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Optimal Forest Investment Decisions Through Dynamic Programming

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OPTIMAL FOREST INVESTMENT DECISIONS THROUGH DYNAMIC PROGRAMMING

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To ALLY

whose patience is one of her finest irrationalities

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Of course, no one except the author can be held responsible for the final text.

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ABSTRACT

A model is set up that covers the whole production process from tree seedling to final product of one or more of the primary forest industries. It consists of a string of revenue and cost functions which together form the objective function to be maximized. Thus it translates the assumed objective of the study: maximization of the discounted present value of the net benefits of alternative possible investments in the forestry sector. What makes the maximization non-trivial is the interdependency of the variables involved. This multistage decision problem is molded into a form that can be solved by dynamic programming. One state and one decision variable, respectively total standing volume and total cut, are specified. To handle volume growth forward recursion is necessary. This, plus the special computational procedure developed, makes it possible after specifying the state and the decision variable in aggregative terms to refind the per unit area values of these two variables and thus to calculate the growth on a per unit area basis. All other variables either are assumed constant or related to the state and/or the decision variable, and differentiated for specific area units. A very desirable sensitivity analysis on the final state variable is automatically implied.

The model answers such questions as whether over-industrial capacity (relative to sustained yield capacity) is a desirable thing in the first stages of development of the forest resources. Thus it enables the forest planner to submit optimal decision rules for any of the targets or constraints that higher policy may dictate. One special case of the model is the Faustman formula case. By maximizing the discounted net benefits from the *whole* production process and by allowing for the alternative of ordering raw material from elsewhere, the model generalizes considerably the traditional soil expectation-sustained yield approach. Another special case is that of the forester managing the forests of a plant subject to its demands. The model itself is a special case of a possible model embracing the whole forestry sector.

I. OBJECTIVE OF THE STUDY

THE USE and development of the forest resource historically has been quite haphazard in most countries. If planning took place at all it generally was done in the following way: foresters concentrated on the tree growing-harvesting aspects while industrialists restricted their attention to the wood conversion or industrial aspects of forestry. Of course tremendous conflicts of interests arose because of these separated approaches. Foresters developed the sustained yield philosophy which, coupled with the Faustman formula or soil expectation approach, formed the main framework within which management decisions were taken. Industrialists eager for quick profits and often overlooking long range raw material supply questions, insisted on much faster harvesting rates and cared little for the renewable aspect of their raw material base.

The assumption implicit in this separated approach to the planning of the development and use of the forest resources is that optimization of all parts of a unit within the forestry sector will lead to the optimization of the whole unit. In other words the implicit assumption generally made is that the conditions of pure competition hold true. Economists, of course, have pointed out the fallacy of this assumption, but have done relatively little in the way of developing alternative decision models for foresters.

The objective of this study will be to develop optimal criteria or rules for action when planning the use and the development of the forest resources, taking into account the tree growing-harvesting activity as well as the wood processing phase. The decision rules must be optimal in the sense that the contribution of the possible forestry activities to the objective of the economy is maximized, given constraints on the availability of resources. The objective of the country will be taken as given.

A mathematical programming model, embracing the whole production process from tree seedling to final product of one or more of the primary forestry industries, will be developed which will maximize over time the contribution of the possible forestry activities to the objective of the economy, subject to constraints. Some of the constraints will be given data determined exogenously or endogenously to the situation, other constraints will be determined jointly by the foresters, economic planners and politicians.

The philosophy behind this approach is that the forester or forest planner should not submit a single or at most a few alternative investment plans to be subjected as a kind of fait accompli to a cost-benefit analysis and then perhaps be accepted or rejected by the final decision makers. This results generally in a single cost-benefit ratio with some marginal comments, which are subsequently forgotten, as to the external effects of the investment plan. Rather the forest planner should submit a model of investment whose performance can be ascertained in respect to any targets or constraints that higher policy may dictate. In this way the desired optimal decision rules are provided by the model for a variety of assumed conditions. A mathematical programming model coupled with a sensitivity analysis seems eminently suitable to handle this type of a problem. The model, finally, will be applied to a particular case using real data where available.

II. A DYNAMIC PROGRAMMING APPROACH TO FOREST INVESTMENT DECISIONS

REVIEW OF LITERATURE

Like every other sector of an economy, the forestry sector should try to maximize its contribution to the objective(s) of an economy subject to given restrictions on the availability of resources. Or, alternatively, it should try to minimize the use of scarce resources per unit contribution to the objectives of the economy. This, of course, is a perfectly general statement.

Several questions arise immediately in connection with the statement above which have to be dealt with before the problems to which this study directs itself can be stated. What are the important characteristics of an economy as they concern the allocation of resources? Which possible forestry activities are considered in this study and to what kind of forestry situation is the study directed?

In the traditional static world of pure competition with all its assumptions of perfect foresight, perfectly divisible resources and commodities, absence of external and internal economies, the resulting price system would assure the optimal allocation of resources among the alternative uses. The allocation of resources would be optimal in the sense of assuring a maximum output at a given social cost or a given output at a minimum social cost. As many economists have pointed out, this optimal allocation of resources would be assured if a policy of profit maximization would be followed: select investment projects according to the profits they are able to provide. Equilibrium would be indicated by the so-called set of marginal equations and when the profits of all activities in use are zero. The price system would make a decentralized decision making process possible. Adam Smith's "invisible hand" would guide everybody to pursue the best interest of the society by pursuing his own best interest. No deliberate economic policy designed to influence the amount and composition of investment would be able to raise national income. And disregarding an ethical value judgment about Personal income distribution, the maximum output obtained would also be a social optimum, even over time.

However, economists found many important departures from this idealized picture of the economy. These departures were found in advanced and underdeveloped economies alike, though they are likely to be especially significant in the latter. Tinbergen (38) indicates at least three areas in which the underdeveloped economies are normally in a position of structural disequilibrium: the market interest rate is likely to be too low because of rationing of loan funds by the banks, the wage rate is apt to be too high because of disguised unemployment and the exchange rate is often overvalued. Other reasons for the occurrence of structural disequilibrium are numerous. To name just a few: indivisibilities, imperfect foresights, the occurrence of monopoly positions, ignorance of demands and technological possibilities, the occurrence of outputs with no readily assignable or fully recoverable market values, etc. External economies and diseconomies form another departure from the idealized picture and are especially important in underdeveloped countries. In advanced countries the establishment of a new plant, firm or industry generally entails only marginal adjustments in the rest of the economy; hence the environment of a project generally changes slowly enough to be taken as given. In an underdeveloped economy many activities are non-existent and the establishment of a new plant in one sector might involve substantial changes in other sectors, which in turn will affect the operation of the new plant.

In short the existing market price system cannot be relied upon as a tool of resource allocation in most economies. Moreover, an investment often cannot be judged under the ceteris paribus conditions of partial equilibrium analysis. Hence, optimizing all parts of an economy, sector, industry or firm does not imply the optimization of the whole as would be the case when the assumptions of pure competition were fulfilled. And this in general is the more true, the more underdeveloped an economy is.

The conclusion of economists has been that especially in underdeveloped economies not only are there likely to occur systematic discrepancies between existing market prices of factors and their true opportunity or scarcity costs, or between present private and social costs and benefits, but future prices, themselves the resultant of the allocation of the available resources, have to be estimated in a general equilibrium model. However, even in the advanced countries the mass of data that would be necessary to apply such a general equilibrium allocation model is often not available. This has led to the elaboration of investment or allocation criteria. The idea of investment criteria is to rank or order alternative potential investment projects or their consequences according to a chosen criterion. The limit between projects that should be, and projects that should not be carried out is then determined by the available resources.

Investment criteria, of course, represent a partial equilibrium approach to the problem of resource allocation. Many theoretical objections to their use have been raised in the literature and it has been shown that there exist additional difficulties when applying the investment criteria to the forestry sector.* In addition, they do not provide much help to answer some of the basic questions of the forestry planner.

This leads us back again to the ideal of the general equilibrium approach for allocation problems. Chenery and Kretschmer (9) point out that a mathematical programming model embracing the whole economy may be of help for this type of an approach to allocation decisions. Such models have actually been constructed (7, 9). However, these models should not work with crude technical coefficients, production figures and economic data because in actual practice these quantities vary with the way of doing things and hence with the alternative possible projects. Therefore, it seems to me that before such a general mathematical programming model can be developed and applied, knowledgeable people in each sector should indicate projects which are alternatives for producing certain commodities. These projects should be alternatives under the different possible assumptions for such things as resource availabilities, technical and economic data to be used over time, etc.

In correspondence with the general interest of forestry planners, the main forestry activities to be considered in this study are:

I) Raw material supply.
   a) Tree growing. This activity produces primarily wood material in the forest. The possible secondary effects of tree growing, often termed forest influences (erosion control, recreation, etc.) will be taken into account only indirectly, at least in so far as they cannot be expressed quantitatively.
   b) Harvesting ("logging") and transport.

2) Raw material processing: the most important pnmary forest industries.
   a) The sawmill industry.
   b) The veneer and plywood industry.
   c) The fibreboard and particleboard industry.
   d) The pulp and paper industry.

It seems fair to say that foresters by and large have restricted their attention to either the tree growing-harvesting phase or to one (or more) of the four pri-

* Schrcuder, G. F. The usefulness of investment criteria in the forestry sector. To be published soon.

mary forest industries. Only rarely has the whole production process from tree growing to final product of the primary forest industry plant been taken into account. This means that generally the implicit assumption is made that maximization of any part of a unit within the forestry sector will lead to maximization of the whole unit. In other words the implicit assumption that generally has been made by foresters is that the conditions of pure competition hold true for any unit in the forestry sector (even for such big units like the U.S. Forest Service). Forest economists, of course, have pointed out the fallacy of this assumption, but have done relatively little in the way of developing alternative decision models for foresters. And certainly no model has been developed that would handle the whole forestry sector.

Consequently one of the main decision models in the tree growing part of the forestry sector has been and still is the soil expectation approach developed originally by Faustman, or one of its modifications. Investments are made in those species, those silvicultural practices and those installations that promise the highest soil expectation (of course once the decision to engage in tree growing-harvesting has been taken many sub-optimization problems in forest management are solved using either the marginal or total benefit minus total cost approach). Gaffney (17) gives an excellent summary, analysis and critique of the theoretically sound (within the objective set) Faustman approach and some of the existing modifications. It is interesting to notice that he does not even mention the fact that the objective set may not be realistic in the case where the assumptions of pure competition are not fulfilled.

The soil expectation model combined with the philosophy of sustained yield has provided the framework within which sound forest investment, harvesting and other management decisions are supposed to be made. It is true that the philosophy of sustained yield has come under fire already for some time, especially because it did not seem to provide the optimal solution in cases where the forest was not fully regulated, i.e. where mature stands, say, were overrepresented in comparison with young stands. Still the unsound forest management decisions of the small forest owner and of some of the primary forest industries which own forests (often very large areas) have been rather universally condemned. Moreover, except for some rather general intuitive guidelines as to what should be done in the case where a forest is not fully regulated (see 13, chapter 7), not much in the way of a model to aid in making decisions has been developed. Finally, such recent studies as the ones by Gould and O'Regan (18) and by Clutter and Bamping (10) have argued or actually showed that the unsound forest practices may not be so unsound after all. True the revenue and cost stream is quite irregular. But on the other hand revenue is increased by 15 to 30% over what it would have been following the sustained yield approach.

Recently forest economists have become interested in operations research techniques and a number of studies applying the principles of linear programming, parametric programming, dynamic programming and simulation to problems in the forestry sector have appeared. An excellent review is given by Hall (20). True to tradition all of these focus either exclusively on the timber growing-harvesting phase or on specific problems of one of the primary forest industries. Some of these do, however, get away from the strict per unit of area approach and at least consider the tree growing enterprise as an entity (for example 10, 18 and 32). These latter studies show that one has to pay some attention to the computational aspects of the model; because of the large areas involved in forestry the number of possible combinations of the different treatments, unit areas, site class conditions, etc. tend to exceed very rapidly the storage capacity of even the biggest (conceivable) computers. If no attention is paid to the computational feasibility one gets results such as (25):

> Three silvicultural treatments, 17 age-site classes and five time periods create a large number of possible harvest alternatives. Because of limited computer capacity only 78 alternatives were identified for the forest.

This study will develop an investment decision model that will consider both the tree growing-harvesting phase and the primary manufacturing part as an entity. As such it can be used for a complete vertically integrated plant or even for a complex of forestry enterprises. The assumed objective is the maximization of the discounted present value of the net benefits of alternative possible investments in the forestry sector. To obtain the optimum, the Bellman type of dynamic programming is used. Given this model the forest planner will be able to submit optimal rules for action for any of the targets or constraints that higher policy may dictate. In this way the model furnishes the necessary data for a possible general equilibrium or mathematical programming model embracing the whole economy, like the one developed by Chenery and Kretschmer (9).

THE MODEL

As mentioned before the objective chosen to be Inaximized is the discounted present value of the net benefit of alternative possible investments in the forestry sector (in so far as these benefits can be expressed in quantitative terms). The justification for choosing this particular objective from a host of others is two-fold. In the first place it assures a maximum total quantity of goods and services available for public and private consumption and investment at a certain point of time. Moreover, Bator (4) showed that under certain restrictive assumptions (primarily that the rate of saving is not related to specific projects, i.e. to the market imputed distribution of income), instantaneous productive efficiency is a necessary condition for obtaining dynamic intertemporal efficiency; or, in other words, that we always must maximize current net output. Secondly, it provides a workable objective which is most closely akin to what the forest manager or the forest planner are likely to have in mind or actually follow in practice. This objective is rather similar to the social marginal productivity investment criterion.

Review of Dynamic Programming

To obtain the optimum, the dynamic programming approach was used. Dynamic programming was developed by Bellman (at least the Bellman type of dynamic programming; sometimes dynamized versions of linear programming are also called dynamic programming). It is a computational approach used to solve complex optimization problems. As such it is fundamentally different from linear programming which basically is a mathematical model. It is more like simulation in this respect; however, simulation does not search for an optimum while dynamic programming does.

What dynamic programming does is to take a model for which an optimal solution is sought and to transform it to a form that has the same optimal solution but that can be optimized more easily. For example, multistage or sequential decision processes containing many interdependent variables are transformed into a series of single-stage problems, each containing one or only a few variables. The problem is then solved recursively (which explains the term recursive optimization as a synonym for dynamic programming). Hence, instead of solving one optimization problem in which all the decisions are interdependent, the optimal decisions are found one at a time.

At each stage of the problem we have a certain input as described by one or more input state variables, and a certain output as described by one or more output state variables. A certain set of decisions can be taken as described by one or more decision variables. Finally, as a result of the input(s), decision(s) and output(s) we have a certain stage return, which has to be a single valued function of the input(s), output(s) and decision(s). The input of one stage is the output of a former stage. The different stages are combined by the stage transformation or stage-coupling function. This stage transformation must be a single-valued transformation, expressing each output state variable as a function of the input state variable(s) (34). The optimal solution (return function) is calculated at each stage for each feasible value of the state variable(s). The transformation itself is based on Bellman's principle of optimality (5):

> An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

This principle is intuitively obvious but as Nemhauser (34) states, it "... can be more appropriately described as powerful, subtle and elusive ..."

Dynamic programming has been used extensively on operations research problems. It has been proved especially effective for inventory control, production smoothing and allocation problems. Still it has not enjoyed the attention of linear programming, which probably is due to the fact that it is difficult to delineate a class of problems amenable to this approach; in fact, the trick largely is to formulate a problem in terms of dynamic programming or to recognize that a certain problem can be transformed into a multistage form. Therefore, applications will depend on the ingenuity of the problem solver. Also, as Howard (24) says:

> Dynamic programming requires considerable insight on the part of the analyst ... Hence it is more likely to be used by the professional analyst rather than by a manager directly.

One of the biggest advantages of dynamic programming is that it can handle both continuous and highly discrete variables and functions. This, perhaps, accounts for its popularity in cases where uncertainty is taken into account through the use of stochastic variables. On the other hand, one has to be careful in using dynamic programming in order not to create computationally infeasible problems. As the number of state variables and/or the number of stages increases, the number of necessary calculations increases very rapidly. In fact, the number of calculations increases exponentially for each additional state variable, while it increases by a multiplicative amount for each additional stage. It is generally true that a state variable is needed at each stage for each constraint that relates state and decision variables (see 34, chapter III, section 10 for exceptions). In some cases computational refinements, such as Fibonacci search, the coarse grid approach, the use of Lagrange multipliers and the one-at-a-time method can alleviate the computational burden (see 34, chapter IV).

Of course not every problem can be formulated as a dynamic program. The necessary and sufficient conditions to use dynamic programming on a multistage decision problem are (34, 31): the condition of separability; the condition of monotonicity. These conditions are quite general. In fact even some min max or max min problems satisfy these conditions (34).

Once a problem is formulated as a dynamic program it generally is solved working backward: one starts from the final (output) state variable(s) and works backward to the initial (input) state variable(s). Individual stage returns are found as a function of the input state variables and the problem is solved as a function of the initial input state variable(s). This is called backward solving or initial state optimization (5, 34, 19). Notice that a sensitivity analysis on the initial (input) state variable(s) is automatically built into the dynamic programming formulation. This is what Bellman (5) calls the "imbedding of the original problem into the whole family of problems."

It is also possible to solve the same optimization problem as a function of the initial input(s) and of the final output(s). This is called the initial-final state optimization (34). This problem generally still is solved working backward.

If, in the stage transformation function, the input state variable(s) can be expressed as a function of the output state variable(s) and the decision variable(s) (this is called state inversion), then a dynamic program can be solved working forward. This is often called final state optimization because the optimum is found as a function of the final (output) state variable(s) (34, 19).

In summary, the basic difference between backward and forward recursive optimization is that in backward recursion the analysis proceeds from the final stage to stage one, and the optimal returns are found as functions of the stage input variable(s). In forward recursion the analysis proceeds from the first stage to the final stage and the optimal returns are found as functions of the stage output state variable(s).

When the choice between input and output variables is arbitrary from a mathematical point of view, the only conceptual difference between forward and backward recursion is the order in which the transformations are made. However, the direction of solving may make a significant difference in the ease of solving the problem and may be crucial in the case of stochastic transitions and/or returns, or in the case of nonserial multistage decisions (see 34). The direction of analysis also turns out to be very important for the forestry problem of this study.

Dynamic programming has been applied to forestry problems by Arimizu (1, 2) and Hool (21,22). Both follow the traditional forestry approach, already commented upon, of optimization per unit area, i.e., of a part rather than the whole forestry enterprise. In this way they get around the difficulty of the large number of combinations of the unit areas and of the states these units can be in at each stage. Burt's (8) paper comes most closely to that which this study will do. However, there are some basic differences between his optimal water resource use problem and a typical forestry problem. In the first place he does not have to worry about the per unit area versus the whole area approach, because only the latter makes sense. Also important, the quantity of resource added to the stock per period is *not* under the decision maker's control, while it (i.e. growth) is to some large extent in forestry. Finally and less basic, his model employs continuous variables and the decision rule obtained works only if the initial stock is large relative to (quantity used minus quantity added) for a "long" period of time.

Casting the Objective in Dynamic Programming Form

Suppose that we have a very large undeveloped forested area. Assume that most of the forest is mature or overmature and virgin (this assumption is just made for the sake of discussion; basically the forest can be in any state). How should this forest area be brought into use and managed so that the discounted present value of the net benefits is at a maximum for society (or for one or more enterprises, if so desired)?

A preliminary forest inventory should be made to indicate the several forest types and site classes, and to give a rough figure about volume available. Some forecasts should be made as to what type and quantity of forest products the local market demands now and over time and at what prices; exports should be considered too. Harvesting costs should be estimated and several levels of forest management should be considered, for example, no management, very extensive and very intensive management. For each level of management, questions concerning possible species to plant, regeneration and management costs, density of the required road network, etc. will have to be answered. Number, possible sizes and locations of sawmills and/or other forest raw material processing plants, including integrated plants, should be discussed. Resource limitations should be considered.

On the basis of these data, many of which are likely to be very incomplete, a number of interdependent decisions have to be taken. Should one invest in the forestry sector at all? If so, how much and in what part of the forestry sector? What should be the rate of cut, the level of management intensity, the species to be planted, the rotation to be followed and the total capacity and type of raw material processing plants to be established? Basically these decisions should be taken in such a way that the following expression is maximized:

$$\sum_{n=1}^{N} \frac{1}{(1+K_n)^n} (r_n - c_n)$$

N = the planning horizon.
K = the discount rate in period n.
r_n = a vector of revenues accruing in period n as a result of decisions taken in and before period n.
c_n = a vector of costs incurred in period n as a result of decisions taken in and before period n.

The model above is perfectly general and does not help the forestry planner much to obtain optimal decisions. The reason mainly is that all the decisions to be taken are interdependent to a large extent. It is, however, a multistage decision model, so dynamic programming may be expected to provide some help. Moreover, the problem can be thought of as an inventory management or production scheduling problem with some peculiarities. One cannot order or produce additional product at will but is constrained by the growth capacity of the forest. (In a way one can be taking other countries or regions than the one under consideration into account; this, however, will entail an extra cost, comparable to the penalty cost of inventory control problems.) Moreover, whether we want it or not, some growth is generally forthcoming. The amount of inventory (timber-volume) conditions largely the re-order or production rate (growth-rate). Finally, in case of emergency it is possible to supply the demand by cutting into the as yet immature stands (i.e., by liquidating part of the machine), thus shortening in effect the rotation. The question now is: How can the above model be recast so that it can be solved by dynamic programming?

Foresters have always realized that their most powerful decision variable is the periodic (yearly) cut. It is this decision variable that regulates primarily the flow of benefits, to a large extent the flow of costs and the amount of investment in the tree growing part. Hence it was decided to take this as a decision variable in the dynamic program. As the accompanying state variable, the total volume as composed of the sum of the per unit area volumes seemed to be a natural choice.

The total amount of capital to be invested in all the forestry activities seems to be a necessary second decision variable. It will determine plant capacity and management intensity. However, when trying to use the total amount of capital to be invested as a decision variable, a number of major difficulties crop up. What part of this amount should go to investment in the raw material processing plants and what part to the tree growing-harvesting phase? Moreover, in the former no new investment (excepting regular operating costs) is needed once an initial investment has been made, until the time of replacement or obsolescence comes; also the problem of replacement is as yet largely unsolved (see (30) for example).

The best way to get around these difficulties is to assume a certain (any number preferred) total plant capacity to be present from the start and to assume a certain replacement schedule. Operating costs, including depreciation, would then vary only with the amount of production, which in turn depends on the cut. Investment in the tree growing-harvesting phase could then be taken as a second decision variable to determine the management intensity.

However, data in the tree growing-harvesting phase are not that exact that a few thousand dollars more or less invested in a large forest area would mean much in terms of increased production. The investment rate as a second decision variable would have to be considered in quite large discrete steps. Moreover, introduction of a second decision variable, coupled with the customary long planning horizon in forestry, increases the necessary computations tremendously. So, although it is very well possible to make the amount of investment (in the tree growing-harvesting phase, given the investment in the plant capacities) a second decision variable, it was decided to follow a computationally easier and probably a not much less satisfactory approach: only a very limited number of alternative possible plant capacities (say over-capacity, sustained yield capacity and under-capacity) and management intensities (say no management at all, extensive forest management and intensive management) are considered and the dynamic program is solved for each one of the (nine) possible cases. This approach seems especially satisfactory because so often the amount of capital available for investment is determined exogenously to the forestry sector. In the example worked out in chapter III, sustained yield plant capacity throughout the planning period and an extensive form of forest management are assumed.

A number of simplifying assumptions was necessary to obtain an operational model. It is assumed that the optimal number, type, size and location of the raw material processing plants (within the total capacity assumed at the outset) can be determined by some sub-optimization procedure. In other words, the model abstracts from the question whether one big plant or several small ones have to be constructed and from the question of their optimal location. The number of suitable plant locations is generally quite small. Linear programming and integer programming have been used to answer these questions in other sectors and could presumably be used in the forestry sector (see (29), for example).

The density and type of road network to be constructed is assumed to be related to the management intensity. Optimal ways of carrying out such silvicultural operations as planting, thinning etc., and the harvesting operations also should be found through sub-optimization methods (the marginal approach of economic analysis (13) and linear programming (15) have been used). It will be assumed that, for every management intensity, the forest planner knows at each year of age of a stand the exact volume that should be maintained per hectare (and hence the amount to be thinned) in order to maximize value growth. Given sufficient data this does not seem too unrealistic an assumption for a given soil climate and tree species (or species mixture). In this way most management costs and revenues can be related to the volume per area unit or to the area unit involved.

The species or species mix to be grown (at least if management is considered at all) and consequently the product or product mix to be obtained must be considered. The type of plants to be constructed at the planning outset will be dictated largely by the raw material base present. As it is possible to change this raw material base over time, it may be desirable to plan also a change in the type of plants and/or plant capacities over time. The particular soil and climate of the forest area under consideration and the management intensity considered will generally severely limit the choice of alternative possible species and hence the possible obtainable products. Because the number of alternatives is likely to be very small the model can be run for each alternative, if so desired.

The Dynamic Programming Model

The general model given before can now be made more specific and transformed into dynamic programming form. Let

$$H = \sum_{n=1}^{N} \frac{1}{(1+K_n)^n} [r_n^{(1)} (Y_n, P_{nj}^{(1)}) + r_n^{(2)} (Y_n - M, P_{nj}^{(2)}) + r_n^{(3)} (Y_n - Y_{n.}, P_n^{(1)}) + r_n^{(4)} (Y_n, V_{nj}, I) - c_n^{(1)} (V_{nj}^{(0)} - Y_{nj}, P_{nj}^{(3)}, K_n) - c_n^{(2)} (L_{nj}, I^{(1)}, K_n) - c_n^{(3)} (Y_{nj}, W_n^{(1)}, O_n^{(1)}, I^{(m)}) - c_n^{(4)} (F_{nj}^{(1)}, D_n^{(1)}, K_n, I^{(1)}) - c_n^{(5)} (Y_{n.}, W_n^{(2)}, V_{nj}, I^{(1)}, K_n, O_n^{(2)}, F_{nj}^{(2)}) - c_n^{(6)} (I^{(2)}, K_n, D_n^{(2)}, F_n^{(3)}) - c_n^{(7)} (Y_n, W_n^{(3)}, O_n^{(3)}, K_n, I^{(2)}) - c_n^{(8)} (Y_n - Y_{n.}, P_n^{(2)}) - c_n^{(9)} (Y_n, V_{nj}, I)]$$

The explanation of the various symbols is as follows:

—The dot notation is used to indicate that summation is carried out over the subscript in whose place the dot stands. For example $Y_{n.} = \sum_j Y_{nj}$

—In contrast with accepted dynamic programming notations the stages are numbered in the normal and not in reverse order. The reason is that forward recursion, as opposed to the traditional backward way of solving, will be shown to be essential for the forestry problem of this study. Thus the stages from the beginning to the end of the planning period are numbered respectively 1, 2, 3, . . . , n — 1, n, n + 1, . . . , N

—Small letters indicate either subscripts or functions. Capital letters represent parameters or variables. The meaning of the capital letters will be explained first. For ease of reference it may be noted here already that r represents a revenue function and c a cost function.

—The planning horizon is supposed to be N periods or N stages long (a period may stand for a month, a year, five years or some other preferred time unit). In the example of chapter III the total planning period is 75 years and composed of 15 stages of five years each.

—The total forest area is supposed to be divided into J area units. It is not necessary that the area units are of equal size. Because it will simplify computations somewhat and does not affect the reality of the model, equal sized area units will be assumed for most of this study. The division should be done in such a way that an area unit always is homogeneous in respect to site class and/or forest cover. Each unit is represented by a subscript $j = 1, 2, \ldots, J$. In the example of chapter III, three site classes are distinguished and each area unit is 1000 hectares.

—M is the maximum volume of wood that the installed plant capacity can process. As explained before, it is assumed to be a constant for a particular case, though different cases may be examined.

—Y_{nj} is the cut in period n from area unit j; $Y_{n.}$ consequently is the total cut in period n and will constitute part of the decision variable. Y_n is the total amount of wood processed by the forestry activities under consideration in period n and will be used as the decision variable. Hence $Y_n - Y_{n.}$, which may be zero but cannot be negative, is the amount of wood ordered from outside the region under consideration. Also $Y_n^{(1)}$ will stand for the total amount of wood processed by the primary forest industries and $Y_n^{(2)}$ for the total amount sold in log form. Of course $Y_n^{(1)} + Y_n^{(2)} = Y_n$. Unless stated otherwise it will be assumed that $Y_n^{(2)} = 0$ as long as $Y_n^{(1)}$ is smaller than M. This means that we assume that no wood will be sold in log form (in unprocessed form) until the installed plant capacity is fully utilized. Of course, other assumptions can be made.

—K_n is the periodic (not necessarily annual) discount rate at stage n. We notice that it need not remain constant over the planning period but may be assumed to rise, fall or to fluctuate wildly. In the example of chapter III the model will be run for three different but constant interest rates.

—$P_{nj}^{(1)}$ is the price per unit volume of timber on the stump on a finished product basis (finished product meaning it has passed the production process of one of the primary forest industries) in period n from area unit j. Notice that if the jth area unit produces wood for lumber purposes while the (j + 1)th unit produces wood for pulp purposes, the two prices are likely to be different. And even if both area units produce wood for the same purpose, the prices may be different due to the fact that the forest cover on area unit j is different from that of unit (j + 1) (a different species or quality mix). There also may be more than one product being produced from timber coming from the jth area unit (for example in the case of integrated plants); then we have to work with a weighted price. A weighted price is also needed in the case where the forest cover on an area unit is made up of a mixture of species, each commanding a different price. Notice also that it is possible to make the price $P^{(1)}_{nj}$ a function of the amount of product produced, i.e. $P^{(1)}_{nj} = p^{(1)}_{nj} (Y_n)$. This is important when the forest enterprise under consideration faces a not perfectly elastic demand curve. It is possible to incorporate to some extent price differentials for different size classes of timber by relating price also to volume per area unit at the moment of cut, i.e. $P^{(1)}_{nj} = p^{(1)}_{nj} (V_{nj}, Y_n)$. The n subscript on the parameter indicates that changes of product prices over time can be taken into account. An allowance for possible quality improvements or deteriorations after the first cut may be made by assuming a different price value when harvesting for the second time (i.e. after one "rotation") from the same area unit. Finally $P^{(1)}_{nj}$ may be made a function of the management intensity $I(m)$, i.e. $P^{(1)}_{nj} = p^{(1)}_{nj} (I(m))$.

—$P^{(2)}_{nj}$ is the price per unit of volume of timber on the stump on a roundwood f. o. b. basis. The remarks made for the $P^{(1)}_{nj}$ price are applicable here. If we want to make $P^{(1)}_{nj}$ and $P^{(2)}_{nj}$ a function of Y_n we must remember that the two are likely to be interrelated because the products on which the two prices are based, are close substitutes for each other. That is $P^{(1)}_{nj} = p^{(1)}_n (Y^{(1)}_n, Y^{(2)}_n)$ and $P^{(1)}_{nj} = p^{(2)}_n (Y^{(1)}_{nj}, Y^{(2)}_{nj})$. But as both are on the same per unit of volume on the stump basis no difficulties arise. If the demand schedules for the two products on which $P^{(1)}_{nj}$ and $P^{(2)}_{nj}$ are based, are different (say because finished products are sold on the domestic market but logs are exported), then the two prices might be independent functions of Y_n. That is $P^{(1)}_{nj} = p^{(1)}_n (Y^{(1)}_n)$ and $P^{(2)}_{nj} = p^{(2)}_n (Y^{(2)}_n)$. Both cases can be handled easily.

—$P^{(3)}_{nj}$ is the price per unit volume of stumpage. Again the remarks made for the $P^{(1)}_{nj}$ and $P^{(2)}_{nj}$ prices are applicable here.

—$P^{(2)}_n$ represents the per unit of cubic volume price of logs that can be bought and brought from elsewhere (from other regions than the one under consideration). It resembles $P^{(2)}_{nj}$ except that an extra transportation price is included. Hence in general $P^{(2)}_{n} \geq P^{(2)}_{nj}$. If so desired $P^{(2)}_{n}$ may be made a function of $Y_n - Y_{n.}$ so that the price to be paid depends on the amount ordered.

—$P^{(1)}_{n}$ is the price per unit of cubic volume finished product on a roundwood basis obtained from the logs brought in from outside the region under consideration. It may or may not be different from $P^{(1)}_{nj}$.

In the example of chapter III, $P^{(1)}_{nj} = P^{(1)}_{n,j+1} = P^{(1)}_{n}$, $P^{(2)}_{nj} = P^{(2)}_{n,j+1}$ and $P^{(3)}_{nj} = P^{(3)}_{n,j+1}$. Also $P^{(2)}_{n} = P^{(2)}_{nj}$ + some nonnegative amount. All three prices $P^{(1)}_{nj}$, $P^{(2)}_{nj}$ and $P^{(3)}_{nj}$ will be assumed to increase over time. Finally $P^{(1)}_{n}$ and $P^{(2)}_{nj}$ are taken to be decreasing functions of respectively Y_n and $Y_n - M$, while it is assumed that $Y^{(2)}_{n} = 0$ as long as $Y^{(1)}_{n}$ is smaller than M.

—$I^{(1)}$ is the amount of fixed investment in the tree growing-harvesting phase of the forestry enterprise. Examples are: investments in roads, drainage ditches, nurseries and buildings. As commented upon before $I^{(1)}$ is fixed for the level of management intensity $I^{(m)}$ under consideration.

—$I^{(2)}$ is the amount of fixed investment in the raw material processing plants. It determines the total plant capacity M and was commented upon before.

—$I = I^{(1)} + I^{(2)}$

—V_{nj} is the cubic volume at the beginning of the nth period of the jth area unit. Consequently $V_{n.}$ is the total volume of the forestry enterprise at the beginning of period n; it will be used as the state variable. A superscript 0 indicates that a stand is under consideration which has cost nothing to grow.

—G_{nj} is the net cubic volume growth of the jth area unit during the nth period. The prediction of growth is one of the most essential things in forest management. Yet no exact growth prediction method has been developed. In this study the development of a stand as a function of site class, age and management intensity will be assumed known (this is a fairly strong, yet quite common assumption in forestry). The growth of a stand on a specific area unit and for a given level of management intensity as a function of the volume per area unit (which is part of the state variable) can then be derived easily. That is $G_{nj} = g(V_{nj}, I^{(m)})$. As will be seen later it is necessary to relate growth to the state variable in order to be able to use dynamic programming. The total growth $G_{n.}$ is found as the sum of the growth of each area unit. In the example of chapter III the growth for each of three different site classes is given as a function of the per unit area volume.

—L_{nj} represents the land value (soil expectation value where available) of area unit j in period n. If fixed investments $I^{(1)}$ are made, this enhances the value of L_{nj}. Hence L_{nj} is not determined exclusively by the general demand and supply for land; it may be increased (or decreased) by some actions of the forest manager. As the n subscript indicates, the price of land may change over time.

—$W_n^{(1)}$, $W_n^{(2)}$ and $W_n^{(3)}$ represent the wage rates in period n of labor employed in respectively the tree growing phase, the harvesting phase and the primary forest industries. Again the n subscript allows for possible changes over time.

—$O_n^{(1)}$, $O_n^{(2)}$ and $O_n^{(3)}$ represent all other costs in period n that vary with the decision variable Y_n, respectively in the tree growing phase, the harvesting phase and in the primary forest industries. They may change overtime. $O_n^{(1)}$ is determined also in large part by $I^{(m)}$, the level of forest management intensity considered.

—$F_{nj}^{(1)}$, $F_{nj}^{(2)}$ and $F_{nj}^{(3)}$ are fixed costs of operation in period n, respectively of the tree growing phase, the harvesting phase and the primary forest industries. Insurance costs and certain taxes are examples.

—$D_n^{(1)}$ and $D_n^{(2)}$ are the rates of depreciation in period n of those fixed investments, respectively in the tree growing phase and in the primary forest industries, that are subject to decay or obsolescence.

The specific meaning of the revenue and cost functions follows.

—$r_n^{(1)}$ represents the revenue received from the sale of the products of the primary forest industries. Unless the $r_n^{(3)}$ term is omitted, this excludes those products based on the raw material from elsewhere. The term $r_n^{(1)}$ depends on the decision variable Y_n only as long as $Y_n \leq M$; $r_n^{(1)}$ will become constant as soon as $Y_n > M$.

—$r_n^{(2)}$ represents the revenue received from the sale of logs. As explained before, it is assumed that as long as $Y_n \leq M$, all the timber will be processed. As soon as $Y_n > M$, the plants cannot process the additional timber any more by the definition of M. The unprocessed amount $(Y_n - M)$ will then be sold in log form. Hence this term will become operative only if $(Y_n - M) > 0$. If the above assumption is not made, a more general notation becomes necessary: replace $Y_n - M$ by $Y_n^{(2)}$ in the $r_n^{(2)}$ function and Y_n by $Y_n^{(1)}$ in the $r_n^{(1)}$ function.

—$r_n^{(3)}$ represents the revenue received from the sale of the final products obtained from processing the logs brought in from outside the region under consideration. The implicit assumption, that it will not pay to buy logs from elsewhere and sell them again in unprocessed form, seems reasonable enough as long as $P_n^{(2)} \geq P_{nj}^{(2)}$. The contrary is hard to imagine in practice but possible in theory; it can be handled just as easily by the model. This term is included only to allow for the possibility that wood imported from elsewhere is of a different quality and/or species mix than the timber obtained locally. If this is the case, the price $P_n^{(1)}$ of the final product obtained from this material will be different from $P_{nj}^{(1)}$. If this is not the case, the term $r_n^{(3)}$ will be omitted and absorbed in the term $r_n^{(1)}$ because $P_n^{(1)} = P_{nj}^{(1)} = P_{n,j+1}^{(1)}$. The latter case is the one considered in the example of chapter III.

—$r_n^{(4)}$ has been included to allow taking into account external economies of the forestry activities. These, of course, can be considered only when some quantitative estimate can be supplied for them.

—$c_n^{(1)}$ represents the costs of holding for an additional period of time those stands that have cost nothing to grow. It is applicable to all stands if no management is practiced. If management is being practiced it is applicable only to those stands that have not been cut for the first time. As the interest K_n and the stumpage price $P_{nj}^{(3)}$ may change over time, this cost may change also over time. In the example of chapter III, this cost is assumed to be applicable only to the original stands (the stands present at the beginning of the planning period).

—$c_n^{(2)}$ represents the periodical land rent or K_n times the land value (or soil expectation value if the land value is not known). It can also be looked upon as an opportunity cost of timber growth foregone and of possible stand improvement foregone in future stands when holding the present forest cover for an additional period of time. It is also a function of $I^{(1)}$ in so far as investments are made that increase the value of the land, such as roads, drainage ditches etc. As the interest rate K_n and the landprice L_{nj} may change over time, this cost may change also over time. In the example of chapter III, land prices are assumed to be different for the three site classes distinguished. Moreover, a constant periodic increase in land values is assumed.

—$c_n^{(3)}$ represents those management costs that vary with Y_n, $W_n^{(1)}$ and $O_n^{(1)}$, such as costs of preparation and planting the cut-over sites, release cuttings and other plantation tending costs. This cost function will depend on the level of management intensity $I^{(m)}$ considered. It may be zero if no management is planned (cut-and-get-out type of operation). If so desired this cost may be assumed to vary with site class, or with any other characteristic that distinguishes one area unit from another. Often this will not be worthwhile or not even possible, as when variable management costs are recorded on a total area basis and not separately per area unit; generally an average variable management cost per area unit will do. As its parameters may change over time, this cost may change also over time. In the example of chapter III a constant cost is assumed as soon as an area unit is clearcut.

—$c_n^{(4)}$ are the fixed per area unit costs of management, such as certain taxes, costs of inventories on a per area unit basis, fire protection costs, interest charges on fixed investments not included in the $c_n^{(2)}$ term etc. Some of these will be incurred even if no management is practiced. This cost may be assumed to increase or decrease over time.

—$c_n^{(5)}$ represents the cost of harvesting the cut Y_n. and of transporting the material to the processing plant(s) or to the f. o. b. place. It is assumed that a time period n is so long that logging equipment is depreciated within the time period (this is not a necessary assumption; it is made for simplicity only, as otherwise a logging and transportation equipment depreciation term has to be separated out from $c_n^{(5)}$). Hence logging costs include all costs from tree on stump to log in the yard of the processing plant or on the f. o. b. site. It does include the construction of temporary logging roads, but not the construction and/or maintenance of permanent roads, which are considered part of the management costs 1(1). Consequently the higher 1(I) is, the lower $c_n^{(5)}$ will tend to be. The logging costs may be differentiated by the area from which the different Y_{oj} are secured (thus taking into account distance of transportation, ease of logging etc.). As such these costs can also be made a function of the per area unit volume V_{nj} from which the cut Y_{nj} is taken. Finally it is also possible to allow for lower logging costs when harvesting for the second time from the same area unit (say after one "rotation"). This cost may be assumed to increase or decrease over time depending on how the parameters move. In the example of chapter III a constant cost per unit of volume cut is assumed as long as the cut comes from the original stands. A lower cost is charged when cutting on subsequent occasions.

—$c_n^{(6)}$ represents such fixed costs as the depreciation of the processing plants, fixed interest charges, insurance, certain taxes etc. The problem of depreciation and replacement appears to be a complicated one (16,30). Eckstein (16) gives basically four different depreciation methods: straight line depreciation, declining balance depreciation, sum-of-the-years-digit and the sinking fund method. Optimal depreciation and replacement strategies differ with the tax system of a region (37), with business attitudes (the desire to use depreciation as a method of internal accumulation of investible funds), with technical progress etc. Perhaps one can abstract from technical progress in the forest industries. Eckstein recommends the sinking fund method of depreciation for public undertakings, like investments in water resources. Any form of the depreciation rate Do assumed, whether continuous or irregularly discrete, can be handled by the model. But the planner has to know whether the plants will be able to continue operations at the end of the depreciation period and whether increased operating costs in this case will result; or whether new investments should be planned at that time. In the example of chapter III the investments in machinery are depreciated within 10 years, the investments in buildings in 20 years. Straight line depreciation is assumed. After 10 years the plant continues to operate with the old investments but the cost of repair and replacement is increased.

—$c_n^{(7)}$ represents the operating costs of the processing plants, including working capital charges, power costs, labor costs etc. It is a direct function of Y_n. The function can be quite irregular in respect to Y_n; for example the operating costs (especially labor costs) will show a jump when Y_n becomes so big that the plants have to operate on two 8-hour shifts a day, instead of on one 8-hour shift. Three 8-hour shifts a day will bring the plants up to the maximum capacity level M defined earlier. Once Y_n has reached the M level, $c_n^{(7)}$ will remain constant; timber will remain unprocessed and will be sold in log form. Only if the amount of investment were to be made a second decision variable, could M be varied also. The form of the function may be quite different for different levels of installed capacity considered. As its parameters may change over time, the value of this function may also change over time. In the example of chapter III, labor costs are assumed to increase over time, while the other variable costs remain constant. The plant can operate on one 12-hour shift or on two 12-hour shifts, and the variable costs are quite discrete. The two 12-hour shifts will mean full capacity utilization. As indicated before, sustained yield capacity is the case considered in the example.

—$c_n^{(8)}$ represents the cost function associated with the buying of raw material outside the region under consideration because the region has run out of appropriate raw material.

—$c_n^{(9)}$ has been included to allow taking into account external diseconomies of the forestry activities. These, of course, can be considered only when some quantitative estimate can be supplied for them.

With the exception of $c_n^{(2)}$, $c_n^{(4)}$ and $c_n^{(6)}$, all the functions depend on the variable Y_n, on the variable $V_{n.}$ or on both. All the other parameters in the parentheses of a function are either constant for any one stage (though not necessarily for the whole planning period, as explained) or are assumed to be related to Y_n and/ or $V_{n.}$. The model can now be stated as follows. Maximize H as a function of $V_{1.}$, $V_{2.}, \ldots, V_{N.}$ and $Y_1, Y_2, \ldots, Y_N$ subject to the following two sets of constraints:

$$V_{n.} + G_{n.} - Y_{n.} = V_{n+1,.} \qquad \text{for } n = 1, 2, \ldots, N.$$
$$Y_{n.} \geq 0 \qquad \text{for } n = 1, 2, \ldots, N.$$

Of course, $c_n^{(2)}$, $c_n^{(4)}$ and $c_n^{(6)}$ do not influence this optimization problem and consequently could have been omitted. They are included for the sake of completeness.

What makes this maximization problem complicated is the first set of constraints, one for each stage. It is this set of constraints that makes the $Y_{n.}$'s interdependent. Because of it, the various revenue and cost items have to be balanced against each other and over time: a large $Y_{n.}$ in any period n will make some costs and revenues in period n big and others small; but it may make some costs so unreasonably high or some revenues so small in subsequent periods that it is better to choose a smaller $Y_{n.}$ in period n. Consequently it is this part of the variable Y_n that is of crucial importance in the optimization problem. If it were not for $Y_{n.}$, we could have optimized separately for each period. The second set set of constraints limits only the range of the $Y_{n.}$'s.

Hence a multistage optimization problem is obtained with at each stage two constraints, one of which connects the stages. In dynamic programming terminology this constraint is the stage transformation function which expresses the state output variable $V_{n+1,.}$ in terms of the state input variable $V_{n.}$ and the stage decision variable $Y_{n.}$ (or conversely, which expresses the state input variable as a function of the state ouput variable and the decision variable). It will be shown later how and why it is advantageous to use Y_n, the total amount of wood processed by the region, as the decision variable instead of $Y_{n.}$, the total amount cut (the difference $Y_n - Y_{n.}$, which may very well be zero, being the amount imported from elsewhere). For the time being Y_n and $Y_{n.}$ will be denoted interchangeably as the decision variable with due regard for the context.

Before obtaining the recurrence equations we have to know whether or not it makes a difference if the traditional backward way of solving is used, or whether perhaps the forward recursive optimization (if at all feasible) is to be preferred.

Forward versus Backward Recursion

The stage transformation function is $V_{n.} = V_{n+1,.} + Y_{n.} - G_{n.}$ or equivalently $V_{n.} + G_{n.} - Y_{n.} = V_{n+1,.}$. The latter version is more familiar to foresters. Now it is possible to make two different assumptions about $G_{n.}$.

1) $G_{n.} = g\ (V_{n+1,.})$, i.e. the growth in period n is a function of the volume at the end of the nth period, that is of the state output variable (actually, as indicated before, $G_{n.}$ is found as $\sum_j G_{nj}$, and each G_{nj} under the above assumption is found as a function of $V_{n+1,j}$). This in fact assumes that the whole cut $Y_{n.}$ is obtained in the first minute of the stage period n.

Generally this assumption would be conservative; it tends to underestimate growth. If so desired only half a period growth on those stands that are cut can be assumed to take place, but this would complicate calculations and does not seem to promise a sufficient increase in accuracy. This assumption would allow backward recursive optimization and not the forward way of solving, because state inversion is impossible.

2) $G_{n.} = g\,(V_{n.})$, i.e. the growth in period n is a function of the volume at the beginning of the nth period (that is of the state input variable). To be theoretically correct this would imply that $Y_{n.}$ is obtained during the last minute of period n. In reality this assumption would tend to overestimate growth to the extent that plantations in their first few years grow slower than the stands that have been cut. As before, only half a period growth on those stands that are cut can be assumed to take place. Much depends also on the length of a stage period. Anyhow this assumption would allow forward recursive optimization because state inversion is possible, i.e. because we can write the coupling function as $V_{n.} + G_{n.} = V_{n+1,.} + Y_{n.}$.

The conclusion is that by changing the assumption on the growth function, we can solve recursively either forwards or backwards. Which way is to be preferred? In planning the development of a forest region the forest planner generally knows $V_{1.}$, i.e. the initial volume. Hence it does not seem worthwhile to optimize as a function of $V_{1.}$, i.e. to solve as an initial state problem. Moreover, it will be all but impossible computationally to solve backwards anyhow, as a specific $V_{N+1.}$ can be composed of the individual $V_{N+1,j}$'s in an infinite number of ways. Some of the ways of composing a given $V_{N+1.}$ by specific $V_{N+1,j}$'s will be impossible in practice or nonsensical economically, but we have no way of knowing beforehand which ones will make sense and which ones not. In short, one would have to consider each possible value of $V_{N+1,.}$ and for each value of $V_{N+1.}$ all the possible ways of composing it of the individual $V_{N+1,j}$'s.

Hool (21, 22) gets around this problem by optimizing on a per area unit (1/5 acre) basis. This avoids having to consider all possible conditions the total number of area units (125 in his case) can be in at the end of the planning horizon. In his case each area unit can be in anyone of 36 states. Assuming that each area unit is different from another one, this would make it necessary to consider 36^{125} different cases when optimizing backwards over the whole enterprise instead of on a per area unit basis (assuming that each area unit is not different from another one reduces this number to 36 X 125 different cases). As remarked before, maximization of each part of an enterprise does not necessarily lead to maximization of the whole. More important, Hool's study does not allow for basic stand structure changes, such as those brought about by clearcutting and replanting, nor for differences in site qualities and/or species compositions. Consequently, growth in the general forest management sense is not really taken into account: once a 1/5 acre plot is reduced to the 0-10 trees' state (and when clearcutting all plots eventually will), it is likely to stay in that state forever.

Hence solving the problem as an initial state or as an initial-final state problem, using backward recursive optimization is computationally infeasible and practically not interesting. Only if volume growth could be related to total standing volume in the whole region in question and if $r_n^{(1)}$, $r_n^{(2)}$, $c_n^{(1)}$, $c_n^{(3)}$, $c_n^{(5)}$ could be related to the aggregative values of the state and/or the decision variable, could the backward approach be used profitably. Obviously this would make for a much cruder (though simpler) model. While it probably would be too crude in the case of even-aged forest management and when large areas are involved, the backward approach in this form might be of some use in the case of all-aged management, especially when small areas are involved.

Obviously forward recursion is the answer. It entails a minimum of computational burden and has the very important feature of solving as a function of V_{N+1}. This, of course, is due to a feature of dynamic programming that at each stage (and thus also at the last stage N) we solve not for one value of the state output variable but for a whole set of values. In other words, a sensitivity analysis on the total volume at the end of the planning horizon is automatically imbedded in the solution.

The recursion equations can now be obtained easily. First H can be rewritten as

$$
\begin{aligned}
H &= h(V_1, V_2, \cdots, V_{N+1}, Y_1, Y_2, \cdots, Y_N) \\
&= h_1'(V_1, Y_1, V_2) + h_2'(V_2, Y_2, V_3) + \ldots + h_N'(V_N, Y_N, V_{N+1})
\end{aligned}
$$

Using the stage transformation we can replace Y_n in the h_n' term by $V_{n+1} + Y_n - G_n$. Once V_{n+1} and Y_n are specified, G_n becomes a constant and can be found easily as will be shown later. Hence, the value of $V_{n+1} + Y_n - G_n$ depends on the two variables V_{n+1} and Y_n and may be replaced by

$h_n''(V_{n+1}, Y_n)$. Rewriting H we obtain

$$
\begin{aligned}
H &= h_1'(h_1''(V_2, Y_1), Y_1, V_2) + h_2'(h_2''(V_3, Y_2), Y_2, V_3) + \ldots + h_N'(h_N''(V_{N+1}, Y_N), Y_N, V_{N+1}) \\
&= h_1(V_2, Y_1) + h_2(V_3, Y_2) + \ldots + h_N(V_{N+1}, Y_N).
\end{aligned}
$$

The absence or presence of primes indicates that different functions are involved.
The maximization problem can now be rewritten as follows

Maximize $[h_1 (V_{2.}, Y_1) + h_2 (V_{3.}, Y_2) + \ldots + h_N (V_{N+1,.}, Y_N)]$

subject to $Y_{n.} \geq 0 \qquad n = 1, 2, \ldots, N.$

$V_{n.} + g (V_{n.}) = V_{n+1,.} + Y_{n.} \qquad n = 1, 2, \ldots, N.$

In words: determine the decision variables $Y_1, Y_2, \ldots, Y_N$ subject to the two constraints, so that H is at a maximum. Because of the discreteness of the parameters involved, especially of the growth term, no neat analytic solution will be possible. This was done deliberately to avoid imposing additional restricting assumptions and to keep the model as general as possible.

Bellman's principal of optimality states that the above interdependent multistage optimization problem can be solved a stage at a time as a function of the state output variable, as long as the two conditions of monotonicity and separability, mentioned earlier, are met. This can actually be proved (see (34), chapter 2, section 9). In accordance with the general dynamic programming theory, the recursion equations consequently become:

1) $f_1 (V_{2.}) = \max h_1 (V_{2.}, Y_1)$
   subject to $Y_{1.} \geq 0$
   $V_{1.} + g (V_{1.}) = V_{2.} + Y_{1.}$

2) For $n = 2, 3, \ldots, N$
   $f_n (V_{n+1,.}) = \max [h_n (V_{n+1,.}, Y_n) + f_{n-1} (V_{n.})]$
   subject to $Y_{n.} \geq 0$
   $V_{n.} + g (V_{n.}) = V_{n+1,.} + Y_{n.}$

The maximization problem should be read as "Determine Y_n so that the expression in the big brackets is maximized for a given value of $V_{n+1,.}$ (and consequently for a given value of $V_{n.}$)"

Hence the first step is to determine $f_1 (V_{2.})$ for each feasible value of $V_{2.}$. Then $f_n (V_{n+1,.})$ is determined for $n = 2, 3, \ldots, N$, the last one being $f_N (V_{N+1,.})$. The optimal decisions $Y^*_N, Y^*_{N-1}, \ldots, Y^*_1$ can be found finally by tracing back the whole process, either for a specific assumed value of $V_{N+1,.}$ or, if this is preferred, by first determining the value of $V_{N+1,.}$, say $V^*_{N+1,.}$, that maximizes $f_N (V_{N+1,.})$.

This calculation process has to be elaborated upon as in practice some rather formidable computational difficulties arise. These difficulties are all associated with the growth term in the stage transformation function. This term makes it impossible to use the ordinary way of solving a dynamic programming problem.

Computational Aspects of the Model

To find $f_1 (V_{2.})$ for a specific value of $V_{2.}$, we first find $V_{1.} + G_{1.}$. It was assumed that the forestry planner knows V_{1j} ($j = 1, 2, \ldots, J$) and thus $V_{1.}$. As $G_{1j} = g(V_{1j})$, it can be determined easily for $j = 1, 2, \ldots, J$. By simple summation $G_{1.}$ is determined. Finally $Y_{1.}$ can be found as $Y_{1.} = V_{1.} + G_{1.} - V_{2.}$. No maximization is necessary in this first step because for each value of $V_{2.}$, the value of $Y_{1.}$ that maximizes h_1 is automatically determined as there is only one value $V_{1.}$.

However, two difficulties crop up already at this first stage. In the first place each forester will ask immediately: which area units should furnish the cut $Y_{1.}$. Secondly, after specifying $V_{2.}$, that is the total volume at the beginning of stage 2, somehow V_{2j} for $j = 1, 2, \ldots, J$ also has to be found if we are to find G_{2j} and $G_{2.}$ at the second stage; and so on for subsequent stages. If we work only with aggregative quantities at the first stage (and thus also at subsequent stages), that is with a set of different $V_{2.}$ values and not also with the respective V_{2j} values, the growth determination at the second and subsequent stages will be impossible or very crude. This is so because then we have $G_{2.} = g(V_{2.})$, that is we would have to find $G_{2.}$ as a function of the total volume of all the area units. As was pointed out, the same total volume can be built up of the individual area unit volumes in many ways. Consequently this would be a very inexact way to determine $G_{2.}$. To be able to determine growth with some accuracy we have to keep calculating everything on a per area unit basis before aggregating.

Cutting Priorities Versus Outside Buying Alternatives

The solution to the above difficulties consists of the following three rules. First let $Y_{1.}$ be made up of all thinnings due (if any). As indicated before, knowing V_{nj} we assumed that the forester not only knows the growth but also when to thin, i.e. that he knows at each V_{nj} the optimal volume to be maintained per area unit (and hence whether to thin or not) in order to obtain maximum growth. Secondly, only if with the total amount of thinnings the total $Y_{1.}$ is not yet reached, we should actually start scheduling some of the area units for a harvest felling. Or thirdly, we should order raw material from outside the region under consideration.

An important point here is to remember that we not only know $V_{1.}$ and $G_{1.}$, but also the individual V_{1j} and G_{1j} for $j = 1, 2, \ldots, J$. So if we know which area units to schedule for a thinning or harvest cutting, we can adjust $V_{1j} + G_{1j}$ for the cut Y_{1j} to obtain V_{2j}. And knowing V_{2j}, we can determine G_{2j} and obtain both $V_{2.}$ and $G_{2.}$. We would then be ready for the optimization at the second stage. The big and only difference at the second (and subsequent) stages is that now, if we try to find $f_2(V_{3.})$ for each specific value of $V_{3.}$, a whole range of $Y_{2.}$'s is possible because we had to consider a whole range of values $V_{2.}$ at the first stage. We have to find the one $Y_{2.}$ that maximizes $[h_2(V_{3.}, Y_2) + f_1(V_{2.})]$ for each different $V_{3.}$. The dynamic programming trick is, of course, that the specification of a $V_{3.}$ and a $Y_{2.}$ entails the specification of $V_{2.} + G_{2.}$, while $f_1(V_{2.})$, that is the maximum with respect to $V_{2.}$ (and consequently also the maximum with respect to $V_{2.} + G_{2.}$), has been determined already at stage 1.

However, we still need to know which area units first to schedule for thinning (if thinning is considered at all) and harvesting; simultaneously we need to know the cut-off point at which it is more economical to order raw material from outside the region rather than to schedule still vigorous stands for harvesting. As to the question of where to thin first, it seems reasonable to assume that all thinnings have to be done when due out of silvicultural considerations; this neatly circumvents all difficulties and does not seem unrealistic. However, it is not necessary to make this assumption; some alternatives, for example, could be to thin the most valuable stands first or to thin the most vigorous stands first. Both alternatives relate back to the state variable V_{nj} and hence an order of thinning urgency can be established easily.

As to the simultaneous questions of which stands to schedule first for a harvest cut and/or whether to revert to ordering the raw material from outside the region under consideration, the following method has been developed. Suppose $P_n^{(2)} - P_{nj}^{(2)} = A_{nj}$, that is buying the raw material outside the region means paying A_{nj} dollars more per volume unit of raw material relative to the price we can get it for from area unit j. Calculate the gross value growth of the stands on each area unit j during stage n as $P_{nj}^{(3)} G_{nj}$. To obtain the net value growth, subtract the incremental costs associated with the same stand or area unit during the same stage as composed of $K_n \times P_{nj}^{(3)} \times V_{nj} + c_{nj}^{(2)}$ and possibly of some parts of $c_{nj}^{(3)}$ and $c_{nj}^{(4)}$; these are all related to the state variable V_{nj}, to the area unit j, or to the stage n and hence can be found easily.

Denote the net value growth by B_{nj}. It should be clear that B_{nj} can be negative. If we rank all area units according to B_{nj} it is obvious that those area units for which B_{nj} is smallest should be harvested first. (To avoid the problem of assigning two area units, for one of which B_{nj} is still increasing while for the other it is already decreasing, the same rank, add a large constant value to all the B_{nj}'s that are still on the increase). As long as $Y_{n.}$ comes from area units for which $B_{nj} \leq 0$ or as soon as $Y_{n.} \geq M$ it will obviously not pay to consider the alternative of ordering raw material from outside the region. But as soon as we have cut all area units for which $B_{nj} \leq 0$ and as long as $Y_{n.} < M$, we do have to consider that alternative. If the latter two conditions hold, we will consider cutting area units for which $B_{nj} > 0$ as long as $B_{nj} \leq A_{nj} \times V_{nj}$. As soon as we come to the first area unit for which $B_{nj} > A_{nj} \times V_{nj}$, we will start ordering from outside the region. In other words, the "marginal" area unit to be considered for harvesting is the one (may be plural) for which $B_{nj} = A_{nj} \times V_{nj}$.

Faustman has developed a theoretically sound (within the objectives and assumptions implied, commented upon before) method of determining the rotation of a stand. It is well known to foresters, often under the name "soil expectation approach", in both its total value and its marginal value version (see (17)). In its marginal value approach version it states that a stand should be cut as soon as its net value growth equals zero. It is this method that the model of this study generalizes considerably, as can be deduced from the cutting versus ordering rules described above.

The above method of establishing cutting priorities is not only theoretically sound but also quite easy to apply in practice. The method of determining when to order from elsewhere by establishing the marginal area to be considered for cutting, is also theoretically sound. In practice, however, it would be somewhat difficult to follow. The cut $Y_{n.}$ is specified at the outset of the computations at stage n. If we now find that part of $Y_{n.}$ would come from sub-marginal area units and (instead of cutting) order this part from elsewhere, we will be in trouble with the dynamic programming computational procedure. Only part of the $Y_{n.}$ specified at the outset will really be cut, another part will be ordered from elsewhere. Still the whole value of $Y_{n.}$ is assumed to be cut when specifying a value for $Y_{n.}$ and $V_{n+1,.}$ at the outset of the computations. This assumption is necessary in order to find $V_{n.} + G_{n.}$ through the coupling function $V_{n+1,.} + Y_{n.} = V_{n.} + G_{n.}$.

There appear to be two possible solutions to this difficulty.

1) We specify at the outset a different decision variable Y_n, made up in some known way by the part $Y_{n.}$ that will be cut and comes only from marginal and supra-marginal area units, and a part $Y_n - Y_{n.}$ that will be ordered from elsewhere. This is a feasible but tedious way, because we would have to make sure each time that the part $Y_{n.}$ specified comes indeed only from marginal and supra-marginal area units.

2) We specify $Y_{n.}$ at the outset and assume that amount to be cut, whether part of it comes from sub-marginal units or not. But in addition (and after the calculations are made with $Y_{n.}$) we consider for each value of $Y_{n.}$ specified, various amounts $dY_1, dY_2, \ldots, dY_k, \ldots$ to be ordered from elsewhere. Denote $Y_{n.} + dY_k$

by $Y_n^{(k)}$, k = 1, 2, ... The first amount dY_1 is taken to be zero to allow consideration of $Y_{n.}$ by itself. Because $P_n^{(2)} \geq P_{nj}^{(2)}$ we consider adding additional amounts only as long as $Y_{n.} + dY_k < M$. As soon as $Y_{n.} + dY_k \geq M$ we find the one $Y_{n.} + dY_k$ (denoted by Y_n) that gave the biggest value $h_n(V_{n+1,.}, Y_n^{(k)}) + f_{n-1}(V_{n.})$. This would be the value recorded for $f_n(V_{n+1,.})$. For each value of $f_n(V_{n+1,.})$ recorded, we would also record both the values of $Y_{n.}$ and Y_n in order to enable us to find the optimal decision variables when tracing back our steps at the end of stage N. In fact, there are two decision variables $Y_{n.}$ and Y_n at each stage. The dynamic programming procedure will automatically eliminate from consideration at stage n + 1 those occasions where we considered cutting sub-marginal area units. Hence, although the marginal area unit to be considered for cutting is not determined explicitly, it is taken into account implicitly. This second solution is eminently practical and quick. It will be followed in the remaining sections and in the example of chapter III.

Reduction of Computations Through the Coarse Grid Approach

The above formulations, modifications and adaptations of the dynamic programming approach are theoretically sound. However, when we try to apply the above reasoning and actually carry out the calculations, a minor but very troublesome difficulty arises. This difficulty is still, in a way, caused by the growth term. It arises as follows. When dynamic programming is used, every feasible value of the state variable and of the decision variable should be considered at each stage. This means in the model that two different values of $V_{n+1,.}$ considered, should differ by only one unit of cubic volume (say by 1 m^3 or by 1 cu ft) and the same would be true for the decision variable $Y_{n,.}$. This requires an enormous amount of bookkeeping of the calculations in the sense that a certain area unit may be scheduled to be cut over a number of stages. In turn, this destroys the simple relation between the growth of an area unit and the volume of that area unit: different parts of the same area unit would be in different stages of development and hence carry different volume densities. Worse, however, the number of state and decision variables to be considered at each stage would be very large indeed, even for a relatively small planning region. Coupled with a possible long planning horizon (say 100 years) the whole problem would be computationally infeasible.

Obviously the solution is to consider at each stage, state and decision variables that differ from each other by more than say, 1 m^3, for example which differ by 1000 m^3. Or that differ by a quantity that is at least as big as the volume on one area unit is likely to be. In fact, what is wrong with calling 1000 m^3 a unit?

This, in effect, proposes to reduce the number of feasible values of the state variable at each stage (and consequently also the number of feasible values of the decision variable). This computational trick is known under the name coarse grid approach (see 34, chapter 4, section 5). If we may assume unimodality of our maximization problem, the maximum will still be obtained. However, it is when applying the coarse grid approach that the difficulty, which can be traced back to the growth term, strikes with full force.

To explain what happens, suppose we try again to find f_n ($V_{n+1,.}$). As explained before $V_{n+1,.}$ is specified and so is $Y_{n.}$. Through the coupling function $V_{n.} + G_{n.}$ is found. The difficulty now is that when using a coarse grid, the value of $V_{n.}$ or of $V_{n.} + G_{n.}$ may not have been considered at stage $n - 1$ when calculating the various f_{n-1} ($V_{n.}$). The following simple example will show this more clearly. Assume for simplicity that each area unit stand grows by 2 m³/hectare/year until there are 100 m³/hectare, after which growth is zero. Assume that the grid considered is 100 m³, and that we have a region of three hectares each with 100 m³ at the first stage. Hence, $V_{1.} = 300$ and $G_{1.} = 0$. If $V_{2.} = 300$ then $Y_{1.} = 0$; if $V_{2.} = 200$ then $Y_{1.} = 100$; if $V_{2.} = 100$ then $Y_{1.} = 200$; finally if $V_{2.} = 0$ then $Y_{1.} = 300$. Assume for simplicity that the three hectares are identical in all respects so that we can start cutting from $j = 1$ etc. As there is no maximization at this first stage we find f_1 (300), f_1 (200), f_1 (100) and f_1 (0) and save them for stage two, while the decisions as a function of $V_{2.}$ are saved till the end; that is we store $Y_{1.}$ (300) $= 0$, $Y_{1.}$ (200) $= 100$, $Y_{1.}$ (100) $= 200$ and $Y_{1.}$ (0) $= 300$. As G_{2j} is a function of V_{2j} only, we can find it easily. If $V_{2.} = 300$ this means that all three hectares still have 100m³ and consequently that $G_{2,j} = 0$ for $j = 1, 2, 3$. Again if $V_{2.} = 200$ this means that by assumption the cut came from $j = 1$; hence $V_{2,1} = 0$ and $G_{2,1} = 2$, while $V_{2,2} = V_{2,3} = 100$ and $G_{2,2} = G_{2,3} = 0$. Similarly if $V_{2.} = 100$ then $V_{2,1} = V_{2,2} = 0$, $G_{2,1} = G_{2,2} = 2$ and $V_{2,3} = 100$, $G_{2,3} = 0$. Finally if $V_{2.} = 0$ then $V_{2,1} = V_{2,2} = V_{2,3} = 0$ and $G_{2,1} = G_{2,2} = G_{2,3} = 2$. So if $V_{2.} = 300$ then $G_{2.} = 0$; if $V_{2.} = 200$ then $G_{2.} = 2$; if $V_{2.} = 100$ then $G_{2.} = 4$; and if $V_{2.} = 0$ then $G_{2.} = 6$. No difficulties arise at this first stage.

At the second stage we want to find f_2 (300), f_2 (200), f_2 (100) and f_2 (0). Let us take f_2 (100) as an example. Here $V_{3.} = 100$ and the $Y_{2.}$'s that must be considered to find $f_2(100)$ are 0, 100 and 200. Let us consider the case $Y_{2.} = 100$. From the coupling function we then find $V_{2.} + G_{2.} = V_{3.} + Y_{2.} = 100 + 100 = 200$. Now h_2 ($V_{3.} = 100$, $Y_{2.} = 100$) can be found easily; however as indicated before, the dynamic programming calculations assume that at this second stage f_1 ($V_{2.}$) $=$ f_1 ($V_{2.} + G_{2.}$) is known. We found $V_{2.} + G_{2.}$ in this case to be 200, which means that $V_{2.}$ must have some value between 100 and 200 (quite close to 200 in fact).

Now the difficulty is clear: at the first stage we did *not* calculate f_1 for any value of $V_{2.}$ between 100 and 200, but only for $V_{2.} = 100$ (exactly) and $V_{2.} = 200$ (exactly) or alternatively, we did not calculate f_1 for values of $V_{2.} + G_{2.} = 200$; the nearest value for which we did calculate f_1 is for $V_{2.} + G_{2.} = 200 + 2 = 202$.

Obviously the error made in this case by taking f_1 $(V_{2.} + G_{2.} = 200) = f_1$ $(V_{2.} + G_{2.} = 202)$ is not likely to be of any significance. However, the error becomes bigger and bigger as we go on to the third and following stages; and the error may not be so insignificant, even at stage two, if the grid used is very coarse (for example in the case where each unit represents 10^4 m^3). However fine or coarse we make the grid, this difficulty arises. Moreover, as indicated before, we cannot make the grid too fine (specifically not much finer than the volume of one area unit), because otherwise we run into additional difficulties of having to consider harvesting an area unit over a number of stages. As explained, the latter increases bookkeeping requirements and hence will tax the computational and storage limitations of the computer.

The way found to get around this difficulty is to take $Y_{n.} - G_{n.}$ instead of $Y_{n.}$ as the decision variable. It may seem awkward to do this, as part of the decision variable (i.e. $G_{n.}$) is only to some (small) extent under the control of the decision maker. This, however, can be taken care of by setting bounds on the values of $Y_{n.} - G_{n.}$ to be considered. After all, such bounds also existed in the case where $Y_{n.}$ is the decision variable. A second possible objection may be that this decision variable can be negative; all programming methods require the decision variables to be non-negative. As the actual decision variable still remains $Y_{n.}$, and $G_{n.}$ is only included for computational purposes, again this objection does not hold in this case. As long as there is a one-to-one relation between decision variable and objective function and as long as the requirements of monotonicity and separability are satisfied, we can use almost any decision variable. Finally, it should be obvious that taking $Y_{n.} - G_{n.}$ instead of $Y_{n.}$ as the decision variable, does not affect in any way the computational procedures developed before to establish cutting priorities and to take into account the alternative of ordering raw material from elsewhere.

Review of the Computational Sequence Developed

The computational sequence developed can be summarized with the following steps. Suppose we want to find f_n $(V_{n+1,.})$ for a general stage n (n = 2, 3, . . . , N). The computational sequence for the first stage (i.e. when n = 1) is basically the same but much more simplified. As indicated before, this is because at the first stage only one value $V_{1.}$ has to be considered, while at any other stage n (n = 2, 3, . . . , N) a whole set of values $V_{n.}$ has to be considered. Thus the specification of a value for $V_{2.}$ at stage 1 automatically fixes the value of $(Y_{1.} - G_{1.})$; specifying a value $V_{n+1,.}$ at a general stage n (n = 2, 3, . . . , N) does not imply any specific value for $(Y_{n.} - G_{n.})$.

Step 1: Determine the possible range of values of $V^{(t)}_{n+1,.}$, where t = 1, 2, . . . , T. The smallest value to be investigated, i.e. $V^{(1)}_{n+1,.}$, is zero. The largest value, i.e. $V^{(T)}_{n+1,.}$, cannot exceed the largest recorded value of $V^{(t)}_{n.}$, i.e. $V^{(T)}_{n.}$, plus the respective growth $G^{(T)}_{n.}$.

Step 2: Determine the possible range of values of $(Y_{n.} - G_{n.})^{(z)}$, where z = 1, 2, . . . , Z. At the lower end $Y_{n.} - G_{n.}$ is restricted by the value (− maximum $G_{n.}$), because $Y_{n.}$ cannot be smaller than zero. At the upper end it is bounded by the value $V^{(T)}_{n.}$ = maximum $V_{n.}$. Hence the first value of $Y_{n.} - G_{n.}$ to be investigated, i.e. $(Y_{n.} - G_{n.})^{(1)}$, equals zero minus a whole number x times the grid unit. (if we investigate the values 0, 1000, 2000, 3000, . . , the grid unit is 1000). Here x is determined in such a way that (0 − x times the grid unit) $\geq$ − max $G_{n.}$ while (0 − (x + 1) times the grid unit) < − max $G_{n.}$.

Perhaps it is not superfluous to indicate that the grid size used on $V_{n+1,.}$ should also be used on $Y_{n.} - G_{n.}$. That is, if the consecutive values of $V^{(t)}_{n+1,.}$ to be investigated differ by 1000 m^3, then the successive values of $(Y_{n.} - G_{n.})^{(z)}$ should differ by 1000m^3.

Step 3: Start with $V^{(1)}_{n+1,.} = 0$.

Step 4: Start with $(Y_{n.} - G_{n.})^{(1)}$

Step 5: $V^{(t)}_{n.} = V^{(1)}_{n+1,.} + (Y_{n.} - G_{n.})^{(1)}$. If $V^{(t)}_{n.}$ is smaller than zero go back to step 4 and take the value $(Y_{n.} - G_{n.})^{(2)}$, which is one grid unit larger than $(Y_{n.} - G_{n.})^{(1)}$ etc., until $V^{(t)}_{n.} \geq 0$.

Step 6: Recover $V^{(t)}_{nj}$, j = 1, 2, . . . , J. Because both the values that $V^{(t)}_{n+1,.}$ (t = 1, 2, . . . , T) and $(Y_{n.} - G_{n.})^{(z)}$ (z = 1, 2, . . . , Z) can take on, are dictated by the grid selected, we automatically obtain a value for $V_{n.}$ which is also a value of the grid. Thus we are sure that the value $V^{(t)}_{n.}$ has been considered at stage n − 1. The irregular number difficulty has been solved. It is true that $Y_{n.}$ and, as will presently become clear, Y_n will be a bit irregular (that is a non-grid number), but there is nothing against that. Of course, $G_{n.}$ will by its very nature always be an irregular quantity. Knowing $V^{(t)}_{n.}$ we also know $V^{(t)}_{nj}$, j = 1, 2, . . . , J. This shows the big advantage of the forward approach being followed: at each stage we know by area unit what happened in the former stage.

Step 7: Determine $G^{(t)}_{nj}$ and $G^{(t)}_{n.}$. Obviously $G^{(t)}_{n.}$ is the growth component of the specified value $(Y_{n.} - G_{n.})^{(1)}$.

Step 8: $Y^{(t)}_{n.} = (Y_{n.} - G_{n.})^{(1)} + G^{(t)}_{n.}$. If $Y^{(t)}_{n.}$ is negative we go back to step 4 and take the value $(Y_{n.} - G_{n.})^{(2)}$ which is one grid unit larger than $(Y_{n.} - G_{n.})^{(1)}$ etc. until $Y_{n.}^{(t)} \geq 0$.

Step 9: Calculate the net value growth of each area unit j.

Step 10: Rank the area units according to their net value growth, smallest one first. We obtain $V_{nji}^{(t)}$ where i indicates the rank according to the net value growth. Of course $i = 1, 2, \ldots, J$, but generally $i \neq j$.

Step 11: Determine $Y_{nj}^{(t)}$. First all the prescribed thinnings (if any) are made, remembering that an area unit j is scheduled to be thinned as soon as V_{nj} has a certain predetermined value. As soon as the thinnings add up to the prescribed amount $Y_{n.}^{(t)}$ (found in step 8) go to step 12. If the thinnings do not add up to the prescribed amount $Y_{n.}^{(t)}$, one starts scheduling stands for a harvest cut beginning with the stand on the area unit showing the lowest net value growth. No more stands are scheduled for a harvest cut as soon as the prescribed amount $Y_{n.}^{(t)}$ is obtained. A harvest cut can either be a clear cut (considered in the example of chapter III) or a partial cut (as practiced in all-aged or unevenaged management). In the latter case the desired remaining volume level should have been specified before, of course; it is presumably determined by silvicultural and other considerations (see, however, section D for a more extensive discussion of this case).

Step 12: $V_{n+1,j}^{(t)} = V_{nj}^{(t)} + G_{nj}^{(t)} - Y_{nj}^{(t)}$ for $j = 1, 2, \ldots, J$.

Step 13: Take $Y_{n}^{(1)} = Y_{n.}^{(t)}$, that is add $dY_1 = 0$ to $Y_{n.}^{(t)}$.

Step 14: Calculate $h_n (V_{n+1,.}^{(1)}, Y_n^{(1)})$.

Step 15: Recover $f_{n-1} (V_{n.}^{(t)})$ from storage; it was found at stage $n - 1$.

Step 16: Find $h_n (V_{n+1,.}^{(1)}, Y_n^{(1)}) + f_{n-1} (V_{n.}^{(t)})$.

Step 17: Check whether $Y_n^{(1)} \geq M$. If so, go to step 20. If not go to step 18.

Step 18: Set $Y_n^{(2)} = Y_{n.}^{(t)} + dY_2$. Here dY_2 is one grid unit larger than dY_1. Note that the grid imposed on dY_k does not have to be the same as the one imposed on $V_{n+1,.}^{(t)}$ and $(Y_{n.} - G_{n.})^{(z)}$.

Step 19: Find $h_n (V_{n+1,.}^{(1)}, Y_n^{(2)}) + f_{n-1} (V_{n.}^{(t)})$ and if $Y_n^{(2)} < M$, repeat steps 18 and 19 with $dY_3, dY_4, \ldots$ until $Y_n^{(k)} \geq M$.

Step 20: Set $f_n^{(1)} (V_{n+1,.}^{(1)})$ equal to the largest value of $h_n (V_{n+1,.}^{(1)}, Y_n^{(k)}) + f_{n-1} (V_{n.}^{(t)})$ obtained for $k = 1, 2, 3, \ldots$. Suppose this maximum was reached for $Y_n^{(1^*)}$. Save $V_{n+1,.}^{(1)}$, $Y_n^{(1^*)}$, $Y_{n.}^{(t)}$, $f_n^{(1)} (V_{n+1,.}^{(1)})$, $G_{n.}^{(t)}$ and $V_{n+1,j}^{(t)}$ for $j = 1, 2, \ldots, J$ temporarily.

Step 21: Go back to step 4, increasing the value of $(Y_{n.} - G_{n.})^{(1)}$ by one grid unit to obtain $(Y_{n.} - G_{n.})^{(2)}$.

Step 22: Repeat steps 5–20, obtaining successively $V_{n.}^{(t+1)}$, $V_{nj}^{(t+1)}$, $G_{nj}^{(t+1)}$, $G_{n.}^{(t+1)}$, $Y_{n.}^{(t+1)}$, $Y_{nj}^{(t+1)}$, $V_{n+1,j}^{(t+1)}$, $f_n^{(2)}(V_{n+1,.}^{(1)})$ and $Y_n^{(2^*)}$.

Step 23: Repeat steps 21 and 22 with $(Y_{n.} - G_{n.})^{(3)}$, $(Y_{n.} - G_{n.})^{(4)}$, . . . , until we obtain a $V_{n.}^{(t+z-1)}$ which is larger than the largest recorded value of $V_{n.}$ at stage n − 1, i.e. larger than $V_{n.}^{(T)}$. Then we go to step 24.

Step 24: Find $f_n(V_{n+1,.}^{(1)})$ as the maximum among the $f_n^{(z)}$ $(V_{n+1,.}^{(1)})$ values, z = 1, 2, . . . , Z, determined by the steps 1–23. It is this value f_n $(V_{n+1,.}^{(1)})$ that is saved for stage n + 1, together with the respective $V_{n+1,j}$ values, j = 1, 2, . . . , J. There may be several $f_n^{(z)}$ $(V_{n+1,.}^{(1)})$ of the same value and thus there may be more than one maximum. In this case there is more than one solution and all should be saved for stage n + 1. The values f_n $(V_{n+1,.}^{(1)})$ and $V_{n+1,j}$, j = 1, 2, . . , J, are not needed any more after stage n + 1 (if so desired, they might be printed out by the computer before eliminating them. This might be useful to obtain detailed insight as to how our enterprise develops over time. In the example of chapter V all f_n $(V_{n+1,.}^{(z)})$ were in fact printed out for z = 1, 2, . . . , Z and for n = 1, 2, . . . , N). The values $Y_{n.}$, $G_{n.}$ and Y_n for which the maximum (maxima) was (were) obtained, are saved until after stage N is finished and we are ready to trace back our steps to find the optimal decision policy.

Step 25: Go back to step 3 and repeat 4–24 with the next higher grid value $V_{n+1,.}^{(2)}$ to find f_n $(V_{n+1,.}^{(2)})$ and the pertaining values $V_{n+1,j}$ (j = 1, 2, . . . , J), $Y_{n.}$, $G_{n.}$ and Y_n. The first two sets of values are again saved for stage n + 1, while the last three are saved until the end.

Step 26: Repeat step 25 with the grid values $V_{n+1,.}^{(3)}$, $V_{n+1,.}^{(4)}$, . . . , $V_{n+1,.}^{(T)}$. Once we have had $V_{n+1,.}^{(T)}$, we are finished with stage n. We have found f_n $(V_{n+1,.}^{(t)})$ for t = 1, 2, . . . , T. For each $V_{n+1,.}^{(t)}$ we have saved the pertaining $V_{n+1,j}$ (j = 1, 2, . . . , J), $Y_{n.}$, $G_{n.}$ and Y_n. We are prepared to start the next stage, i.e. stage n +1.

Step 27: Start again at step 1, replacing the value n by the value n + 1. We obtain f_{n+1} $(V_{n+2,.}^{(t)})$, t = 1, 2, . . . , T, and for each t we find one set of the pertaining values of $V_{n+2,j}$ (j = 1, 2, . . . , J), $Y_{n+1,.}$, $G_{n+1,.}$ and Y_{n+1}.

Step 28: Repeat step 27 for n + 2, n + 3, . . . , N. At stage N we find f_N $(V_{N+1,.}^{(1)})$, f_N $(V_{N+1,.}^{(2)})$, . . . , f_N $(V_{N+1,.}^{(t)})$, . . . , $f_N(V_{N+1,.}^{(T)})$. Only now we can find the optimal decision policy.

If we want the optimal plan for a specific value $V_{N+1,.}$, say V_{Final} (where V_{Final} has to be a number of the grid used, of course), we proceed as follows. The maximum discounted net return of the whole operation over the planning period is f_N (V_{Final}). At stage N the optimal decision policy was

1) to cut the amount $Y_{N.}$ pertaining to V_{Final}, say $Y_{N.}^*$

2) to obtain a growth of $G_{N.}^* = G_{N.}$ pertaining to V_{Final}

3) to order the amount $Y_N^* - Y_{N.}^*$ from elsewhere, where again Y_N^* is the amount specified for the value $V_{N+1,.}^{(t)} = V_{Final}$.

We also can find the individual per unit area volumes $V_{N+1,j}^*$ for $j = 1, 2, \ldots, J$, that is the total forest volume composition pertaining to the value V_{Final} specified. To find the optimal decision at the stages $N - 1, N - 2, \ldots, 1$ we note that $V_{Final} + Y_{N.}^* - G_{N.}^*$ gives us a certain value $V_{N.}$ which is, of course, a value of the set of grid values $V_{N.}^{(t)}$, $t = 1, 2, \ldots, T$, investigated at stage $N - 1$. As mentioned in step 27, not only are these values saved until the end but the pertaining values $Y_{N-1,.}$, $G_{N-1,.}$ and Y_{N-1} are also saved. Hence we find $Y_{N-1,.}^*$, $G_{N-1,.}^*$ and Y_{N-1}^* after having found $V_{N.}^* = V_{Final} + Y_{N.}^* - G_{N.}^*$. But this in turn enables us to find $V_{N-1,.}^* = V_{N.}^* + Y_{N-1,.}^* - G_{N-1,.}^*$ and consequently $Y_{N-2,.}^*$, $G_{N-2,.}^*$ and Y_{N-2}^*. Continuing back to stage 1 we obtain consequently $V_{n+1,.}^*$, $Y_{n.}^*$, $G_{n.}^*$ and Y_n^* for $n = N, N - 1, \ldots, 1$.

If the optimal policy is desired for another value at the end of the planning period, say for V_{other}, we proceed in exactly the same way as was indicated for V_{Final}. This is why a sensitivity analysis on the final total amount of volume left at the end of the planning period, is automatically implied in the solution.

If we want the optimum optimorum, the best we can possibly do, we determine at stage N the one (or more) $V_{N+1,.}^{(t)}$ that gives the largest f_N value. Then we trace back the optimal decisions in the same way as was indicated for V_{Final}, starting however with this different value of $V_{N+1,.}^{(t)}$.

It should be remarked that the above summary of the computational sequence obviously can be used to construct a computer flow chart. However, as it was presented mainly to clarify the working of the model some shortcuts are possible. Some of the computational results need not be saved as long as indicated. Neither need every element of h_n be recalculated every time a different value dY_k is added to $Y_{n.}^{(z)}$ to give a new value $Y_n^{(k)}$. Good programming will reduce both storage requirements and computational time.

Concluding Observations on the Model

The model developed has been shown to be able to arrive at optimal decision rules for the whole production process from tree seedling to one or more of the products of the primary forest industries, under alternative possible, and generally exogenously imposed, amounts and patterns of capital investments. A few special possibilities of the model will be indicated in this section. Some of these have been hinted at, or can be inferred from the explanation of the symbols used.

As indicated before $P_{nj}^{(2)}$ (the price of the logs sold) can be made a function of

$Y_{n\cdot}$, while $P_n^{(2)}$ (the price to be paid for logs ordered from elsewhere) can be made a function of quantity ordered. The method given to determine which stands to harvest first is not affected by the specific size of $P_{nj}^{(2)}$; only an ordering of stands is involved and any price will do. The "marginal" area unit to be considered for cutting is affected though by the relative sizes of $P_{nj}^{(2)}$ and $P_n^{(2)}$. Each different size of $Y_{n\cdot}$ considered and each different quantity ordered may affect $P_{nj}^{(2)}$ and $P_n^{(2)}$, i.e. their relative sizes, and so change the "marginal" area unit. As remarked before, the computational method developed does not require the explicit determination of the "marginal" area unit. As the concept is accounted for implicitly through the natural dynamic programming selection process, it makes no difference by which parameters it is influenced as long as these parameters are in the model. And $P_{nj}^{(2)}$ and $P_n^{(2)}$ are in the model, whether or not we make them a function of $Y_{n\cdot}$

It may, of course, be undesirable to cut timber before it has reached a certain size. This can be handled in either of two ways. Establish a volume level (= age) below which no area unit should be considered for cutting; this level, which may be different for the different site classes and/or management intensities, mayor may not override the previously determined "marginal" area unit. Alternatively we might fix $P_{nj}^{(1)}$ and $P_{nj}^{(2)}$ at a very low level as long as the volume is not high enough, i.e. as long as the desired sizes are not yet present on the area unit. This last way is demonstrated with the example in chapter III.

It cannot be stressed sufficiently that although the optimization is carried out over the whole enterprise, and although the state and decision variables are in the first instance specified in aggregative terms, all calculations are done on a per area unit basis. The model, the forward recursive solution and the computational procedure were all designed to make this possible. It means that most of the traditional advantages of working on a per area unit basis are maintained. Growth is determined per area unit as a function of stand volume present and, indirectly, as a function of age; site is taken into account. If only x% of the stand volume is merchantable, as is often the case in virgin or old second growth natural forests, this is easily taken care of by the fact that Y_{nj} is determined on a per area unit basis. Differential logging and silvicultural costs and price differentials for species, timber qualities and sizes produced, can all be taken into account by tying them back to the specific area units. For example, after a changeover from no management to extensive management, the percentage of the stand volume that is merchantable may rise (as it does in the example in chapter III), the species composition may change, the quality of timber may increase, logging costs per volume unit harvested may decrease, etc. The model can be made to handle all this.

As can already be inferred from the explanation of the symbols, the model is not static. It will handle changes over time in prices, interest rates, wages, landprices and other costs. Of course the forest planner must be given these data and their changes over time. These data may be very unreliable, but this is not a fault of the model. Shadow prices may be used whenever they are available. A decrease in the forest area under consideration may be foreseen. Certain area units may be more suitable for say—agriculture or pasture. Or population pressure may be expected to force some area units out of its forest use. It is possible to include this as a modifying factor into the model. All that has to be done is designate these areas with known j-numbers and drop them at the stages specified. Depending on the time period at which these areas are planned to revert to another use, it will be advantageous to schedule them for early harvesting or for cutting just before the date due.

Historically most regions have witnessed over-capacity of the plants erected, or at least over-exploitation of their forests, relative to what sustained yield would have dictated. While most foresters would probably have favored a somewhat greater than sustained yield cut, at least as long as there is a relative over-abundance of mature forests, they are generally rather alarmed by the very rapid rates of exploitation often experienced in the first stages of development. The model developed not only supplies optimal decision rules to the forest planner, but also can be used to find out whether overexploitation and/or over-capacity in the beginning is economically desirable or not. Running the model for a number of different capacity situations, the most economical one can be found. As solutions are found as a function of the total volume at the end of the planning horizon, any desired weight to the future can be given (over and above the weight expressed by the discount rate used). One could imagine running the model for cases such as: over-capacity throughout the whole planning period, over-capacity only in the beginning with sustained yield capacity later on, sustained yield capacity throughout (as assumed in the example of chapter III), undercapacity in the beginning etc. The model will indicate whether a policy of over-exploitation, sustained yield exploitation or even under-exploitation should be followed. For example, the model might indicate that the most economical thing to do is to start with under-capacity coupled with over-exploitation in the beginning (implying the sale of unprocessed raw material in the beginning) and to revert to sustained yield only later on. This approach really implies a simulation of the dynamic programming model.

The model can take account of external economies and diseconomies only in so far as they can be given some quantitative measure and only in so far as they can be related to the state variable $V_{n+1,.}$ (and $V_{n+1,j}$) and/or to the decision variable Y_n (and Y_{nj}). The second requirement will by and large be easy to fulfill, but the first one might be impossible to satisfy or at least will involve considerable uncertainty. Again by obtaining the solution as a function of $V_{N+1,.}$ (the total volume at the end of the planning horizon) some externalities can be accounted for by specifying a certain desired $V_{N+1,.}$.

Some Special Cases of the Model Developed

SPECIAL CASE 1:

The decision variable Y_n can be constrained in a number of ways without increasing computations, if so desired. All we have to do is add a constraint like $Y_n \leq X_n$, $Y_n \geq X_n$ or $X'_n \leq Y_n \leq X''_n$ to the model. Here X_n is a constant in period n, which however, may be different in period $n + 1$. As this constraint limits only the range of feasible values that Y_n is allowed to take on, it presents no difficulties. Rather it diminishes the computations. Of course, we might not get the optimum but only a constrained optimum. The following examples show the possible uses of such a constraint. It may be decided that the forest activities should give employment to at least x persons. This would mean that Y_n must be at least equal to a certain number. This number may be increased or decreased over time depending on whether the forestry activities are, or are not, expected to absorb additional labor with the onset of economic development. The capacity of the transportation system may also constrain Y_n; again this constraint may be changed over time. Constraints by the amount of investment available are taken into account already, as explained before. There may exist a labor constraint, either for all activities or for some. By constraining Y_n and/or the maximum capacity of the plants to be erected (= M), this will be taken into account. Limitations imposed by certain types of labor (entrepreneurs, for example) are handled the same way.

SPECIAL CASE 2:

Unless stated explicitly otherwise, it was assumed until now that clearcutting and hence the even-aged type of forest management was being practiced. This is, after all, the most commonly practiced form of management to date and in the foreseeable future. And if the length of a stage period is taken to be about five years or longer, the shelterwood type management can be accommodated too. Actually the all-aged type of forest management and hence the selective cut, can be handled by the same model. All we need to do is to change the harvesting rules as follows.

First, there are no thinnings to consider separately. Second, the area units are considered to supply the desired cut in order of their B_{nj}, starting with those for which B_{nj} is smallest. As explained before B_{nj} is the net value growth of the jth area unit during stage n (a marginal concept). However, this time we do not clearcut, but leave that residual volume which the forester considers optimal (as determined by a suboptimization process or otherwise). Again as long as the cut $Y_{n.}$ comes from units for which $B_{nj} \leq 0$, or as soon as $Y_{n.} \geq M$, it will not pay to consider the alternative of ordering raw material from outside the region. But as soon as we have cut all area units for which $B_{nj} \leq 0$ and as long as $Y_{n.} < M$, we do have to consider that alternative. In fact, if the latter two conditions hold, we will consider cutting into the residual volumes of those area units for which $B_{nj} > 0$. However, one should never go below the lowest volume $V_j^{(min)}$ that is necessary silviculturally or otherwise. This is an absolute minimum level. Economically speaking it may or may not pay to cut down to that level. In fact, these considerations dictate cutting down to that volume level $V_{n+1,j}$ for which the net value growth B_{nj} equals $A_{nj} \times (V_{n+1,j} - V_j^{(min)})$. As defined before, A_{nj} is the additional amount to be paid for raw material imported from elsewhere over and above the price of local raw material, i.e. it equals $P_n^{(2)} - P_{nj}$.

The "marginal" area unit is now replaced with the "marginal" residual volume level to be left. As long as $Y_{n.} < M$ we will continue considering cutting down to the marginal residual volume level until we have had all area units. Once that point is reached, and assuming $Y_{n.}$ is still smaller than M, we will consider the alternative of ordering from outside the region. Again it is never necessary to determine actually the "marginal" residual volume level. As indicated in the review of the computational procedure, it is taken into account implicitly. In this case of selective cutting it may be desirable to constrain Y_{nj} from below, that is to establish minimum amounts cut per area unit, in order to make harvesting operations economically feasible. Better even is to make the harvesting cost a function of Y_{nj}, as indicated in the model already.

SPECIAL CASE 3:

In the discussion of the cutting order and the harvesting rules, the words generally used are area units *considered* for cutting, not area units that *will be* cut. Only when retracing our steps will $Y_{n.}^*$ and Y_n^* be determined and *only then* shall we know what *will* happen. Only then shall we know for each investment pattern whether we indeed have to harvest stands for which $B_{nj} > 0$, that is financially immature stands according to the Faustman rotation. And only then shall we know whether it pays to run into shortages at some stages because we overcut earlier; and whether in this case it pays to leave some plant capacity idle, or whether to order raw material from outside the region. Finally, if we consider various investment levels and patterns, we shall know whether it pays to start with overcapacity (possibly making it necessary to order raw material from elsewhere at some stages), under-capacity (possibly implying the sale of unprocessed raw material at some stages), or sustained yield capacity. It is in this way that the Faustman or Soil Expectation approach to the determination of the rotation is generalized to take into account the demands of the plant capacity established and the outside-the-region buying and selling alternatives; and alternatively to take the growth potential of the forestry region into account when considering the total plant capacity to be established.

Hence, the Faustman formula is a special case of the model developed. To show this more clearly, let

$$\begin{aligned} H = \sum_{n=1}^{N} \frac{1}{(1+K_n)^n} \, [& r_n^{(2)} (Y_{nj}, P_{nj}^{(2)}) - c_n^{(1)} (V_{nj}^{(0)} - Y_{nj}, P_{nj}^{(3)}, K_n) - \\ & - c_n^{(2)} (L_{nj}, I^{(1)}, K_n) - c_n^{(3)} (Y_{nj}, W_n^{(1)}, O_n^{(1)}, I^{(m)}) - \\ & - c_n^{(4)} (F_{nj}^{(1)}, D_n^{(1)}, K_n, I^{(1)}) - \\ & - c_n^{(5)} (Y_{nj}, W_n^{(2)}, V_{nj}, I^{(1)}, K_n, O_n^{(2)}, F_{nj}^{(2)})] \ . \end{aligned}$$

The problem can be stated as: For each separate $j = 1, 2, \ldots, J$ determine Y_{nj}, $(n = 1, 2, \ldots, N)$ so as to maximize H, which is a function of V_{nj} and Y_{nj}

subject to
$$V_{nj} = V_{n+1,j} + Y_{nj} - G_{nj} \qquad \text{for } n = 1, 2, \ldots, N$$
$$Y_{nj} \geq 0 \qquad \text{for } n = 1, 2, \ldots, N.$$

Note that Y_n is not included any more, because no outside buying alternatives are considered. Neither are allowances made for factors related to industrial plants. The optimization is on a per area unit basis, not for the whole forestry enterprise. We have in fact J different optimization problems. The cost and revenue functions and such parameters as $P_{nj}^{(2)}$ and $P_{nj}^{(3)}$ are not a function of $Y_{n.}$. Consequently, perfectly elastic demand curves are assumed. We have the straight forward maximization of the soil expectation of an area unit. The costs of holding the present forest stands for another period, inclusive of such things as taxes and insurance (at least in so far as these are related to the value of the standing volume), plus the value of timber growth or any other benefits of future (new) forest stands and rotations foregone in holding the present forest stand for an additional period (this is an allowance for the site rent), are balanced against the growth in value of the present forest stands. Possible reductions or increases in logging costs are taken into account also.

Finally, if we impose the additional restriction of a fairly constant total periodic cut $Y_{n.}$ in order to assure a rather constant benefit and cost stream over time, the traditional sustained yield framework is obtained. Hence $X_1 \leq Y_{n.} \leq X_2$, where X_1 and X_2 represent respectively the lower and upper bound between which $Y_{n.}$ is allowed to fluctuate. In fact, if we do not add this constraint, H would be maximized by cutting almost all the area units in the first period already (unless the cost and revenue functions are made also a function of $Y_{n.}$, which the Faustman formula does not do).

SPECIAL CASE 4A:

Another special case of the model is that of the forester managing the forests for an established plant. Most forest plants do not own sufficient forests to supply their whole demand. In fact, many plants keep forests mostly to strengthen their bargaining position and/or to protect their capacity over the long run. The forest is called upon to supply wood only at irregular intervals, indicated primarily by the needs of the plant and only secondarily by financial considerations of the forest part itself. The question is, what is the optimal cutting strategy in this case? On the one hand the plant wants to keep the forests as a kind of insurance to safeguard its continuous operation. On the other hand, it does not want to manage its forests too far from what is the optimum according to the Faustman formula approach.

Of course, if absolute uncertainty as to the future exists, the model will not be of much help (nor will any model probably for that matter). Suppose however, that on the basis of regional or national supply forecasts, the plant is able to indicate over a number of years that part of its total requirements that its own forest is expected to provide for. It is also able to attach a penalty or shortage cost for each unit by which the requirements are not met. This penalty cost, which can be made a function of the amount by which the plant's demands are not met, can represent the increase in price which a plant has to pay for raw material purchased and shipped in from more distant suppliers; or, if that alternative is not available, the cost of operating the plant at only a part of its capacity.

The model, changed to cover this case, may look as follows. Again let

$$\begin{aligned} H = \sum_{n=1}^{N} \frac{1}{(1+K_n)^n} \big[& r_n^{(2)} (Y_n, P_{nj}^{(2)}) - c_n^{(1)} (V_{nj}^{(0)} - Y_{nj}, P_{nj}^{(3)}, K_n) - \\ & - c_n^{(2)} (L_{nj}, I^{(1)}, K_n) - c_n^{(3)} (Y_{nj}, W_n^{(1)}, O_n^{(1)}, I^{(m)}) - \\ & - c_n^{(4)} (F_{nj}^{(1)}, D_n^{(1)}, K_n, I^{(1)}) - \\ & - c_n^{(5)} (Y_{n.}, W_n^{(2)}, V_{nj}, I^{(1)}, K_n, O_n^{(2)}, F_{nj}^{(2)}) - c_n^{(10)} (Q_n - Y_n, S_n) \big] \end{aligned}$$

Then we have to determine Y_n, $(n = 1, 2, \ldots, N)$ so as to maximize H

subject to $\quad V_{n.} = V_{n+1,.} + Y_{n.} - G_{n.} \quad$ for $n = 1, 2, \ldots, N.$

$\quad Y_{n.} \geq 0 \quad$ for $n = 1, 2, \ldots, N.$

Here $c_n^{(10)}$ is the shortage cost, which is a function of $Q_n - Y_n$ and of S_n. The parameter Q_n represents the plant's demand on its own forests in period n; it may or may not be also the plant's total requirements for raw material in period n. The term S_n represents the cost per unit of volume by which the plant's demand is not met. Obviously, S_n may be a function of $Q_n - Y_n$, i.e. $S_n = s\,(Q_n - Y_n)$. Note that $c_n^{(10)} = 0$ if $Q_n - Y_n \leq 0$. The alternative of ordering raw material from elsewhere is accounted for by $Y_n - Y_{n.}$. The "marginal" area unit to be considered for cutting is now determined in exactly the same way as before, except that instead of working with A_{nj} we use S_n if $S_n > A_{nj}$ and A_{nj} if $S_n \leq A_{nj}$. Although Y_n may be larger than Q_n in any one or more periods, this presumably will occur rarely because the plant is assumed not to own sufficient forests to supply its own demand. Thus this problem can be solved through dynamic programming in exactly the same way as explained for the general model.

Although this last problem might seem reminiscent of a production smoothing or inventory control problem as described by Scarf et al (36) and by Hadley (19, chapter 10, 11), it is basically different in some important respects. Additional quantities of raw material cannot be produced at will (though they can be ordered at an additional cost A_{nj}); the growth capacity of the forest and the standing volume present are binding constraints. Moreover, it does not make sense to assume that demand can be back ordered: if the plant is to do without the necessary raw material in stage n, it most probably cannot make up for this in stage n + 1. Finally it may be advantageous in the forestry problem above to make Y_n in any one period larger than Q_n, at least as long as the amount by which $Y_n > Q_n$ can be disposed of just as profitably as when $Y_n \leq Q_n$. This does not seem to be an unrealistic assumption. In standard production scheduling-inventory control problems, however, an inventory cost would be associated with this case where $Q_n < Y_n$. This special case model can be made a bit more realistic by assuming a stochastic demand Q_n with a known probability density function.

SPECIAL CASE 4B:

As explained before, because dynamic programming can handle almost any kind of variable it is often used in the case where variables are stochastic. This generally makes the model much more complicated computationally. Basically three cases are still amenable to solution:

1) Only the return function is stochastic.

2) The stage transformation function (or the output state variable) is stochastic, and consequently the return is uncertain.

3) The stage transformation is stochastic, but the return is deterministic.

The assumption, that generally has to be made, is that the random variables of the different stages are independent (if this assumption is not made, additional state variables are needed to account for the dependence). However, the expected value of the random variable may vary from stage to stage. This assumption may be doubtful in many instances in forestry. It would, for example, have to be true that when growth is assumed to be stochastic, the fact that it is very low in one year (due to, say, an insect attack, fire, extra dry or wet season, etc.) will have absolutely no influence on the growth of next year.

Hool (22) has worked out an example for the second case based on the above mentioned assumption. In his study the development of a stand is assumed to be stochastic and described by a special case of stochastic processes, a Markov process. (He actually uses an even more specialized case, that of a stationary Markov process, but this is not essential for his model.)

Because it is impossible in the last two cases to link a decision with a specific value of the output state variable(s), the forward approach cannot be used and backward recursive optimization must be employed. Furthermore, because the values of the output state variable(s) are uncertain, we cannot determine the decisions at the stages two to N (or at the stages one to N-1 when the stages are numbered reversely as is customary in backward solving). That is, we cannot determine the sequence of optimal decisions, except the decision at the first stage. For the remaining stages only a set of decision *functions* can be obtained. Except for the decision at stage one, the resulting optimal decision policy is itself stochastic and the optimal decisions cannot be expressed deterministically until the stochastic elements that precede them are revealed. In this sense, an N stage stochastic optimization yields incomplete results (see for example Hool (22) tables 4 and 6).

However, when only the return is stochastic while the stage transformation functions (and hence the state output variables) are deterministic, it seems that we can still use the forward recursive approach. This is the case in our problem if we assume the demands Q_n in the different periods ($n = 1, 2, \ldots, N$) to be independent random variables, which are either continuously or discretely distributed. If the demand Q_n for raw material in period n is assumed to be continuously distributed according to the probability density function $u_n^{(c)}(Q_n)$, we have to replace the $c_n^{(10)}$ term in the expression for H by the stochastic term

$$\int_{Y_n}^{\infty} c_n^{(10)} (Q_n - Y_n, S_n)\, u_n^{(c)} (Q_n) d\, Q_n$$

If Q_n in period n is assumed to be distributed according to the discrete probability density function $u_n^{(d)}$ (Q_n), we work with

$$\sum_{Q_n = Y_n}^{\infty} c_n^{(10)} (Q_n - Y_n, S_n)\, u_n^{(d)} (Q_n)$$

Otherwise the problem remains exactly the same. We have to determine Y_1, $Y_2, \ldots, Y_N$ so as to maximize H subject to

$$V_{n.} = V_{n+1,.} + Y_{n.} - G_{n.}$$
$$Y_{n.} \geq 0.$$

In the actual calculations the infinity sign will be replaced by some large value M, the maximum amount of raw material the plant can process in a period. The stage transformation function in this formulation of the problem is in no way affected by what the demand actually turns out to be. Hence our forward approach of solving, developed before, can be employed. Note that the subscript n on the probability density functions indicates that these functions may change over time. As before, it has been assumed that the amount by which $Y_n > Q_n$ can be disposed of just as profitably as when $Y_n \leq Q_n$. This does not seem to be unrealistic.

It might be felt that the above formulation of the problem, when demand is stochastic, is not completely realistic in the sense that it would be better to obtain optimal decisions Y_n, $(n = 1, 2, \ldots, N)$ as a function of what the demands turned out to be in the stages before the nth stage. This would entail associating an inventory cost with overproduction, and a shortage cost when requirements of the plants are not met. This is being done in standard inventory control problems (see 19, 36). While this would be possible it would abstract from the possibility of selling the amount $Y_n - Q_n$ also to the plant (so that the plant will have to buy less elsewhere), or on the open market. It also would make the demand a part of the stage transformation function. This in turn would make the output state variable stochastic. As a result it would be impossible to use the forward approach developed in this study. It would seem that generally the objective function as used in this study is the most realistic one, but one can imagine situations where other objective functions are more appropriate.

It is of course possible, that the beginning volume $V_{1.}$ and the growth together cannot fulfill the demands and that the beginning volume is steadily being drawn down. This could be seen in the calculations by the fact that the cut $Y_{n.}$ is in no period bigger than the demand Q_n, but in most periods has to be increased by the amount $Y_n - Y_{n.}$. This would provide an early warning to the plant that, if it is to stay in business, it must buy more forest land and/or look for additional raw material suppliers. Anyhow, it must rely less on its own forests for the necessary raw material than it planned to do.

Regional Versus National Considerations

After having pointed out the above special cases of the model, it is well to indicate that the general model presented in this study itself can be viewed as a special case. Basically the model presented pretends to be of help on the regional forest development planning level (or on lower levels as indicated by the special cases described). When the region is taken big enough, or when the country is so underdeveloped as to have no forestry sector yet to speak of, then the factors taken into account by the regional forest planner tend to be the same as those considered by the national forest planner. Still, even in those cases and especially in view of the long planning period considered, many additional factors should be considered at the national forest planning level.

The trouble is that at the national level so many factors enter, which are completely outside the competence of the forest economist, which are non-quantifiable, or about which prospects are so uncertain that probably no model can ever be expected to incorporate them all without becoming non-operational. Yet, in so far as these additional factors might profoundly influence the regional view point, decisions have to be made about these factors and their possible bearing on the regional viewpoint must be analyzed. If considered necessary, the regional viewpoint, and consequently the optimal decision rules developed by the model presented in this study, should be modified accordingly. Some of the considerations which are important at the national level will be reviewed now.

Some considerations related to the production factor labor are likely to be weighted more carefully on the national than on the regional level. The national planner might specify certain shadow prices for labor which are different from the current market prices. Through subsidies, special taxes or otherwise, the national planner might try to enforce the use of these shadow prices in order to influence employment in the region by the forestry activities. The national planners might even specify minimum employment levels in the sense that the combined forestry activities are expected to employ x persons. As indicated before this could be handled by the model (see special case 1).

The model developed assumed that labor can be laid off or rehired at will, possibly implying that labor can be moved around within and between regions without incurring any extra costs. On the national level the social cost involved is bound to be weighted more heavily than on the regional level. As the optimal decisions are only determined after the maximum to the problem has been obtained and we are tracing back our steps, it is only after the calculations are made that we find out how the decision variable Y_n, and consequently the labor force employed, will fluctuate from period to period.

Thus the model in the form presented cannot handle this consideration. However, it would be possible to include at stage n the decision variable at stage n-1 (i.e. Y_{n-1}) as an additional state variable. This would solve the problem because the size of the labor force employed is related to Y_n. The model would become a two state/one decision variable model. A cost function associated with labor force changes would be included in the expression for H and might have the form $c_n^{(11)}(Y_n - Y_{n-1})$. Otherwise the problem remains the same: to maximize H subject to the two constraints. Computationally there is a difference, however. At stage n, for each possible value of $V_{n+1,.}$ *and* each possible value of Y_{n-1}, we would determine the Y_n that maximizes the expression set up by the model. Computations would increase, but the model remains feasible. Actually, if the stage periods are made large enough (which anyhow is desirable computationally if the total planning period is of the order of 75–100 years or longer), say 5 years each, then the costs associated with fluctating employment levels can possibly be decreased considerably by advanced planning.

Another point, which is likely to be very important on the national planning level, is the supply picture over time, both domestically and on the world market. Many countries want to maintain a certain degree of self-sufficiency in such a strategic raw material as timber. Questions as to possible substitutions of forest products for or by other products will have to be considered. To some extent the self-sufficiency criterion and related questions can be accommodated in the model by specifying a certain $V_{N+1,.}$ at the end of the planning horizon, and/or certain minimum $V_{n+1,.}$ ($n = 1, 2, \ldots, N$) levels at the end of each stage below which cutting is prohibited.

On the demand side there are likely to be also some factors which are of more specific interest to the national planner than to the regional planner. Quantities of products demanded and prices are generally considered to be given factors for the regional planner, often to the extent of assuming perfectly elastic demand curves for the region's products at existing prices. At the national level demand curves are almost never perfectly elastic, domestic as well as export markets are considered, product substitution possibilities must be studied, etc. Also it is known that prices often do not reflect true social costs. Any or all of these considerations may make it necessary to reconsider the regionally obtained optimal decision rules. It may be desired to make alternative runs of the model under different demand schedule assumptions, especially in view of the enormous uncertain demand and supply forecasts over time.

The regional planner may not have the same attitude towards foreign capital and business as the national planner. He is not likely to be as much concerned with foreign exchange constraints and the necessity of earning foreign exchange. Considerations of the availability of investment capital in general, and foreign exchange in particular, may make it desirable to run the model under different assumptions in order to test various policy alternatives.

The power of the authorities in charge to tax away, or to force the reinvestment of any desired percentage of the benefits obtained from the forestry undertakings may not be large in the beginning stages of development. It generally may be expected to increase substantially over time. Some ways of doing things may be expected to result in a higher reinvestment than others. It may be desirable to assume population growth to be, to some extent, an endogenous variable in the model. All these considerations may again profoundly affect the optimal regional decision rules.

External effects of the tree growing-harvesting phase and of the primary forest industries may be weighted more heavily on the national than on the regional level. These externalities have been pointed out duly in the forestry literature: recreational uses of the forests, watershed and erosion control effects of the forests, backwood character and educative effects of the primary forest industries, enhancement of the productivity of the agricultural labor force, high social costs of destructive exploitation of the forests resulting in severe erosion, etc. Some of the externalities may be very important in the beginning stages of development, others become more important as development progresses and/or forests become scarcer. An example is the shift that has taken place in many advanced countries from exclusive emphasis on the timber production role of forestry to its multiple use aspects.

It is mainly the task of the national planners to survey the picture of the whole country and to indicate which areas should remain in forestry and which should revert to other uses. For those area units which are destined to remain in forest cover, they should set up broad usage priorities. These priorities should be established on the basis of a careful examination of the timber production, the watershed, the recreational and all the other values and functions of the forests, as well as of the demand for these goods and services over time. The model contains terms to take account of the externalities. In so far as they often cannot be given quantitative measures, however, they will tend to be deemphasized more on the regional than on the national level, where a broader view must be taken.

Many institutional factors are nationally determined and can be changed, if at all, only at the national level. To the extent that they are variables at the national level, they might profoundly affect the regional optimal decision rules.

Perhaps a final word is necessary about the philosophy of planning, the use of the model and the relevance of the optimal decision rules obtained. As indicated repeatedly in this study, the model is basically directed to a planning situation. On the basis of the data used in the model, the forest planner obtains decision rules which are optimal within the objective assumed and the constraints employed. The data which are available and used originally probably are rather crude and inaccurate. Often they will be just educated guesses, or they may have been taken from elsewhere where conditions are or are not rather similar. The quality of the data may be expected to ilnprove over time. The constraints employed may change over time and new ones may be added. Factors which previously did not influence the region under consideration and its actions, may become important. In short, it is obviously impossible to plan once and for all the forestry activities over any length of time.

As better data become available and situations and prospects change, replanning becomes necessary. Consequently the model has to be rerun periodically to reorient planning and to obtain new decision rules which are optimal for the changed situation. In this way planning and the optimal decision rules are dynamic and can and should be changed as the situation dictates. The model, including the computational procedure developed, is only the tool used for each new situation to make some of the many necessary decisions when planning the forestry activities.

III. AN EXAMPLE

JUST TO show how a real life problem will be handled by the model and to give some idea about the computational procedures to be followed, a concrete example will be given in this chapter. The example is based upon plausible data. Some of the data were extracted from the literature as indicated, others were educated guesses or obtained through personal contact with existing enterprises. As no specific area with the necessary data available could be found, the example is purely illustrative. An initial situation with overmature, stagnant forest is envisioned (although as was indicated in chapter II, the model may be employed at any stage of the development or use of the forest resource). As this situation, coupled with significant departure from the assumptions of pure competition, is most likely to be encountered in underdeveloped regions and because these regions are so often situated in the tropics, the data reflect as much as possible tropical conditions (although again non-tropical situations could have been assumed).

The Data

Assume we have to plan the development of the forestry activities in a region with about 100000 hectares in forest cover. A preliminary survey is made to get an impression about volume/ha, species composition, merchantable volume, and site quality (as determined by some crude method and expressed as: good = site class I, medium = site class II and poor = site class III).

The results are as follows:

20 000 ha of site I with 400 m^3/ha of which 40% is merchantable
60 000 ha of site II with 400 m^3/ha of which 30% is merchantable
20 000 ha of site III with 400 m^3/ha of which 20% is merchantable

These figures are typical for a tropical rain forest (see 39, 44, 54 and 52). Assuming that the survey was based on a 2% sample, an average per hectare inventory cost of $0.50 seems reasonable, i.e. a total inventory cost of $1000.

The region has only recently become economically or otherwise accessible. Consequently we are dealing with naturally grown forests which have cost nothing to grow in terms of human effort (as is still the case with many forests in the tropics, such as those situated in the Amazon region). The forests consist of old and probably virgin stands and although biologically speaking growth is taking place, net volume growth is zero. Harvesting is technically and economically feasible. In view of the resources available, especially the amount of investment capital and the species composition of the forests, an extensive type of forest management is considered as well as the establishment of some sawmill capacity. The forestry planner wants to obtain decision rules over a planning period of 75 years.

On the basis of experience elsewhere, the forestry planner expects a stand on site II under some type of extensive management to develop as follows.

Age in Years	*Volume in m^3/ha*	*Age in Years*	*Volume in m^3/ha*
0	nursery stock	55	288
5	9	60	336
10	18	65	366
15	33	70	384
20	51	7S	396
25	72	80	402
30	96	85	405
35	123	90	408
40	153	95	409
45	192	100	410
50	237	>100	410

This implies an average annual increment of 5m^3/ ha over a period of 80 years which seems to be attainable (39, 40 and 41). Converting this to a stand volume -volume growth relation, the following table is obtained

Stand Volume in m^3/ha	*Volume Growth in m^3/ hal5 Years*	*Stand Volume in mS/ha*	*Volume Growth in m^3/ ha/5 Years*
0	9	237	51
9	9	288	48
18	15	336	30
33	18	366	18
51	21	384	12
72	24	396	6
96	27	402	4
123	30	405	3
153	39	408	1
192	45	409	1
		≧410	0

A similar stand is expected to grow 4/3 times as fast on site I and 2/3 times as fast on site III (see 53).

Extensive management is assumed to entail the following. Harvesting is carried out with some minimum attention being paid to the silvicultural requirements. Regeneration will be done by one or another variety of the enrichment system (40, 42). Specifically, this will be assumed to imply some minimum site preparation such as the poisoning and girdling of the undesirable trees, the planting of some 500 young trees per hectare and about four release cuttings during the first five years. No plantation care thereafter. No thinnings. Representative management costs are given by Yoho et al (59), Catinot (40) and Martyn (47) and may look as follows (note that they all are incurred during the first five years after clearcutting).

Poisoning, girdling and some other minimal site preparations	$1O/ha
Planting (500 trees/ha), inclusive of nursery charges and transportation of plants	$15/ha
Four release cuttings during the first five years of the plantation	$45/ha
Total	$70/ha

It seems reasonable to assume that some 20 Km of roads are needed for every 400 Km^2 (see 46, 48, 49). This means that in average one is never more than 10 Km from a road. Assuming it costs $10,000 to construct one Km of road (inclusive of simple bridges, draining systems etc), in total 50 Km of road have to be constructed at a cost of $500,000. Another $100,000 is assumed to be needed for buildings and nurseries (see 51). No depreciation costs are assumed on roads, buildings and nurseries, only maintenance costs.

Fixed yearly costs, irrespective of the amount of wood harvested, for such charges as taxes, road maintenance, insurance etc. may come to $1 per hectare per year (see 51).

Land values will be put at $0.50, $1 and $1.50 per hectare for site III, II and I quality land respectively (see 50). In correspondence with experiences in Europe and the United States these will be assumed to increase by 5 %(compound interest) per year (which corresponds to 27.6% compound interest per five years).

The costs of logging, including felling, cross cutting, yarding, rigging, loading, hauling, unloading, opening up (temporary) logging roads, depreciation of tools and equipment (tools and equipment are assumed to depreciate within five years), are put at $10 per m) roundwood (see 48, 49, 58). These costs are assumed to be the same for all area units. When harvesting for the second time from the same area unit, these costs will be put at $8 per m}. Logging costs will be assumed to remain constant. Thus it will be assumed that rising labor costs will be offset by increased mechanization and advances in technology.

To bring all costs and revenues back to the same point of time an annual discount rate of 6% will be used. This corresponds to a 33.33% rate on a five year basis. This discount rate will serve as the alternative rate of return. It is assumed to be the same for all parts of the production process, i.e. for the tree growing part, for the harvesting phase, and for the sawmill operation. The discount rate is assumed to remain constant during the whole planning period.

The average growth rate of the forests is 5 m^3 per hectare per year. Assuming the forestry planner wants to obtain optimal decision rules for sustained yield sawmilling capacity throughout the planning period, we have to construct one or more sawmills with a total maximum capacity of 5×10^5 m^3 per year on a roundwood basis. As explained before, whether one big miU or several small ones should be constructed is a question that has to be answered through a suboptimization process. The following cost data for the indicated sawmill capacity were obtained from existing plants in the United States (see also 50).

Number of 12 hour shifts a day for 250 days a year	two	one
Yearly production in m^3, roundwood basis	5×10^5	2.5×10^5
Investment		
Machinery (inclusive of engineering costs, freight and erection)	$1,800,000	same
Buildings, storage space, yard, transport sidings, etc.	$1,000,000	same
Depreciation (straight line)		
Machinery 10% per year over 10 years		
Buildings 5% per year over 20 years		
Operating costs		
Power costs: per hour of operation (or $0.36 per m^3 roundwood sawed)	$ 30	same
Materials for maintenance and repair: cost per year	$ 600,000	$ 380,000
Number of personnel (inclusive of maintenance, repair, lumber yard, sawmill, etc.)	200	110
Average annual salary of personnel	$ 5,000	$ 5,000
Property taxes and insurance, per year	$ 300,000	$ 200,000
Working capital (raw material stocks, in-process inventory, product inventory, credit, funds for wages, etc.)	$5,000,000	$3,000,000

The machinery is assumed to continue to be used after having been depreciated, but the costs for maintenance and repairs are expected to increase to $750,000 for the two shifts a day operation and $500,000 for the one shift a day operation. They are assumed to remain constant thereafter. It is assumed that the average salary will rise by 1% compound interest per year. These data work out to a minimum sawmill processing cost of $5.22 per m^3 roundwood basis (or to about $20. per 1000 bd. ft. lumber scale basis), attainable when operating on full capacity on the two shifts a day basis; they amount to $6.52 per m^3 roundwood basis when operating on full capacity on the one shift a day basis.

The following product prices seem reasonable (see 44, 50, 51, and especially 54). Stumpage sells at $4 per m^3 roundwood basis. This is assumed to increase 2% compound interest per year (for justification see 43, 55, 56 and 57). The price per m^3 roundwood either f.o.b. shipside or f.o.b. the sawmill is $15 for the first

600,000 m^3 sold in a year and \$1 less for each additional 100,000 m^3 sold per year. This price is assumed to increase by the same amount the stumpage price is expected to increase. The price per m^3 sawnwood on a roundwood basis is \$25 for the first 100,000 m^3 sold per year and \$1 less for each additional 100,000 m^3. This price is again assumed to increase by the same amount by which the stumpage price will increase. No differences in wood quality are assumed. Hence the same prices are paid for timber from the three site qualities and for the sawn wood produced from the raw material imported from elsewhere.

Finally it is assumed that in case of shortage of raw material unlimited amounts of roundwood can be obtained at an additional current per m^3 roundwood f.o.b. sawmill cost of \$3, i.e. at a current price of \$18 per m^3 roundwood f.o.b. sawmill (see 54).

If we were to work with stages of a duration of one year, we would have 75 stages. To reduce computations five year long stages will be used, so that we have a total of only 15 stages. In terms of chapter II we have $n = 1, 2, \ldots, 14, 15\ (=N)$.

For similar reasons we will divide the total area of 10^5 ha into 100 area units of 1000 ha each, so that $j = 1, 2, \ldots, 99, 100\ (=J)$. Assume that the first 20 are the site quality I area units, the next 60 the site quality II ones and the last 20 the poorest or site quality III area units.

The symbols and terminology of the model in chapter II will now be translated in terms of the above data for a general stage n, where $n = 1, 2, \ldots, 15$.
$K = K_n = 0.3333$ or 0.06 on an annual basis.

$V_{1j} = 1000 \times 400 \times 0.4 = 16 \times 10^4\ m^3$ for $j = 1, 2, \ldots, 20$
$V_{1j} = 1000 \times 400 \times 0.3 = 12 \times 10^4\ m^3$ for $j = 21, 22, \ldots, 80$
$V_{1j} = 1000 \times 400 \times 0.2 = 8 \times 10^4\ m^3$ for $j = 81, 82, \ldots, 100$
$V_{1.} = 12 \times 10^6\ m^3$
$V_{nj} = V_{n+1,j} + Y_{nj} - G_{nj}$ (all terms in m^3 units)

$$V_{n.} = \sum_{j=1}^{100} V_{nj} \text{ (all terms in } m^3 \text{ units)}$$

In what follows all cost and revenue functions are calculated for a five year period. This explains the multiplicative factor five used in both these functions and in the determination of the demand dependent constants X_2 and X_3.
If V_{nj} equals the value indicated in the table, then G_{nj} equals the corresponding table value.

V_{nj} (in 10^3 m^3)	G_{nj} (in 10^3 m^3)	V_{nj} (in 10^3 m^3)	G_{nj} (in 10^3 m^3)
0	$9 \times X_1$	$237 \times X_1$	$51 \times X_1$
$9 \times X_1$	$9 \times X_1$	$288 \times X_1$	$48 \times X_1$
$18 \times X_1$	$15 \times X_1$	$336 \times X_1$	$30 \times X_1$
$33 \times X_1$	$18 \times X_1$	$366 \times X_1$	$18 \times X_1$
$51 \times X_1$	$21 \times X_1$	$384 \times X_1$	$12 \times X_1$
$72 \times X_1$	$24 \times X_1$	$396 \times X_1$	$6 \times X_1$
$96 \times X_1$	$27 \times X_1$	$402 \times X_1$	$3 \times X_1$
$123 \times X_1$	$30 \times X_1$	$405 \times X_1$	$3 \times X_1$
$153 \times X_1$	$39 \times X_1$	$408 \times X_1$	$1 \times X_1$
$192 \times X_1$	$45 \times X_1$	$409 \times X_1$	$1 \times X_1$

$G_{nj} = 0$ for every other V_{nj}. This assumption is made for computing purposes only; if desired, interpolations could be made in the above table.

Consequently $G_{nj} = 0$ if $V_{nj} = 160 \times 10^3$ m^3, if $V_{nj} = 120 \times 10^3$ m^3 or if $V_{nj} = 80 \times 10^3$ m^3. These cases correspond to the initial state of the virgin and stagnant forest with 400 m^3/ha, of which only a certain percentage is merchantable. Hence $G_{1j} = 0$ for $j = 1, 2, \ldots, 100$ and $G_{1.} = 0$.

Furthermore $X_1 = 4/3$ for $j = 1, 2, \ldots, 20$
$X_1 = 1$ for $j = 21, 22, \ldots, 80$
$X_1 = 2/3$ for $j = 81, 82, \ldots, 100$

$r_n^{(1)} = Y_n \times (P_{nc}^{(1)} - X_2)$, where

$P_{nj}^{(1)} = P_{nc}^{(1)} = P_n^{(1)} = P_{n-1,c}^{(1)} + 0.104 P_{n-1,c}^{(3)}$ for $j = 1, 2, \ldots, 100$
(the c subscript indicates that the price is the same for all area units).

$P_{1j}^{(1)} = P_{1c}^{(1)} = P_1^{(1)} = 25.16$ for $j = 1, 2, \ldots, 100$)the average sawn wood price during the first five years).

$P_{1j}^{(3)} = P_{1c}^{(3)} = 4.16$ for $j = 1, 2, \ldots, 100$ (the average stumpage price during the first five years).

$P_{nj}^{(3)} = P_{nc}^{(3)} = (1.104)^{n-1} \times 4.16$ for $j = 1, 2, \ldots, 100$.

$X_2 = 0$ if $0 \leq Y_n \leq 5 \times 10^5$

$X_2 = 1$ if $5 \times 10^5 < Y_n \leq 10 \times 10^5$

$X_2 = 2$ if $10 \times 10^5 < Y_n \leq 15 \times 10^5$

$X_2 = 3$ if $15 \times 10^5 < Y_n \leq 20 \times 10^5$

$X_2 = 4$ if $20 \times 10^5 < Y_n \leq 25 \times 10^5$

$r_n^{(1)} = 25 \times 10^5 \times (P_{nc}^{(1)} - 4)$ if $Y_n > 25 \times 10^5$

$r_n^{(2)} = 0$ if $Y_n \leq 25 \times 10^5$

$r_n^{(2)} = (Y_n - 25 \times 10^5)\ (P_{nc}^{(2)} - X_3)$ if $Y_n > 25 \times 10^5$

$P_{nj}^{(2)} = P_{nc}^{(2)} = P_{n-1,c}^{(2)} + 0.104\ P_{n-1,c}^{(3)}$ for $j = 1, 2, \ldots, 100$

$P_{1j}^{(2)} = P_{1c}^{(2)} = 15.16$ (the average price of logs during the first five years).

$X_3 = 0$ if $25 \times 10^5 < Y_n \leq 30 \times 10^5$

$X_3 = 1$ if $30 \times 10^5 < Y_n \leq 35 \times 10^5$

$X_3 = 2$ if $35 \times 10^5 < Y_n \leq 40 \times 10^5$

$X_3 = 3$ if $40 \times 10^5 < Y_n \leq 45 \times 10^5$

$X_3 = 4$ if $45 \times 10^5 < Y_n \leq 50 \times 10^5$

$X_3 = 5$ if $50 \times 10^5 < Y_n \leq 55 \times 10^5$

$X_3 = 6$ if $Y_n > 55 \times 10^5$

$r_n^{(3)}$ is included in $r_n^{(1)}$ because the sawn wood produced from the imported raw material is assumed to fetch the same price as the sawn wood produced from locally grown raw material (no quality differences are assumed). That is, because $P_{nc}^{(1)} = P_n^{(1)}$.

$r_n^{(4)} = 0$ because no external economies are taken into account.

$c_{nj}^{(1)} = 5 \times 0.06 \times P_{nc}^{(3)} \times (V_{nj} - Y_{nj})$ if
$V_{nj} = 160 \times 10^3$ and $j = 1, 2, \ldots, 20$
$V_{nj} = 120 \times 10^3$ and $j = 21, 22, \ldots, 80$
$V_{nj} = 80 \times 10^3$ and $j = 81, 82, \ldots, 100$

$c_{nj}^{(1)} = 0$, otherwise

$$c_n^{(1)} = \sum_{j=1}^{100} c_{nj}^{(1)}$$

$c_{nj}^{(2)} = 5 \times 0.06 \times (1.276)^{n-1} \times 1.66 \times 10^3$ for $j = 1, 2, \ldots, 20$

$c_{nj}^{(2)} = 5 \times 0.06 \times (1.276)^{n-1} \times 1.11 \times 10^3$ for $j = 21, 22, \ldots, 80$

$c_{nj}^{(2)} = 5 \times 0.06 \times (1.276)^{n-1} \times 0.55 \times 10^3 \qquad$ for $j = 81, 82, \ldots, 100$

Note that 1.66, 1.11 and 0.55 are the average per hectare land prices during the first five years (i.e. during stage 1) respectively for site quality I, II and III land.

$$c_n^{(2)} = \sum_{j=1}^{100} c_{nj}^{(2)}$$

$c_{nj}^{(3)} = 70 \times 10^3$ if $V_{n+1,j} = 0$; $c_{nj}^{(3)} = 0$ otherwise. Hence planting and tending costs are incurred only if the land has been clearcut.

$$c_n^{(3)} = \sum_{j=1}^{100} c_{nj}^{(3)}$$

$c_{nj}^{(4)} = 5 \times (5 + 1) \times 0.06 \times 10^3 + 5 \times 10^3$ (i.e. the costs associated with the investments in roads and buildings plus the other fixed yearly costs).

$$c_n^{(4)} = \sum_{j=1}^{100} c_{nj}^{(4)} = 6.8 \times 10^5$$

$$c_{nj}^{(5)} = 10\, Y_{nj} \qquad \begin{array}{l} \text{if } V_{nj} = 160 \times 10^3 \text{ and } j = 1, 2, \ldots, 20 \\ \text{if } V_{nj} = 120 \times 10^3 \text{ and } j = 21, 22, \ldots, 80 \\ \text{if } V_{nj} = 80 \times 10^3 \text{ and } j = 81, 82, \ldots, 100 \end{array}$$

$$c_{nj}^{(5)} = 8\, Y_{nj} \qquad \text{otherwise}$$

$$c_n^{(5)} = \sum_{j=1}^{100} c_{nj}^{(5)}$$

$$c_n^{(6)} = 5 \times (18 \times 10^4 + 5 \times 10^4) \qquad \text{if } n = 1, 2$$

$$c_n^{(6)} = 5 \times 5 \times 10^4 \qquad \text{if } n = 3, 4$$

$$c_n^{(6)} = 0 \qquad \text{if } n = 5, 6, \ldots, 15$$

$$c_n^{(7)} = 5 \times 2 \times 10^5 \qquad \text{if } Y_n = 0$$

Otherwise $c_n^{(7)}$ consists of property taxes + power costs + maintenance costs + wages + interest on working capital as follows.

$$c_n^{(7)} = 5 \times 2 \times 10^5 + Y_n \times 0.36 + 5 \times 38 \times 10^4 + $$
$$+ 5 \times 110 \times (1.051)^{n-1} \times 5101 + 5 \times 0.06 \times 3 \times 10^6$$
$$\text{if } n = 1, 2 \qquad \text{and} \qquad 0 < Y_n \leq 5 \times 2.5 \times 10^5$$

$$c_n^{(7)} = 5 \times 3 \times 10^5 + Y_n \times 0.36 + 5 \times 60 \times 10^4 + \\ + 5 \times 200 \times (1.051)^{n-1} \times 5101 + 5 \times 0.06 \times 5 \times 10^6$$
$$\text{if } n = 1, 2 \quad \text{and } 5 \times 2.5 \times 10^5 < Y_n \leq 5 \times 5 \times 10^5$$

$$c_n^{(7)} = 5 \times 3 \times 10^5 + 5 \times 5 \times 10^5 \times 0.36 + 5 \times 60 \times 10^4 + \\ + 5 \times 200 \times (1.051)^{n-1} \times 5101 + 5 \times 0.06 \times 5 \times 10^6$$
$$\text{if } n = 1, 2 \quad \text{and} \quad Y_n > 5 \times 5 \times 10^5$$

$$c_n^{(7)} = 5 \times 2 \times 10^5 + Y_n \times 0.36 + 5 \times 50 \times 10^4 + \\ + 5 \times 110 \times (1.051)^{n-1} \times 5101 + 5 \times 0.06 \times 3 \times 10^6$$
$$\text{if } n = 3, 4, \ldots, 15 \text{ and} \quad 0 < Y_n \leq 5 \times 2.5 \times 10^5$$

$$c_n^{(7)} = 5 \times 3 \times 10^5 + Y_n \times 0.36 + 5 \times 75 \times 10^4 + \\ + 5 \times 200 \times (1.051)^{n-1} \times 5101 + 5 \times 0.06 \times 5 \times 10^6$$
$$\text{if } n = 3, 4, \ldots, 15 \text{ and } 5 \times 2.5 \times 10^5 < Y_n \leq 5 \times 5 \times 10^5$$

$$c_n^{(7)} = 5 \times 3 \times 10^5 + 5 \times 5 \times 10^5 \times 0.36 + 5 \times 75 \times 10^4 + \\ + 5 \times 200 \times (1.051)^{n-1} \times 5101 + 5 \times 0.06 \times 5 \times 10^6$$
$$\text{if } n = 3, 4, \ldots, 15 \text{ and} \quad Y_n > 5 \times 5 \times 10^5$$

Note that 5101 is the average wage of factory personnel during the first period.

$c_n^{(8)} = (Y_n - Y_{n.})\ (P_{nc}^{(2)} + 3)$, where $P_{nc}^{(2)} + 3 = P_n^{(2)}$ in the terminology of chapter II.

$c_n^{(9)} = 0$, because no external diseconomies are taken into account.

All costs and revenue functions are expressed in dollar terms. The dollar sign has been omitted for ease of notation.

The net value growth, which is used to establish the cutting priorities of the area units, is denoted by VG.

$VG_{nj} = P_{nc}^{(3)} \times G_{nj} - 5 \times 0.06 \times P_{nc}^{(3)} \times V_{nj} - c_{nj}^{(2)}$, for $j = 1, 2, \ldots, 100$.

It was found that the net value growth was still increasing as long as:

$$V_{nj} < 15 \times 10^3 \text{ m}^3 \text{ for } j = 1, 2, \ldots, 20.$$
$$V_{nj} < 10 \times 10^3 \text{ m}^3 \text{ for } j = 21, 22, \ldots, 100.$$

Hence if these volumes were present, a large constant was added to VG_{nj} in order to increase the rank of these area units (and thus to decrease the cutting priority) assigned on the basis of their VG_{nj}.

Clearcutting is assumed. Consequently
$Y_{nj} = V_{nj} + G_{nj}$ and $V_{n+1,j} = 0$ if the area unit j is scheduled to be harvested.

$Y_{nj} = 0$ and $V_{n+1,j} = V_{nj} + G_{nj}$ if the area unit j is not to be harvested.

Finally Y_{nj} may be smaller than $V_{nj} + G_{nj}$ if the predetermined amount $Y_{n.}$ is not yet obtained with the amounts

$Y_{n1} + Y_{n2} + \ldots + Y_{n,j-1}$, but is exceeded when we add Y_{nj}. In this case $Y_{nj} = Y_{n.} - (Y_{n1} + Y_{n2} + \ldots + Y_{n,j-1})$ and $V_{n+1,j} = V_{nj} + G_{nj} - Y_{nj}$. A partially harvested area unit is immediately assigned a very small (negative) net value growth VG in order to assure the harvesting of the residual volume at the next stage n + 1.

In all the foregoing terms the simplifying, but neither necessary nor realistic assumption is made that at every stage all costs and revenues accrue in five equal yearly parts. This implies that if $Y_n = 5$ units, we process the equivalent of 1 unit durent each year of stage n. In reality we might prefer to process all in one year. Also, instead of using annual prices and wages, an average number is used for each parameter during a given stage n. These assumptions simplify the form of the cost and revenue functions and especially the discounting procedure. In fact, we can discount at each stage by multiplying each term by

$$\frac{1}{5} \times \frac{(1.06)^5 - 1}{(1.06)^5 \times 0.06} \times \frac{1}{(1.06)^{5(n-1)}}$$

The following grid on the state variable $V_{n+1,.}$ and on $(Y_{n.} - G_{n.})$ will be used. The maximum value that $V_{n+1,.}$ can attain is 410×10^5 m³. A grid of 10×10^5 m³ means that we have to consider at most 42 different values of $V_{n+1,.}$ at each stage n. As pointed out before, $Y_{n.}$ is at least zero; it is at most 41×10^6 m³. Finally $G_{n.}$ can at most be 51×10^5 m³, but it is at least zero. Thus $(Y_{n.} - G_{n.})$ is always greater than -51×10^5 m³ but smaller than 41×10^6 m³. Hence for each value of the state variable, we have to consider at most 47 different values of $(Y_{n.} - G_{n.})$ when employing a grid of 10×10^5 m³. As indicated in chapter II, a different grid can be imposed on the dY_k. Because an increase in the number of dY_k's considered does not increase the computational burden too much, a much finer grid of 2.5×10^5 m³ was imposed on these quantities to be ordered from elsewhere. There was no special reason behind the choice of these grids, except that the limited amount of available computer time acted as a constraint.

A computer program was written in the Fortran IV (version 13) language. The computer flow diagram followed quite closely the steps described in the review of the computational procedure developed for the model in chapter II. Using the data described in this chapter, it took the IBM 7090-7094 about 11 minutes to obtain and print out the solutions. If the grid employed is made finer, the required computational time will increase proportionally faster. A somewhat finer grid than the one employed in this example seems desirable. A grid, using intervals of 5×10^5 m^3 instead of 10×10^5 m^3, will certainly be sufficiently fine as it is smaller than the total growth $G_{n.} = 9 \times 10^5$ m^3 pertaining to a total volume $V_{n.} = 0$. This is desirable as it allows considering a zero cut $Y_{n.}$ in period n after a complete 100% harvest in period n−1. Naturally this is impossible with grids larger than 9×10^5. Considering the magnitude of the problem and the values involved, the time required to obtain a solution on the computer does not seem to be excessively high.

In view of the nature of the data and the objective of this example, which was purely illustrative, no sensitivity analysis on the parameters was planned. Still the following different runs were made (the run based on the data presented before is denoted by the name standard run).

1. Employing an interest rate $K_n = 8\%$, instead of the 6% used above.
2. Assuming $P_n^{(2)} = P_{nc}^{(2)} + 1$, instead of the $P_{nc}^{(2)} + 3$ employed above. That is, the price differential between locally obtained raw material and the raw material imported from elsewhere was taken to be \$1 per m^3 roundwood instead of \$3.
3. Assuming the $P_{nj}^{(2)} = P_{nj}^{(1)} = 0$ if Y_{nj} was obtained from an area unit with less than 100 m^3/ha. In effect, this prohibits cutting stands which are less than 30 years old.
4. Assuming $P_n^{(2)} = P_{nc}^{(2)}$ and an interest rate K_n of 4%, instead of respectively $P_n^{(2)} = P_{nc}^{(2)} + 3$ and $K_n = 6\%$ as used in the description before.

The Results

The following tables were part of the computer output (for the standard run) and will give an impression about the type and form of the results. As before:
n indicates the stage number
$f_n(V_{n+1,.})$ indicates the maximum stage return as a function of $V_{n+1,.}$. Hence it is the maximum discounted net revenue at the end of stage n, the determination of which is the objective of this study.
$V_{n+1,.}$ = the total volume at the end of stage n (or beginning of stage n + 1) and represents the state variable.
$Y_n = Y_{n.} + dY$ represents the decision variable.
dY represents the amount of raw material ordered from elsewhere.

$Y_{n.}$ indicates the total volume cut from the region under consideration during stage n. Of course $Y_n \geq Y_{n.}$.
$G_{n.}$ indicates the total volume growth during stage n.

TABLE 1. STANDARD RUN; OPTIMUM OPTIMORUM FOR $V_{16.} = 0$

n	$f_n(V_{n+1,.})$ ($1000)	$V_{n+1,.}$ (1000m³)	Y_n (1000m³)	$Y_{n.}$ (1000m³)	$G_{n.}$ (1000m³)
15	31325	0	1948	1948	948
14	30961	1000	1194	944	944
13	30719	1000	3360	3360	1360
12	29567	3000	2796	2796	1796
11	28387	4000	1160	660	1660
10	28191	3000	1134	134	1134
9	28386	2000	1056	56	1056
8	28733	1000	3443	3443	1443
7	25359	3000	2774	2774	1774
6	21893	4000	1173	673	1673
5	21364	3000	1100	350	1350
4	21588	2000	1006	1006	1006
3	19979	2000	3546	3546	546
2	12245	5000	3294	3294	294
1	5870	8000	4000	4000	0

TABLE 2. STANDARD RUN; THE STAGE RETURNS AT THE END OF THE 15TH STAGE.

$f_{15}(V_{16.})$ ($1000)	$V_{16.}$ (1000m³)	$f_{15}(V_{16.})$ ($1000)	$V_{16.}$ (1000m³)
31325	0	28107	9000
31113	1000	27229	10000
30773	2000	26354	11000
30467	3000	25765	12000
30229	4000	25321	13000
29948	5000	23222	14000
29710	6000	14637	15000
29386	7000	9733	16000
29071	8000	negative	17000

Perhaps it is not superfluous to indicate that a table like table 1 could have been obtained for each value of $V_{16.}$ in table 2. And similar tables were available for each different computer run, i.e. for the runs employing a different interest rate and/or price as described before. Also available from the computer output for each run but not reproduced here, were the volume distributions over the area units (i.e. $V_{n+1,j}$ for $j = 1, 2, \ldots, 100$) pertaining to each $V_{n+1,.}$, as well as the corresponding Y_{nj} for $j = 1, 2, \ldots, 100$.

Suppose for example that for some reason it had been desired to have a volume of 8×10^6 m^3 at the end of 75 years, employing the values of the standard run. From table 2 we see that the optimal discounted net return in that case is \$29,071,000. This, as could be expected, is less than the optimum optimorum of \$31,325,000 obtainable when we cut and get out at the end of 75 years, i.e. for $V_{16.} = 0$. A table similar to table 1 would be extracted from the computer output, giving $f_n(V_{n+1,.})$, $V_{n+1,.}$, $Y_{n.}$, Y_n and $G_{n.}$ for $n = 1, 2, \ldots, 15$. For each $V_{n+1,.}$ the volume distribution over the area units, i.e. $V_{n+1,j}$, would be obtained from the computer output, as well as Y_{nj}. For example for $V_{16.} = 8 \times 10^6$ m^3 the following values were found for $V_{16,j}$ (in 1000 m^3): 164 for $j = 1, 2, \ldots, 13$; 128 for $j = 14$; 12 for $j = 15$; 408 for $j = 16$; 0 for $j = 17$; 270 for $j = 18$; 512 for $j = 19, 20$; 18 for $j = 21, 22, \ldots, 52$; 9 for $j = 53$; 123 for $j = 54, 55, \ldots, 80$; 6 for $j = 81, 82, \ldots, 100$. Moreover the cut $Y_{15.} = 754 \times 10^3$ m^3 was obtained from only two area units: $Y_{15,17} = 512 \times 10^3$ m^3, $Y_{15,18} = 242 \times 10^3$ m^3, $Y_{15,j} = 0$ for $j \neq 17$ or 18. Note that area unit 17 has been clearcut while area unit 18 has been harvested only partially; $V_{15,18} + G_{15,18}$ was 512×10^3 m^3 of which 242×10^3 m^3 is cut, leaving $V_{16,18} = 270 \times 10^3$ m^3. Under our regime of management area unit 18 will be clearcut at the next stage. Because Y_n was found to be 1004×10^3 m^3, we have to import from elsewhere 1004 000 minus 754000 or 250 000 m^3. Thus the results indicate exactly when, where and how much to cut and how much to buy and import from elsewhere.

The results were definitely very reasonable. The following appeared to be the most striking features.

1. In all runs the original forest was cut down as fast as possible. This is imminently logical: only in this way can the considerable $c_n^{(1)}$ cost term be escaped. Moreover, the net volume growth of this forest is zero. While it did not pay to cut all volume in the first stage, presumably because of the assumed price-quantity relation (the demand-schedule), the cut during the first 15 years was considerably above the yearly amount of 5×10^5 m^3 which the strict sustained yield approach would indicate. And this was true even when the interest rate was assumed to be only 4%, so that the future was discounted less heavily (run 4).

2. A comparison of the Y_n and $Y_{n.}$ columns in the computer output and for the different runs indicated at which stages the alternative of ordering raw material from elsewhere has been used. As could have been expected, it has been used especially in the case when the interest was 8% (run 1), or when the price paid for the imported raw material was the same as that of the locally grown wood (run 4).

3. Like many actual companies, this forestry enterprise has its ups and downs. See for example the $f_n(V_{n+1,.})$ column in table 1. After an initial financially favorable start, it loses money during the 5th stage and again during the 8th, 9th and 10th stages. These losses are incurred during periods when the forest is being built up, as can be deduced from the $V_{n+1,.}$ column or alternatively comparing the $Y_{n.}$ and $G_{n.}$ columns. Losses were largest in the possibly most realistic case where we assigned a zero price to the timber cut from stands less than 30 years old (run 3). In this case the 6% interest rate was apparently too high to afford waiting that long for a stand to grow up after the initial investment has been made and hence to maintain high volumes per hectare. Without imposing such a lower age cutting limit but employing a discount rate of 8%, we do little less than to cut as soon as there is something to cut in order to escape the heavy interest charges on the initial investments. This was evident from the very low, mostly zero values for the $V_{n+1,.}$ term at the various stages of run 1. Employing an interest rate which many foresters would consider more reasonable, i.e. 4% ,the enterprise succeeded nicely. Profits occurred at every stage of run 4 and the total forest volume was maintained at a rather high level. Another point to be made is that a heavy cut, followed by some lean years to build up the forest again, was the optimal thing to do. This seems to be in contrast with what many foresters would advocate in a similar situation.

4. The optimum optimorum, that is the best we can possibly do, over 75 years was in all cases except one obtained for the value $V_{16.} = 0$. The one exception pertains to the case where a zero price was assigned to the timber cut from stands less than 30 years old (run 3). These results are completely in line with expectations: if we do not care what happens after 75 years, we cut all and get out. If we do care, we have to specify the desired $V_{16.}$ for which we want to obtain a set of optimal decisions, like $V_{16.} = 8 \times 10^6$ m^3 for example. This is the built-in sensitivity analysis referred to in chapter II: no additional calculations are necessary, just a back tracing for a different value of $V_{16.}$.

5. A rotation in the accepted forestry sense (the age of a stand at which net value growth drops down to zero, or at which the soil expectation is at a maximum) is not present. When a stand is being cut depends largely on the supply and demand situation of the moment, as it should be. Sometimes it pays to cut about 10^6 m^3 timber when we have only 2×10^6 m^3 of wood on all the area units together (as at stage 3 to 4 in table 1), at other times it is optimal to cut only 134×10^3 m^3 in the same situation (as at stage 9 to 10

in table 1). The volume distribution over the individual area units provide us with a more detailed explanation of what went on.

To be able to formulate general conclusions, more runs should be made assuming different industrial capacities, forest management intensities, depreciation rates, etc. A somewhat finer grid is desirable. However, as observed at the beginning of this chapter, the example was meant to be purely illustrative. Viewed in that light, the objective has been reached: the working of the model was illustrared, the computational procedure developed in chapter II was tried out on the computer and optimal decision rules were obtained. Specifically the questions: when, where and how much to cut and/or to order from elsewhere were answered.

IV. CONCLUSIONS AND SUMMARY

IN PLANNING the forestry activities of a region, the objective was assumed to be the maximization of the discounted present value of the net benefits of alternative possible investments in the forestry sector. This objective was assumed throughout the study. Instead of assuming one specific set of constraints, it was desired to develop a model that would provide optimal (in the sense of the objective defined above) decision rules for a wide variety of exogenously and endogenously imposed contraints and variables.

The study was directed mainly to a situation characterized by significant departures from the assumptions of perfect competition. This creates problems in the allocation of resources as the existing market price system cannot be expected to do an efficient job. Consequently, optimizing all parts of an economy, sector, industry or firm does not imply the optimization of the whole. For forestry purposes this means that an integrated planning approach, taking into account the tree growing/harvesting phase as well as the wood conversion or forest industry activities, to the allocation of resources in the forestry sector has to be taken. This is in contrast with the present practice of concentrating rather exclusively either on the tree growing activity, the harvesting activity or the wood conversion activity. This integrated approach would be a necessary first step toward the theoretic ideal of a general equilibrium approach to the allocation of resources.

The initial situation envisioned by this study can be any stage in the development or use of the forest resource, from the completely non-regulated all-aged or even-aged forest with any age or volume class over or underrepresented to the situation of a fully regulated all-aged or even-aged forest. Likewise the forest industries mayor may not yet exist and, if present, their combined capacity may or may not equal the traditionally advocated long run sustained yield capacity of the forest base.

In analogy with the efforts of economists to develop a programming model embracing the whole economy, a model is set up that covers the whole production process from tree seedling to final product of one or more of the primary forest industries. It consists basically of a string of revenue and cost functions which together form the objective function to be maximized. Thus it translates the assumed objective of the study.

What makes this maximization problem complicated is the time interdependent nature of the cost and revenue functions. The objective function includes all the conflicting and competing variables and objectives and most of the constraints, which all have to be balanced against each other and over time. Some variables may be high in some periods, thus forcing other variables to be low, but this might imply an undesirable disproportionate reduction or increment of these variables in subsequent periods.

The model is then conceived of as a multistage decision problem and molded into a form that can be solved by dynamic programming. Because planning horizons are long in forestry, the number of stages is large. Consequently, for computational reasons it was decided to work with only one state and one decision variable, respectively the total periodic standing volume and the total amount of wood material processed. But as indicated, other variables such as the investment rate or the amount of wood processed in the preceding stage, could be used as a second state or decision variable. All other variables are either assumed to be constant at anyone stage or to be related to the state and/or to the decision variable.

In order to properly handle the volume growth term the forward way of solving dynamic programming was chosen. This, plus the special computational procedure developed, made it possible after specifying the state and the decision variable in aggregative terms to refind the per unit area values of these two variables and thus to calculate the growth on a per unit area basis. Those variables that were assumed to be constant at anyone stage or to be related to the state and/or to the decision variable, consequently could also be differentiated according to the different area units if desired. In this way most of the traditional advantages of working on a per area unit basis were maintained while the optimization was carried out over the whole forestry production process.

The sequence in which the forest stands on the different area units should be considered for harvesting is determined by their net value growth, a marginal concept. Those stands showing the lowest (possibly negative) net value growth should be harvested first. This, of course, corresponds to what the Faustman formula tells us. But by taking into account the demands of the industrial capacity installed and by allowing for the alternative of obtaining raw material elsewhere, possibly at an additional cost, the traditional method of determining when and where to cut is shown to be generalized considerably. By the same token, the model is shown to be useful to the forester who has to manage the forests of a plant subject to its demand. The demand is allowed to be stochastic.

Throughout the discussion of chapter II and more specifically in the example of chapter III, it is shown that the model provides optimal decisions or rules for action to the forestry planner. It tells the forester when, where and how much to cut and/or to order from elsewhere. When solved for different situations, it will answer such questions as whether over-industrial capacity relative to sustained yield capacity is a desirable thing in the first stages of development. Or whether sustained yield capacity throughout the planning period is more desirable in terms of the objective. Whether the region under consideration can afford to practice intensive, extensive or no forest management and care at all. Finally, these answers can be studied in the light of any value for such variables as the the discount rate, the price-quantity schedule, landprices, wage rates, transportation constraints, number of laborers employed, etc., which the forestry planner might care to specify. External effects can be taken into account where quantifiable. The interests of the future can be considered (other than through the interest rate) and their costs evaluated, by specifying a certain value for the final state variable. As was shown, a sensitivity analysis on the final state variable is automatically implied in each solution to the model. Thus the objective of supplying optimal decisions and rules for action for the wide variety of conditions, which the factors exogenous or endogenous to the forestry sector may impose, has been reached.

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