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Detecting trends and patterns in reliability data over time using exponentially weighted moving-averages

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A simple, easy-to-use graphical method is presented for use in determining if there is any statistically significant trend or pattern over time in an underlying Poisson event rate of occurrence or binomial failure on demand probability. The method is based on the combined use of both an exponentially weighted moving-average (EWMA) and a Shewhart chart. Two nuclear power plant examples are introduced and used to illustrate the method. The false alarm probability and power when using the combined procedure are also determined for both cases using Monte Carlo simulation. The results indicate that the combined procedure is quite effective in rapidly detecting either a small or large step increase in the Poisson rate or binomial probability over time.

1 INTRODUCTION

One important class of reliability data analyses concerns the statistical identification of trends and patterns inherent in the data over time. Following Atwood *et al.*,[1] a trend means a steady increase or decrease over time in a reliability quantity of interest, such as a failure rate (or probability). On the other hand, a pattern means any deviation from a stable state or condition resulting from some assignable cause more fundamental than those producing the mere randomness of the data while in the stable state. An example might be the significant step increase in the failure rate in a certain year due to the discovery of a generic problem.

We present a simple, easy-to-use graphical method for use in determining if there is any statistically significant trend or pattern over time in either an underlying Poisson event rate of occurrence or a binomial probability of failure on demand.

Suppose that we have $m > 1$ independent Poisson or binomial data sets $\{(x_i, t_i), i = 1, 2,..., m\}$ or $\{(y_i, n_i), i = 1, 2,..., m\}$, respectively, which are ordered over time. In the Poisson case, the ith data set consists of the number of events x_i (typically failure events), which have occurred in a given operating or exposure time t_i. For operational Poisson data, t_i is the total exposure time for which the event of interest is at risk of occurring and may or may not denote total calendar time. In the binomial case, the ith data set consists of the observed number of failures y_i in a total number of demands n_i. For binomial data, n_i typically represents both true unplanned demands and various kinds of scheduled tests on a given system/component. Atwood[2] discusses several issues regarding exposure time and demands. Also, it is implicitly assumed here that i indexes equally spaced calendar time periods; for example, annually reported data.

The Poisson or binomial data sets are said to be balanced if $t_i = t$ or $n_i = n$, respectively, for all i and unbalanced otherwise. For the ith data set, let λ_i denote the Poisson failure rate and let p_i denote the binomial probability of failure per demand. Recall that the maximum likelihood (ML) estimator of λ_i is $\hat{\lambda}_i = x_i/t_i$, while the (ML) estimator of p_i is $\hat{p}_i = y_i/n_i$.

Consider the following Poisson data from 1987–1992 ($m = 6$) for the failure to start (FTS) of a certain system in a particular subset of US commercial nuclear power reactors:

Example 1. Reactor System Failures to Start, 1987–1992

i	Year	Number of FTS Events, x_i	Reactor-years, t_i
1	1987	4	4.31
2	1988	5	4.06
3	1989	3	4.02
4	1990	5	5.07
5	1991	5	5.23
6	1992	4	5.02

We also consider the following binomial example. Atwood *et al.*[1] present binomial auxiliary feedwater (AFW) turbine train failure data for the years 1987–1991 ($m = 5$) at US commercial nuclear power plants.

Example 2. Turbine Train Failure Data, 1987–1991

i	Year	Turbine Train Failures, y_i	Demands, n_i
1	1987	6	62
2	1988	2	40
3	1989	7	32
4	1990	3	35
5	1991	2	25

2 AN EWMA CONTROL CHART

The exponentially weighted moving-average (or EWMA) control chart was first introduced by Roberts.[4] Hunter[5] and Montgomery[3] give good introductory discussions of the EWMA. It is well known (see Hunter[5] and Montgomery[3]) that the EWMA control chart is a good alternative to the historical Shewhart chart when we are interested in detecting small shifts in the reliability parameter of interest. We also demonstrate this in Section 4.

We now develop the EWMA control chart for the general case of unbalanced Poisson and binomial data. Suppose that time period i is the current time period of interest. For $k = 1, 2,..., i$, let $\hat{\theta}_k = \hat{\lambda}_k$ when we are considering Poisson data, and let $\hat{\theta}_k = \hat{p}_k$ for the case of binomial data. The EWMA is defined as

$$z_i = \gamma\hat{\theta}_i + (1-\gamma)z_{i-1}, \quad i = 1,2,...,m, \tag{1}$$

where $0 < y \leq 1$ is a user-specified exponential decay constant which controls the weight given to the sequence of ML estimates $\hat{\theta}_k$, $k = 1, 2,..., i$, when calculating z_i. Note that the well-known Shewhart control chart is obtained when $\gamma = 1.0$; that is, the EWMA depends only on the most recent ML estimate $\hat{\theta}_i$. The statistic z_i, when plotted along with its corresponding control chart limits, constitutes what is known as an EWMA control chart. A starting value z_o is required when $i = 1$ which may be empirically calculated from the data as

$$z_0 \equiv \bar{\theta} = \bar{\lambda} = \sum_{i=1}^{m} x_i \Big/ \sum_{i=1}^{m} t_i, \text{ for Poisson data}$$

$$= \bar{p} = \sum_{i=1}^{m} y_i \Big/ \sum_{i=1}^{m} n_i, \text{ for binomial data.} \tag{2}$$

If we treat z_o as a constant (thus ignoring the dependency of z_o on all the available data), then the EWMA z_i is a weighted average of all previous ML estimates $\hat{\theta}_{i-k}$, $\theta \leq k \leq i-1$. This may be seen by substituting for z_{i-1} on the right-hand side of (1) and continuing this recursive process to finally obtain

$$z_i = \gamma \sum_{k=0}^{i-1} (1-\gamma)^k \hat{\theta}_{i-k} + (1-\gamma)^i z_o. \tag{3}$$

because the weights $\gamma(1-\gamma)^k$ decrease geometrically with k, the EWMA is also sometimes called a geometric moving average (or GMA).

Now let us consider the mean and variance of z_i under the null hypothesis that there has been no change in the Poisson rate or binomial probability underlying the m data sets; that is, under the assumption that $\theta_i = \theta_o$ for all $i = 1, 2,..., m$ for some unknown value θ_o. Upon taking the expectation of (3) and simplifying, we find that $E(z_i) = \theta_o$. Because $\hat{\theta}_o = \dot{\theta}$ *under* H_o, $\dot{\theta}$ *is the sample estimate for* θ_o that we use as the centerline of our EWMA chart.

Similarly, as a consequence of the assumed independence of θ_i, upon taking the variance of (3) and considering z_o to be constant, we find that, under H_o, the variance of z_i is estimated as

$$\hat{V}(z_i) = \bar{\lambda}\gamma^2 \sum_{k=0}^{i-1} \left[\frac{(1-\gamma)^{2k}}{t_{i-k}}\right], \text{ for Poisson data}$$

$$= \bar{p}(1-\bar{p})\gamma^2 \sum_{k=0}^{i-1} \left[\frac{(1-\gamma)^{2k}}{n_{i-k}}\right], \text{ for binomial data.} \tag{4}$$

Now, suppose that we define

$$K_i(\bullet,\gamma) \equiv K_i(\mathbf{t}_i,\gamma) = \begin{cases} \gamma^2 \sum_{k=0}^{i-1} \left[\frac{(1-\gamma)^{2k}}{t_{i-k}}\right], & 0 < \gamma < 1 \\ \frac{1}{t_i}, & \gamma = 1, \text{ for Poisson data} \end{cases} \tag{5}$$

where $\mathbf{t}_i = (t_1, t_2,... t_i)$ is the (i x 1) vector of exposure times. Similarly, we define $K_i(\bullet, \gamma) = K_i(\mathbf{n}_i, \gamma)$, where $\mathbf{n}_i = (n_1, n_2,... n_i)$ is the (i x 1) vector of sample sizes, when we are considering binomial data.

The corresponding EWMA c-σ upper and lower control chart limits for use at time period i (denoted by UCL_i and LCL_i, respectively) are

$$\text{UCL}_i = \bar{\lambda} + c\bar{\lambda}^{1/2}K^{1/2}{}_i(\bullet,\gamma), \text{ for Poisson data}$$

$$= \bar{p} + c\bar{p}^{1/2}(1-\bar{p})^{1/2}K^{1/2}{}_i(\bullet,\gamma), \text{ for binomial data,}$$

and

$$\text{LCL}_i = \bar{\lambda} - c\bar{\lambda}^{1/2}K^{1/2}{}_i(\bullet,\gamma), \text{ for Poisson data}$$

$$= \bar{p} - c\bar{p}^{1/2}(1-\bar{p})^{1/2}K^{1/2}{}_i(\bullet,\gamma), \text{ for binomial data.} \tag{6}$$

Although c can be taken to be any small positive number, here we will consider $c = 1$, 2, and 3; that is, 1-σ, 2-σ, and 3-σ control chart limits.

As stated earlier, when $\gamma = 1$ the EWMA depends only on the most recent ML failure rate estimate $\hat{\theta}_i$; thus, there is no smoothing of the ML estimates. This is the Shewhart procedure. On the other hand, the smaller the value of γ, the greater the weight that is given to the past estimates $\hat{\theta}_k$, and the greater the smoothing of the ML estimates. For small values of γ, the EWMA thus has a long memory and performs like the CUSUM statistic which places equal weight on all the past ML estimates (see Montgomery[3]). Montgomery[3] states that it has been found that values of γ in the interval $0.05 \leq \gamma \leq 0.25$ work well in practice, with $\gamma = 0.08$, $\gamma = 0.10$, and $\gamma = 0.15$ being popular choices. We have likewise found that the EWMA with $\gamma = 0.1$ performs well in detecting small shifts, but does not react to large shifts as quickly as the Shewhart chart.

A good way to further improve the sensitivity in detecting large shifts without sacrificing the ability to detect small shifts is to use both an EWMA and a Shewhart chart. Such a combined procedure signals an anomalous condition if either statistic fails outside its respective control chart limits. We have found that the combined procedure is effective against both large and small shifts and is easily implemented because the Shewhart chart is a special case of the EWMA when $\gamma = 1.0$. Thus, we propose this combined procedure here. In accordance with this combined procedure, in the remainder of the paper we consider only the two values $\gamma = 0.10$ and $\gamma = 1.0$; however, in practice, any other desired value (or values) for γ can be used by the analyst.

The Shewhart control chart limits, corresponding to $\gamma = 1$, are

$$\text{UCL}_i = \bar{\lambda} + c\sqrt{\frac{\bar{\lambda}}{t_i}}, \text{ for Poisson data}$$

$$= \bar{p} + c\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}, \text{ for binomial data,}$$

$$\text{LCL}_i = \bar{\lambda} - c\sqrt{\frac{\bar{\lambda}}{t_i}}, \text{ for Poisson data}$$

$$= \bar{p} - c\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}, \text{ for binomial data.} \quad (7)$$

Further simplification is possible in the case of balanced data. In this case, $K_i(\bullet, \gamma)$ has the closed form

$$K_i(\bullet,\gamma) = \frac{1}{t}\left(\frac{\gamma}{2-\gamma}\right)\left[1-(1-\gamma)^{2i}\right], \text{ for Poisson data}$$

$$= \frac{1}{n}\left(\frac{\gamma}{2-\gamma}\right)\left[1-(1-\gamma)^{2i}\right], \text{ for binomial data.} \quad (8)$$

For large values of i, further simplification is also possible; namely,

$$K_i(\bullet,\gamma) \equiv K(\bullet,\gamma) \approx \frac{1}{t}\left(\frac{\gamma}{2-\gamma}\right), \text{ for Poisson data}$$

$$\approx \frac{1}{n}\left(\frac{\gamma}{2-\gamma}\right), \text{ for binomial data.} \quad (9)$$

Recall that an EWMA control chart consists of a graphical plot of the EWMA from (1) and corresponding upper and lower control limits from (6) as a function of time period i. The EWMA chart provides a graphical indication of any trend or pattern in the underlying Poisson failure rate over time. Apart from any trend or pattern in the EWMA (which may be of interest in itself), a statistical indication of a trend or pattern occurs when the EWMA fails outside a corresponding 2-σ or 3-σ control chart limit at some time period i. We will refer to this situation as an anomalous condition or signal corresponding to those limits. The use of 1-σ limits serve here as an alert to a potential anomalous condition; these will be further discussed in Section 4 and Section 5.

3 EXAMPLES

Along with the EWMA (the solid dots connected by the solid line), the corresponding 1-σ, 2-σ, and 3-σ control chart limits for Example 1 are plotted in Fig. 1

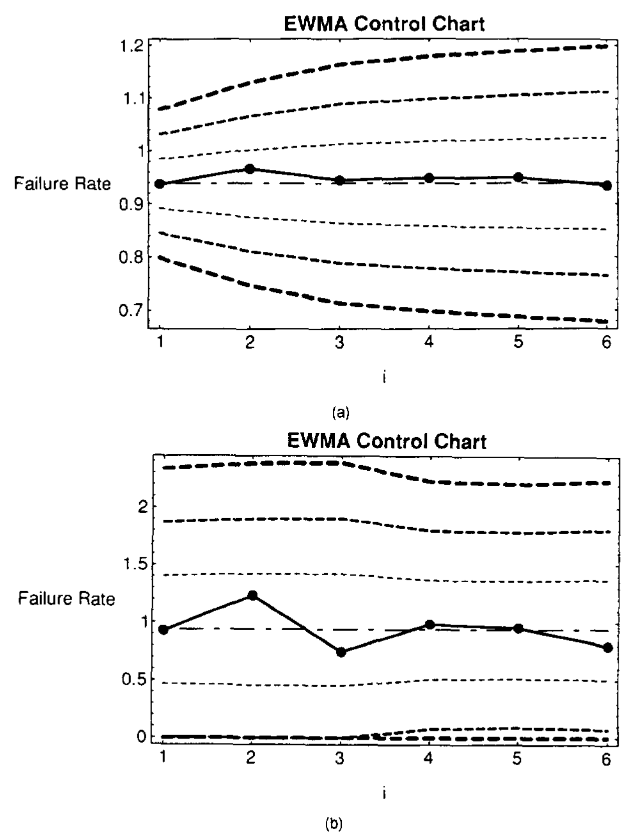

Fig. 1. The EWMA and Shewhart control charts for Example 1 (a) EWMA ($\gamma = 0.1$) and (b) Shewhart ($\gamma = 1.0$).

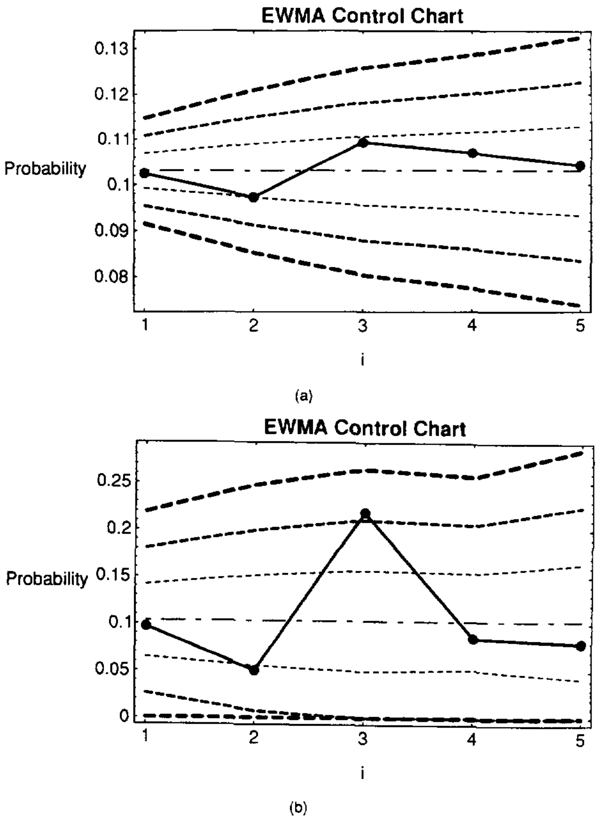

Fig. 2. The EWMA and Shewhart control charts for Example 2 (a) EWMA ($\gamma = 0.1$) and (b) Shewhart ($\gamma = 1.0$).

as a function of time period i. The centerline ($\bar{\lambda}$) of the EWMA chart is also shown. We see from Fig. 1 that there clearly is no statistically significant trend or pattern over time in the Poisson FTS rate for Example 1. The corresponding Shewhart control chart is also shown in Fig. 1. There clearly is no statistically significant indication of a trend or pattern on this chart either.

The corresponding EWMA and Shewhart control charts for Example 2 are shown in Fig. 2. We see that, while there is no indication of any significant trend or pattern in the probability of an AFW turbine failure to start on demand on the EWMA chart, there appears to be an anomaly for 1989 on the 2-σ Shewhart chart.

4 PERFORMANCE ASSESSMENT

We expect an increase in the power for detecting trends and patterns when using a combined EWMA and Shewhart control chart procedure. However, we must carefully determine the accompanying increase in the probability of a false alarm, which is the price we must pay in order to obtain increased power. We use Monte Carlo simulation with 10,000 replications to examine both the false alarm probability and the power when using this combined procedure. Also, the sampling error associated with the calculated false alarm probabilities and power is generally 0.01 or less. We assess the power of the combined procedure in detecting a sudden step increase over time in both a Poisson event rate and a binomial probability for the case of both balanced and unbalanced data.

In the case of balanced Poisson data, let λ_1 denote the event rate prior to the shift and let λ_1 denote the rate after the step shift occurs. The corresponding performance is completely determined given specified values for $k_1 = \lambda_1/\lambda_o$, the factor increase in event rate that is of interest to be detected, and k_2, the expected number of Poisson events in exposure time t when the failure rate is λ_o. Thus, $k_2 = t\lambda_o$.

In order to assess the performance of the combined procedure in detecting both small and large step increases in failure rate, we consider $k_1 = 1$ (corresponding to a false alarm), 1.25, 2.0, 5.0 and 10.0. This range represents step increases ranging from a factor of 1.25 (a 25% increase) through a factor of 10 (an order-of-magnitude increase). We examine the performance for a broad range of exposure times by considering values of $k_2 = 1$, 5, 10 and 25.

The performance of such a combined procedure against a step shift in the failure rate is shown in Figs 3, 4 and 5. The rectangular 'box' in each cell contains the cumulative probabilities of detecting a factor of k_1 step increase in the event rate by 0, 1,..., 5 time periods after the increase first occurs. Thus, time period 0 corresponds to the time at which the failure rate increase first occurs. These six probabilities thus represent the cumulative power (except when $k_1 = 1$) of the combined procedure for detecting the indicated step increase. The column labeled 'U(1,25)' in each

k_1 \ k_2	1	5	10	25	U(1, 25)
	0 1 2 3 4 5				
1.00*	.37,.60,.74,.84,.90,.93	.34,.57,.71,.81,.87,.92	.34,.57,.72,.82,.88,.92	.34,.56,.72,.82,.88,.92	.38,.61,.75,.84,.89,.93
1.25	.39,.63,.76,.87,.92,.95	.42,.66,.80,.88,.92,.95	.47,.71,.84,.91,.95,.97	.59,.83,.92,.97,.98,.99	.56,.81,.91,.95,.97,.99
2.00	.52,.77,.89,.95,.96,.99	.80,.95,.99,1.0,1.0,1.0	.94,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	.99,1.0,1.0,1.0,1.0,1.0
5.00	.91,.99,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0
10.00	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0

*False alarm

Fig. 3. Cumulative probabilities for a 1-σ combined EWMA and Shewhart control chart procedure for detecting a step increase in the Poisson event occurrence rate.

	k_2 = 1	5	10	25	U(1, 25)
	0 1 2 3 4 5				
1.00*	.06,.12,.17,.22,.27,.31	.05,.10,.15,.20,.24,.29	.06,.11,.16,.21,.26,.30	.06,.11,.17,.21,.26,.31	.06,.12,.17,.21,.26,.31
1.25	.10,.19,.26,.33,.39,.44	.12,.22,.32,.40,.47,.54	.16,.29,.40,.50,.58,.65	.26,.45,.59,.70,.78,.85	.22,.40,.53,.58,.68,.74
k_1 2.00	.26,.43,.56,.65,.73,.78	.54,.77,.88,.95,.97,.99	.79,.95,.99,1.0,1.0,1.0	.98,1.0,1.0,1.0,1.0,1.0	.94,1.0,1.0,1.0,1.0,1.0
5.00	.82,.96,.99,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0
10.00	.99,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0

*False alarm

Fig. 4. Cumulative probabilities for a 2-σ combined EWMA and Shewhart control chart procedure for detecting a step increase in the Poisson event occurrence rate.

figure gives the results for the case of unbalanced Poisson data in which the value of k_2 is randomly selected from a uniform distribution on the range 1 to 25 for each time period.

In Fig. 3, we consider 1-σ control limits. A point failing outside the 1-σ limits when using the combined procedure can be used to alert a potential anomalous condition; that is, a situation worthy only of continued or additional study or monitoring but not strong definitive action. In the case of an alert, we see from Fig. 3 that the false alarm probability at the initial stage of the shift is between 34 and 38%, a clear reason for no strong action.

A combined procedure using 2-σ and/or 3-σ control limits can be used to provide a stronger statistical indication of a trend or pattern in the Poisson rate. Figure 4 gives the results for a combined 2-σ procedure. We observe that the false alarm probabilities at the first time period after the shift occurs are around 5–6%, which are sufficiently small for a 2-σ procedure. Similarly, Fig. 5 gives the cumulative detection probabilities for a stronger 3-σ combined procedure. We see that the false alarm probabilities for the first time period after the shift occurs are now around 1–2%. Figures 4 and 5 indicate that the combined 2-σ and 3-σ procedure is quite powerful in rapidly detecting step increases in a Poisson event rate.

In the case of balanced binomial data, the performance of the combined chart depends upon k_1, k_2 and p_o, the value of the binomial probability prior to the shift. Thus, three parameters (in contrast to two for the Poisson) are required to summarize the performance in the binomial case. Analogous to the Poisson case, let $k_1 = p_1/p_o$, the factor increase in the binomial probability that is of interest to be detected. Similarly, let k_2 denote the expected number of binomial failures in n demands when the binomial parameter is p_o; that is, $k_2 = n\ p_o$.

For illustration here, we consider only a single value of p_o; namely, $p_o = 0.1$. In order to assess the performance of the EWMA chart in detecting small through large step-increases in the binomial probability, we likewise consider $k_1 = 1$ (false alarm), 1.25, 2.0, and 5.0. In order to examine the performance for a broad range of demands, we consider values of $k_2 = 1$, 5, 10 and 25.

As in Figs 3–5, Figs 6–8 give the binomial simulation results when using the 1-σ, 2-σ, and 3-σ limits for

	k_2 = 1	5	10	25	U(1, 25)
	0 1 2 3 4 5				
1.00*	.02,.04,.05,.07,.08,.10	.01,.01,.02,.03,.04,.05	.01,.01,.02,.03,.03,.04	.01,.01,.02,.02,.03,.04	.01,.01,.02,.02,.03,.04
1.25	.03,.06,.09,.12,.15,.17	.03,.06,.09,.12,.15,.19	.04,.08,.12,.16,.21,.26	.07,.14,.21,.30,.39,.49	.06,.13,.18,.20,.27,.33
k_1 2.00	.12,.21,.28,.36,.43,.49	.30,.48,.63,.76,.86,.92	.53,.77,.89,.96,.99,1.0	.91,.99,1.0,1.0,1.0,1.0	.82,.97,.99,1.0,1.0,1.0
5.00	.64,.85,.94,.98,.99,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0
10.00	.98,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0

*False alarm

Fig. 5. Cumulative probabilities for a 3-σ combined EWMA and Shewhart control chart procedure for detecting a step increase in the Poisson event occurrence rate.

	k_2 = 1	5	10	25	U(1, 25)
	0 1 2 3 4 5				
1.00*	.37,.61,.76,.85,.90,.93	.34,.56,.71,.82,.88,.92	.35,.58,.72,.82,.88,.92	.33,.56,.71,.82,.89,.93	.46,.65,.79,.86,.91,.94
1.25	.40,.64,.79,.87,.92,.96	.41,.66,.79,.88,.93,.95	.48,.72,.84,.91,.95,.97	.62,.85,.94,.97,.99,.99	.66,.81,.89,.95,.97,.98
k_1 2.00	.55,.80,.90,.95,.97,.99	.83,.97,.99,1.0,1.0,1.0	.96,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0
5.00	.97,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0	1.0,1.0,1.0,1.0,1.0,1.0

*False alarm

Fig. 6. Cumulative probabilities for a 1-σ combined EWMA and Shewhart control procedure for detecting a step increase in the binomial probability when $p_o = 0.1$.

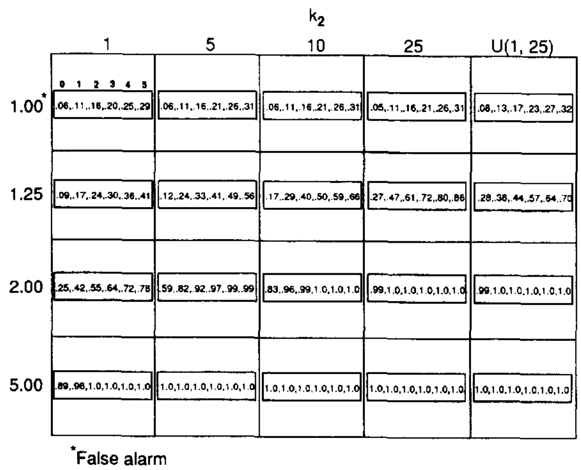

Fig. 7. Cumulative probabilities for a 2-σ combined EWMA and Shewhart control chart procedure for detecting a step increase in the binomial probability when $p_o = 0.1$.

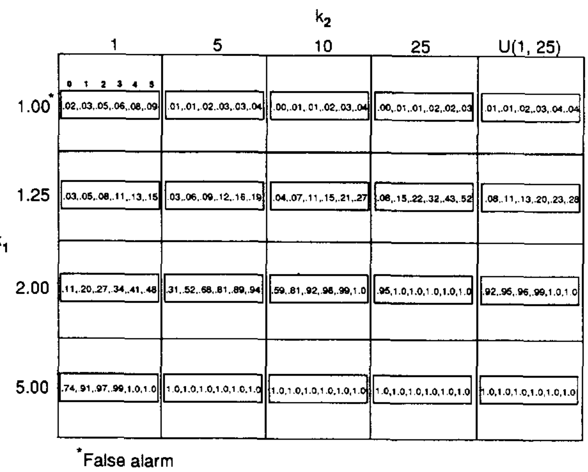

Fig. 8. Cumulative probabilities for a 3-σ combined EWMA and Shewhart control chart procedure for detecting a step increase in the binomial probability when $p_o = 0.1$.

the combined procedure for all combinations of the values of k_1 and k_2 stated above and $p_o = 0.1$. Note that the false alarm probability for time period 0 is roughly 33–46% for the 1-σ limits, 5–8% for the 2-σ limits, and 0.5–2% for the 3-σ limits. The power for detecting large step shifts is quite high, particularly for large values of k_2. In general, the performance is quite similar to the performance for the Poisson case.

5 CONCLUSIONS

The results in Section 4 indicate that the combined use of an EWMA control chart in which $\gamma = 0.1$ and a Shewhart control chart ($\gamma = 1.0$) is quite effective in detecting both small and large step increases in either a Poisson rate or a binomial probability over time. Such a combined procedure signals an anomalous condition if either statistic fails outside its respective control chart limits. The 1-σ limits can be used as an alert to a potential anomalous condition, while the false alarm probabilities for the 2-σ and 3-σ limits are sufficiently small such that these limits can be used as a signal of an actual anomalous condition. The use of such a combined procedure should be particularly useful to regulatory agencies, such as the US Nuclear Regulatory Commission, whose goal it is to statistically detect such anomalies in nuclear power reactor system/component failure rates of occurrence and failure on demand probabilities as rapidly as possible using operational reliability data.

It is interesting and useful to compare the power of the combined procedure with the corresponding power of the individual EWMA and Shewhart charting procedures. For example, Fig. 9 gives the false alarm probabilities and power of all three of these procedures for detecting a step increase in the Poisson event occurrence rate when using 2-σ charts. The combined chart results from Fig. 4 are reproduced in Fig. 9 along with the corresponding EWMA results (indicated in boldface) and Shewhart results (indicated in italics). Because of the 'either or' manner of using the combined procedure, the false alarm probabilities of the individual charts in the first row of Fig. 9 are significantly less than those of the combined procedure, while the power of the combined approach is significantly greater in many cases. However, because of these differences in the false alarm probabilities, it is not possible to directly compare the power of the three methods. On the other hand, because the cumulative false alarm probabilities are nearly the same for the individual

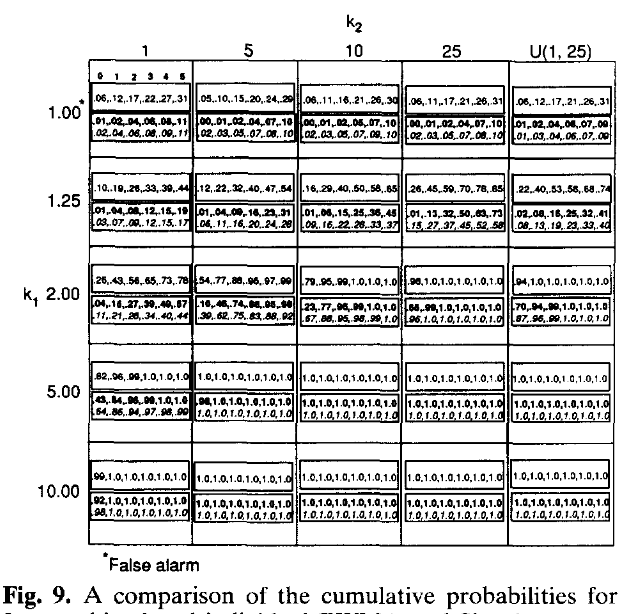

Fig. 9. A comparison of the cumulative probabilities for a 2-σ combined and individual EWMA and Shewhart control chart procedures for detecting a step increase in the Poisson event occurrence rate: combined, plain; EWMA, boldface; Shewhart, italics.

EWMA and Shewhart charts, it is possible to compare the power of these two charts. Note that the power of the Shewhart chart generally exceeds that of the EWMA chart in detecting step increases in the event rate of occurrence soon after the increases occur, while the EWMA is generally more powerful in ultimately detecting such increases.

This analysis, however, does not address another real question: Given that an anomalous condition has been identified for a given time period, what is the probability that this corresponds to a false alarm, as opposed to a true step shift, in the underlying rate or probability? To answer this question requires an unconditional prior probability distribution on the rate or probability. This additional distribution is required in order to apply Bayes' theorem, which is used to determine the desired inverse probabilities. Martz & Tietjen[6] illustrate these calculations for the case of emergency diesel generator reliability.

Finally, all of the required EWMA calculations, as well as the graphical control charts in Figs 1 and 2, were obtained using Mathematica[7] software. Martz & Kvam[8,9] describe this software, listings of which are available from the authors upon request.[3]

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