. Both Bayesian and frequentist prediction methods are presented, and if non-informative prior distributions are applied, the upper prediction bounds for the generators are the same.

Key Words: Bayesian Prediction, Poisson Distribution, Upper Prediction Bounds

1 Introduction

External events that can cause the simultaneous failure of a network of components are called common cause failures (CCFs). These failure events play a crucial role in maintaining high standards of system reliability for nuclear power plants. Because CCFs occur only rarely in a nuclear power plant's day to day operations, simultaneous failure data are sparse and difficult to quantify for probabilistic safety assessments (PSAs). This can be a serious consequence in some plant assessments because high target reliabilities are sometimes achieved by building redundancy into a system; e.g., if an emergency diesel generator (EDG) does not meet target reliability for the plant's safety goals, one or more additional generators might be brought in to provide stand-by backup, in case the main EDG fails to start or run properly. Obviously, the potential improvement due to built in redundancy can be negated if the system of EDGs is susceptible to common cause failures.

The size of the system under study (i.e., the number of components in the system) can actually affect the common cause failure probabilities for various reasons, so problems exist if the size of the system under study is different from groups represented in the available CCF data. If such a difference exists, one might choose to translate the existing CCF data so it appears as if the size of the system represented in the data is the same as the size of the system under study in the PSA. This procedure, called mapping, is not uncommon in reliability studies for power plants. See Mosleh, et al. [10], for example.

As an example, suppose the PSA includes plans for a working set of two EDGs, but existing failure data exist from a nuclear power plant in which three EDGs are used to back up the existing

power supply. In the event that two out of three EDGs failed to start in a test at that plant, it is logged as a common cause failure with three items on test and two failing simultaneously. If we imagine only two of the three EDGs were chosen to represent this group (matching the number in the PSA), then one or two failures would have occurred, depending on which EDGs were selected. If the EDGs were chosen randomly, then the probabilities of recording one or two failures are 2=3 and 1=3, respectively.

This is the essence of data mapping, but general procedures for mapping data are not so straightforward. In the case where the data originate from a common cause group larger than the group in the PSA, mapping equates to a simple application of the hypergeometric distribution. We call this mapping down. However, if the PSA group is larger, the mapping procedure is essentially extrapolating, and more information is needed to make the transformation sensible. This is called mapping up. Problems in mapping data were ¯rst recognized by Doerre [4] and Poucet, et al. [12]. Mapping algorithms for small common cause groups are listed in Chapter 3 of Mosleh, et al. [10] and more re¯ned mapping techniques are outlined in Kvam [7].

A primary goal of the PSA is to predict the frequency of core damage in future operations at the power plant. The future number of component failures is more important to the PSA than the parameters that characterize their failure distributions, which are arguably meaningless by themselves, especially if lifetimes are modeled with common reliability distributions such as Weibull, Log-normal or Inverse-Gamma. Additionally, one might in practice be tempted to substitute parameter estimates in the failure distribution and to then use the resulting estimated distribution for predictive inferences. Though intuitively appealing, this specious approach ignores uncertainty related to the parameter estimate.

Rather than drawing inference on these distributions that underlie common cause failures,

prediction regions for future simultaneous failures should be a factor in PSA. In this paper, we rely on mapped CCF data to make predictive inference (e.g., upper prediction bounds) on the number of future failures at a power plant. Although some exact prediction bounds can be derived for simpler reliability problems (see Kvam and Miller [8], for example), in general no closed form solutions exist.

2 Prediction of CCF events

For data from operating power plants, failures are aggregated over ¯xed time intervals. The failure rates of most components in the PSA are extremely low and stay relatively constant between maintenance operations. Because of this, analysts commonly model the failure times with homogeneous Poisson processes. Statistical prediction with Poisson data is elementary; if $Y \sim P(\lambda)$, then $\hat{Y}_{new} = \hat{\lambda}$, where the hat-notation represents an estimate. Prediction uncertainty is not as easy to obtain. We consider the following case from Kvam and Miller [8].

From a homogeneous Poisson process with unknown recurrence rate $\mu_1 > 0$, let X represent the number of occurrences in time interval $(0; T_1)$, and denote Y the future number of occurrences of an independent process with unknown recurrence rate $\mu_2 > 0$ in the time interval $(T_1; T_1 + T_2)$. It is our goal to predict $Y \sim \text{Poisson}(\mu_2 T_2)$ using $X \sim \text{Poisson}(\mu_1 T_1)$, assuming the processes have identical recurrence rates; i.e., $\mu_1 = \mu_2$. Our solution approach is best understood in terms of a related hypothesis test. Consider the test based on $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_1 < \mu_2$. The rejection region for the α-level test of H_0 has form $\{(x; y) : y > U(x)\}$, where $U(x)$ serves as a $(1 - \alpha)$ upper prediction bound (UPB) for Y, given X. Lower prediction bounds are constructed similarly by switching the inequality in the alternative test hypothesis.

Under H_0, the conditional distribution of X given $N = X + Y$ is distributed binomial with parameters $N = n$ and $p = T_1/(T_1+T_2)$, thus $X|N = n$ forms a pivotal quantity, and we can use this binomial distribution to construct exact prediction intervals for Y. Because exact $1 - \alpha$ confidence levels are generally not achievable with discrete random variables, the prediction intervals will have confidence levels that are no less than $1 - \alpha$. The rejection region for the one-sided hypothesis test consists of all pairs $(x; y)$ for which y is considered too large relative to x. For a fixed α, the lower bound of this rejection region is the smallest value of y such that $P(Y \geq y|X + Y = n) \leq \alpha$. If we denote this lower bound as $y^* + 1$, then y^* is the upper bound of the acceptance region for the test. Since the size of the rejection region does not exceed α, the probability measure of the acceptance region can be no less than $1 - \alpha$. That is, under the null hypothesis, $P(Y \leq y^*) \geq 1 - \alpha$ which holds for all $\mu = \mu_1 = \mu_2$. (It is important to keep in mind that both Y and y^* are random variables in the probability expression.) Since $y^* + 1$ is the smallest value of y in the rejection region, $[0; y^*]$ is the narrowest interval such that $P(Y \in [0; y^*]) \geq 1 - \alpha$. Thus, from a frequency theory viewpoint, we can take y^* as a $1 - \alpha$ UPB on Y given $X = x$ has been observed.

As a simple example, suppose that in one (reactor) year, a group of containment cooling fans at a power plant experiences no failures or significant degradation during continuous service. The competing risks for cooling fans might include broken blades, fan belt slippage and loss of motor-bearing lubrication which causes degradation and low flow. We can form a 95% UPB for the number of future failures experienced by the cooling fans in the next (reactor) year if we assume the failure rate remains the same (that is, the Poisson process for failure events is homogeneous). If we denote $X = 0$ as the number of observed failures and Y as the (Poisson distributed) number of future failures, the pivotal statistic is distributed binomial with $n = 0 + Y$, and $p = T_1/(T_1 + T_2) = 0.5$. Because $P(Y \geq 4|X + Y = 4) = 0.0625 > \alpha$ and $P(Y \geq 5|X + Y = 5) = 0.0313 \leq \alpha$, $y = 5$

is the lower bound of the rejection region which means we take $y^{\alpha} = 4$ to be the 95% UPB.

Table 1 below lists 95% upper prediction bounds for future (Poisson) failure counts given we observe x failures in a past time interval of the same length. Other prediction methods for PSA are described in detail in Kvam and Miller [8]. General methods for standard prediction limits are described further in various sources for the reliability practitioner, including Nelson [11] and Cox and Hinkley [3], where quicker, approximate methods are usually prescribed. A normal approximation, presented in Chapter 6 of Nelson [11], is not complicated and sometimes effective for this Poisson prediction problem. For example, the $(1 - \alpha)$-UPB for Y in the case $T_1 = T_2$ is $x + z_{1-\alpha}\sqrt{2x}$, where z_q is the q^{th} quantile of the standard normal distribution. However, the bounds are not accurate unless the mean values $\mu_i T_i$ are large, which is uncommon with typical CCF analyses.

3 Prediction with mapped data

In this paper, we consider the problem of predicting future CCF frequency for a system of components by using mapped failure data from similar systems of equal size or larger. The failure counts are from an aggregated data set involving simultaneous failures of EDGs in domestic (U.S.) power plants. The failure data for CCF events between 1980 and 1995, listed in a report for the U.S. Nuclear Regulatory Commission, IEEL [5], are summarized in Table 2 below. Failures include natural disasters, shared design flaws, or machine maintenance errors, to name a few. The database also lists incipient failures, which include observations where a single diesel failed, but it was determined that its cause of failure could potentially cause the failure of other EDGs in the near future. In such cases, these potential failures are conservatively listed as actual CCFs. Component group sizes range from 2 to 5. Each group size refers to one or more nuclear power plants, and they

are not considered identical.

To show how prediction bounds are generated with mapped data, we assume the PSA is for a group of two EDGs, so the data will be mapped down (or not at all). The consequences of extrapolation due to mapping up data present a serious problem to PSA in cases where parametric failure models cannot be justified; see, for example, Kvam [7].

The time on test for each component group is not available for public disclosure. As a consequence, we do not investigate the overall failure frequencies for the different systems. Instead, we naively assume the rates are identical for every group (this is almost certainly not the case) and assume the total time on test for the group of systems with k components is T_k reactor years. For illustrative purposes, we will seek a 95% and 99% UPB for a future ($T = 1$) reactor year. Any future CCF will refer to the simultaneous failure of both EDGs from this group of two diesels in the PSA.

Each failure frequency can be mapped to an expected number of CCFs of both EDGs using the mapping technique described in the last section. In general, suppose we have observed failure data for a group of size m that we wish to map to a PSA group of size $k < m$. Using the method described in Kvam [7], denote the observed data with the $m \times 1$ impact vector $\mathbf{y}^{(m)}$ and the estimated or mapped expectations with the $k \times 1$ vector $\hat{\mathbf{y}}^{(k)}$. We can then take

$$\hat{y}_j^{(k)} = E\left[y_j^{(k)} | \mathbf{y}^{(m)}\right] = \sum_{r=1}^{m} C_{j;r}\, y_r^{(m)},$$

for $j = 1, \ldots, k$ where

$$C_{j;r} = \frac{\binom{j}{0}\binom{k-j}{1}}{\binom{m}{k}} I(j \le r \le m - k + j).$$

The third column in Table 2 lists these expected CCFs based on the mapping procedure, which sum to 35.54. In the ¯rst row, no data transformation is necessary, but in the next three rows, where common cause group sizes are (3,4,5), the data are mapped down to the PSA group of size two using hypergeometric probabilities. For example, a multiple failure of two EDGs in a group of four is mapped to (2=4)(1=3) = 1=6, so from 10 such failures, we expect 10=6 = 1:67 failures of both EDGs from an equivalent group of size two.

From the mapping procedure, the estimated number of failures is a simple linear combination of Poisson random variables. If the failure counts are independent, it is noted in Kvam [7] that the variance of the linear estimate is less than the actual Poisson variance based on an equal failure rate parameter. Obviously, to make prediction bounds for the future failure estimate, the simple Poisson prediction interval method described in the last section is inadequate for a linear estimator that is not distributed as Poisson.

To compute an upper prediction bound, mapped failure counts cannot be used in a practical way. Instead, we will manipulate the time on test and ¯x the number of failures to what was observed in the data. For Poisson distributed data, this has intuitive appeal because of the constant rate of the Poisson process that generates the failure data. That is, a process with rate μ measured over time span T_a is equivalent to a process with rate $T_a\mu=T_b$ measured over time span T_b. For mapped data we can use this to conclude that mapping $y_r^{(m)}C_{j;r}$ common cause failures to the interval T_k is equivalent to mapping $y_r^{(m)}$ failures to the interval $T_k=C_{j;r}$.

To illustrate this mapping procedure, consider the ¯ve observed simultaneous failure events involving 2 out of 3 components listed in Table 2. In the same time frame, the expected number of future CCFs would be 5/3. Equivalently, for a group of two EDGs, we would expect ¯ve such CCFs in a time interval three times larger than the time frame used for the systems with three

components (i.e., $T = 3T_3$). Rather than aggregating the expected number of CCFs based on the observed time on test, we sum the expected time on test (implied by the mapping procedure) required to observe the actual number of simultaneous failures.

To predict future failures using the mapped test times, we count 54 failure events in the total time aggregated from the ¯fth column of Table 3:

$$T = T_2 + 4T_3 + 9T_4 + 16T_5:$$

The predicted number of CCFs equals 54/T, which is the same answer one would get with the mapping procedures described in Mosleh, et al. [10] and Kvam [7]. More importantly, an upper prediction bound on the future number of CCFs can be calculated using the method described in Section 2.

For example, suppose $T_2 = 75$ reactor years, T_3 and $T_4 = 50$ reactor years, and $T_5 = 10$ reactor years (these are completely ¯ctitious). Then T = 885 reactor years, and we predict 0.0610 CCFs per year for the system of two EDGs. Note that if we used only data from two-component systems, we predict 14/75 = 0.1867 CCFs. A 99% UPB for this point estimate, based on T = 885, is equal to one (the 95% UPB is also equal to one). For this example, the approximations in Chapter 6 of Nelson [11] yield the same answers.

4 Bayesian Prediction

Up to this point, we have presented the frequentist viewpoint for predictive inference with mapped data. In many probabilistic safety assessments, Bayesian methods have been preferred over frequentist methods, in part because Bayesian methods allow a convenient avenue for the input of expert

opinion into the data analysis. Unlike the frequentist prediction procedure, the Bayesian approach to the prediction problem is straightforward. Given the density $f(x|\mu)$ of the observed failure counts and the prior distribution $\pi(\mu)$ that characterizes our uncertainty about the parameter, the updated posterior distribution for the unknown parameter μ is computed as

$$\pi(\mu|x) = \frac{\pi(\mu) f(x|\mu)}{\int_{\Omega} \pi(\mu) f(x|\mu) d\mu},$$

where Ω represents the parameter space for μ. The predictive density of a new observation Y from $f(x|\mu)$ is then defined as

$$P_{Y|X}(y|x) = \int_{\Omega} f(y|\mu)\pi(\mu|x) d\mu.$$

The posterior $\pi(\mu|x)$ serves as a mixing distribution which combines the updated parameter uncertainty with the original prediction uncertainty.

For Poisson data, if we specify a Gamma prior (integer shape parameter = r, scale parameter = γ) for the failure rate parameter μ, it is possible to show the predictive density of Y (given x failures) is negative-binomial with parameters $r + x$ and $p = (\gamma + T_x)/(\gamma + T_x + T_y)$, where T_x and T_y are test times, respectively, for the observed and predicted number of failures. If we choose a more general Gamma prior in which r can be non-integer, the predictive distribution is Poisson-Gamma, which can be found in Section 8.3 of Johnson, Kotz and Kemp [6]. In this case the Poisson-Gamma parameters are $r + x$, $\gamma + T_x$ and T_y. Of course, other reasonable prior distributions can be chosen for μ; the Gamma distribution is emphasized here only because it is a conjugate family for the Poisson distribution; that is, by using a Poisson likelihood with a Gamma prior, the posterior distribution of μ is also Gamma.

Unlike prediction intervals outlined in the previous section, one-sided Bayesian prediction bounds are made simply by using appropriate percentiles from the predictive density. Two-sided prediction intervals are constructed in the same manner as regular Bayesian credible sets. If one can apply prior information with a sensible conjugate prior distribution, the resulting Bayesian analysis is simple and analytically convenient. However, nonconjugate priors are sometimes more appropriate, including in PSAs of domestic nuclear power plants where log-normal distributions are sometimes used for Poisson rate parameters. Without conjugacy, we lose the simple and elegant solutions for predictive densities, but given available computing algorithms (i.e., Markov chain Monte Carlo, numerical integration methods), the selection of a prior from the more general class of proper distributions no longer precludes the feasibility of Bayesian inference.

With little or no prior information about μ, a noninformative prior distribution can be used to represent our knowledge of μ. For the Poisson case, μ is a scale parameter, which has a Je®ries noninformative prior distribution of $¼(\mu) = 1=\mu; \quad \mu > 0$. See Berger [1] for further explanation of noninformative priors, including Je®ries priors. Although $¼(\mu)$ is not a proper distribution, as long as at least one failure is observed (i.e., $X \geq 1$), the resulting predictive probability density of Y is negative binomial with parameters x and $p = T_x=(T_x + T_y)$. However, if we observe no failures in the observed data, the improper prior will produce an improper predictive distribution for Y. This can limit Bayesian methods in safety assessments for which zero-failure experiments are common.

In our example, if we choose an improper Bayesian prior distribution, the predictive density is Negative Binomial with $x = 54$ and $p = 885=(885 + 1)$. The 95% UPB, based on the 95^{th} percentile of the predictive distribution, equals 1. The 99% UPB also equals 1, which means we are over 99% con¯dent of observing no more than one CCF in the upcoming reactor year, despite observing 54 in the data. In the case in which noninformative priors are applied, the frequentist and Bayesian

methods produce identical answers.

5 Discussion

Mapping procedures allow practitioners to combine different sources of data without making strong assumptions about the failure mechanisms and their corresponding probability distributions. In this case, we avoid having to characterize the simultaneous failures with a parametric model, such as the binomial-failure rate model suggested by Vesley [13]. Unlike past mapping procedures, we suggest transforming the test time instead of the failure frequency. Although the point estimation method is the same, upper prediction bounds can be more easily generated from the mapped data.

Our analysis is more intricate if we consider the non-homogeneous failure environments at the different power plants where failure data was solicited. Although the uncertainty associated with plant to plant variability has an effect on estimated and predicted reliability, we have set this up as a separate problem. Complexities of combining plant data to estimate component reliability was approached with an empirical Bayes framework in Martz, et al. [9], and a similar problem was worked out using a hierarchical Bayes approach in Chen and Singpurwalla [2].

Predictive inference is not a standard approach in the engineering industry, but upper prediction bounds tend to have more credence than confidence statements about unknown parameters or distribution functions. Clearly, prediction intervals have a place in probabilistic safety assessment. An account of needs and methods for predictive inference with discrete data (e.g., Poisson, Binomial) can be found in Kvam and Miller [8].

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Table 1.

failures	0	1	2	3	4	5	6	7	8	9
95 % UPB	4	6	8	9	11	12	14	15	17	18
failures	10	11	12	13	14	15	16	17	18	19
95 % UPB	19	21	22	23	25	26	27	29	30	31
failures	20	21	22	23	24	25	26	27	28	29
95 % UPB	32	34	35	36	37	39	40	41	42	44

95 % Upper Prediction Bounds for Poisson Events.

Table 2.

Group Size	Impact Vector	Expected Failures
2	(17,14)	(0,14)
3	(9,5,6)	(0, 1.67, 6)
4	(11,10,7,6)	(0, 1.67, 3.5, 6)
5	(2,2,1,2,1)	(0, 0.2, 0.3, 1.2, 1)

Common Cause Failure Data for Emergency Diesel Generators.

Table 3.

type of observed multiple failure	frequency	time on test	P(CCF)	translated time on test
1 out of 2	17	T_2	0	
2 out of 2	14	T_2	1.0	T_2
1 out of 3	9	T_3	0	
2 out of 3	5	T_3	1/3	$3T_3$
3 out of 3	6	T_3	1.0	T_3
1 out of 4	11	T_4	0	
2 out of 4	10	T_4	1/6	$6T_4$
3 out of 4	7	T_4	1/2	$2T_4$
4 out of 4	6	T_4	1.0	T_4
1 out of 5	2	T_5	0	
2 out of 5	2	T_5	1/10	$10T_5$
3 out of 5	1	T_5	3/10	$10T_5/3$
4 out of 5	2	T_5	3/5	$5T_5/3$
5 out of 5	1	T_5	1.0	T_5

Transformed time on test for EDG failure data.