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On Temperature and Heat Flow in Tree Stems

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ON TEMPERATURE AND HEAT FLOW IN TREE STEMS

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INTRODUCTION

INTEREST in the study of the microclimate of vegetated areas by analysis of their energy budgets has been increasing in recent years. This type of study requires keeping track of the amounts, locations and forms of energy within the "system" and its inter and intra system transport. The energy can be in the form of a sensible or latent heat, chemical bonds, physical displacements, etc. Energy can be transported within the system by physical transport (movement of water, air, etc.) by conduction or by radiation. Complex transformations between the energy forms also take place. One locus of such processes'which has received little attention is the stems of trees and small plants.

The purpose of this study was to investigate the part played by tree stems in the thermal energy budget of the forest stand. Heat flows between the stem surface and the environment by radiative, convective, and conductive modes of heat transfer. This flux and the resultant storage of thermal energy amounts to 5 to *100/0* of the net radiation incident upon the stand (Geiger, 1965). Although the underlying purpose of this study was the understanding of this particular locus of thermal activity in relation to the forest as a whole, several preliminary questions have to be answered first. These are: (1) how are stem surface temperatures related to air temperatures, (2) how can the temperature field in a stem be characterized, and (3) is the stem a passive thermal (physical) system? This report deals mainly with these preliminary questions.

The best approach to this problem is that of mathematical physics. Although this method is a powerful tool in the physical sciences it is a relative newcomer to the field of biology. In his book, *The Physics of Plant Environment,* Van Wijk (1963) states:

> The essential characteristic of such an analytical or physical approach is that it leads to a quantitative theory of the studied phenomena, expressable in mathematical language. In doing so generalizations can be made from a limited amount of experimental data, i.e. the theory can be applied to circumstances differing from those encountered in the original experiments. This can rarely be done when a problem is approached in a purely empirical way.
>
> In addition the analytical method can often help in considerably reducing the required number of experiments. In this connection it may be useful to point out that the application of mathematical statistics to the design and interpretation of experiments does not change their empirical character. Although these techniques can also serve to reduce the necessary number of experiments, they differ essentially from the physical methods referred to here.

In this study the tree stem is taken to be a physical system which may be expected to respond (internal temperatures) to a given stimulus (the variation of air temperature) in a predictable way. A little reflection will convince the reader that this problem can become extremely complex. For this reason certain basic restrictions are made at the outset. These are: (1) the stem is assumed to be in a completely closed stand, that is; the net exchange of radiant energy between the stem and its surroundings is zero, and (2) the stem is assumed to be an infinite circular cylinder. It is noted here that further simplication of the problem will be required.

Throughout this study certain terms and concepts are used which are either modifications of familiar terms or likely to be unfamiliar to many readers. These are outlined below:

The term *moisture fraction* is defined as the ratio of the weight of water in a unit weight of *dry wood.* The term *wet wood* is used throughout to indicate wood with a moisture fraction greater than the fiber saturation point.

The temperature and heat flow variations in time are treated as sinusoidial waves or sums of sinusoidial waves throughout this work. In all cases these waves are characterized by their *angular velocity* ω and an amplitude ${}^{m}T$. ω is found from:

$$\omega = 2\pi/P \tag{1}$$

where P is the period of the wave. All periods are normalized to 2π radians. A sinusoidial wave is given by:

$$T(t) = {}^{m}T \sin \omega t \tag{2}$$

where T(t) is the temporal variation of temperature. This wave is shown in figure (1).

A second wave is shown in figure (1) which is lower in amplitude and peaks later in time than the first. This second wave is related to the first by a *phase lag* or *epoch* angle, $\phi = \omega t_1$ where t_1 is the lag in real time, and an amplitude ratio or *gain,* $\delta = {}^{m}T_a/{}^{m}T_b$. Thus the second wave may be written as:

$$T_b\,(t) = \delta\,{}^{m}T_a \sin\,(\omega t - \phi_b) \tag{3}$$

The vector representation to the right in figure 1 shows more clearly the meaning of phase angle and angular velocity.

The study is broken into several main sections. Following the literature review, the tree stem is discussed in terms of a physical system. This is followed by the development of an analytic solution to the simplified problem. Testing of this

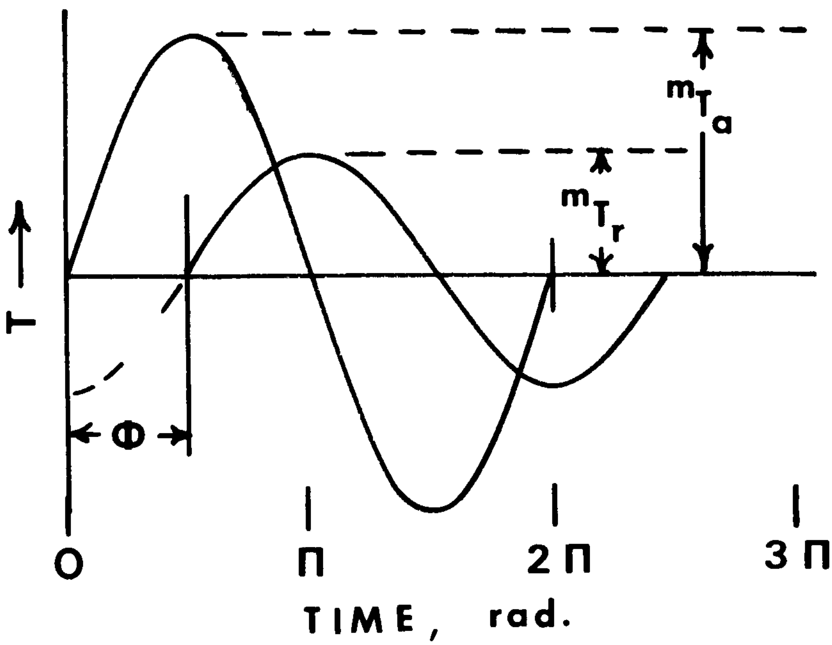

FIGURE 1. Relationships between sine waves.

model against measurements in a real stem is followed by the development of an analog computer model to attempt to include some of the complicating factors which could not be included in the analytic model. The discussion brings together the various aspects of the problem and indicates how the results may be applied to groups of stems.

PREVIOUS MEASUREMENTS OF STEM TEMPERATURE

THE EARLIEST measurements of tree temperatures on record are probably those of Hunter (1775, 1778). There were a large number of measurements of the temperatures of various plant parts during the next 100 years. These are well summarized in Gerlach (1929) and Koljo (1950). In early work interest was centered on heat production in plant material, which at that time was a subject of thriving controversy. More recent work (Leick, 1910; Gerlach, 1929) found that no temperature rise due to respiration could be detected in tree stems.

Although a fairly large number of measurements have been made recently, many are open to criticism on several fronts. In almost every case the temperatures were measured by the radial insertion of a thermometer, thermocouple, or thermistor into the stem. The theory of heat conduction indicates that this is not an acceptable way in which to measure temperatures (Jakob, 1957). Since heat flow is normal to the isotherms in a body, the radial insertion of measuring probes places the probe in a position to conduct a maximum of heat toward or away from the point of measurement. This flow of heat results in an unwanted change in temperature at the point of measurement (Donaldson, 1959; Jakob, 1957). This effect was clearly shown by Eggert (1946) who found that radial placement of very fine thermocouple wires (30 gauge) in twigs caused errors of up to 6°C at a radial depth of ¼ inch. It would be expected that the magnitude of this type of error would increase with the diameter of the probe and decrease with the depth of insertion. The latter effect was observed by Eggert. The thermal properties of the probe material will also affect the magnitude of this error, the error being higher for materials with higher conductivity and lower volumetric heat capacity.

Reynolds (1939) comments on this type of error but still placed his rather large probes (resistance thermometers) in radial holes. For measurement of cambial temperature he inserted the probe along a diameter of the stem, in effect approaching the cambium from the inside. This is an improvement, but the probe is still normal to the expected isotherms.

Morowitz (1955) places the following restrictions on sensing probes:

> (1) Probes measuring quantities, which can be represented by scalar or vector fields, must be small in comparison with dimensions in which appreciable change of the field occurs.

(2) The perturbation of the scalar or vector field by the probe must be small in comparison to the quantity measured.

Although (1) has usually been satisfied, (2) usually has not, especially when thermometers are used for temperature measurements.

A second criticism, and perhaps the more important one from a scientific point of view, is the approach taken to the problem.

Usually temperature measurements were made with little or no study of the mechanisms of heat transfer. In the literature cited, authors rarely related their measurements to the physical properties of the stem. Usually the stem is identified by species, age, and other indefinite denominations. Thus it becomes difficult to generalize the results in any but a nebulous and gross way. Usually no attempt was made to measure temperatures for a full day; only Koljo (1950), for three days, and Reynolds (1939), intermittently for four years, have done so.

Despite the above criticism, however, certain general results are noted:

1. Temperature waves in the interior of tree stems are reduced in amplitude and maximums and minimums occur later than on the stem surface. (Gerlach, 1929; Reynolds, 1939; Koljo, 1950; Haarløv and Petersen, 1952.)

2. Insolation of the tree causes the southern side of the stem to experience higher maximum temperatures, while on overcast days the surface temperatures are uniform (Ihne, 1883; Gerlach, 1929; Koljo, 1950; Haarløv and Petersen, 1952).

3. Smaller stems and branches were found to respond more quickly than larger ones to variation in surface or air temperatures (Gerlach, 1929).

4. Temperatures in stems vary with height, the base being cooler in spring and warmer in fall, due to heat transfer between the stem base and the ground (Gerlach, 1929).

5. A vertical temperature gradient in the stem due to fluid flow (transpiration stream) has been noted by Gerlach (1929) and Rouschal (1939). Both these workers found the effect to decrease rapidly with height. The effect is more pronounced in ring porous species than in diffuse porous or coniferous. Hartig (1874) concluded that a living oak was cooled by the transpiration stream.

6. Gerlach (1929) found wind to have no effect on the temperatures of larger trees, whereas Koljo (1950) found convectional cooling when the surface of the tree was raised above the ambient air temperature by insolation.

Reynolds (1939) carried out a long term study of the temperatures of a cottonwood tree. The measurements were made on a cross section 30 feet above the ground (diameter 25.4 inches). His continuous measurements of air, cambial and central temperatures are the most voluminous yet collected. As mentioned above, the resistance thermometers he used as probes were large and were placed normal to the isotherms. Nevertheless his results are interesting. During the winter the central stem temperature dropped to -1.5°C and remained there for considerable periods. Reynolds ascribes this to the freeZing and thawing of the water in the wood. During mild weather he observed the expected amplitude reduction and lagging of the temperature waves. During hot dry weather, however, the stem temperature tended to remain relatively constant, although the air temperature showed its usual diurnal cycle. Reynolds explains this phenomenon as being a result of (l) the absorption of heat by stretching of the water columns and (2) vaporization of water into the air spaces in the stem wood. No calculations or estimates of either the amounts of heat that would have to be absorbed in order to account for the steady temperatures or the amount of heat that could possibly be absorbed by these mechanisms are presented.

The recent work of Derby and Gates (1966), actually carried out after the experimental portion of my study was complete, must be mentioned here. Their model, which is a digital finite differences approximation to the heat transfer processes within the stem, does allow the incorporation of both the radiant and convective modes of heat transfer between the stem and its surroundings. Freezing and thawing processes can also be included. Their model was able to predict the surface temperatures of an aspen to a reasonable degree of accuracy. Measurements of stem temperatures support the general conclusions made above.

THE TREE AS A THERMAL SYSTEM

THIS SECTION deals with the analysis of the terms in the generalized heat flow equation for cylindrical bodies and the boundary conditions which are applied in arriving at a solution. It is in effect the elimination of terms and boundary conditions which can be shown to be of little *probable* importance in their influence on the temperature at a point in the stem.

Because of the obvious importance of the thermal properties of wood and bark these will be treated first. Following this the complete equation for heat flow in cylindrical bodies will be introduced and analyzed term by term.

Thermal Properties of Wood and Bark

Although there is a relatively large amount of information pertaining to the thermal properties of dry wood, that is, wood with a moisture content below the fiber saturation point (see Wangaard, 1940; Kollman, 1951; Kollman and Malmquist, 1956; Kuhlmann, 1962; Rowley, 1933; and Maku, 1951) there is very little data concerning the thermal conductivity of wet wood. MacLean (1941) has carried out the most complete study on the thermal conductivity of wet wood. He found that the conductivity was given by significantly different regression equations above and below the fiber saturation point. In the moisture fraction range 0.41 to 1.29 and specific gravity range 0.33 to 0.59 and at an average temperature of 30°C the conductivity was given by

$$k = (S_2\,(4.78 + 13.0\,m) + 0.568) \times 10^{-4}\ \text{cal cm}^{-1}\ \text{sec}^{-1}\ {}^{\circ}\text{C}^{-1} \qquad (4)$$

where S_2 is the specific gravity based on green volume and oven-dry weight and m is the moisture fraction. For specific gravity based on oven-dry weight and volume (S_1) we have

$$S_2 = S_1\,(1 - \alpha) \qquad (5)$$

where α is the coefficient of volumetric green-to-oven-dry shrinkage. The temperature dependency of the thermal conductivity can be found by a simple linear equation for moderate temperature ranges. Estimates of the coefficients for wet wood could not be found, however.

The thermal conductivity of a stem is neither homogeneous nor isotropic. It can be seen from equation (4) that the conductivity is a function of both specific gravity and moisture content and these quantities are known to be functions of not only position in the stem but also of time (see page 12). The conductivity also varies with the direction of heat flow, being roughly 2 to 2.5 times greater in the longitudinal than in the radial or tangential directions.

Although there is some question as to the validity of using a linear regression to predict the conductivity of wood, I decided to use equation (4). An equation based on modeling of the heat flow in the microstructure of the wood would be preferable (see, for example, Kuhlmann, 1962), but the testing of such models for wet wood was beyond the scope of this work.

The specific heat of dry wood has been investigated in detail by Dunlap (1912) and his results are expressed by the relation

$$C_o = 0.266 + 0.00116T \text{ (cal gm}^{-1}\text{)} \tag{6}$$

where C_o is the specific heat of dry wood at O°C and T is the temperature (°C). Dunlap's results are in general agreement with those reported by Kuhlmann (1962), Ward and Skaar (1963), Hearmon and Burcham (1956) and Martin (1963). His values are, however, about 30% lower than those reported by the Russian workers (see Chudinov, 1954). Dunlap's values are used in this work in light of agreement with the majority of other reports.

The specific heat of a mixture of *inert* material and water is given by

$$C'_m = (C_o + m)\,(1 + m)^{-1} \tag{7}$$

where C_o is the specific heat of dry wood and m is the moisture fraction. This simple relation does not hold for hygroscopic material, however, since the heat of wetting of the hygroscopic material, which is a function of temperature, must be included. For hygroscopic material Kirchoff's thermochemical equation

$$(1 + m)^{-1}\,\frac{dW}{dT} = C_{meas} - C'_m = \Delta C \tag{8}$$

should hold. Here W is the heat of wetting and ΔC the excess specific heat. Hearmon and Burcham (1956) found that this was valid for beech (European) sawdust. The specific heat of wet hygroscopic material is then

$$C_m = \frac{C_o + m}{1 + m} - \frac{dw}{dt} \tag{9}$$

or with (8)

$$C_m = \frac{C_o + m}{1 + m} + \Delta C_m\,(T,m)$$

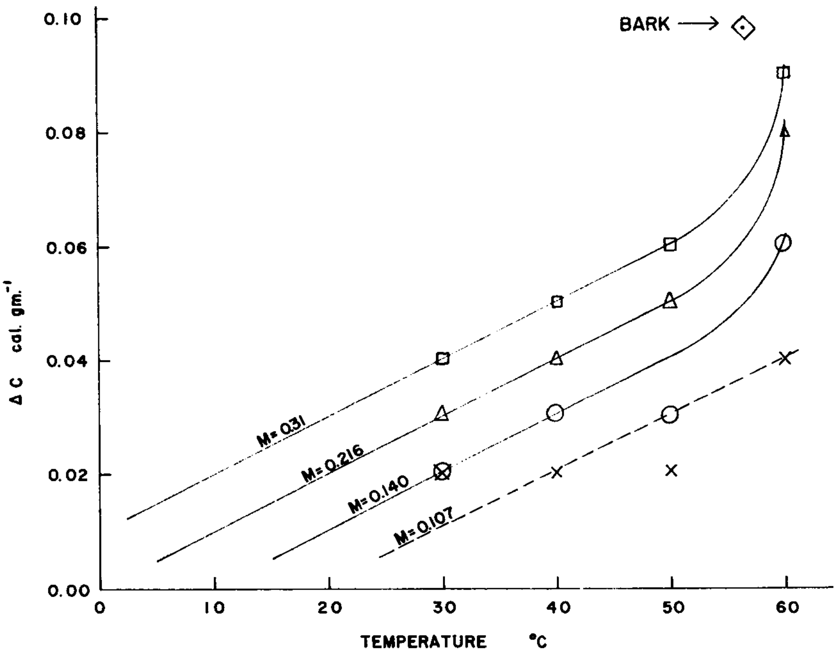

FIGURE 2. The excess specific heat of wood as a function of moisture fraction and temperature. The data is from Hearmon and Burcham (1960). Also shown is a value for bark as determined by Martin (1960).

where ΔC_m is a function of temperature and moisture fraction. For wood with a moisture content above fiber saturation, where the simple weighted sum may be used, we have

$$C_m = \frac{C_o + m + 1.3\ \Delta C(T, 0.3)}{1 + m} \tag{10}$$

where T is the temperature and a fiber saturation point of 0.3 is assumed. Figure 2 shows ΔC as a function of moisture fraction and temperature as reported by Hearmon and Burcham.

The volumetric heat capacity (C_v) is given by

$$C_v = C_m S_3 \tag{11}$$

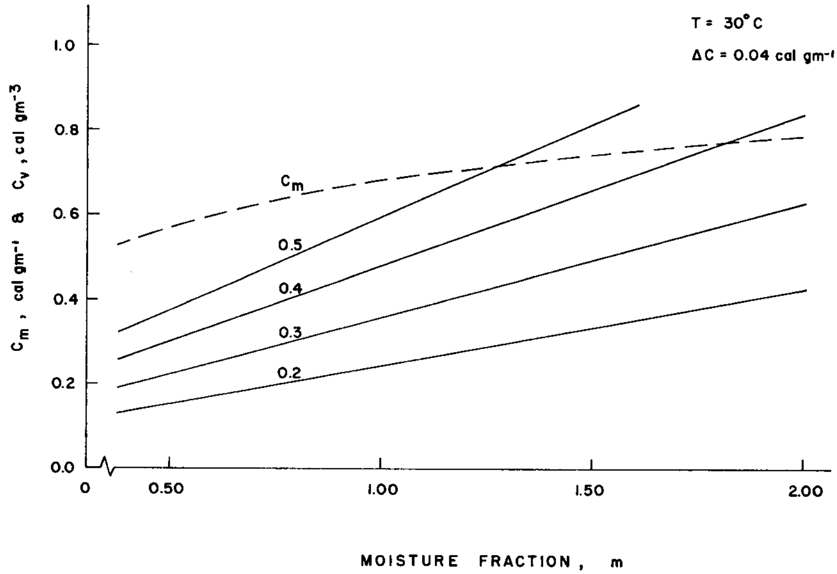

FIGURE 3. The specific heat and heat capacity of wood as a function of moisture fraction and density.

where

$$S_3 = S_1 (1 - \alpha)(1 + m) \tag{12}$$

Substitution of (9) into (11) gives

$$C_v = S_1 [C_o + m + 1.3\Delta C (T, 0.3)](1 - \alpha) \tag{13}$$

This equation with the data of Hearmon and Burcham (1956) is used in this study to compute C_v, which is shown in figure (3) as a function of moisture fraction and density.

The diffusivity of wood is given by

$$\kappa = k/C_v \tag{14}$$

and is shown in figure 4 as a function of moisture fraction and density. Note that at high moisture contents there is little change of the diffusivity with moisture fraction.

The thermal properties of bark have been studied intensively by Martin (1963) and Reifsnyder, *et al.* (1967).

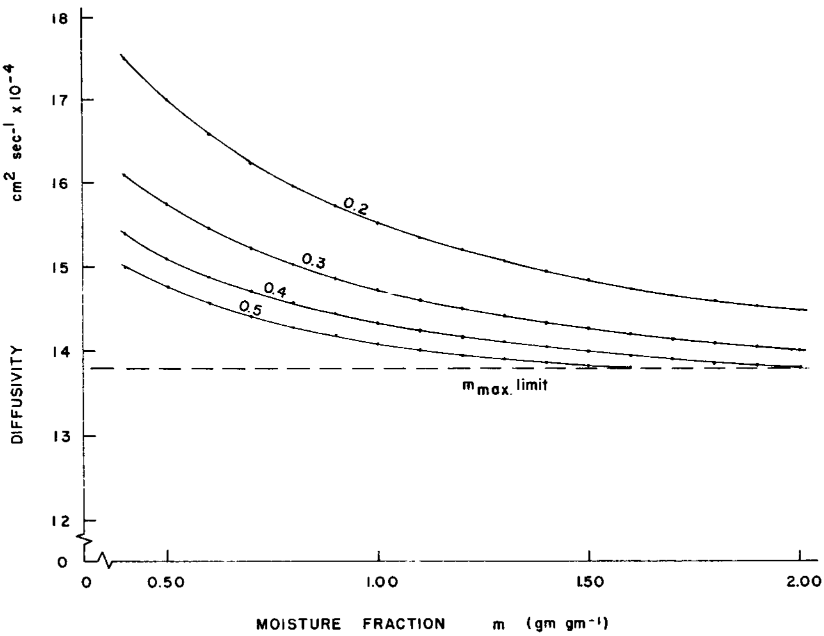

FIGURE 4. The thermal diffusivity of wood as a function of moisture fraction and density. The limit due to saturation of the wood is also shown (max limit).

Martin found the conductivity of bark at 25°C to be given by

$$k = 0.2689 + 5.329\ S_3 + 10.266\ m\ (1 + \alpha) \tag{15}$$

while Reifsnyder finds for the average of Longleaf, Shortleaf and Red pine

$$k = 0.232 + 4.964\ S_3 + 5.558\ m\ (1 + \alpha) \tag{16}$$

The coefficients in these equations are nearly the same except for the term involving the moisture fraction. Since outer bark moisture contents are low (Reifsnyder *et al.*, 1967), this discrepancy is of little practical importance.

Both Reifsnyder and Martin recommend, on the basis of experimental data, that Dunlap's relation (6) be used for the specific heat of dry bark.

The excess specific heat of bark has been measured by both Reifsnyder *et al.* (1967) and Martin (1963). At a temperature of 80°C and $m >$ fsp Reifsnyder *et al.* find a value of approximately 0.125 cal gm^{-1} dry bark for both Longleaf and Shortleaf pine. Martin gives this value as 0.089 cal gm^{-1} dry bark as an average of several species. These values are very close to those given by Hearmon and Burcham for beechwood. Since both these values are close to those for wood, equation 13 was used to compute the volumetric heat capacity of the bark. Dunlop's relation, equation 6, for dry wood was used to compute the specific heat of dry bark.

Variation of Thermal Properties

The thermal properties of a stem are neither isotropic nor homogeneous in space or time. Since variations in these properties will influence temperature variations at a point in the stem they must be studied and, if necessary, incorporated into the models. It has been well established that the thermal conductivity of wood is 2 to 2.5 times greater in the longitudinal direction (parallel to the gain) than in the radial or tangential directions (Wangaard, 1943; Kellog and Ifju, 1962; Kollmann and Malmquist, 1956). The conductivity differs only very slightly between the radial and tangential directions (see Wangaard, 1940). This information pertains only to wood at or below the fiber saturation point. It might be expected that the longitudinal-tangential conductivity ratio might approach unity for increasing moisture contents and decreasing specific gravities.

The only isotropy of consequence is then that between the transverse plane and the longitudinal axis.

In the section dealing with the estimation of the thermal properties of green wood and bark it was shown that the conductivity, the volumetric heat capacity, and the diffusivity are functions of the specific gravity, the moisture content, and the temperature of the wood or bark. The influence of temperature is relatively small, and cannot be estimated with any confidence for wet wood and will not be considered here.

Both the moisture content and the specific gravity distribution in tree stems are quite variable. To some degree the specific gravity will be determined by species but the variation around these levels is quite large and a complex function of heredity, site, water supply, microclimate, etc. (Hall, 1963; Brown, *et ale* 1952).

In hardwood species the radial distributions can be quite irregular (Brown, *et al.*, 1952) but may show a systematic decrease toward the bark (Wangaard, 1950). In softwoods the specific gravity is generally low near the pith and increases to a more or less constant value as the bark is approached. In both hardwoods and softwoods there may be variation of the specific gravity with angular position, especially if the tree is leaning or has an uneven crown. Specific gravity usually decreases with height.

The specific gravity of bark is also variable but there are insufficient data for any general conclusions (Spalt and Reifsnyder, 1962).

The specific gravity distribution (oven dry weight and volume) will be a function of space alone. The moisture content, however, will be a function of both space and time. R. P. Gibbs has spent a good portion of his life studying the moisture patterns in trees, and his major conclusion is that these patterns show a variability which is hard to "explain" (Gibbs, 1958).

The patterns of the spacial and temporal variations of moisture content are too complex for discussions here. The reader is referred to Gibbs (1958) and Kramer and Kozlowski (1960) for more thorough discussions.

The thermal properties of wood and bark have been found to be predicted by specific gravity and moisture content. The volumetric heat capacity is a function of temperature, specific gravity and moisture content. Because of the variation of specific gravity and moisture in the tree stem the thermal properties will be functions of radius, angular position, height, and time. For a particular tree at a particular time it would be virtually impossible to predict the thermal properties without some knowledge of the distribution of the specific gravity and the moisture content.

HEAT FLOW IN THE STEM

In the following sections the heat flow equation is introduced and analyzed term by term. From this analysis a reasonable model and set of boundary conditions can be defined.

As we mentioned in the introduction, the stem is specified as a circular cylinder with no radiation heat exchange on the surface. The coordinate system to be used in this and following discussions is shown in figure 5. Any point in the stem can be defined by specification of its axial coordinate (z), radial coordinate (r), and angular coordinate (8).

Conduction in Cylindrical Coordinates

The heat conduction equation for cylindrical bodies in which there is heat production at the point (r, z, *0)* at the rate A (cal cm^{-3} sec-I) is

$$C_v \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + A\ (r,z,\theta) \qquad (17)$$

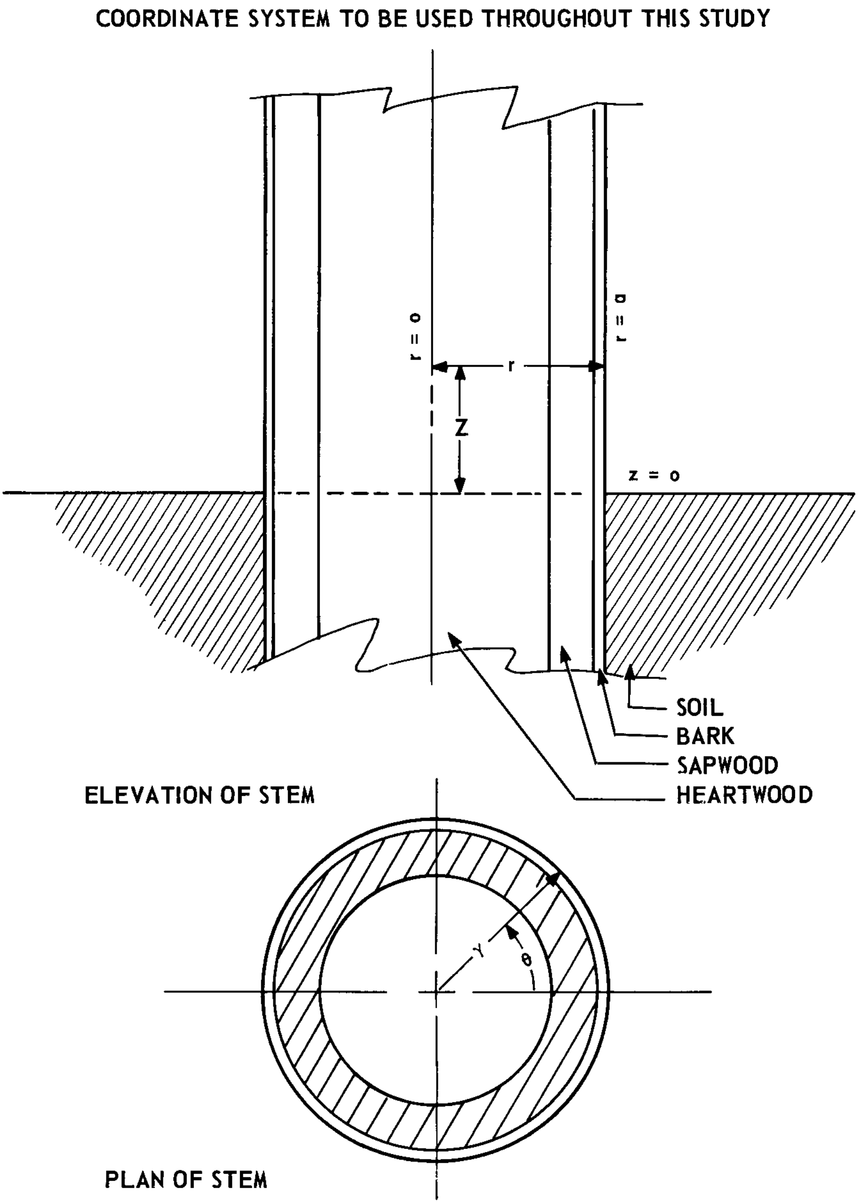

FIGURE 5. Coordinate system.

where T = temperature, °C
t = time, sec.
C_v = volumetric heat capacity, cal cm^{-3}
k = thermal conductivity, cal cm^{-1} sec^{-1} °C^{-1}

This form of the conduction equation assumes that the thermal properties of the cylinder are isotropic, homogeneous and invariant with temperature. As was shown in the preceding chapter this is not the case here. Equation (17) is, however, the simplest form and, as will become evident, its use here as an approximation to the actual case is acceptable even though wood is non-isotropic. The variation of the thermal properties in space is most easily considered in the boundary conditions which must be applied to equation (17). This aspect of the problem will be considered later.

As the primary boundary condition the surface temperature is specified as a periodic wave of angular velocity ω, amplitude ${}^{m}T_a$, and average temperature T_o:[1]

$$T(a,t) = T_o + {}^{m}T_a \sin \omega t \text{ for all } t \tag{18}$$

where $\omega = 2\,\pi\,/P$, and P is the period of the wave.

Returning to equation (17) it can be seen that the temperature at time t and some point (r, z, θ) is a function of the fluxes of heat in the axial ($\partial^2T/\partial z^2$), tangential ($r^{-2}\partial^2T/\partial\theta^2$), and radial ($\partial^2T/\partial r^2 + r^{-1}\partial T/\partial r$) directions, plus any effect due to local heat generation (A). A discussion of the fluxes of heat in these directions and their probable significance follows.

The Longitudinal (Axial) Flux

If a vertical temperature gradient exists in the stem there will be a flux of heat in the axial direction. With the assumption that the radial flux, and thus the surface temperature, will have the largest effect on temperatures in the stem, this factor reduces to that of vertical stem surface or air temperature gradients.

On open ground steep vertical gradients of air temperature are established due to insolation during the day and net long-wave radiation to cold skies during the night. In the forest, however, the temperature gradients are not large since the active surface is in the canopy. Geiger (1957), and Baumgartner (1956), have presented data which indicate that the temperature gradients in the stem space of the forest is usually less than 3°C. Both W. E. Reifsnyder[2] and R. Leonard[2] have

[1]In all temperature notation the superscript m indicates the amplitude of a wave and the subscripts a and r refer to, respectively, the surface and some radial point (r).

[2]Personal communication.

found gradients of the same order. It would be reasonable to assume that under conditions where there is no insolation of the stem surface, that the surface temperature of the stem will be quite uniform with height.

The stem is considered here to be a cylinder set into the ground and there will be some exchange of heat between the stem and the ground. Petrov (1955) found, for example, that for a spruce tree near Moskava (USSR) the stem was usually a little warmer at a height of 1 m than it was at 10.5 m in December and January. His measurements were made with ordinary thermometers inserted radially into the stem and therefore the data are questionable. Saharov (1952) reports that mean daily temperatures on the South side of a pine stem at breast height were 5°C higher than the mean at the ground level during July. For an oak 9 cm in diameter the temperature was 1°C higher at 1 meter than at 0.5 meters. However, since readings were taken only three times a day (0700, 1300 and 1900 hours) the results are hard to interpret. Again, radially placed thermometers were used.

Even though the conductivity of wood is greater in the longitudinal than in the transverse direction, it is my opinion that in closed stands the effects of vertical conduction will be negligible at heights over a meter. In any case, this effect would probably be overshadowed by the convection of heat in the sap stream.

Thus, if the study is restricted to a zone above some critical height the term for vertical heat flow ($\partial^2 T/\partial z^2$) can be dropped from equation (17).

Tangential Heat Flux

If the gradient $dT/d\theta$ exists, there will be a flow of heat in the tangential direction. With the assumption that there is no vertical flux of heat of any significance, the factors that will cause a tangential gradient are: (1) variation of the surface temperature with θ in addition to its variation in time, and (2) variation of the thermal properties of wood and bark with θ. As was noted earlier, thermal properties of wood are a function of its specific gravity and moisture content. There seems to be little data on the spatial variation of specific gravity in tree stems. Langenkamp (1931) found considerable asymmetry in the specific gravity distribution of a short bolt of beech. However, many studies of the distribution of specific gravity on a vertical cross section (Trendelenburg, 1937; Volkert, 1941 or 1956; see also Kollmann, 1951) indicate that a high degree of axial symmetry exists on straight trees with no pronounced lean. This would be expected if the theory that wood is laid down in such a manner that the stem forms a beam of uniform resistance to horizontal loads is accepted (Schniewind, 1962; Bryant, 1951; and others). It can be further assumed that for trees with a uniform water supply and crown that the moisture content of the stem will not vary to a large extent with angular position. The effect of moisture content variation is of less importance at high moisture contents than at low as was pointed out on page 10 (see figure 4, page 11).

Assuming that the surface temperatures will be uniform in respect to θ, it would be reasonable, as a first approximation, to eliminate the term for tangential flux from the heat conduction equation (17).

Heat Production and Removal

There are two possible sources of heat in the stem: (1) heat generation due to respiration of living cells in the wood, bark and cambium, and (2) that due to addition or removal of heat from the cross section under consideration by the transpiration stream.

The amount of energy released by the oxidation of glucose to carbon dioxide and water is 112×10^3 cal. per mole of oxygen taken up or carbon dioxide released. Since this is the energy by which the "machinery" of the cell is kept functioning and with which new materials are formed, it would seem that only a portion of it would be given off as sensible heat. It would be expected then, as pointed out by Meyer and Anderson (1952), that in rapidly growing tissue only a small amount of the energy released is given off as sensible heat while in mature tissue close to 100% of the energy is released as heat.

The amount of heat released can be estimated, assuming 100% of the energy of respiration is lost as heat, from the maximum rate of oxygen production given by Goodwin and Goodard (1940): 300 $mm^3 O_2\ hr^{-1}$ gm (wet material)$^{-1}$. Calculation indicates that a maximum of approximately 2.8×10^{-4} cal sec^{-1} cm^{-3} (green wood) will be released. To arrive at an idea of the order of magnitude of the effect which a source of this strength would have on stem temperatures the relation

$$T(r) = A(a^2 - r^2)/4k \qquad (19)$$
$$\text{with } T(a) = 0 \text{ for all } t$$

can be used (Carslaw and Jaeger, 1959). Here A is the strength of a *uniformly distributed source* in cal cm^{-3} sec^{-1}, a is the radius of the stem, and k is the thermal conductivity. Consider a stem with a radius of, say 10cm, in which heat is produced at the above maximum rate (2.8×10^{-4} cal cm^{-3} sec^{-1}). With k as 5×10^{-4} cal cm^{-1} sec^{-1} $°C^{-1}$ and r equal to zero, the central temperature is raised by some 3°C. This calculation was based on the maximum rate observed by Goodwin and Goodard. As pointed out above, the energy released by actively growing tissue is probably relatively small. Using a value of 0.28 cal cm^{-3} sec^{-1} (oxygen absorption of 10 mm^3 hr^{-1} gm^{-1}), results in a temperature excess of only 0.5°C at the center. Since these estimates are for a distributed source, it is not surprising that Gerlach (1929) and Leick (1910) were not able to detect any increase in stem temperature attributable to respiration.

The vertical flow of fluid in the transpiration stream poses a more difficult problem. In terms of a cross section of the stem this fluid flow can be considered as a source if the fluid is coming from a warmer region or as a sink if it comes from a colder region. To complicate matters further it must be noted that the flow will be roughly periodic in nature (Kramer and Kozlowski, 1960). Hartig (1873) concluded from temperature data taken at a height of 1 meter on a live oak and on an oak log set into the ground that the transpiration stream cooled the living stem. There are, however, many other factors which could have caused the differences on which he based his conclusions.

Gerlach (1929) noted that in the spring when the ground was colder than the air that the transpiration stream had a cooling effect on the stem. This effect decreased with height in a birch stem 22 cm. in diameter. At 0.5 meters above the ground the temperatures were lowered slightly in relation to the temperatures at the same radial points at a height of 1.75 meters. The effect was strongest at a radial depth of 6 cm. Study of his curves indicates that at 1.75 meters (height) the stem was unaffected by the fluid flow from the colder ground. Gerlach also found that this effect was marked during the rise of sap.

Rouschal (1939) was able to show that the cooling effect as measured in the region of the cambium was greater in ring porous than in diffuse porous and coniferous trees. He was able to detect a cooling effect at a height of 3 meters in ring porous species. In diffuse porous and coniferous stems no effect was detectable at this height. For the latter, the temperatures were reduced approximately 1°C (maximum) at a height of 1 meter. In both of these studies temperature measurements were made via radial holes, although small instruments were used.

Summary of Internal Effects

The results of the above discussion can be summarized briefly as:

(1) The effects of conduction from the ground and fluid flow can probably be neglected if the study is restricted to a zone above some critical height.

(2) If the surface temperatures are independent of θ, then the interior temperatures can be expected to be independent of θ for healthy straight trees without abnormal specific gravity or moisture content distributions.

(3) The contributions of heat from living tissue can be neglected.

(4) The radial flow of heat, and thus surface temperature, can be expected to make the major contribution to the temperature in the stem.

As a first approximation, then, the heat conduction equation for the tree can be written as

$$C_v \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \tag{20}$$

Boundary Conditions

Surface Temperatures

Since radiant heat exchange has been excluded from the study, the surface temperatures will be related to the air temperature. In order to have heat flow between the air and the surface of the stem, a temperature difference must exist and a flow of heat will take place over some "thickness" of air. Since the flow of heat to the surface must be the same as the flow of heat into the surface, we have[1]

$$k \left. \frac{\partial T}{\partial r} \right|_{r=a} = h_m (T_a - T_c) \tag{21}$$

where T_a is the surface temperature, T_e is the environmental air temperature, h_m is the coefficient of surface heat transfer (surface conductance) in cal cm^{-2} sec^{-1} °C^{-1}. The reciprocal of h_m is the boundary resistance.

For engineering applications many determinations have been made of the value of the surface coefficient of heat transfer for flow normal to cylinders (pipes and tubes). Carslaw and Jaeger (1959) give the relation

$$h_m = 8 \times 10^{-5} (V/D)^{1/2} \text{ cal cm}^{-2} \text{ sec}^{-1} \text{ °C}^{-1} \tag{22}$$

for cylinders in turbulent flow (air). Here D is the diameter of the cylinder (cm) and V is the velocity of the air flow (cm sec^{-1}). Note that the resistance to heat flow, $1/h_m$, decreases with the square root of the velocity.

The usefulness of such relations in the present study is questionable even though the range of Reynolds numbers for trees (in flows from 1 to 450 cm sec^{-1} the Reynolds number varies from ca 1 to 150,000) is within the range for which data were taken. The main difficulty is that in these determinations every effort is made to keep the approaching flow non-turbulent and smooth cylinders

[1]This is known as the "radiation type" of boundary condition and can be used for any form of heat transfer between a body and its surroundings.

are used. Since the wind in the forest will be turbulent, and since several studies have shown that an increase in turbulence caused an increase in h_m (McAdams, 1954; Jakob, 1957), it would be expected that the predictions of equation (22) would be low.

Since there are, as far as I can determine, no data concerning the surface coefficient of heat transfer for tree stems, and since there is good reason to conclude that the predictions of the above relations will be low by an unknown amount, the problem here must be limited to consideration of the interior temperatures of stems as related to the stem surface temperature and not the air temperature.

It might be noted here that the extensive study of Reynolds (1939) discussed above was based on comparison of interior temperatures with air temperatures. There may have been some factors not reported by him related to the surface conductance which would account for some of the unexpected responses he found at high environmental temperatures.

It is reported in the engineering literature (see Jakob, 1957; McAdams, 1954) that the surface coefficient of heat transfer is not uniform over the surface of a cylinder. On smooth tubes a maximum of heat transfer takes place on the upstream and downstream parts of the cylinder, while minimums occur on the sides. Although there is a possibility that this situation may occur on tree stems the effect on the angular variation of h_m would be small due to the turbulent nature and low velocity of the flow.

Assuming that the surface temperatures will not vary with height or angular coordinate we have as the surface temperature boundary condition

$$T(a,t) = T_o + {}^{m}T_a \sin\omega t \tag{23}$$

Radial Variation of Thermal Properties

The simplest way to introduce the radial variation of the thermal properties into the problem is to consider the stem to be made up of j concentric annuli of differing properties. We have then, as an additional boundary condition at the boundary (r′) between the j th and the j th + 1 annuli, that

$$k_j \frac{\partial T_j}{\partial r'} = (k_{j+1}) \frac{\partial T_{j+1}}{\partial r'} \tag{24}$$

and

$$T_j\,(r') = T_{j+1}\,(r') \tag{25}$$

Boundary conditions (24) hold only when the annuli in question are in intimate contact (Carlslaw and Jaeger, 1959). This requirement holds for the interior of the stem since a continuous variation of the thermal properties is being approximated by division into annuli. At the bark-xylem[1] interface this may not be true. If there exists a contact coefficient, h_c (cal cm^{-2} sec^{-1} °C^{-1}) between the bark and the xylem, then (24) is replaced by

$$- k_j \frac{\partial T_j}{\partial r} = h_c \left[T_1(r) - T_2(r)\right] \tag{26}$$

where the j th annulus is the bark; now the heat flow is proportional to the temperature difference between the inner surface of the bark and the outer surface of the xylem.

Summary

It has been shown that as a first approximation to the problem of heat flow in the tree stem only the radial flux of heat need be considered. The surface temperatures of the stem can be expected to be a function of time alone and are directly related to the air or environmental temperature. However, since no reasonable estimate can be made as to the magnitude of the coefficient of surface heat transfer, the problem must be treated in terms of the surface temperature.

The boundary conditions relating to the variation of thermal properties with radius and the possibility of a contact coefficient between the bark and the xylem have been presented.

[1]Here "bark" indicates all tissue from the xylem outward.

THE ANALYTIC SOLUTION

BECAUSE of the complexity of biological situations, the problem usually has to be simplified further than can be conveniently rationalized. In what follows, it will become apparent that equation 20 will have to be drastically simplified if any tractable solution is to be reached. However, the knowledge gained from working with such simplified problems often allows sense to be made from what otherwise might be useless data. From the simplified analytic attack on the problem a way of representing the data which is *not* restricted to the simplified solution is found.

The preceding chapter has stated the problem in its most simplified form. In what follows the possible solutions are presented and the simplified analytic solution defined and discussed.

Possible Solutions

In the preceding chapter the problem was defined as being that of an infinite cylinder with thermal properties which are a function of radius. The surface temperature variation, dependent on circumferential position, is the forcing function. A review of the literature showed that (1) an analytic solution to the problem had not been found, and (2) that if one were found the solution would be so complex that it would be very difficult to make any sense of the results. The second of these points is based on the fact that solutions in the literature for the cases similar to that sought here due to Vodica (1956 a,b) and Lowell and Patton (1955) rapidly become intractable and recourse must be made to numeric solutions for specific cases.

Further simplification is necessary. Therefore, the boundary condition relating to radial variation of the thermal properties is relaxed.

Analytic Solution

A solution to the characteristic equation

$$\frac{1}{\kappa}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial^2 r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \tag{27}$$

is sought. The appropriate boundary condition for surface temperature drive is

$$T\,(a,t) = {}^{m}T_a \sin \omega t \text{ for all } \theta \tag{28}$$

The average temperature term T_o has been dropped for simplicity. A solution of the form

$$T\,(r,t) = \delta(r)\, {}^{m}T_a \sin\,(\omega t + \phi(r)\,) \tag{29}$$

is assumed where $\delta(r)$ is the gain, defined as the ratio of the amplitude of the temperature wave at radius r to that of the surface wave; that is

$$\delta(r) = {}^{m}T_r \,/\, {}^{m}T_a \tag{30}$$

and $\phi(r)$ is the phase difference, in radians, between the surface and interior waves;

$$t_1 = \phi(r) \,/\, \omega \tag{31}$$

and t_1 is the phase lag in units of time. These relations have been described in the introduction (see figure 1).

In arriving at the analytic solution a change is made in the independent variable r so that the solution will be in terms of dimensionless ratios. The application of dimensional analysis yields the following relations:

with

$$\lambda^2 = (\omega/\kappa) \tag{32}$$

then

$$\rho = \lambda r$$
$$\xi = \lambda a$$

where ρ is the dimensionless radial coordinate and ξ is the dimensionless cylinder radius. The angular velocity of the applied wave and the diffusivity of the material determine the parameter λ.

The manipulative complexity of finding a solution has been reduced; the original four variables (r, t, ω, κ) have been replaced by two (t, ρ), and it is now apparent that all cylinders which are characterized by the same parameter λ will have exactly similar responses.

It was assumed that the temperature field in the stem would be given by a relation of the form,

$$T(\rho,t) = {}^{m}T_\xi\, \delta(\rho) \sin\,[\omega t + \phi(\rho)] \tag{33}$$

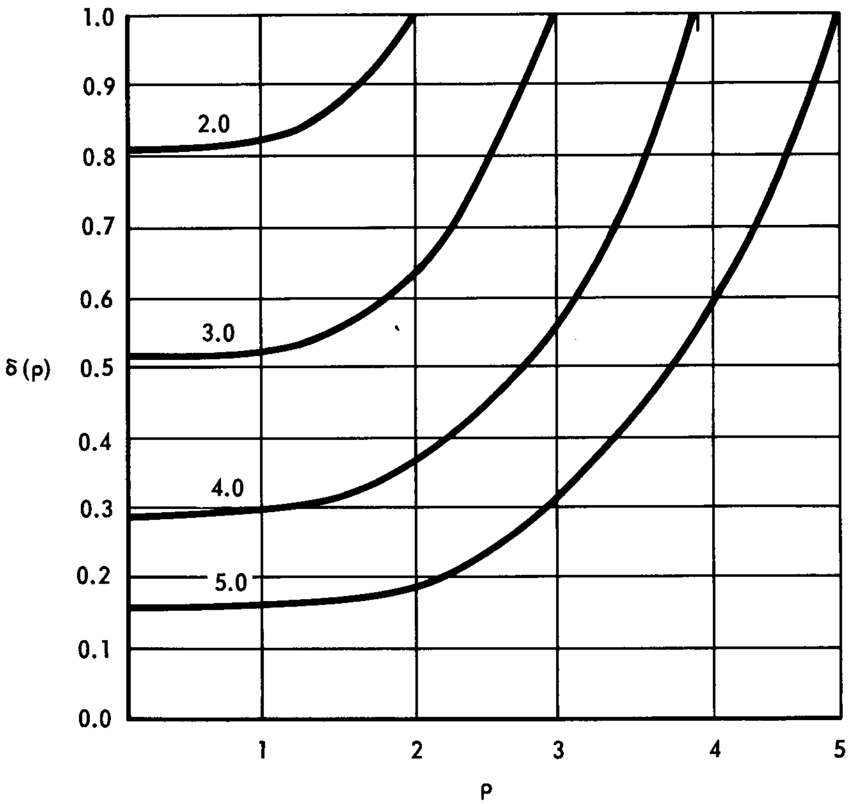

FIGURE 6. Gain as a function of radial coordinate and radius.

Solution[1] of the characteristic equation (eq. 27) with these boundary conditions and the assumed form of solution gives the following relations for the gain and the phase angle:

$$\delta(\rho) = \frac{{}^{m}T_{\rho}}{{}^{m}T_{\xi}} = \left[\frac{\mathrm{Ber}^2\rho + \mathrm{Bei}^2\rho}{\mathrm{Ber}^2\xi + \mathrm{Bei}^2\xi}\right]^{1/2} \tag{34}$$

[1]The derivation of this solution may be found in Herrington (1964). It is a modification of the solution to a similar problem solved by Reismann (1958). A second solution is given by Lowell and Patton (1955) and may also be found in Carslaw and Jaeger (1959).

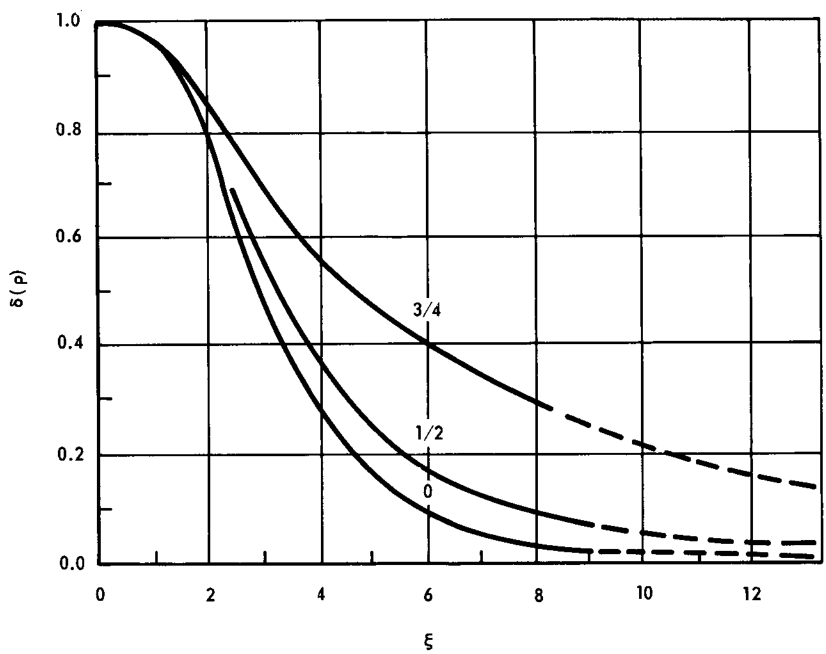

FIGURE 7. Gain at fractional radii as a function of radius.

and

$$\omega t_1 = \phi\ (\rho) = \tan^{-1}\left[\frac{\text{Bei}\xi\ \text{Ber}\rho - \text{Ber}\xi\ \text{Bei}\rho}{\text{Ber}\xi\ \text{Ber}\rho + \text{Bei}\xi\ \text{Bei}\rho}\right] \qquad (35)$$

where t_1 is the phase lag in seconds ($\phi(\rho)$ in radians). The functions Ber (x) and Bei (x) are, respectively, the real and imaginary parts of the Bessel functions of complex argument (see McLachlan, 1934; Watson, 1944; or Bowman, 1958). These functions are tabulated in McLachlan (1934) and Lowell (1959). A short table of Ber and Bei and related functions is given in Appendix B.

Figure 6 shows the gain δ as a function of the dimensionless radial coordinate ρ for cylinders of differing dimensionless radius. As would be expected the amplitude of the wave is progressively reduced as it moves toward the center of the stem. The amplitude is reduced sharply in the outer portion of the stem and the rate of reduction decreases as the center is approached, becoming very small close to the center. The curves for the stems of differing radius appear to be of

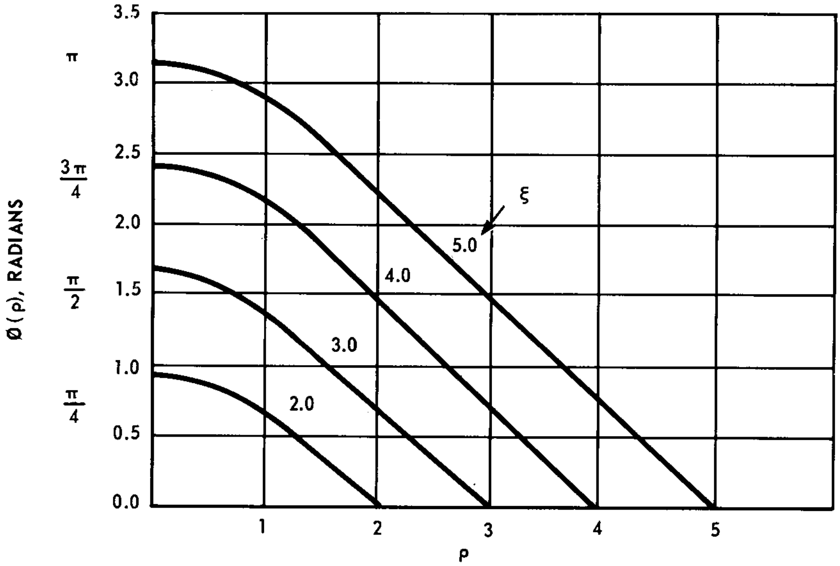

FIGURE 8. Phase lag as a function of radial coordinate and radius.

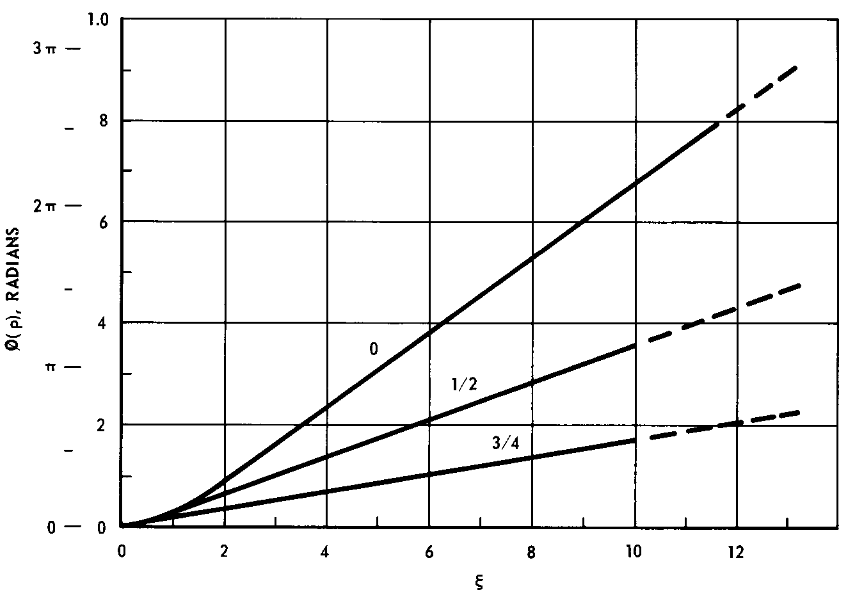

FIGURE 9. Phase lag at fractional radii as a function of radius.

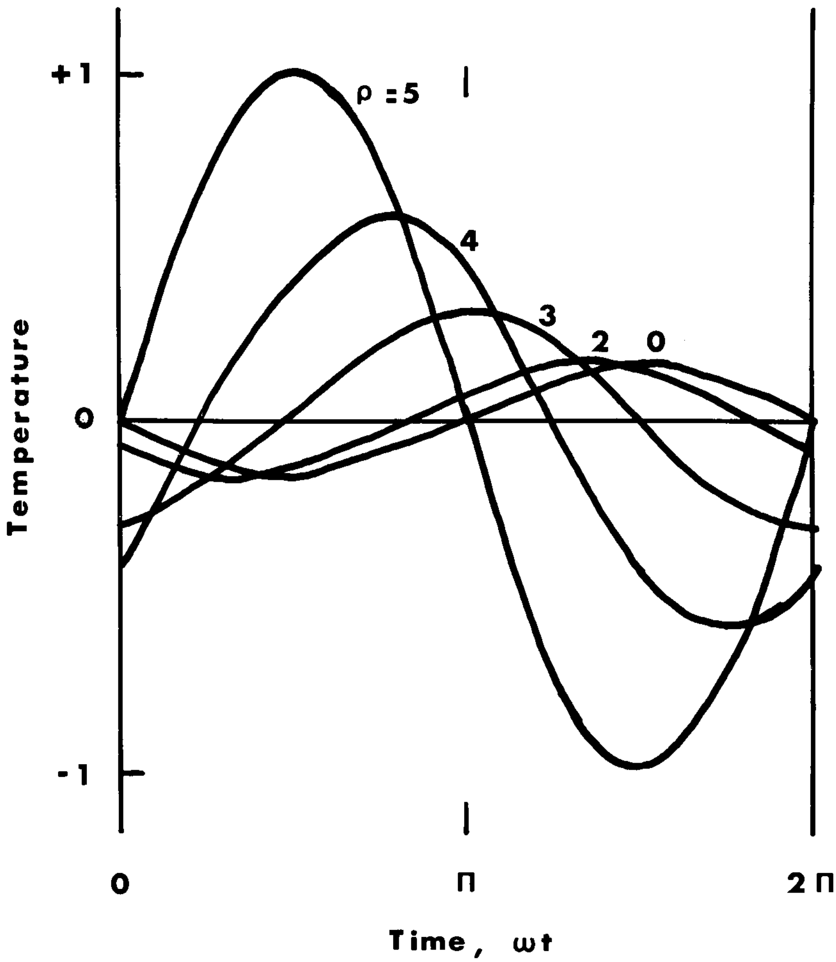

FIGURE 10. Temperature history for several radial points within the stem. The dimensionless radius is 5.

slightly different shape. This is supported by the fact that differentiation of equation (34) in respect to ρ yields:

$$\delta'(\rho) = \frac{\text{Ber}\,\rho\,\text{Ber}'\,\rho + \text{Bei}\,\rho\,\text{Bei}'\,\rho}{(\text{Ber}^2\xi + \text{Bei}^2\xi)^{1/2} + (\text{Ber}^2\rho + \text{Bei}^2\rho)^{1/2}} \tag{36}$$

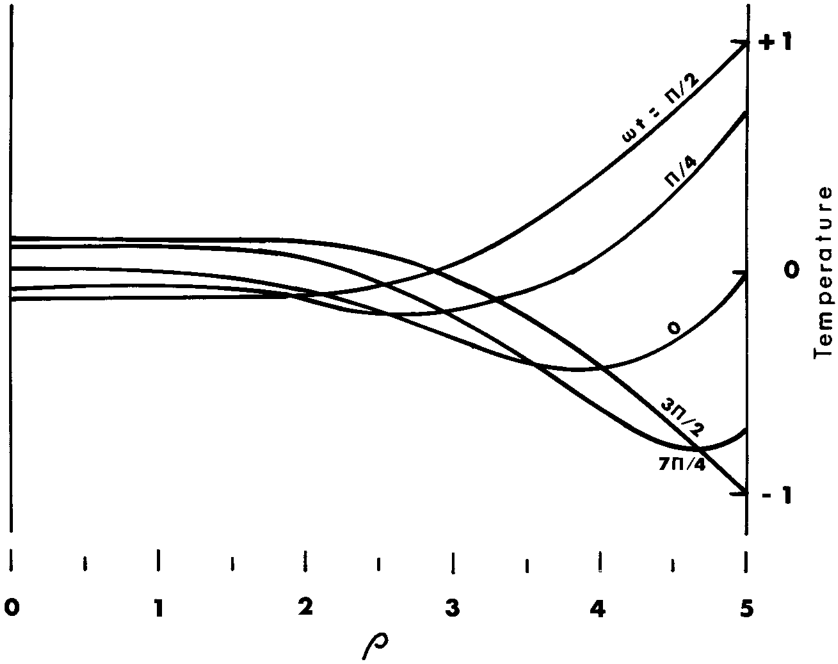

FIGURE 11. Temperature as a function of radial coordinate at various times.

Thus the slope of $\delta(\rho)$ is a function of both ρ and ξ and the relation cannot be generalized further.

The gain plotted as a function of ρ is shown in figure 7. It is near unity for very small cylinders, decreases rapidly for $1 < \xi < 6$, and then approaches zero asymptotically.

The phase lag is shown as a function of ρ for cylinders of different radii in figure 8. The lag increases linearly with a decrease in ρ and increases with ξ for any given ρ (figure 9) except at small values of ρ and ξ. The curves of figure 8 appear to be very similar in shape.

Differentiation of equation (35) in respect to ρ yields:

$$\phi'(\rho) = \frac{\text{Bei}\rho\ \text{Ber}'\rho - \text{Ber}\rho\ \text{Bei}'\rho}{\text{Bei}^2\rho + \text{Ber}^2\rho} \tag{37}$$

Thus the slope of the lag is a function of ρ alone and a single curve may be used for any size cylinder.

TABLE 1. MAGNITUDE OF PARAMETERS AND RESPONSES OF STEM AND TWIG

PARAMETERS			RESPONSE			
Period of Applied Wave	ω Sec^{-1}	λ cm^{-1}	*Twig* a = 0.5 cm		*Stem* a = 10.0 cm	
			$\delta(0)$	$\theta(0)$	$\delta(0)$	$\theta(0)$
1 min	1.05×10^{-1}	8.4	0.4	$2\pi/3$ (1/3 min)	(0)	very large
1 day	7.27×10^{-5}	0.2	1.0	(0)	0.4	$2\pi/3$ (8 hrs.)
1 year	1.99×10^{-7}	1.2×10^{-2}	1.0	(0)	1.0	(0)

The temperature history for several radial points of a theoretical stem is shown in figure 10. The effect of the abrupt reduction in amplitude near the surface combined with the nearly linear increase in phase lag is apparent. Figure 11, showing the radial temperature distribution at intervals of $\pi/4$ radian (3 hours for daily wave) shows the nature of these fluctuations more clearly.

The term response is defined so that an *increase* in response corresponds to a *decrease* in the phase lag and an increase in the gain. If the response of the stem is characterized by the central gain and phase angle, then the response is a function of ξ alone. Referring to the similarity relations

$$\xi = \lambda a; \lambda^2 = \omega/\kappa; \omega = 2\pi/P$$

it follows that:

1. The response decreases with an increase in ξ, which means that the response decreases with an increase in nondimensional radius or frequency.
2. The response increases with an increase in the diffusivity κ or period P.
3. Referring to figure 4 it can be seen that an increase in moisture content will increase the response while a stem with a high specific gravity will respond less than one with a low specific gravity.

These characteristics are shown clearly in table 1 in which the order of magnitude of the various parameters and responses are tabulated for cylinders of two sizes.

Both the real time differences and the similarity of the response of the twig and stem are apparent. The response of the twig, a = 0.5 cm, to the daily wave is virtually uniform and instantaneous throughout while the response of the twig to the wave with a period of one minute is similar to that of the stem for a period of 1 day. The response of both the twig and the stem to the yearly wave is nearly uniform and relatively instantaneous, although the lag for the stem is about 8 hours in real time.

SURFACE HEAT FLUX

The flux of heat across a unit area of the model stems surface is given by:

$$q_a\,(t) = k\,\frac{dT}{dr} \tag{38}$$

Differentiation of equation (38) with $\rho = \xi$ and $\delta(\xi) = 1$ yields

$$q_a = K^m T_a\,\lambda\,[\delta'(a)\sin\omega t + \phi(\xi)\cos\omega t] \tag{39}$$

which can be written as

$$q_a = K^m T_a\,\lambda\nu\,\sin(\omega t + \beta) \tag{40}$$

where

$$\nu^2 = [(\delta'(\xi))^2 + (\phi'(\xi))^2] \tag{41}$$

and

$$\beta = \tan^{-1}[\phi'(\xi)/\delta'(\xi)] \tag{42}$$

In these equations the term $k^m T_a\,\nu\lambda$ is the amplitude of the heat flow and β is amount by which the heat flow wave *leads* the temperature wave. This phase lead is given by

$$t_2 = \beta/\omega \tag{43}$$

The half cycle heat flux is the amount of heat which flows across the unit surface in one direction, that is

$$Q_a = k\int_{t_1}^{t_2} \left.\frac{dT}{dt}\right|_{r\,=\,a} dt = 2\,\lambda^m T_a \nu\,(\omega)^{-1} \tag{44}$$

where the limits of integration are the times when $\omega t + \beta = n\pi$, $n = 0, 1, 2, \ldots\ldots\ldots$

ν is a nondimensional parameter which determines, in essence, the ratio of the flux of heat across the surface of the cylinder to that across the surface of an infinite slab. This can be seen by comparing the heat flux for the slab

$$q_s\,(t) = k\,{}^m T_s\,\lambda \sin(\omega t + \pi/4) \tag{45}$$

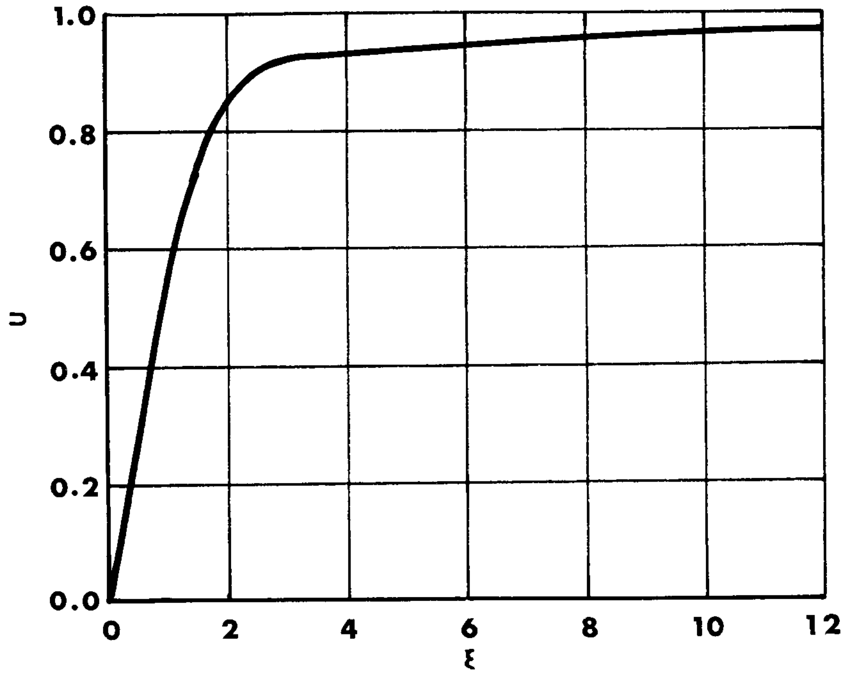

FIGURE 12. Nondimensional heat flow parameter as a function of radius.

to (40). As ν approaches unity the heat flux across the surface of the cylinder approaches that of the infinite slab with the same properties and boundary conditions. ν is shown as a function of ξ in figure 12. It can be seen that the surface heat flux for the cylinder increases very rapidly for ξ less than 3, whereupon the curve abruptly changes slope and approaches 1 asymptotically. Between $\xi = 0$ and 4 the phase lead of the heat flow wave, β, decreases rapidly from a value of $\pi/2$ at $\xi = 0$ as shown in figure 13 and approaches the limiting value for the infinite slab, $\pi/4$ (equation 45).

The half cycle heat flux also can be shown to be identical to that for an infinite slab when $\nu = 1$. The surface heat flux and half cycle heat flow for cylinders are essentially those for an infinite body when ξ is greater than, say, 3.0. Assuming an average diffusivity of 15×10^{-4} cm^{-2} sec^{-1} °C^{-1} for green wood, the half cycle heat flow and the instantaneous heat flux can be approximated by treating the

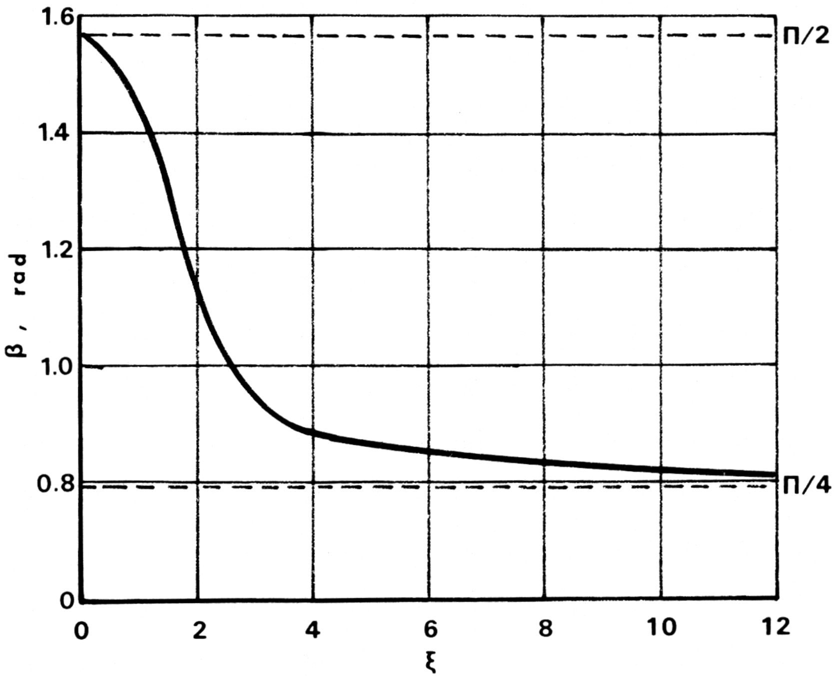

FIGURE 13. Surface heat flux phase lead as a function of radius.

surface area of the stem(s) and an identical area of an infinite slab for stems with a radius of 14 cm or greater. For smaller stems figures 13 and 14 can be used to find the appropriate reduction factor ν and phase lead β.

APPLICATION OF FOURIER SERIES TO THE SOLUTIONS

In nature the temperature waves at the surface will not be purely sinusoidial. However, periodic functions, with few exceptions, can be represented by the sum of an infinite series of sine and cosine functions with periods which are harmonics of the fundamental period. Thus, for the temperature waves here, we have the Fourier Series

$$T(t) = T_o + \sum_{n=1}^{N} [B_n \sin(n\,\omega t) + A_n \cos(n\,\omega t)] \tag{46}$$

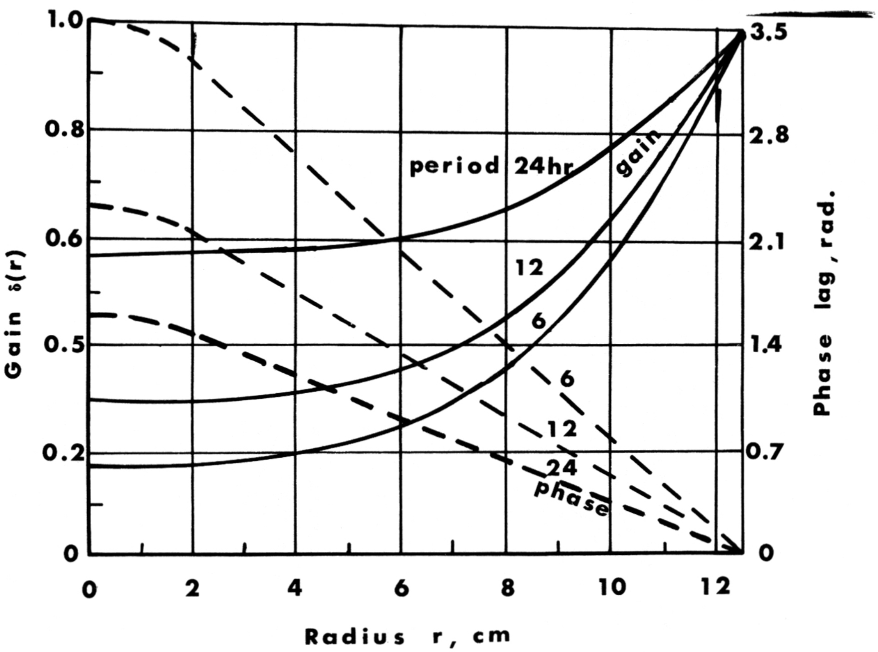

FIGURE 14. Amplitude envelopes and phase lags for waves of various period.

where n = 1, 2, 3, 4 . . . . . B_n and A_n are known as Fourier coefficients. Here they are the amplitudes of the temperature waves of period $2\pi/n\omega$. For naturally occurring temperature waves the amplitude of the fundamental will usually be much larger than that of any of the higher harmonics.

Equation (46) can be expressed identically as

$$T(t) = T_o + \sum_{n=1}^{N} C_n \sin(n\,\omega t + \eta) \tag{47}$$

where C_n is the amplitude of the n^{th} wave, which is shifted by η radians. We have

$$\begin{aligned} C_n &= (B_n^2 + A_n^2)^{1/2} \\ \eta &= \tan^{-1}(A_n/B_n) \end{aligned} \tag{48}$$

Since we are dealing with a linear system, a body which has a surface temperature drive expressable as a Fourier Series will respond as if the waves of differing frequency were applied separately and the responses summed. If the surface temperature is specified by (48) the solution (equation 29) will then be of the form

$$T(\rho,t) = \sum_{n=1}^{\infty} \delta_n(\rho)\, C_n \sin[n\,\omega t + \phi_n(\rho) + \eta_n] \qquad (49)$$

where now the gain and the phase lag are those for waves of period of $2\pi/\eta\omega$ Since the response of the cylinder, as was shown above, is a function of the period of the wave, these components of the surface temperature wave will behave differently in the stem or cylinder. It was seen that the response of the stem decreases with a decrease in the period of the wave. If the surface temperature is a wave that is essentially a sine wave of a period of 24 hours and the higher harmonics are of relatively small amplitude, it would be expected that the temperatures within the stem would become more and more nearly a sine function of time as the wave moves deeper into the stem. This can be seen in figure 14, which shows the amplitude envelopes and phase relations for waves of various periods.

Since these waves of differing frequency behave differently, the composite wave in the stem may not bear much resemblance to the surface wave if the components of higher frequency have rather large amplitudes. This effect is shown in figure 15.

As was noted at the start of this section, these discussions have been based on *steady periodic waves.*

When the waves are not steady, a transient response is evoked. The transient response (Carslaw and Jaeger, 1959; Equation 14, p. 201) of cylinders to the daily variation of surface temperature is such that the steady periodic condition is reached in one day. This condition will further decrease the response of interior points in the stem to rapid fluctuations of surface temperature.

This transient effect complicates the experimental analysis of stem temperature waves. Analysis should not be initiated until the stem has been subjected to one or preferably more days with similar daily temperature waves.

SURFACE TEMPERATURES

Grober (1928), as reported in Lowell and Patton (1955), obtained a solution to the problem of heat flow in an infinite cylinder with uniform and homogeneous thermal properties in which the air temperature is the boundary condition. As mentioned earlier this boundary condition is expressed by

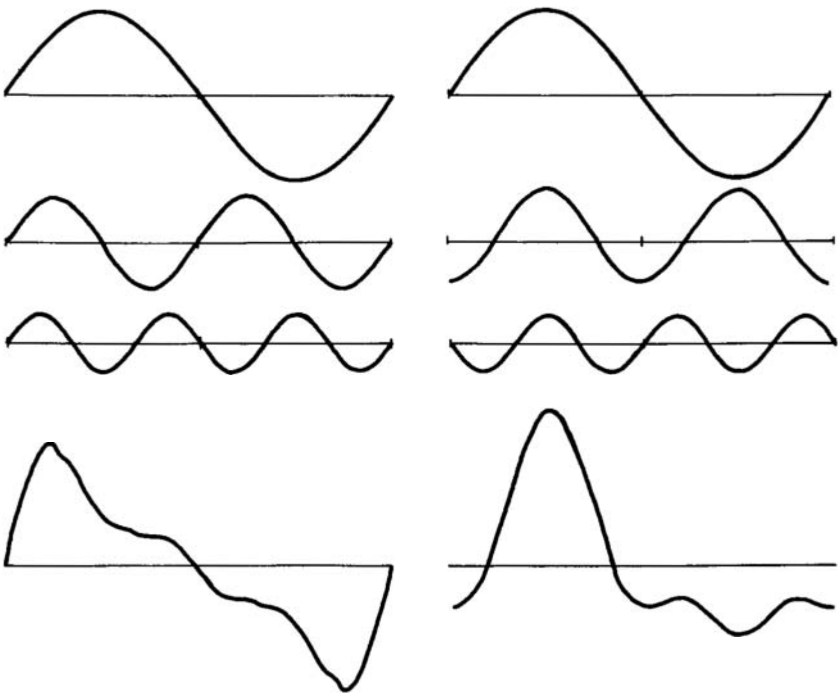

FIGURE 15. Effect of relative phase angle on complex wave shape. **In** each set the top three waves are summed to form the bottom wave.

$$\text{at } r = a \quad T_a = T_e - ((h^*)^{-1}\, dT/dr] \tag{50}$$

and

$$T_e = T_o + {}^{m}T_e \sin \omega t \tag{51}$$

where $h^* = h_m/k$ and T_e is the air or environmental temperature, and h_m has been previously defined as the surface coefficient of heat transfer. Grober's solution is expressed in terms of a gain and a phase lag, as are the other solutions above. Figures 16 and 17 show the gain and the lag, respectively, as a function of the parameters

$$Fo^* = 2\pi\, h_m / (\omega a^2) \tag{52}$$

and

$$Ja^{*2} = 2\pi\, h^2{}_m / (\omega k C_v) \tag{53}$$

which are the notation of Lowell and Patton.

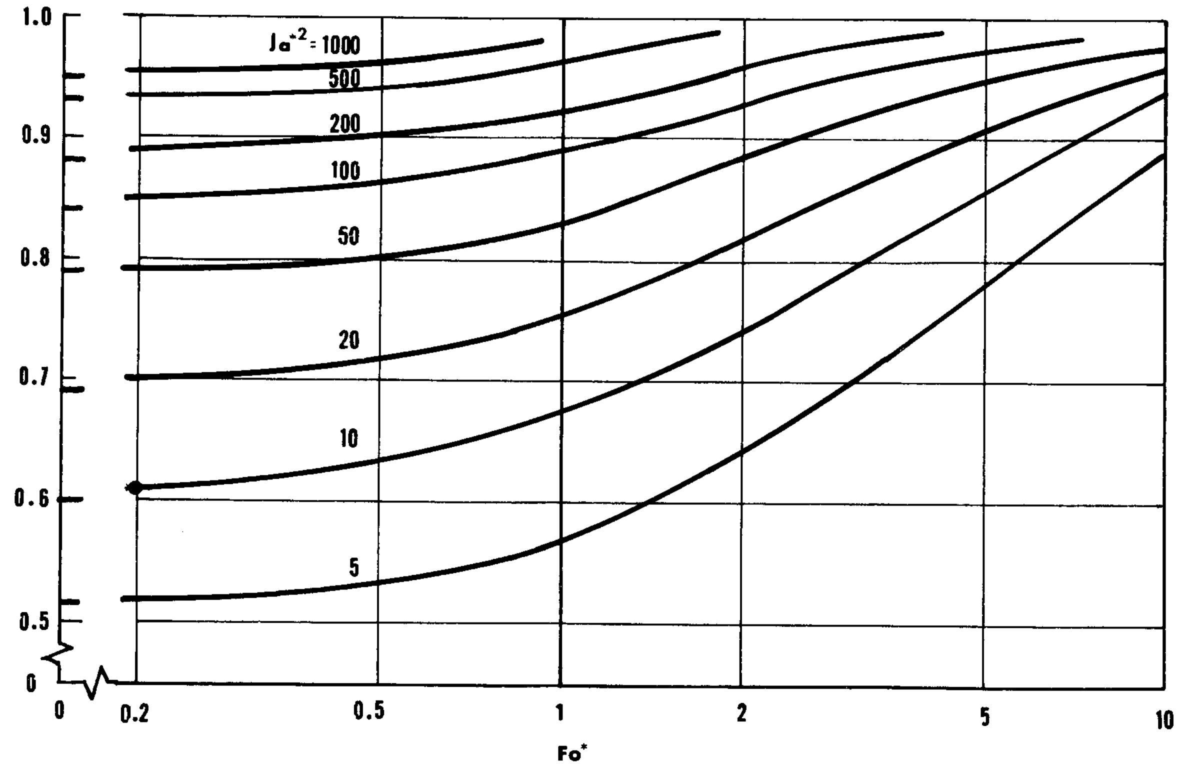

FIGURE 16. Ratio of surface temperature amplitude to air temperature amplitude. Adapted from Lowell and Patton (1955).

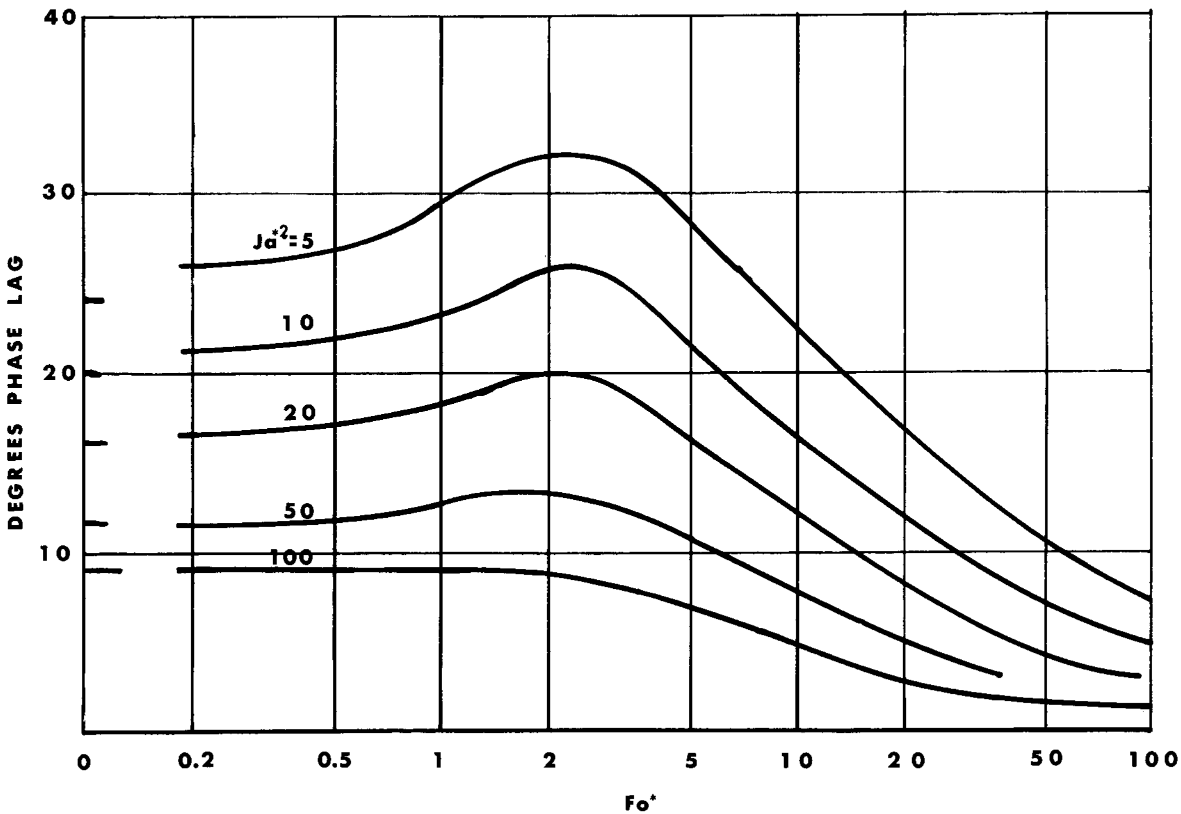

FIGURE 17. Phase relationship of surface and air temperature waves. Adapted from Lowell and Patton (1955).

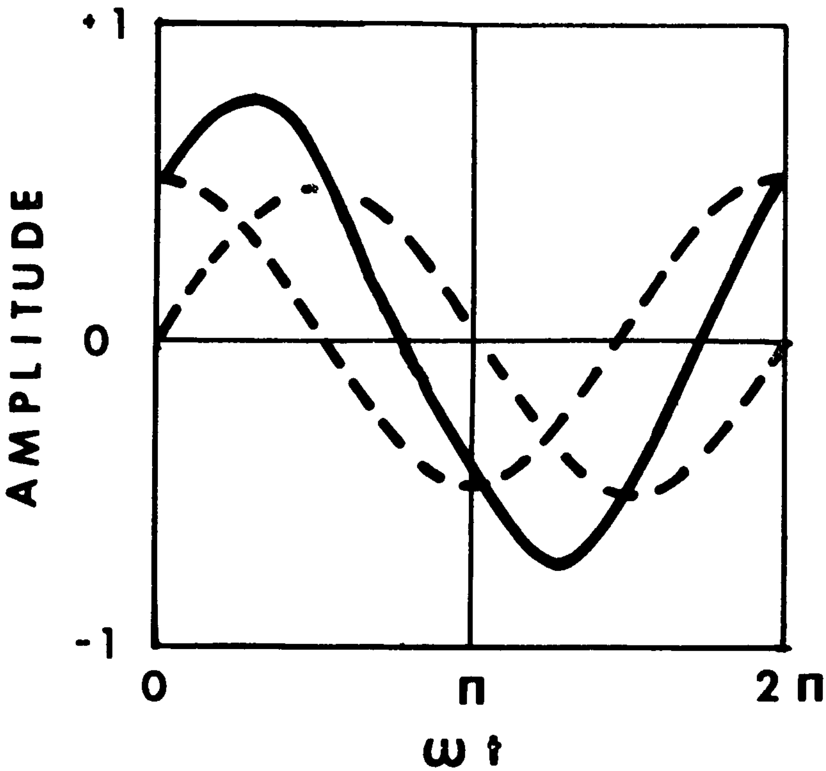

FIGURE 18. Illustration of ½ sin ωt + ½ cos ωt as 0.707 sin (ωt + π/4).

Returning to the two examples used in the discussion of the response of a twig and a stem (table 1) the surface temperature gain and phase lag can be found in relation to the air temperature from figures 18 and 19 and are shown in table 2 with the parameters Fo* and Ja^{*2}.

TABLE 2. THE ORDER OF MAGNITUDE OF THE SURFACE DAMPING RATIO AND PHASE LAG FOR A TWIG AND A STEM.[1]

	Radius cm	h_m cal cm^{-2} sec^{-1}	Fo*	Ja^{*2}	(δ)	φ(a) *rad.*	φ(a) *hrs.*
stem	10.0	2×10^{-4}	0.002	20	0.7	2π/12	1
twig	0.5	8×10^{-4}	300.0	300	1.0	0	0

[1] $k = 5 \times 10^{-4}$ cal/cm sec °C, $C_v = 0.33$ cal/cm³, $\omega = 7.27 \times 10^{-4}$ sec⁻¹; equation (32).

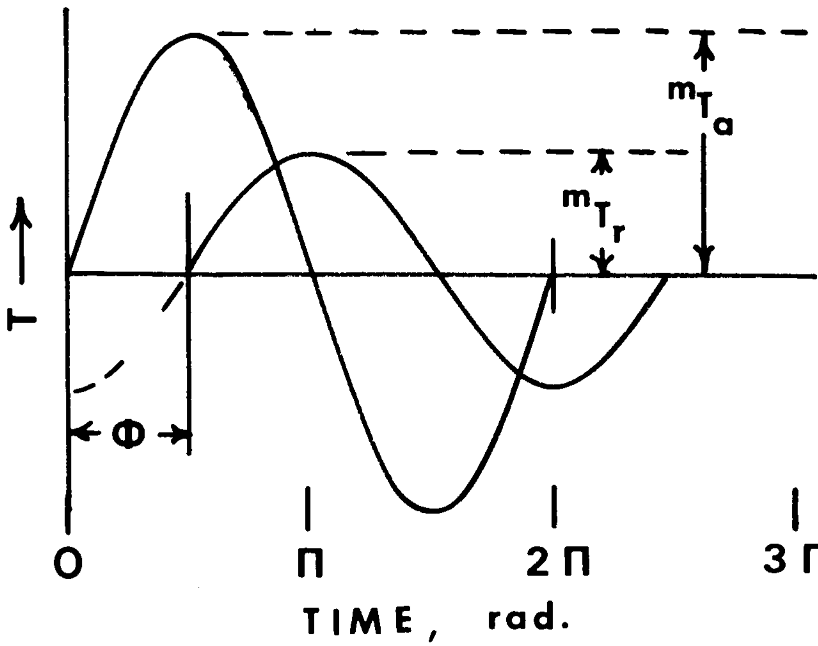

FIGURE 19. Graphic presentation of gain and phase lag.

As might be expected intuitively, the surface temperature of the small twig follows almost exactly that of the air while the amplitude of the surface temperature wave for the stem is approximately seven-tenths that of the air and the surface maximum occurs approximately an hour later than does that for the air.

SUMMARY

It has been shown that considerable simplification of the problem of heat transfer in tree stems is necessary in order to apply the techniques of mathematical analysis with success. The most drastic of these simplifications is that the thermal properties of the stem are not functions of radial position.

The analytic solution for the simplified problem shows that the interior temperatures, assuming the steady periodic condition has been reached, will be reduced in amplitude and maxima (or minima) will occur later in time as the temperature wave moves into the stem. This response is shown to be a function of the radius of the stem, its thermal properties, and the frequency or period of the applied wave. The parameter $\lambda^2 = \omega/k$ was introduced as the dimensionless number indicating similarity of response in different cylinders.

Application of a temperature drive expressed as Fourier Series was introduced and the solution presented in Fourier Series form. The possible effect of differential damping and lag between waves of differing frequency was discussed in terms of loss of higher frequency components of the surface drive. Mention was also made of the transient response and its effects of short-term periodic fluctuations.

Surface temperatures were found to be functions of the surface coefficient of heat transfer, radius of the cylinder, frequency of the temperature wave, and the parameter kC_v.

MEASUREMENTS OF TEMPERATURE PATTERNS IN A RED PINE STEM

IN ORDER to evaluate the usefulness of the analytic model (analytic solution) and to determine the effects of fluid flow (transpiration stream), local heat production, and variation of the thermal properties of the wood in time and space, temperature measurements were made in a living red pine stem.

Procedures

Selection of the Tree

The recording system used in this work was set up in a 48-year old red pine (*Pinus resinosa*) plantation in connection with a study of the energy budget of forest stands (Reifsnyder, 1962). The plantation is located at the north end of Lake Dawson in Woodbridge, Connecticut, and is less than 10 feet above the level of the lake.

In selecting the stem the following criteria were observed: (1) bole as nearly as possible a circular cylinder, (2) no lean or other evidence suggesting the formation of reaction wood, (3) healthy appearance, (4) uniformly distributed crown, and (5) a diameter at breast height of at least 20 cm. In addition, since the study did not include the radiant portion of the energy balance it was required that a minimum of solar radiation reach the stem surface.

Temperature Measurement

Measurement of the temperature field within the stems required that the thermocouple measuring junctions be placed at known locations in the stem. To this end a device (Herrington, 1965) was fabricated which permitted the mapping of a cross section the stem and the insertion of the thermocouple junctions at predetermined locations. The junctions were installed via holes drilled at 45° to the longitudinal axis of the stem. This procedure minimized errors due to conduction of heat along the thermocouple wires.

Surface temperatures are almost impossible to measure accurately with probes, especially when the surface is of a material with a relatively low volumetric heat capacity and poor conductivity.

Jakob (1957) discusses several methods of measuring surface temperature and the reader is referred to his work for details. In this study the surface measurements were made by pressing a small thermocouple into a depression in the bark. Conduction errors due to the leads were reduced by laying the leads on the bark surface for about 10 cm. Tests at the end of the study with a Barnes[1] radiation thermometer indicated that any errors were less than the accuracy of the Barnes instrument (±1°F).

Measuring Junctions

The measuring junctions used in this study were made from Leeds and Northrup 24 gauge duplex thermocouple extension wire. The junctions used for interior measurements were calibrated to be ±0.01°C of the National Bureau of Standards thermocouple tables (Shenker, *et al.*, 1955). The bead type of junction used for surface measurements proved to be very fragile and were not calibrated. The junctions used for air temperature measurements were installed before this study began and were not calibrated. Since the calibrated junctions were either well within the specified limit of error (±0.01C) or showed very large deviations it was assumed that the uncalibrated junctions were within the manufacturer's tolerance of ±0.8°C.

Analysis of the errors in the thermocouple system (see Herrington, 1964) indicated that the signals presented to the input of the data acquisition system were, in terms of allowable error (2.02 times the standard deviation) ±0.22°C for the calibrated junctions and ±0.81°C for the uncalibrated junctions.

Recording of Temperature Data

Temperature data were recorded with a millivolt digitizing and recording system which has been described in detail by Reifsnyder (1962). The digitized thermocouple output voltage is calculated and then converted to temperature by the relation,

$$T = Rt - (22.54mv + 0.55mv^2 + 0.02mv^3) \tag{54}$$

where Rt is the reference junction temperature (65.56°C) and mv is the millivoltage. The polynomial in equation (54) was derived from tables supplied by the reference junction manufacturer. These tables are based on the National Bureau of Standards Copper-constantan thermocouple tables (Shenker, *et al.*, 1955).

Recorder–Computer Calibration

Analysis of the components of variance indicated that at the 90% level the time variation of a single thermocouple-reference junction pair was ±1.0°C and the total variation approximately 1.6°C. The between-thermocouple varia-

[1]Barnes Engineering, Inc.

tion is due mainly to offsets in the reference junction temperature. Thus the absolute temperatures are subject to more error than are temporal and difference measurements.

Analysis of Temperature Data

To permit concise description of the temperature fields in tree stems it is desirable to reduce the position-time-temperature data to gains and phase lags. Without this type of concise description it would be almost impossible to compare the temperature waves in different trees or different portions of the same tree in a quantitative manner.

As was pointed out in the previous chapter (Analytic model) amplitude ratios and phase lags can be computed from maxima and minima and their relative time of occurrence if the surface wave(s) are very nearly sine waves. In terms of the Fourier Series, if the higher harmonics have significantly large amplitudes this simple technique may lead to errors.

The most logical way to measure the gain and phase lag would be to express all the time-temperature curves as a Fourier Series and compare the fundamental and harmonics individually. This was the technique used here.

Rewriting the Fourier Series equation of the last chapter (equation 46) in terms of a period of $n2/\eta$, where n = 1,2,3, ... is the integer specifying the harmonic, we have

$$T(t) = T_o + \sum_{n=1}^{m} \left[A_n \sin \frac{(n2\pi t)}{P} + B_n \cos \frac{(n2\pi t)}{P} \right] \tag{55}$$

The immediate problem is to evaluate the amplitude coefficients A_n and B_n from the measured data.

It can be shown by the application of the theory of least squares (Carslaw, 1930), that when the temperatures are given as

$$T_o, T_1, T_2, T_3 \ldots . T_j \ldots . T_{w-1}$$

at the times

$$0, t, 2t, 3t, 4t \ldots (w-1)t$$

where $\Delta t = 2\pi/w$ = constant[1]

that the amplitudes A_n and B_n and the average T_o are given by

[1]This relation transforms the data to a period of 2π for simplification of the calculations.

$$T_0 = \frac{1}{w} \sum_{j=0}^{(w-1)} T \tag{56}$$

$$A_n = \frac{2}{w} \sum_{j=0}^{(w-1)} T_j \cos\,(2\pi nj/w) \tag{57}$$

$$B_n = \frac{2}{w} \sum_{j=0}^{(w-1)} T_j \sin\,(2\pi nj/w) \tag{58}$$

where subscript j is the time interval and the subscript n is the index identifying the harmonics.

When A_n and B_n are found the Fourier series expression for the temperature variation can be written

$$T(t) = T_0 + \sum_{n=1}^{N} C_n \sin\,(nwt + \eta_n) \tag{59}$$

where C_n is the amplitude of a sine wave which is shifted by η radians from the start of the analysis period, $t = 0$ (see figure 18). We have that

$$C_n^2 = A_n^2 + B_n^2 \tag{60}$$

$$\eta = \tan^{-1}\,(A_n/B_n) \tag{61}$$

Each temperature wave is defined by two parameters, C_n and η. To compare a surface wave with a wave measured at some interior point the relations

$$\delta_n\,(r) = {}^{a}C_n/{}^{r}C_n \tag{62}$$

$$t_{1,n} = \frac{\phi n(r)}{nw} = ({}^{r}\eta_n - {}^{a}\eta_n) \tag{63}$$

where now the superscripts a and r are used to indicate the surface and the interior respectively.

Although the Fourier analysis (harmonic analysis) presented above is quite satisfactory for steady periodic waves, there are several limitations which must be taken into consideration when it is applied to cases where the periodic temperature is not exactly steady. The first of these has been mentioned before. If the surface or air temperatures immediately prior to the use of this analysis have not been quite similar, there will be a transient response. For this reason the stem should be in a nearly steady periodic state for at least one day before any analysis is attempted. The second point to be emphasized is that the analysis of the

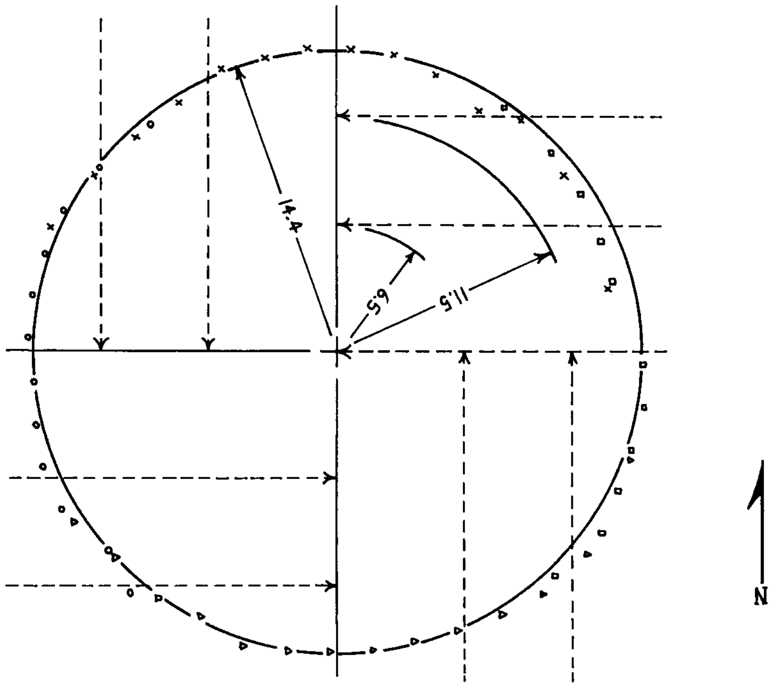

FIGURE 20. Cross section of stem showing thermocouple positions. Points on stem surface indicated by symbols (□, north side; Δ, east side; ○, south side; ×, west side). Dashed lines are plane projections of thermocouple leads.

interior and surface waves cannot be made over the same time period. If preliminary analysis indicates that an interior wave is, say, five hours behind the surface waves ($t_1 = 5$ hrs.) then the best estimate of the damping ratio and the phase lag will be obtained if the time of day of $t = 0$ for the analysis of the interior waves is shifted to the neighborhood of five hours later than the time of $t = 0$ for the surface wave. That is, if $t = 0$ for the surface wave is taken at 0700 hours, then $t = 0$ for the interior waves should be taken near 1200 hours. If this is not done, as can be seen in figure 19, the analysis will include a fairly large part of the wave of the "previous day."

Results

Thermocouple Location

Figure 20 shows the map of the cross section of the stem and the desired location of the nine interior thermocouples. When the tree was felled (February 1964) at the end of this study the actual location of the thermocouples was estimated from the cross section by measurement. Table 3 shows the estimated errors. It is felt that these measurements are accurate only to ± 0.5 cm.

TABLE 3. RADIAL ERRORS IN THERMOCOUPLE PLACEMENT

Radius	*Ouetr Ring*	*Inner Ring*	*Center*
N	0.0	0.0	0.0
E	0.6	0.6	
S	0.0	0.6	
W	0.0	0.0	

Thermal Properties

Table 4 shows the moisture content and specific gravity distribution along two increment cores taken from the stem in early April, 1963. Data were taken only for the wood since the bark samples would have been too small to give meaningful information. These cores were taken on North and South radii from the internode above the one in which the temperature measurements were made. The cores were taken from approximately the same internodal position as the plane of temperature measurement. Table 4 also shows the radial distribution of thermal conductivity, volumetric heat capacity, and diffusivity as computed from the sectional averages of the cores. The moisture content distribution, high in the sap wood and decreasing to nearly the fiber saturation point in the center, seems to be normal. The specific gravity distribution is not unusual.

The overall average thermal properties are:

$$\overline{C}_v = 0.485 \text{ cal cm}^{-3}$$
$$k = 6.93 \times 10^{-4} \text{ cal cm}^{-2} \text{ sec}^{-1}\ ^\circ\text{C}^{-1}$$
$$\kappa = 14.32 \times 10^{-4} \text{ cm}^2 \text{ sec}^{-1}$$

$\overline{C}_v$ was computed on an area-weighted basis while k was computed (MacLean's equation) from the area-weighted specific gravities and moisture contents.

Martin (1963) found that the diffusivity of bark is approximately $13 \times 10^{-4} \text{cm}^2 \text{sec}^{-1}$. Using this figure and the average value for the wood given above, the area-weighted average diffusivity is $12.23 \times 10^{-4} \text{cm}^2 \text{sec}^{-1}$.

TABLE 4. MOISTURE CONTENT AND SPECIFIC GRAVITY, THEIR AVERAGES, AND THE COMPUTED THERMAL PROPERTIES OF THE STEMS

Inner Radial Depth[3] cm	*Specific Gravity*[1] *Radius* 1	*Specific Gravity*[1] *Radius* 2	*Specific Gravity*[1] Aver.	*Moisture Content*[2] *Radius* 1	*Moisture Content*[2] *Radius* 2	*Moisture Content*[2] Aver.	k* $\frac{\text{cal}}{\text{cm sec °C}}$	C_v** $\frac{\text{cal}}{\text{cm}^3}$	κ $\frac{\text{cm}^2}{\text{sec}}$	Area of Annuli cm^2
0.5 (bark)	—	—	—	—	—	—	—	—	13.00×10^{-4}***	14.15
2.5	0.461	0.456	0.458	1.307	1.209	1.258	9.127×10^{-4}	0.632	14.44×10^{-4}	63.25
4.5	0.490	0.466	0.478	1.053	1.080	1.066	8.452×10^{-4}	0.597	13.16×10^{-4}	41.60
6.5	0.410	0.400	0.405	0.884	1.007	0.945	6.667×10^{-4}	0.447	14.91×10^{-4}	33.60
8.5	0.344	0.338	0.341	0.791	0.661	0.726	4.862×10^{-4}	0.311	15.63×10^{-4}	25.60
10.5	0.345	0.337	0.341	0.353	0.355	0.354	3.401×10^{-4}	0.200	17.00×10^{-4}	17.60
12.5	0.362	0.380	0.372	0.350	0.345	0.347	3.625×10^{-4}	0.214	16.94×10^{-4}	11.56

[1]Oven dry weight and volume.
[2]Grams water/grams dry wood.
[3]Depth from surface to inner boundary of section.
*Calculated from equation (4), x = 11.5%.
**Calculated from equation (13), T = 10°C.
***From Martin (1963).

We find then that for the daily wave, P = 24 hrs.

$$\lambda = (0.7272/14)^{1/2} = 0.227 \text{ cm}^{-1}$$

and for

$$a = 14.4,\ \xi = 3.3.$$

This value of ξ was used to compare the real tree to the analytic model.

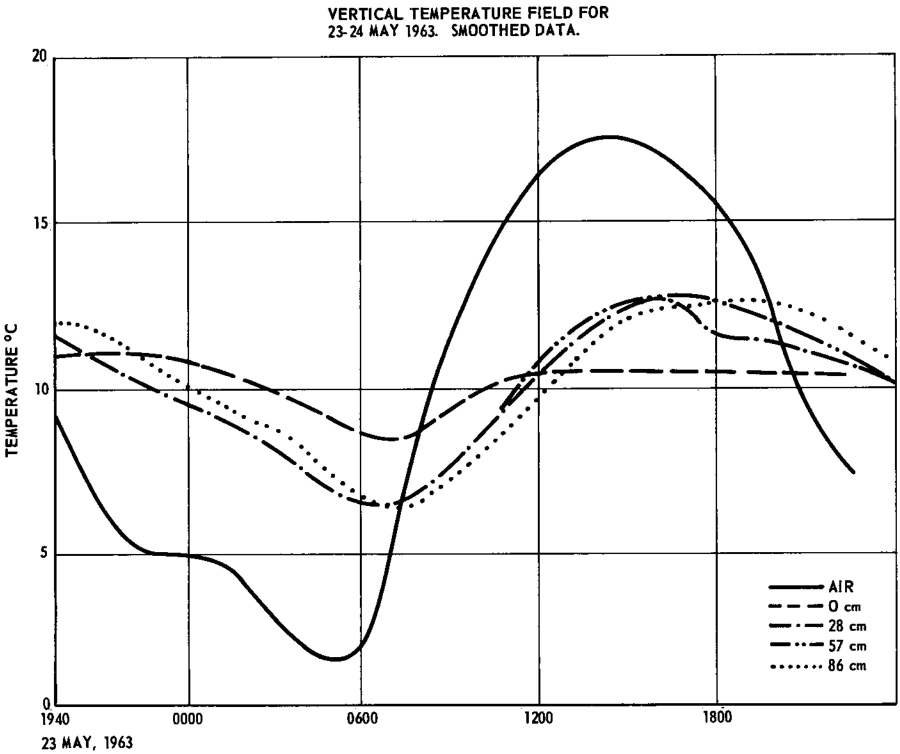

FIGURE 21. Vertical temperature field for 23–24 May 1963.

Vertical Temperature Field

Typical time-temperature curves for the thermocouples placed at heights of 0, 28, 57, and 86 cm at a depth of approximately 4 cm on the north side of the stem are shown in figures 21 and 22 for 23–24 May 1963 and 15–16 August 1963.

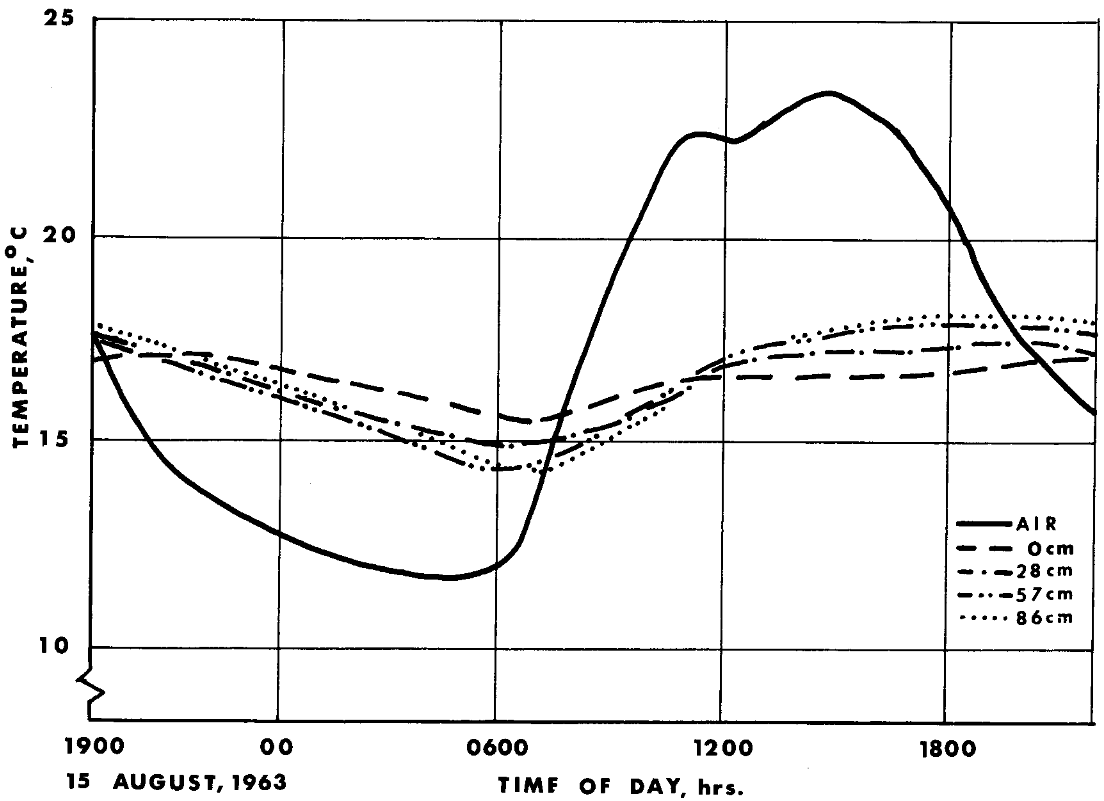

FIGURE 22. Vertical temperature field for 15–16 August 1963.

The effect of the colder ground (yearly wave effect) is quite noticeable in the data for 23 May. At the ground level the temperature is almost uniform showing only a slight dip in temperature in the early morning. This effect is not apparent at the 57 and 86 cm levels. The data for 15-16 August shows almost the same patterns except that the differences are not as great. Here again the curves for 57 and 86 cm lie almost atop one another.

The fact that the curves for 57 and 86 cm in the August data are slightly flattened might indicate that the transpiration stream is lowering the maximum temperatures. Noting that the data are confounded by vertical variation of thermal properties and inaccuracies in thermocouple placement, we can conclude from these data, with reasonable certainty, that the effects of the ground (conductive effects), and tentatively that the effects of the transpiration stream, are negligible above heights of one meter in this stem.

Horizontal Temperature Field

Computer-produced plots of the time-temperature data indicated that in most cases the temperatures on the surface and in the interior of the stem were not, within the limits of experimental error, functions of angular position *(fJ)*. The data for the outer ring of thermocouples (r = 11.6 cm) during the period 12-16 April 1963 showed the only serious deviation from this observation. A typical time-temperature curve is shown in figure 24.

The data for the period 13-16 April 1963 were subjected to Fourier analysis in order to evaluate the temperature uniformity of the fundamental waves. The results of this analysis are shown in table 5. Note that To and ffiT_c are within ±O.5°C of their means (over the radial points). There does not seem to be any real correlation between the maximum values of To and roT_r along anyone radii or circle of constant radius (variation with *fJ*). *Fot* some unknown reason the maxima of To and roT_r of the surface waves occur most often on the north side. There may be a slight correlation here with the day's average wind direction, but this is not conclusive. The effect is small in any case.

Figure 23 shows the values of To as a function of time for April. Note that the radial differences in To are nearly constant. The To's for the interior temperatures are very close to one another, the range being approximately O.2°C. The differences between the To of the surface wave and those of the interior waves are larger, but still less than O.5°C. I believe that these differences are due to reference junction offsets since the variation in time is consistent with the air temperatures variation measured on site and at the New Haven Airport.[1] No

1 Local climatological data. New Haven Municipal Airport, April, 1963.

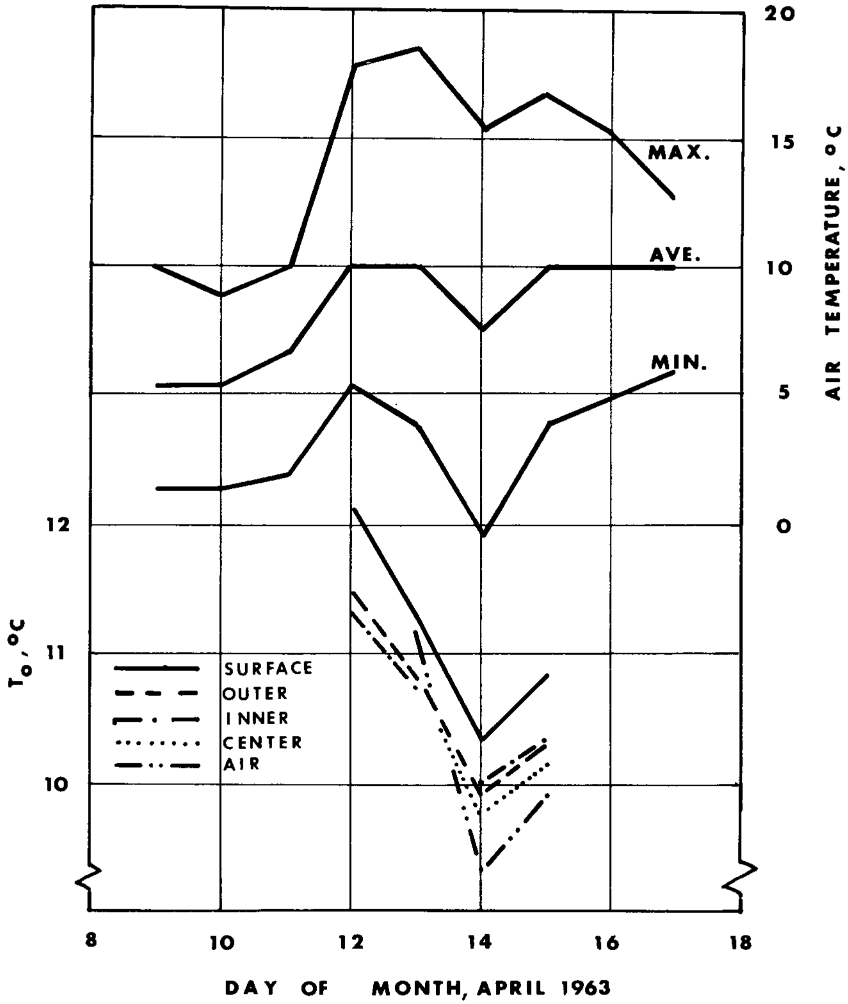

FIGURE 23. Variation of T_0 in time and space.

TABLE 5. RESULTS OF FOURIER ANALYSIS[1]—ANGULAR UNIFORMITY OF TEMPERATURES

Radial Position cm	Radius	T_o °C	^mT_r °C	Radians
Date: 13 April 1963. Time Start 0600. Wind NW at 10 mph.				
14.4	N	13.55*	3.48*	−0.52
	W	12.80	2.83	−0.59*
	S	13.17	3.27	−0.51
	E	13.21	3.32	−0.44
11.6	N	12.54	1.07	−1.99*
	W	12.39	1.71*	−1.54
	S	13.10*	1.38	−1.24
	E	12.37	1.56	−1.26
6.8	N	12.52	1.14*	−2.31
	W	12.08	0.78	−2.51*
	S	12.60*	1.03	−1.72
	E	12.45	0.88	−2.25
Date: 14 April 1963. Time Start 0600. Wind NW at 10 mph.				
14.4	N	16.47*	8.76*	−0.60
	W	15.66	7.66	−0.62*
	S	14.51	8.21	−0.57
	E	16.16	8.62	−0.52
11.6	N	14.71	2.76	−1.56*
	W	15.43*	3.50*	−1.34
	S	14.58	3.46	−1.35
	E	15.38	3.45	−1.07
6.8	N	13.90	2.88*	−2.32*
	W	15.03*	2.62	−2.18
	S	14.81	2.20	−1.94
	E	14.37	2.77	−2.01
Date: 15 April 1963. Time Start 0600. Wind NW at 10 mph.				
14.4	N	13.56*	5.14*	−0.59
	W	13.02	4.42	−0.68
	S	13.02	4.88	−0.61
	E	13.30	4.80	−0.57
11.6	N	12.45	1.95	−1.66*
	W	12.86	2.50	−1.45
	S	12.38	1.49	−1.47
	E	13.04*	2.73*	−1.18
6.8	N	12.04	1.21	−2.89*
	W	12.93*	1.75*	−2.13
	S	12.75	1.31	−1.81
	E	12.43	1.02	−2.38
Date: 25 May 1963. Time Start 0940. Wind S at 6 mph.				
14.4	N	11.91	4.15	−0.29
	W	12.24*	4.30	−0.38
	S	11.89	4.42	−0.40
	E	11.50	4.47*	−0.42*
Date: 16 August 1963. Time Start 0600. Wind W at 5 mph.				
14.4	N	18.22	3.74*	−0.65
	W	18.37*	3.62	−0.69
	S	17.84	3.55	−0.63
	E	17.65	3.68	−0.70*

[1]Fundamental wave only. *Indicates maximum.

TABLE 6. SUMMARY OF FOURIER ANALYSIS—FIRST HARMONIC ONLY

	T_o °C	^mT_r °C	t_1 *Hours*	$\delta(r)$	t* *Hours*
12 April					
1000					
Surface	12.17	5.85	—	—	0
Outer	11.50	2.64	−3.3	0.45	4
Inner	11.39	1.86	−6.3	0.32	4
Center	11.34	2.05	−7.9	0.35	8
Air	—	—	—	—	0
13 April					
1000					
Surface	11.31	7.36	—	—	0
Outer	10.85	3.56	−3.5	0.48	4
Inner	11.00	2.87	−5.7	0.39	4
Center	10.83	3.04	−8.1	0.41	8
Air	11.25	10.56	+0.9	1.43	0
14 April					
1000					
Surface	10.36	6.46	—	—	0
Outer	9.93	3.07	−3.2	0.47	4
Inner	10.01	2.26	−5.8	0.35	4
Center	9.79	2.54	−7.7	0.39	8
Air	9.36	7.81	+0.4	1.21	0
15 April					
1000					
Surface	10.89	6.50	—	—	0
Outer	10.35	3.07	−3.3	0.47	4
Inner	10.33	2.23	−5.9	0.34	4
Center	10.17	2.51	−7.8	0.38	8
Air	9.86	7.95	+0.5	1.22	0
16 April					
1000					
Surface	12.39	5.76	—	—	0
Outer	11.52	2.39	−3.5	0.41	4
Inner	11.40	1.71	−6.4	0.30	4
Center	11.22	1.88	−8.7	0.33	8
Air	11.74	7.34	+0.5	1.27	0
25 May					
0940					
Surface	13.88	5.94	—	—	0
Outer	13.59	3.37	−1.81	0.57	2:40 min.
Inner	13.21	2.65	−3.7	0.47	2:40 min.
Center	12.76	1.75	−7.6	0.26	6
Air	13.21	8.02	+0.6	1.35	0
15 August					
1000					
Surface	18.02	3.67	—	—	0
Outer	17.54	1.07	−3.5	.37	4
Inner	17.21	1.32	−6.6	.36	4
Center	17.58	0.93	−9.0	.25	8
Air	17.98	5.25	+0.7	1.50	0

*Time by which analysis of interior waves was lagged.

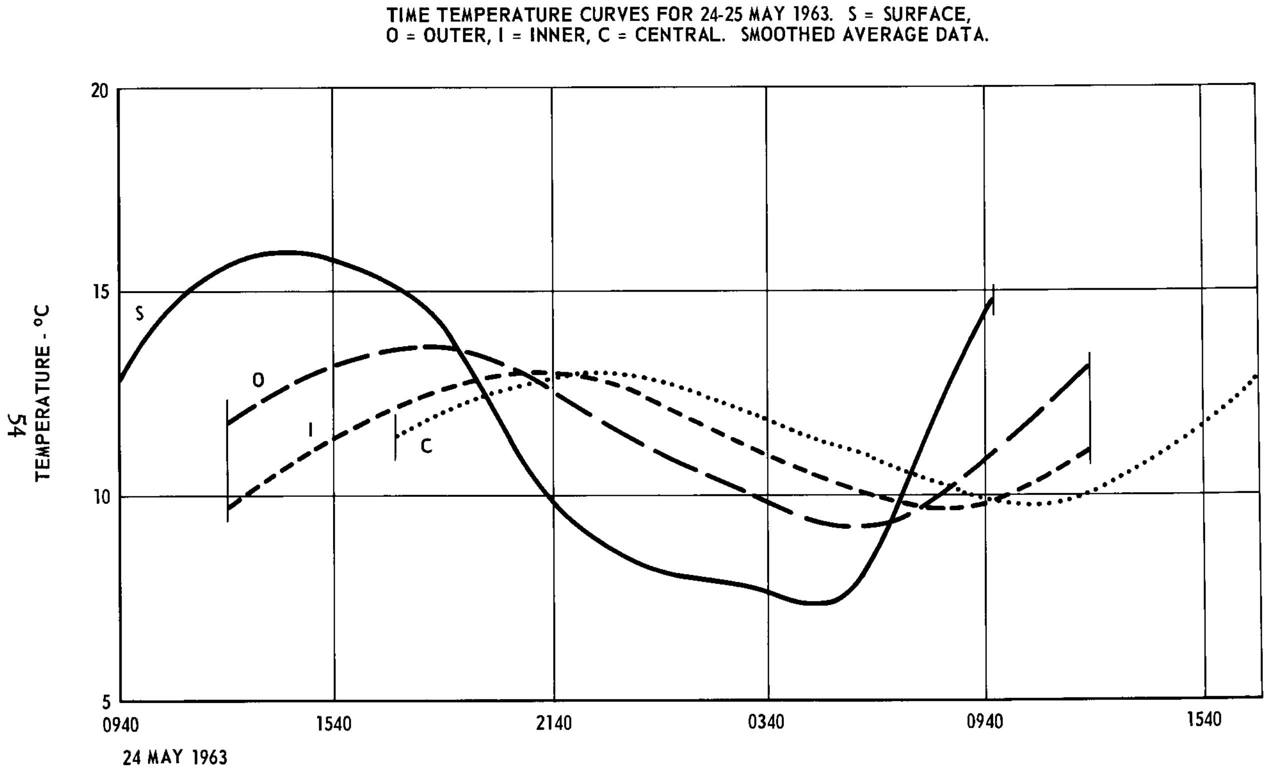

FIGURE 24. Typical time-temperature curve 24–25 May 1963. S=surface, O=outer, I=inner, C=center. Smoothed averaged data.

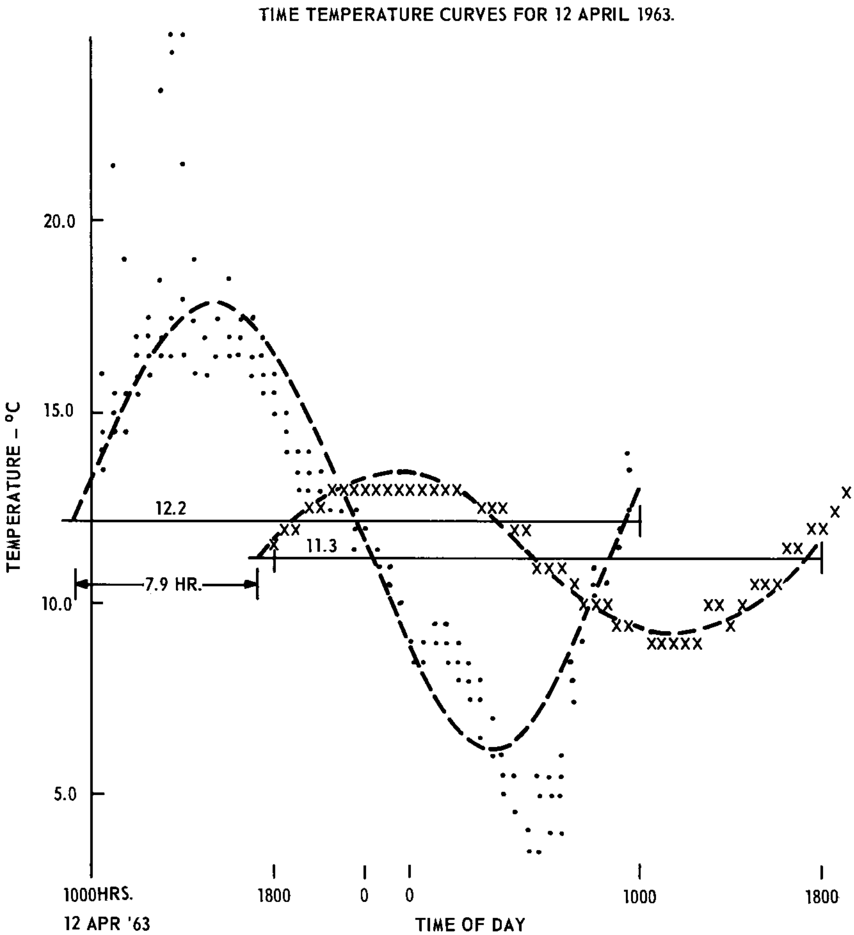

FIGURE 25. Fourier series fit for surface and central temperatures of 12 April 1963.

evidence could be found in the temperature waves which would indicate that this decrease in T_0 is due to transpiration.

Since the temperatures are essentially functions of radius alone the average of the waves at the four radial points were used in evaluation of the phase lag and gain as a function of radius. The results of the Fourier analysis of the temperature

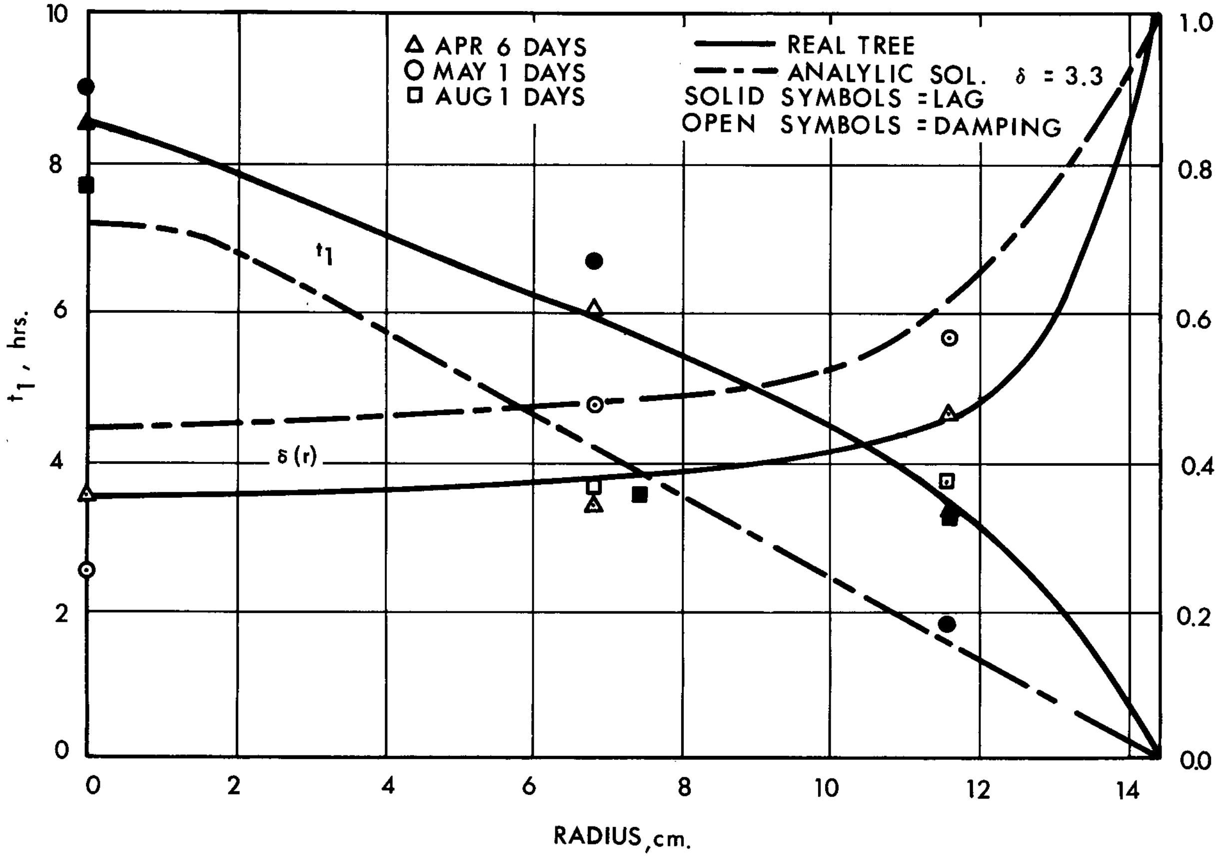

Figure 26. Measured and predicted phase angles and gains.

waves for the seven days that met the requirements set out in the discussion of the application of this technique, and which had relatively low noise levels, are presented in tables 6 and 7. Figure 24 is typical of the type of time-temperature curves which were analyzed. These curves are smoothed and were traced from the computer-produced plots. Figure 25 shows the surface and central waves for 13 April with the first term of the Fourier series fitted to the data.

The average damping ratio and phase lags for the four outer and inner and the one central thermocouple are shown in table 7 and figure 26. The data for April are very consistent, while the data for May and August show some variation from the April data. Figure 26 shows the measured and predicted phase lag and gain as a function of radius. As can be seen in the figure, the analytic solution predictions in terms of response are low. This is not surprising since the analytic solution assumed uniform thermal properties. The estimated shape of the measured damping curve is essentially the same shape as the predicted curve. This is not true for the phase lag curve.

TABLE 7. MEASURED GAIN AND PHASE LAG

	$\delta(r)$				t_1			
Date	11.6	6.8	0.0	Air	11.6	6.8	0.0	Air
12 April	.45	.32	.35	—	—3.3	—6.3	—7.9	—
13 April	.48	.39	.41	1.43	—3.5	—5.7	—8.1	0.9
14 April	.47	.35	.39	1.21	—3.2	—5.8	—7.7	0.4
15 April	.47	.34	.38	1.22	—3.3	—5.9	—7.8	0.5
16 April	.41	.30	.33	1.27	—3.5	—6.4	—8.7	0.5
Average	.456	.34	.352	1.25	—3.36	—6.01	—8.52	0.58
25 May	.57	.47	.26	1.35	—1.8	—3.7	—7.6	0.6
15 August	.37	.36	.25	1.50	—3.5	—6.6	—9.0	0.7
Overall average	.444	.364	.331	1.33	—3.33*	—6.11*	—8.68*	0.60

*Not including data 25 May.

Summary

The data obtained in the experimental portion of this study indicated the following:

1. Within the limits of experimental error (±0.5°C) the temperatures within and on the surface of the stem are functions of radius alone. This is especially true for the first harmonic of the Fourier series expressing the data.

2. The response of the stem is overestimated by approximately 20% by the analytic model.

THE ANALOG MODEL

IT WAS expected at the initiation of this study that the analytic model would not predict the temperature patterns in the stem with great precision. Thus it was desirable to investigate other methods of predicting or modeling the temperature field. When analytic methods fail to produce the desired results, recourse is usually made to one of the methods based on replacement of the differential equation by a finite-differences approximation. There are several ways of solving the simultaneous equations which result. Of these the numeric, active[1] analog and passive analog[2] methods were applicable here. The numeric method was not considered due to the complex nature of the programming which would be required. Both analog methods were suitable. Since the active analog computer which was available was relatively small and modeling by this method would require a large number of computing elements, this method was discarded in favor of the passive analog method. The passive analog has the advantage that any of the complicating factors such as heat sources, non-uniform thermal properties, fluid flow, etc., could easily be included. In addition passive models are easily understood intuitively while the others are not. The reader is referred to Ingersoll, *et al.* (1954), Schneider (1957), or Tribus (1958) for general discussion of these techniques.

Analog Modeling of Heat Flow

Although the concepts presented in this section can be derived mathematically, the physical approach will be used for the sake of clarity. For mathematical derivation the reader is referred to Karplus (1958).

Electric analog models are based on the similarity of the equations of heat flow and electric current flow. The characteristic equation describing the flow of heat in a bar insulated at the sides and with time dependent temperatures is

$$\frac{\partial T}{\partial t} = \frac{k}{C_v} \frac{\partial^2 T}{\partial x^2} \tag{64}$$

where x is the distance from the origin. Similarly, the equation giving the potential in a noninductive electric cable with resistivity R_e and capacity C_e is

$$\frac{\partial V}{\partial t} = \frac{1}{R_e C_e} \frac{\partial^2 V}{\partial x^2} \tag{65}$$

[1]Differential analyzer, analog simulator.

[2]Network analog, direct analog computer.

The similarity of these equations is obvious, especially if the reciprocal of k, the thermal resistivity, is substituted for k in equation (64). Thus the flow of heat in a rod may be investigated by study of the flow of electric current in the cable.

Consider the bar mentioned above to be divided into a number of equal volumes (figure 27). The total heat capacity is the product of the volume and the volumetric heat capacity:

$$C_t = \Delta x\ \Delta y\ \Delta z\ C_v\ \text{cal}^\circ\text{C}^{-1} \tag{66}$$

and is "lumped" at the center of the model volume. These central points, or nodes, are connected by thermal resistances; given by

$$R_t = \frac{\Delta x}{\Delta y\ \Delta z\ k}\ \sec^\circ\text{C cal}^{-1} \tag{67}$$

The electric circuit which is analogous to this bar is shown in figure 27. Electric resistors simulate the dissipative effect of thermal resistance while the elec-

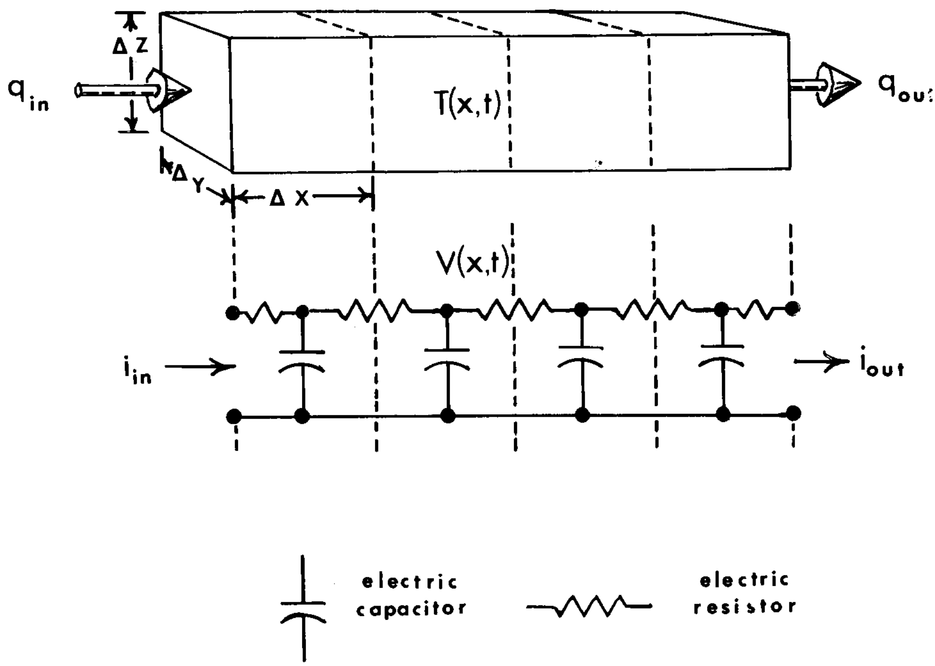

FIGURE 27. Analogous thermal and electric systems.

tric capacitors, connected from each node to the reference potential (V_o), simulate the heat storage capacity of the elemental volume. Transient boundary conditions, obtained when the temperatures in the thermal system and the voltages in the electric analog vary with time, require that there must be a correspondence not only between the electric and thermal capacitance and resistance but also between the products of electric and thermal resistance and capacitances.

The ratio of $R_e C_e$ to $R_r C_r$ determines the relation between the time variables in the analog and the thermal system.

We have then, that

$$t_e = T_F t_r \text{ sec. model/sec. real} \tag{68}$$

$$R_e = R_F R_r \text{ (ohm sec °C/cal)} \times 10^6 \tag{69}$$

$$C_e = C_F C_r \text{ farad/(cal/oC)} \times 10^6 \tag{70}$$

where the subscripts are t, thermal; e, electric; F, factor.

In order to connect voltage and temperature

$$V = V_F T \text{ volt/C} \tag{71}$$

Table 8 lists additional scaling relations which are useful. Not only can physical models be formed but time can be expanded or contracted on the model as desired.

This is the basis on which the analog model of the tree stem was made. The reader interested in further details is directed to Paschkis (1955), Paschkis and Baker (1942), Karplus (1958), Karplus and Soroka (1959), and Tribus (1958).

PROCEDURE

The analog modeling was done on the Heat and Mass Flow Analyzer of Columbia University, which has been described by Paschkis (1955).

The cross section of the stem, taken to be a circular cylinder with a radius of 14.4 em was divided (lumped) as shown in figure 28. The woody portion of the stem was lumped into 20 sections, with a half section at the cambium and at the center. The radial length of each full section is 0.67 em.

Values of electric resistance and capacitance for each node were computed using modifications of equations 66 and 67. The thermal properties used were those found for the real stem and scaling factors were selected to result in reasonable values of R_e and Ceo The resistors and capacitors were then set up in the circuit shown in figure 28.

TABLE 8. COMPARISON OF ANALOGOUS QUANTITIES IN THERMAL AND ELECTRIC CIRCUITS (ADAPTED FROM KARPLUS, 1958)

	Electrical			*Thermal*			
Variable or Parameter	*Description*	*Symbol*	*Unit*	*Description*	*Symbol*	*Unit*	*Scale Factor*
Across variable	voltage	v	volt	temperature difference	T	°C	$V = V_F T$
Through variable	current	i	amp	heat flux	q	cal/sec	$i = V_F/R_F$
Integral of through variable	stored charge	q_e	coulomb	heat energy	Q_t	cal	$q_e = C_F Q_t$
Time	electric time	t_e	sec	thermal time	t_t	sec	$t_e = T_F t_t = R_F C_F t_t$
Dissipating or damping parameter	resistance	R_e	ohm	reciprocal of conductivity x area	R_t	°C sec/cal	$R_e = R_F R_t$
Potential energy reservoir	capacitance	C_e	Farad	capacitance	C_t	cal/°C	$C_e = C_F C_t$

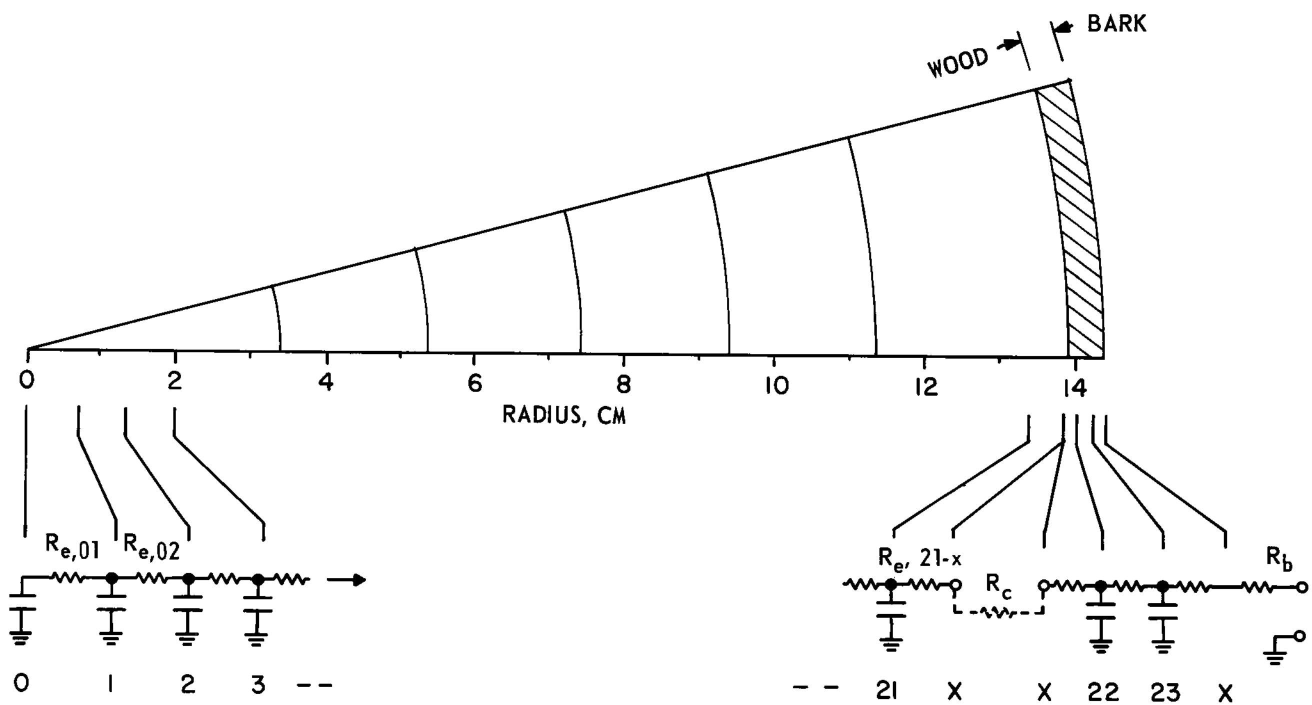

FIGURE 28. Schematic of analog model.

The resistors and capacitors used were accurate to $\pm 1\%$ (tolerance). Once the model was set up the capacitance at each node was checked with a General Radio capacitance checker to make sure that each node of the circuit had the correct capacitance. The resistance portion of the model was then checked by placing a known voltage across the string of resistors and measuring the voltage drop at several nodes. These voltage drops were compared with calculated voltage drops and if the measurements and calculations checked to within $\pm 1\%$ the resistance portion of the model was accepted.

A sinusoidial voltage, corresponding to surface temperature, was impressed upon the "surface" of the model and when the steady periodic state was achieved voltage-time measurements were made at various nodes with three multiple-point recorders. Since any current drain by the recorders would be analogous to a heat sink, special buffer amplifiers with a very high input impedance were used.

The voltage-time curves were analyzed simply by measuring the amplitudes of the surface and interior waves. From these measurements the gain and the average voltage, V_o (analogous to T_o) could be calculated. The phase lag was measured as the relative time between the intersections of the surface and interior waves with their own average "temperature." The ratio of this time to the time required for one period times 24 hours gives the phase lag directly in hours.

In order to measure heat flow the electric current flowing into and out of the analog model was measured. This was done by measuring the voltage drop across part of the outermost resistance. These measurements are none too accurate, however, due to the instability of the recorder when its input was floating (neither side of input grounded).

THE MODELS

The thermal properties used in forming two models are shown in table 9. The values of resistance and capacitance used are shown in table 10. The scale factors used were:

$$R_F = 4.828 \times 10^{-4} \text{ megohm cal/(°C sec)}$$
$$C_F = 3.425 \text{ microfarad/(cal/°C)}$$
$$T_F = 16.536 \times 10^{-4} \text{ sec elect/sec real (1 real day = 2.4 min)}$$

The first model, model A, was set up using equations (4) and (13) for the thermal properties of wood. The bark properties were estimated from the data available at the time.[1] After the model had been set up and the data analyzed, it was

[1]Reifsnyder, unpublished.

TABLE 9. ANALOGS A AND B

		Model A		*Model B*	
Node[1] (p)	r cm	C_e *Microfarads*	R_e *Kilohms*	C_e *Microfarads*	R_e *Kilohms*
0	0	0.25	424.0	0.26	456.0
1	0.67	2.00	141.0	2.1	152.0
2	1.34	4.00	84.9	4.2	91.3
3	2.01	6.00	60.6	6.25	65.1
4	2.68	8.00	47.2	8.3	50.7
5	3.35	11.00	40.8	10.0	43.9
6	4.02	11.2	34.7	11.6	37.3
7	4.69	13.1	30.1	13.6	32.4
8	5.36	19.1	19.1	19.4	20.5
9	6.03	26.5	16.6	27.2	17.8
10	6.70	29.5	15.0	30.2	16.1
11	7.37	45.9	10.2	39.5	11.0
12	8.04	50.9	9.2	52.0	9.9
13	8.71	55.1	8.5	58.5	9.1
14	9.38	68.9	6.3	70.0	6.8
15	10.05	82.7	5.9	84.0	6.3
16	10.72	88.2	5.5	90.0	5.9
17	11.39	104.7	4.8	106.0	5.2
18	12.06	108.5	4.5	111.0	4.8
19	12.73	114.5	4.3	117.0	4.6
20	13.40	152.0	3.1	154.0	3.3
x	13.90		R_c		
x			3.7		2.7
21	14.02	19.5	7.1	13.0	5.22
22	14.27	19.9	3.5	13.1	2.58
x	14.40		R_b		
x	"air"				

[1] x indicates a resistor junction point.

discovered that an error had been made in the calculation of the volumetric heat capacity of the wood. This error resulted in the volumetric heat capacity being computed for 1°C instead of 10°C (averaging 2.5% lower than correct values) as was intended. 10°C was selected since this was approximately the average temperature of the real stem during the measurements made in the early spring of 1963. The conductivity was computed for 30°C since that was the temperature at which MacLean (1940) made his measurements.

The decision was made to set up the model again, with the correct values of C_v. Since Martin's (1963) paper had been published during the interim, the second model, Model B, was to have bark properties as predicted by his relations. It was also planned, for Model B, to make an adjustment in the thermal con-

TABLE 10. THERMAL PROPERTIES OF MODELS

Section No.	*Outer Radius* cm	*Moisture Content* *% Dry Wt.*	*Density*			*Thermal Properties*			
						Model A		*Model B*	
			S_1 gm cm^{-3}	S_2 gm cm^{-3}	S_3 gm cm^{-3}	$k \times 10^{-4}$ cal/cm sec°C	C_v cal cm^{-3}	$k \times 10^{-4}$ cal/cm sec°C	C_v cal cm^{-3}
Bark	14.4	25%	0.360	0.311	0.389	1.900	0.260	2.650	0.171
1	13.9	125.8	0.958	0.405	0.915	9.127	0.625	8.488	0.632
2	11.4	106.6	0.478	0.423	0.874	8.452	0.571	7.860	0.579
3	9.4	94.5	0.405	0.358	0.697	6.677	0.439	6.210	0.447
4	7.4	72.6	0.341	0.302	0.521	4.862	0.305	4.522	0.311
5	5.4	35.4	0.341	0.302	0.409	3.901	0.193	3.163	0.200
6	3.4	34.7	0.372	0.329	0.443	3.625	0.207	3.371	0.214

ductivity of the wood for a reduction in temperature from 30°C to 10°C. A reduction of 7% was made. This very rough figure is based on the reduction of the conductivity of wood at the fiber saturation point between 30°C and 10°C of approximately 5% (Kuhlmann, 1962) and 9% for water over the same temperature range.[2] Although plans were made to make solutions with and without this reduction of the conductivity, this became impossible due to instrument malfunction.

Thus Model A is a solution based on the conductivity at 30°C, the volumetric heat capacity at 1°C, and a rough estimate of the bark properties. For Model B, the conductivity is reduced by 7%, the volumetric heat capacity is at 10°C, and the bark properties were estimated from Martin's relations.

An early practice run on the computer indicated that a contact resistance between the wood and the bark might be needed in order to achieve results that were similar to the measured lags in the tree. Thus in different trials the contact resistance shown in table 11 were included.

TABLE 11. CONTACT RESISTANCES USED IN THE ANALOG SOLUTION

$R_{e,c}$ kilohms	h_c cal/cm²sec °C
0	
5	11.1×10^{-4}
10	5.5×10^{-4}
20	2.8×10^{-4}

It was desired to estimate the coefficient of surface heat transfer (h_m) for the stem by recording the analog air and stem-surface temperatures (voltages) for different values of h_m. Trials were made with the values of h_m shown in table 12 with $R_{e,c}$ = 20 kilohms.

TABLE 12. VALUES OF THE COEFFICIENT OF SURFACE HEAT TRANSFER USED IN THE ANALOG SOLUTION

$R_{e,b}$ kilohms	h_m cal/cm²sec °C
0	
10	5.34×10^{-4}
30	1.78×10^{-4}
60	0.89×10^{-4}
100	0.53×10^{-4}

[2]Chemistry and Physics Handbook, Chemical Rubber Publishing Co., Cleveland, Ohio. 40th Edition, 1958.

Results and Discussion

The results for the interior temperature field are shown in figures 29 and 30, the gain and phase lags respectively. In these figures the solid lines are the results for Model A and the dashed lines for Model B. In each case the results for Model A with the indicated contact resistance are shown.

It was expected that the minor changes in wood properties would have little, if any, effect on the temperature field in the stem. The rather large changes in bark properties were expected, however, to have some effect. The results for the two different models are for all practical purposes, the same. The largest deviations are near the surface. In these figures the average data for April and the data for May and August are shown. For both the gain and the phase lag the fit is quite good for the case where the 20 kilohm contact resistance has been included. This corresponds to a thermal contact coefficient of 2.8×10^{-4} cal cm^{-2} sec^{-1} °C^{-1}. The indicated presence of a contact resistance was first thought to be associated with the initiation of cambial growth and the start of the "sap peeling" season. The loose bark might explain the contact resistance effect. August, however, is well past the sap peeling season and the data for the one day during August, even though less reliable, are far removed from the response curves for zero contact resistance. Thus there is no real explanation for the indicated requirement of a contact resistance.

The surface temperature gain and phase lead are shown in figure 31 as a function of the coefficient of surface heat transfer, h_m. The solid lines indicate the prediction of analog Model A. The dashed lines are for the parameters as predicted by Grober's solution for two cases:

1.	$k = 1.9 \times 10^{-4}$	cal cm^{-1} sec^{-1} °C^{-1}
	$C_v = 0.26$	cal cm^{-3}
	$kC_v = 0.49$	cal^2 cm^{-4} sec^{-1} °C^{-1}
2.	$k = 6.93 \times 10^{-4}$	cal cm^{-1} sec^{-1} °C^{-1}
	$C_v = 0.48$	cal cm^{-3}
	$kC_v = 3.45 \times 10^{-4}$	cal^2 cm^{-4} sec^{-1} °C^{-1}

Case 1 is for a cylinder of bark, and Case 2 is for a cylinder of wood (see table 10). As would be expected the analog result lies between the two analytic predictions. The average phase lead between air and surface temperature on the real tree is 0.6 hours, while the average gain is 0.75 (table 7). As shown in figure 31 this indicates surface resistances of 25 to 20 kilohms, which correspond to approximately 2.0×10^{-4} cal cm^{-2} sec^{-1} °C^{-1} for a value of h_m. Application of equation (22) for h_m gives 1.6×10^{-4} cal cm^{-2} °C^{-1} for a velocity of 67 cm sec^{-1}.

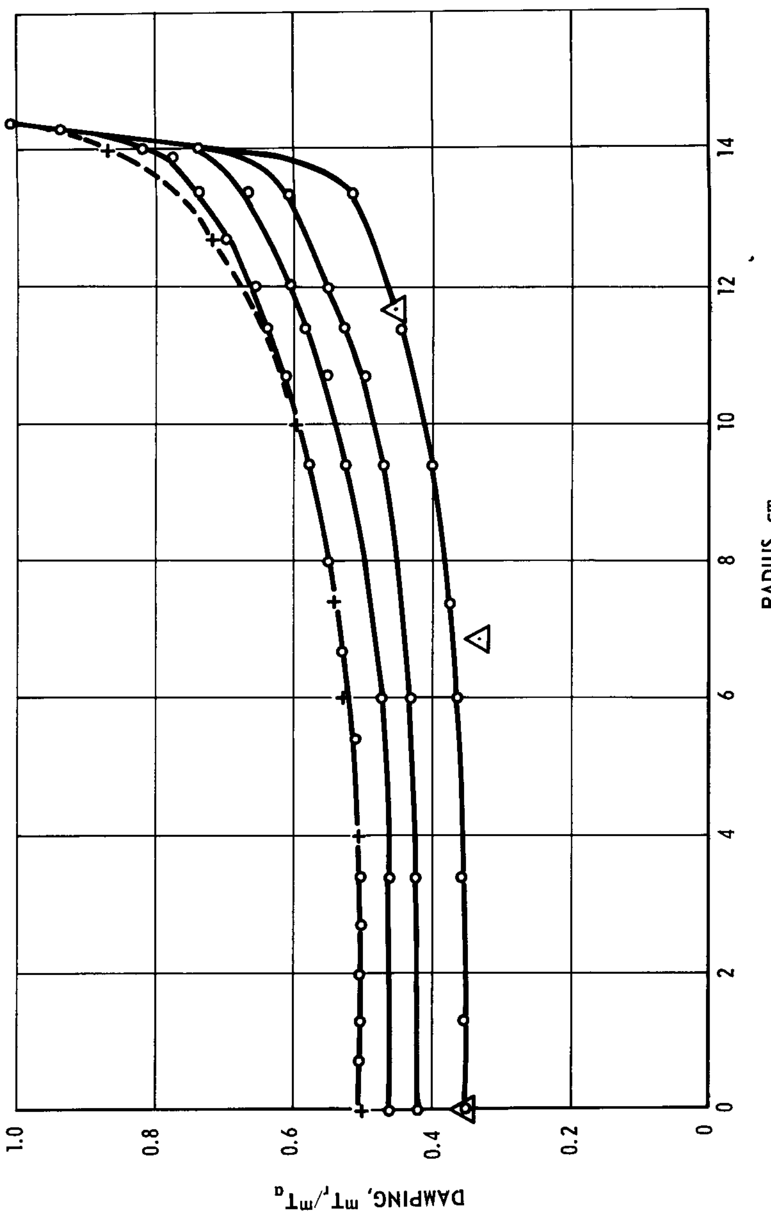

FIGURE 29. Measured and predicted gain.

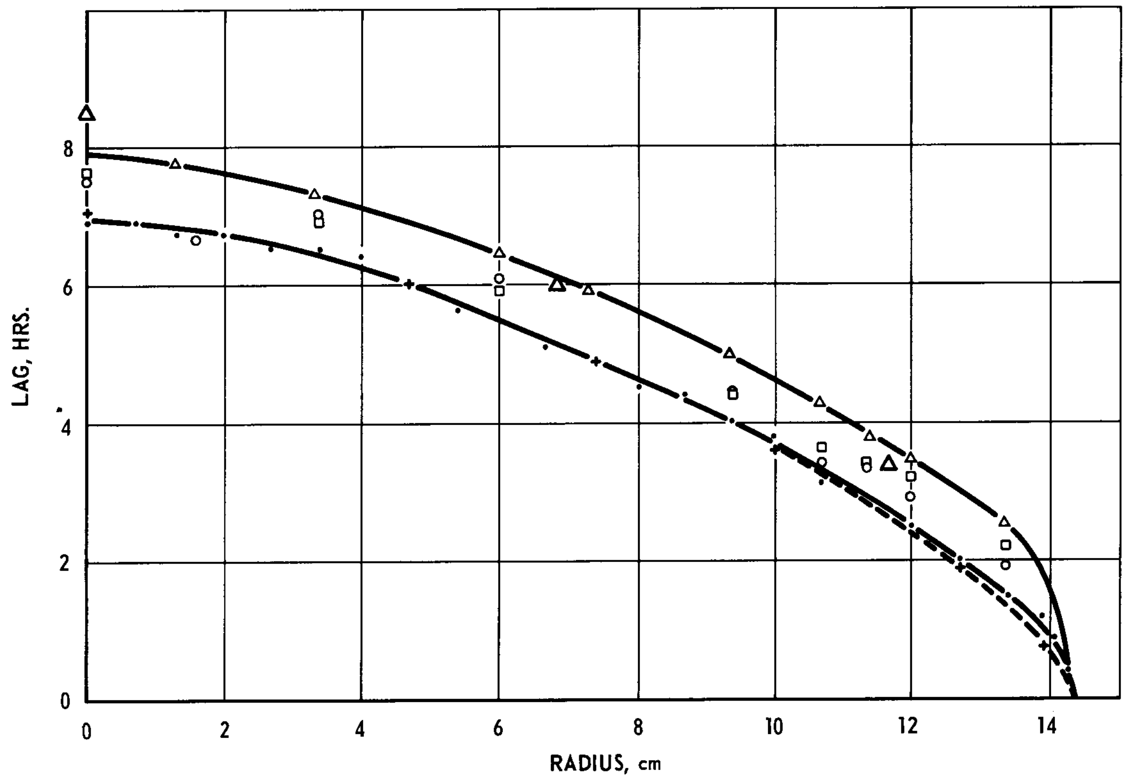

Figure 30. Measured and predicted phase lag.

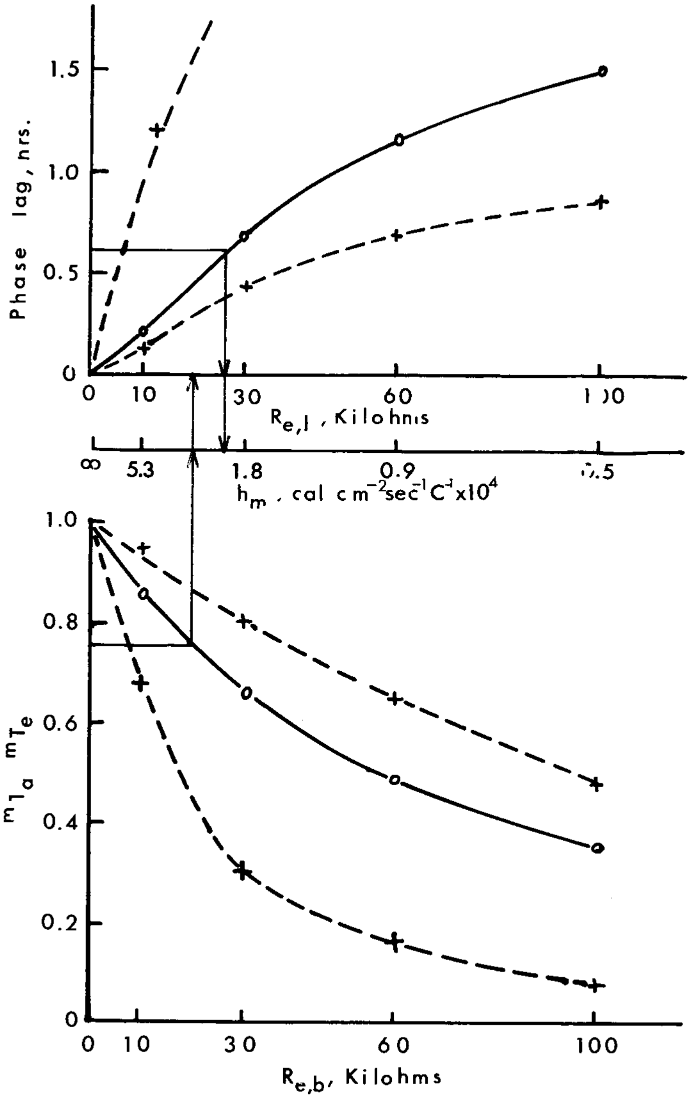

FIGURE 31. Phase lag and gain for two homogeneous models compared with analog model.

The average amplitude of the electric current into the analog model was 556 microamperes ($h_c = 2.8 \times 10^{-4}$ cal cm^{-2} sec^{-1} °C^{-1}). This corresponds to a surface heat flux amplitude, $^{m}q_{a}$ of 0.82 cal cm^{-2} sec^{-1}, for $^{m}T_{a} = 1$°C. The phase lead of the analog surface wave was 2.7 hours. The analytic solution gives, for case 1 above, 0.47 cal cm^{-2} sec^{-1} ($a = 14.4$ cm, $\xi = 4.6$, $\lambda = 0.32$, $\nu = 0.9$), and for case 2, 1.37 cal cm^{-2} sec^{-1} ($a = 14.4$, $\xi = 3.3$, $\lambda = 0.227$, $\nu = 0.9$) for an $^{m}T_{a}$ of 1°C. Again the analog results lie between the two cases. In this case, however, the analog result is closer to the analytic solution which assumes the stems to have the average conductivity of the wood.

Summary

The analog model was able to predict the temperature field in the stem more accurately than the analytic model. It was discovered that a "contact resistance" existed between the bark and the xylem. It was also noted that the solution was not sensitive to small changes in the thermal properties of the wood or to relatively large changes in the thermal properties of the bark.

The surface coefficient of heat transfer was estimated to be approximately 2×10^{-4} cal cm^{-2} sec^{-1} °C^{-1}, and it was noted that in the real stem there was little variation in this estimate. The analytic solution was shown to be unable to predict with any real accuracy the relations between the environmental temperature waves and the stem surface waves. If the analytic solution is to be used for this purpose it is best to assume the stem to be a body with the thermal properties of the bark. The opposite situation was found in the case of surface heat flux. Here the stem should be assumed to be made of material with thermal properties characteristic of the wood.

SUMMARY AND CONCLUSIONS

IN THIS chapter the various aspects of this study of the thermal nature of tree stems will be summarized, and the applicability discussed of the various modeling techniques and other methods used.

The Analytic Solution

The importance of the analytic solution to the understanding of the thermal processes in a tree stem has been demonstrated. This solution for a greatly simplified form of the general problem indicated that the temperature field in the stem can be quantitatively described by a pair of numbers, the gain and the phase lag. In addition, it forms a basis on which the data from a limited number of experimental trials can be generalized to cover a wide range of situations. Since the analytic model was able to predict the internal temperatures with a fair degree of success, it can be assumed that the extensions of the model will be correct at least in a qualitative way. The important findings for the temperature field were:

1. The amplitude of an applied wave is decreased and the maxima (or minima) occur later in time as the wave moves toward the center of the tree.
2. The response of the stem or cylinder is a function of its dimensionless radius, which is a function of the radius of the stem, the thermal properties of the stem, and the frequency of the applied wave.
3. Any stem or cylinder with the same dimensionless radius, has the same dimensionless response to the applied wave.

The relations for heat flow, although they did not match the analog heat flow with any great accuracy, can be used for future analytic modeling concerning the storage of heat in tree stems as a part of the energy budget of the forest. In this regard two points are of interest. First, for very large trees or stands of large trees the relatively simple equation for the flow of heat in an infinite slab can be used. Secondly, the solution indicates that the thermal response of the canopy, which is made up of small twigs and leaves, will be very fast, showing a maximum of heat flow some 12 hours before the maximum of the daily air temperature curve.

The analytic predictions for the relations between the air and stem surface temperatures were not successful but did show how the analog data could be analyzed to provide an estimate of the coefficient of surface heat transfer.

SUMMARY AND CONCLUSIONS

THE ANALOG MODEL

Analog modeling of the real stem proved to be a very flexible and useful tool for this type of study. The only drawback is the expense of using general purpose instruments.

The analog model was able to predict the temperatures in the stem much more accurately than the analytic model. This was because neither the radial variation of the thermal properties nor the indicated presence of a "contact resistance" between the bark and the xylem could easily be included in the analytic solution. The important findings obtained from the analog model are:

1. A "contact resistance" was indicated to be present between the bark and the xylem. This resistance could not be attributed entirely to the loosening of the bark during the spring.

2. The average coefficient of heat transfer was estimated to be 2.0×10^{-4} cal cm^{-2} sec^{-1} $°C^{-1}$.

THE REAL TREE

Measurements of the temperature field in the real tree indicated that:

1. Surface temperatures were uniform in respect to angular position and, within the limits of experimental error, no effect of wind could be found.

2. Internal temperatures were found to be, for all practical purposes, independent of angular position.

3. Within the limits of experimental error, no effects traceable to respiration of living tissues in the stem or fluid flow were found at a height of 1.4 meters.

4. In a qualitative way the predictions of the analytic solution were shown to be valid.

The fact that the surface temperatures (in terms of the fundamental waves) were uniform was not too surprising, since wind speeds in the forest are relative!y low (Reifsnyder, 1955), and the flow is turbulent. The roughness of the bark would also tend to reduce any differences.

The apparent angular uniformity of the interior temperatures indicated that the thermal properties of the stem were not functions of angular position. When the tree was felled at the end of the study measurements of moisture content and specific gravity distribution in a cross section immediately below the plane of measurement show that this was the case. The variation (average error) with angular position averaged 10% for the three annuli into which the stem was divided.

EXPERIMENTAL METHODS

Drill Rig

The drill rig proved itself to be a fairly accurate device for the location of temperature sensors at desired points within a stem. Improvements in the design of this device should allow for better accuracy.

Thermocouples

The measurement of temperatures with thermocouples is not to be advocated due to the relatively small amplitudes being measured. With modern techniques much more accurate results can be obtained with thermistors, which are much more sensitive than thermocouples.

Fourier Analysis

Fourier Analysis should be used whenever complex but roughly sinusoidial temperature variations are to be analyzed. By this technique qualitative comparisons can be replaced with quantitative analysis.

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APPENDIX A

TABLE OF NOMENCLATURE (Frequently Used Symbols)

A	rate of heat production	cal/(cm sec)
a	radius of cylinder	cm
C	specific heat (or electric capacity)	cal/gm (microfarad)
C_m	specific heat, wet wood	cal/gm (microfarad)
C_o	specific heat, dry wood	cal/gm (microfarad)
C_v	volumetric heat capacity	cal/gm^3
D	diameter of cylinder	cm
Fo*	parameter	N.D.
h_m	coefficient of surface heat transfer	cal/(cm^2sec °C)
h_b	contact coefficient	cal/(cm^2sec °C)
Ja	parameter	N.D.
k	thermal conductivity	cal/(cm sec °C)
P	period of applied wave	sec.
Q	half cycle heat flow	cal/cm^2
q	heat flux	cal/(cm^2sec)
R	electric resistance	megohms
r	radial coordinate	cm
S_1	specific gravity, oven-dry weight, volume	N.D.
S_2	specific gravity, green volume oven-dry weight	N.D.
S_3	specific gravity, green weight, green volume or density	N.D. gm/cm^3
T	temperature	°C
t	time	sec.
t_l	lag	sec.
V	velocity	cm/sec.
z	axial coordinate	cm.
α	fractional volumetric shrinkage of wood, green volume to oven—dry volume	N.D.
β	phase lead	radians
δ	gain	N.D.
η	epoch angle	radians
θ	angular coordinate	radians
κ	thermal diffusivity	$cm^2\ sec^{-1}$
λ	parameter = $(\omega/\kappa)^{1/2}$	cm^{-1}
μ	viscosity	$gm\ cm^{-1}sec^{-1}$
ν	amplitude of heat flux	N.D.
ξ	dimensionless radius = λa	N.D.
ρ	dimensionless radial coordinate = λr	N.D.
ϕ	phase lag	sec rad^{-1}
ω	angular velocity = $2\pi/P$	rad sec^{-1}
Subscripts		
a	surface	
e	electric	
F	factor	
r	radial point	
t	thermal	
o	average (time wise)	
Superscripts		
m	amplitude	

APPENDIX B

TABLE OF BER, BEI, AND RELATED FUNCTIONS[1]

x	ber (x)	bei (x)	ber′ (x)	bei′ (x)
0.1	0.99999 84375	0.00249 99996	−0.00006 25000	0.04999 99740
0.2	0.99997 50000	0.00999 99722	−0.00049 99993	0.09999 91666
0.5	0.99902 34640	0.06249 32184	−0.00781 20761	0.24991 86211
1.0	0.98438 17812	0.24956 60400	−0.06244 57521	0.49739 65115
2.0	0.75173 41827	0.97229 16273	−0.49306 71247	0.91701 36134
3.0	−0.22138 02496	1.93758 67853	−1.56984 66322	0.88048 23241
4.0	−2.56341 65573	2.29269 03227	−3.13465 39628	−0.49113 74406
5.0	−6.23008 24787	0.11603 43816	−3.84533 94733	−4.35414 05148
6.0	−0.88583 15966+1	−0.73347 46541+1	−0.02930 79967+1	−1.08462 23329+1
7.0	−0.36329 30243+1	−2.12394 02580+1	1.27645 22560+1	−1.60414 88888+1
8.0	2.09739 55611+1	−3.50167 25165+1	3.83113 25701+1	−0.76603 18414+1
9.0	7.39357 29858+1	−2.47127 83169+1	6.56007 70999+1	3.62993 84423+1
10.0	1.38840 46594+2	0.56370 45855+2	0.51195 25839+2	1.35309 30172+2

[1]From Lowell (1959).

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