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Abstract

In estimating the reliability of a system of components, it is ordinarily assumed that the component lifetimes are independently distributed. This assumption usually alleviates the difficulty of analyzing complex systems, but it is seldom true that the failure of one component in an interactive system has no effect on the lifetimes of the other components. Often, two or more components will fail simultaneously due to a common cause event. Such an incident is called a common cause failure (CCF), and is now recognized as an important contribution to system failure in various applications of reliability. We examine current methods for reliability estimation of system and component lifetimes using estimators derived from the binomial failure rate model. Computational problems require a new approach, like iterative solutions via the EM algorithm.

Introduction

Simultaneous failures of components due to the same cause or initiating event are called common cause failures. Industries that require low risk and high reliability, like nuclear power plants, depend on highly reliable components and redundancy built into the system to maintain a high overall reliability. In this setting, common cause failure is an important contributor to risk, since the advantages of redundant component configurations can be negated by a single common cause event. Examples of such events include natural disasters, like earthquakes or lightning strikes that can fail an entire group of components that were designed to work independently. Components that inherit the same design flaw may also be stochastically dependent if a common cause event exploits the flaw by failing the group simultaneously.

Formulation and analysis of CCF models are essentially rooted in statistics, but for the most part, common cause problems have been overlooked by the statistics community. Marshall and Olkin (1967) derived a multivariate exponential model with an added "shock" variable that allows dependence between the exponential random variables. This model permits lifetimes to be stochastically dependent by adding a random shock event that will either fail a random number of components in the group (called a non lethal shock) or will fail all the components at once (called a lethal shock). Later, Vesley (1977) adapted the shock model to applied problems in the nuclear industry. To illustrate Vesley's model, consider a group of m identical components with exponential lifetimes, each of which possesses a common failure rate (λ) reflecting the frequency of single component failures that are determined to be independent of the other component lifetimes in the group. An additional non lethal common cause shock occurs to the system at a Poisson rate (μ) independent of the individual component failure probabilities. Once a common cause shock occurs, each of the m components can fail according to the results of an independent Bernoulli trial with unknown parameter p. As a result, the number of components failing due to a common cause shock is distributed binomial, hence this was termed the *binomial failure rate* (BFR) model.

The three-parameter BFR model contributed greatly to reliability inference problems for complex systems of components. In many systems, however, the model failed to adequately describe the underlying reliability. Applications that were not modeled well included systems for which shocks and CCF events occurred with different intensities. That is, some systems may typically withstand frequent minor shocks that contribute to the simultaneous failure of few or no components, but can also persevere a rare event that will likely fail all the components in the same CCF group. Atwood (1986) resolved this problem by adding an independent lethal shock variable modeled with a Poisson rate (ω). This four-parameter BFR model has been more readily accepted as a means to reliability estimation in the nuclear industry. In terms of this updated model, the failure rate of components within the group is written as

$m\lambda + \mu mp(1\text{-}p)^{m-1}$ for failures of a single component

$\mu \binom{m}{k} p^k(1\text{-}p)^{m-k}$ for simultaneous failures involving k-out-of-m components ($2 \le k \le m-1$)

$\omega + \mu p^{\,m}$ for simultaneous failure of all components.

The overall failure rate of one or more components is $\theta = m\lambda + \omega + \mu(1-(1-p)^m)$. Notice that θ reflects our inability to record non lethal shocks that fail no components. More complex models can be derived from the basic BFR model. For instance, if data originate from different plants, plant-to-plant variability may be viewed as an important feature in the study. Also, if common cause events are distinguishable and meaningful to the reliability planner, they can be parameterized as separate shock events, given an ample amount of reliability data.

Typical reliability studies of a nuclear power plant are limited to simple attribute data that reflects single and multiple failures of components under study. We assume that data are generated from the same system or the same type of systems with m identical components, and a simultaneous failure of k units out of m fail is represented using an mx1 vector with a 1 in the kth position (and zeros placed elsewhere), and is called an *impact vector*. The sum of the impact vectors is denoted $\underline{n} = (n_1, n_2, \ldots, n_m)$ where n_k = number of failure events in which k out of m components failed simultaneously. In some data sets, n_0 may also be available if failures are tabulated for fixed time lengths, so impact vectors can be of length m+1. If shocks that cause component failure can be distinguished, we can further partition the impact vectors by defining a = number of failure events involving a single component that are caused by independent shocks, b_k = number of failure events involving k components caused by non-lethal shocks ($1 \le k \le m$), and c = number of failure events affecting all m components due to lethal shocks. Notice that $n_1 = a + b_1$ and $n_m = c + b_m$. If n_0 is observable, b_0 is defined and d denotes the number of times no shock occurs in a fixed interval of time (so $n_0 = d + b_0$). Let N and B as the sum of all failure and CCF events, respectively.

Non-shock models

The goal of this paper is to examine and extend the methodology of the BFR model. Since its introduction as a four-parameter model, very little research has focused on BFR model theory for use in CCF analysis, mainly because the nuclear industry, which is a major beneficiary of CCF research, has accepted alternative models for most of its CCF investigations of component failures in nuclear power plants. The alternatives include the beta-factor estimator, the alpha-factor estimators and the multiple Greek letter estimators. The beta-factor estimator, simplest among the three, was developed by Fleming (1975), and gave rise to the other two. All three methods are derived in a similar way, and differences among the three are subtle compared to contrasts with the BFR model and corresponding estimators. We will highlight only the beta-factor estimator, since it is the simplest among all non shock models. The other models are discussed in length in Mosleh, et. al. (1988). The beta-factor method addresses the special case in which m = 2, and the model consists of just two parameters :

λ^* = overall component failure rate

β = proportion of a component failure rate shared by the other component.

This method produces conservative (biased) estimates of parameters if m>2, unless the only possible common cause failures are lethal.

Analyses involving non shock models (NSMs) are usually simple and short, giving the experimenter one or two estimators he or she considers critical in the assessment. If the component group size is kept fixed (i.e., m is a constant), these methods allow a general distribution on the number of common cause events that can occur, while the BFR model constrains the failure count from non lethal shocks to an augmentation of the binomial distribution. However, if complications arise, the NSM estimators have possibly severe shortcomings. Because the NSM parameters are component based and no modeling of shocks occurs in the estimation method, it is unknown how exactly CCF events may change for varying component group sizes. Interpolation and extrapolation schemes based on the BFR model are commonly used in these non BFR models when results do not accommodate the particular component group sizes of interest to the experimenter. These ad hoc procedures, called "mapping rules" (see chapter 3 of Mosleh, et. al. (1988)) are used to estimate conditional probabilities for group sizes (m) different than group sizes available in the data.

The BFR model, on the other hand, allows interpretation of failure events independent of the component group size. This claim is contrary to statements made in Mosleh, et. al. (1988) and Atwood (1986), but the assumption that the shock rate and concurring failure probability can be independent of group size is hardly erroneous or unrealistic. This quality makes the BFR model especially useful in reliability studies for nuclear power plants in which plans include the use of redundancy configurations unlike those in existing plants, or for plants in which components are grouped in contrast to other plants that produce much of the CCF data.

Estimation using the BFR Model

As mentioned earlier, we assume independent Poisson processes determine the occurrence of shocks, and we observe the system over a fixed time period (T) for which repair time and imperfect repair are negligible. For the likelihood based on observing $\underline{n}$, the four basic parameters of the BFR model are not necessarily identifiable. Maximum likelihood estimation leads to estimates outside the parameter space for several combinations of failure data. For example, if $\omega = 0$ and p is fixed, the MLE for λ is negative when

$$\frac{n_1}{n_2 + \cdots + n_m} < \frac{p_1}{p_2 + \cdots + p_m} \quad \text{where}$$

$$p_j = \binom{m}{j} p^j (1-p)^{m-j},\ j = 1,\ldots,m.$$

With no information about n_0 or b_0, the likelihood can be increased dramatically by choosing a value of p close to zero, which allows μ to become larger. From this example we can see that MLEs and corresponding Bayes estimators (without sharp priors) should not be used directly to estimate the basic parameters of the BFR model in this case.

To sidestep this problem, Atwood (1986) substituted a parameter $\lambda_+ = \mu(1-(1-p)^m)$ for the non shock parameter μ. This indirect inference of the modified parameter set uses Bayes techniques, assuming the joint distribution for the parameter set is independent. The resulting methods require knowledge of (a, $\underline{b}$, c) in the estimation scheme. The paper also provides an overview of related inference problems involving the BFR model, including estimation of other parameters that are of interest to practitioners in the nuclear industry. However, very little research on the BFR model exists outside Atwood's paper.

If n_0 or b_0 are observed, estimation is straightforward and easy using maximum likelihood or other classical techniques. The MLEs are

$$\hat{\lambda} = \frac{a}{mT},\ \hat{\mu} = \frac{B}{T},\ \hat{\omega} = \frac{c}{T},\ \hat{p} = \frac{1}{mB}\sum_{k=1}^{m} k\, b_k,$$

and several available methods can be used to construct confidence intervals or hypotheses tests.

<u>*EM Algorithm :*</u> If the non failure data is not observable, direct estimation of the basic parameters is more difficult. The earlier example exploited problems inherent in the MLE. Even in situations where the MLE can be solved (e.g., if m, p are large and λ is small relative to μ), a direct solution from the likelihood equations is unlikely. If explicit solutions cannot be derived, the EM algorithm (see Dempster, Laird, and Rubin, (1977)) can be used to solve for the MLEs.

If b_0 is missing, the EM algorithm provides a two-step procedure from which we alternate estimating b_0 using parameter estimates (from the previous step) and then maximizing the likelihood using our estimate of b_0 in place of the missing data. By the assumptions of the shock model,

$$\hat{b}_0 = \hat{E}(b_0 \mid b_1, b_2, \ldots, b_{m-1}, b_m) = T\hat{\mu}(1-\hat{p})^m.$$

The performance of the iterative estimator is promising, except for certain combinations of parameters and for small data sets.

Method-of-Moments : As an alternative to maximum likelihood, we can use a method-of-moments scheme to estimate the parameters. By equating the statistics

$$\left(\sum_{k=0}^{m} kb_k,\ \sum_{k=0}^{m} k^2 b_k,\ a,\ c\right)$$

with their respective expected values $(\mu mpT,\ \mu mp(1+(m-1)p)T,\ \lambda mT,\ \omega T)$, we derive moment estimators

$$\hat{\lambda} = \frac{a}{mT},\ \hat{\omega} = \frac{c}{T},\ \hat{p} = \frac{\sum_{k=0}^{m} k(k-1)b_k}{(m-1)\sum_{k=0}^{m} kb_k},\ \hat{\mu} = \frac{\sum_{k=0}^{m} kb_k}{m\hat{p}}$$

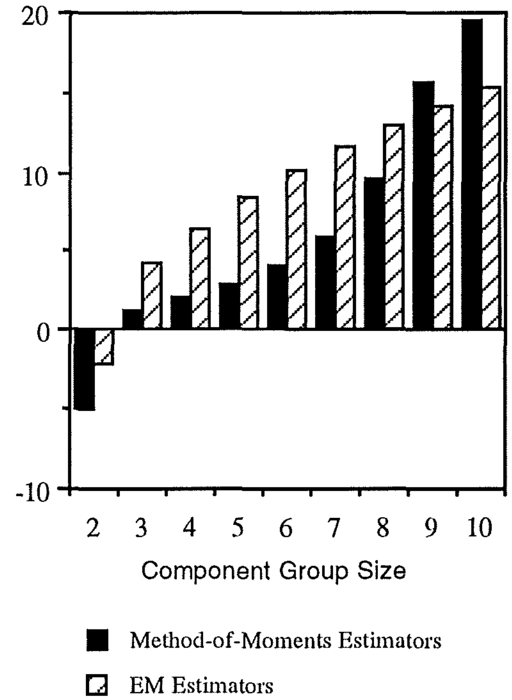
For small samples, we sustain the same bias problems commonly found in quotient estimators. Behaviors of estimatotors for p and μ are erratic. The bias causes very few problems in larger samples, where μT is much larger than one.

In the figure below, comparisons between the method-of-moments estimator, the iterative form of the MLE and the (NSM) beta-factor estimator are made with respect to estimating the parameter β, as defined in the section on NSM alternatives. As a function of the BFR model parameters,

$$\beta = \frac{\omega + \mu p^2}{\omega + \mu p + \lambda}$$

for the case m=2. Model adequacy is measured in terms of mean squared error; values of the BFR model parameters were chosen from typical values found in component data sets from a nuclear power plant. For group sizes (m) larger than two, the data are mapped down to accommodate the beta-factor estimator. For each sample size (m), 10,000 simulations were run.

Mean Squared Error : % Decrease
Beta-Factor vs BFR Estimators
(shocks distinguishable)



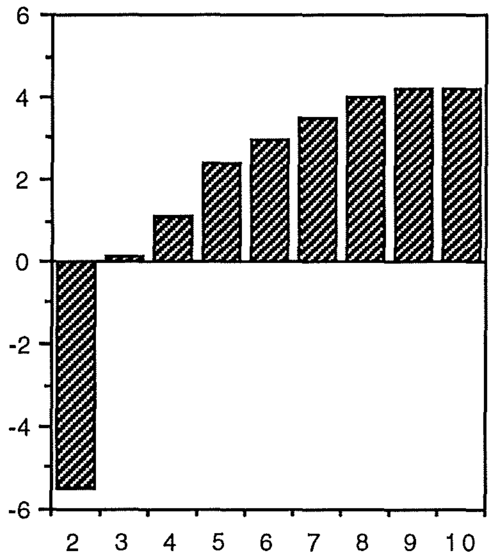
If only $\underline{n} = (n_1, n_2, \cdots, n_m)$ is observable (so cause of failure is not distinguishable in the data), the estimation problem becomes even more difficult. The method-of-moments estimators are not applicable, and the EM algorithm converges even less frequently to a sensible solution. In this case, four additional statistics need to be estimated :

$$\hat{a} = \hat{E}(a \mid n_1, b_2, \ldots, b_{m-1}, n_m) = \frac{n_1\hat{\lambda}}{\hat{\lambda} + \widehat{\mu p}(1-\hat{p})^{m-1}}$$

$$\hat{c} = \hat{E}(c \mid n_1, b_2, \ldots, b_{m-1}, n_m) = \frac{n_m\hat{\omega}}{\hat{\omega} + \widehat{\mu p}^m}$$

with estimates of b_1, b_m determined by the constraints $b_1 = (n_1 - a)$ and $b_m = (n_m - c)$. Though the direct iterative solution performs inconsistently, we can greatly enhance the iterative estimators by adjusting the missing data estimates. By shrinking b_0 toward zero (so that B is more stable), resulting estimators typically converge to satisfactory solutions. It is not certain, however, what amount of shrinking is optimal, given the parameter set. The improvements possible with this iterative method are demonstrated in the final figure below. Again, MSE is recorded for both the NSM estimator and the quasi-EM result, using a fixed shrinkage amount. Although the results here are not conclusive, we have shown that alternative solutions to the problem of parameter estimation for the BFR model are feasible, and further research is warranted.

Percent Decrease in MSE
Beta-factor vs EM-estimator
(shocks not distinguishable)



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