COLLEGE OF EDUCATION | UNIVERSITY OF NEBRASKA AT OMAHA

JOURNAL OF CURRICULUM, TEACHING, LEARNING, AND LEADERSHIP IN EDUCATION

Volume 2 | Issue 2 Article 4

December 2017

Are English Teachers the First Math Teachers? A Comparison of English and Mathematics Syntax

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ARE ENGLISH TEACHERS THE FIRST MATH TEACHERS? A COMPARISON OF ENGLISH AND MATHEMATICS SYNTAX

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Abstract: *The narrative presented hereafter will demonstrate the similarities of syntactic systems in formal written English and in generalized systems of standard computational algebra. The argument is made that algebraic systems can be taught using generalizations from written English syntax. This argument will be characterized in three parts: 1) describing the development of axiomatic systems in mathematics, 2) demonstrating the similar structure of the foundational elements used in English and mathematics, and 3) illustrating how mathematical process may be characterized by providing practical examples from written English.*

Educators share a common notion that, as learners, we spend the first three to five years of our academic lives learning to read and the remaining years of our lives reading to learn. Of course the issue is much more complex but this notion suggests, by its very form, that once the basic syntax of written language is understood, useful information can be shared by translating the syntax (rules) of written systems into semantics (meaning). Moreover, the notion suggests that we can increase what we know about reading and writing by reading and writing. In essence, the reading and writing processes are self-enriching and self-actualizing if properly facilitated as children develop cognitively. Ultimately, then, gaining mastery of reading and writing over time in a formal language such as English is not simply a matter of processing syntactic rules, but of a purposeful and experimental manipulation of symbols and algorithms to create different, and often more sophisticated, ways to communicate and learn. Within this paradigm, it can be argued that the rules of written language have an underlying mathematical structure because symbol manipulation is often identified with mathematical processes. Just as the simple elements of communication can be modified to include increasingly complex ideas with more sophisticated rules, so can the processes of mathematics, and in such a way that uses the rules of communication. In fact, we propose herein that the rules of written language and the rules of computational mathematics overlap significantly at the most basic levels in both form and function.

Within much of the contemporary research that connects language and mathematics, a perfunctory assumption has been made that each subject area has its own unique way of using communication to construct knowledge (Etsy, 2014). However, it becomes apparent that this paradigm is potentially flawed (or at least incomplete) when we consider the obvious interdependence of language and mathematics. In fact, it stands that either system is incomplete without the other (Ostler, 2015). Furthermore, it is important to investigate this interdependence because there exists a unique potential to improve pedagogy in both disciplines by using content models from each discipline within the other. Yet, in an education world concerned with content integration, it is not sufficient to simply show incidental connections between subject disciplines; therefore, the integration model described herein will consider syntactic overlap at a much deeper level. Intentional and purposeful connections will be established to illustrate a practical pedagogical connectivity between the disciplines so that each becomes a legitimate support mechanism for the other. In the narrative hereafter, we will argue that generalized computational (algebraic) *syntax* can be directly taught through the structure and mechanics of written English. This argument will be characterized in three parts: 1) describing the development of axiomatic (syntax) systems in communication based on mathematics, 2) demonstrating the relationship of the functional elements used in English and mathematics, and 3) illustrating how mathematical process may be characterized by providing practical examples from written English.

Conceptual Framework

Combining mathematics and communication, or more precisely, defining communication as a substrate of mathematics is not a new idea. Research in mathematical communication and language has been popular for some time (Halliday, 1978), and has resulted in a number of avenues for understanding the underlying connections between language and mathematics; however, from a formal education standpoint, the research on communication was not widely accepted until the late 1980s. The National Council of Teachers of Mathematics (NCTM) was the first professional education organization to formally include a Standard of mathematics learning defined as "Communication" in the early Standards movement (NCTM, 1989). As a result, pedagogical exemplars of communication in mathematics education have existed for nearly three decades. Though mathematics teachers have struggled a bit since then to determine the exact nuances of what is meant by "mathematics as communication" there has been a clear purpose in mathematics classrooms to embrace many aspects of communication within traditional mathematics education.

Since the release of the Curriculum and Evaluation Standards for School Mathematics (NCTM) in 1989, research involving mathematics as communication has been focused on elements of mathematical vocabulary, defining mathematical meaning within word problems and graphical representations, and on the aspects of written language that better inform students about the deep communication structures within mathematics (Schleppegrell, 2007). These investigations have examined the uses of mathematical terminology and words, the differences in words and phrases that are unique to mathematics versus those that have unique meanings in English, key words and phrases that correspond to algorithmic thinking and logic, defining mathematical ideas within symbols, charts, and graphs; and even defining mathematical precision (Fortescue, 1994; Huang, Normandia, & Greer 2005; Lager, 2004; MacGregor, 2002; Raiker, 2002; & Veel, 1999). These are all reasonable and important ideas; however, from a pedagogical practice standpoint, we believe that one of the most important aspects of mathematical communication has been overlooked and that is the *syntactic overlap* between mathematics and standard written language. Syntax in writing refers to the way in which words are formed into sentences such that meaning is conveyed whereas syntax in mathematics refers to the way in which we arrange variables and logical operators for the same purpose. Friedrich and Friederici (2009) suggest that a similar structural hierarchy is seen in language and in simple equations or algebraic expressions. They note that what the mathematical disciplines have in common with natural language is that the formation of the logic hierarchy in mathematical expressions is not arbitrary, but obeys strict rules, which apply not only to the generation equivalent expressions but also to their interpretation, which is also rooted in *semantics*. They refer to this mathematical logic within language as "language-mathematics interface." However, despite the underlying functional similarities of written communication in English and the rules that govern how mathematics is processed, very little research has been done to investigate how the comfortable and expressive nature of symbol manipulation in written English can be leveraged to teach the presumably lesser understood mathematical processes.

We previously alluded to a popular belief about the language of mathematics, which suggested that mathematical communication structure is typically utilized only by mathematicians, scientists and engineers (Etsy, 2014). As a result, few, if any, formal attempts have been made to contextualize the *syntax* or functional structures of mathematics within the existing mathematical or syntactic structures of written English. So, while the research findings over the past forty years may have merit within traditional pedagogical models, we believe that both English language and mathematical processes operate under similar syntactic systems and are, therefore, practically interchangeable systems within a pedagogical context. It should be noted that this argument will describe the interplay of written English and mathematics beyond grammatical syntax and patterns. Instead we intend to closely examine the similarities in the functional structure of each discipline, and in a way that allows syntactic models from each discipline to be used as illustrative models for the other.

Axiomatic Systems in Mathematics

The rules of algebra and geometry can easily be conceptualized as communication protocols. In fact, many non-mathematics (and even some mathematics) teachers are surprised to learn that the act of "doing" a mathematical problem is deeply rooted in communication. A mathematical problem, as found in a common algebra textbook for example, is not *done* as much as it is continually *reorganized* or *restated* until the expression or equality is presented in its simplest form. In using the term “reorganization” we are suggesting that our final answer is a simplified or more convenient version of the original expression or equation rather than some unique “solution.” That is to say, a mathematical result is *communicated* in a simpler way but has the same basic substance or message as the expression or equation from which it came. This perception of *doing* mathematics, could essentially be called the process of mathematical *editing*. Thus, we uncover our first logical connection between mathematics and the communication structure of language. The fundamental question we need to ask, then, is not how do we *do* math; but rather, what are the *editing* rules we must follow to reorganize a mathematical expression in such a way that the final expression is a simplified or convenient version of the original? To answer this question, it is useful to explore how mathematical principals are defined and communicated specifically within the language and notation of mathematics.

Every system of mathematics was built using a set of *postulates* or *axioms*. In mathematics, a postulate or axiom is a statement that is assumed to be true without a formal proof or derivation, usually because a number of basic assumptions are necessary to define a starting point for the mathematical system. Postulates are also often self-evident and so have no need for formal proof. For example, the ancient mathematician Euclid built a self-perpetuating system of two-dimensional (often called *plane*) geometry, which is still studied and taught worldwide, based on five simple postulates. Every subsequent geometric concept and formula in two-dimensional (plane) geometry was then derived from a unique assembling of the postulates, much the same way that complex machines are all constructed from combinations of the six *Simple Machines*.

Given the axiomatic nature of a mathematical system, we can now establish the congruence between mathematical and written English syntax by defining a similar system of postulates for written English. Ostler (2015) proposes four postulates that equate mathematical and English systems.

Postulate 1: Written language uses permutations of a finite set of alphabetic symbols (letters), which exist in a hierarchical form to express meaning (words and sentences). Mathematics uses permutations of a finite set of numeric symbols (digits) that exist in a hierarchical form to express quantity (place value and exponential notation).

The arrangement of recognized symbols within each system is critical to the operation of the systems. For example, the letters in the word "cat" give us a completely different meaning when rearranged as "act" or no identifiable meaning when arranged as "tca" because a specific meaning or idea has not been universally assigned for the arrangement "tca." Likewise, the arrangement of the digits 123 give us a different value than if we rearrange them as 231, or no identifiable value if rearranged as 1^32 because exponential notation within an array of digits has not been universally defined.

Postulate 2: The symbols of written language include a specific set of delimiters (punctuation) to organize thoughts into manageable subsections and provide order and nuance to ideas. The symbols of mathematics include a specific set of delimiters (mathematical operators) to organize expressions and provide order and nuance to quantities.

Postulate two can be illustrated with some very basic punctuation rules. For example, there is an old joke about how a comma can save a life: "Let's eat Grandma." versus "Let's eat, Grandma." One small delimiter changes the meaning of an otherwise identical permutation of letters. The mathematical system is a bit more obvious. For example, it is clear that 314 has a different value, and by extension a different meaning than 3.14. Again, a small delimiter completely changes the value of an otherwise identical permutation of digits.

Postulate 3: Written language uses different symbol combinations (e.g. words or phrases) that exhibit a degree of congruence or equality to other words or phrases. These equivalent word and phrase combinations can be substituted for one another to simplify communication or clarify meaning. Mathematics uses different symbol combinations (called expressions) that may exhibit congruence or equality and can be substituted for one another to simplify communication or clarify meaning.

Because we have defined words and phrases in such a way as to have multiple meanings, and multiple words and phrases to have the same meaning, we have tremendous power in written English to substitute phrases and words in such a way that they become more descriptive or interesting. For example, we might substitute the phrase, "Those shoes stink." for the phrase, "Those shoes smell bad." We may do this to make the phrase more efficient (four words to represent the idea versus three words for the same idea) or to emphasize an aspect of our idea (the intensity of the smell of the shoes). Substitution, as it happens, is fundamental to mathematical processes as well. In mathematics, this idea is captured by a formal mathematical property called the *transitive property of equality*. This mathematical property states that if A = B and B = C, then A = C, and allows the three variables to be used interchangeably. An extremely simplified example of this property might be illustrated by substituting an improper fraction (e.g. 5/4) into an expression in place of a mixed number (e.g. 1¼). The symbol representations look different but hold the same value and can, therefore, be used interchangeably.

The ability to substitute equivalent values (expressions) within a communication system is perhaps the most critical operational structure of both of these systems. For instance, expository writing is not simply a matter of choosing appropriate words, but choosing the best words to convey the writer's meaning. A mathematician does the same thing by distilling a complex expression through a set of successively simplified equivalences. This will be demonstrated in more detail in the next section.

Postulate 4: Language and numeric systems are interdependent. We use numbers and mathematical constructs in written language and letters and words in numeric systems.

Clearly no kind of quantity or ordinal relationship can be expressed in written language without the underlying mathematical concepts supporting them. Conversely, generalizable algebraic and geometric relationships cannot not be stated without the use of letters as variables, or without the description of conditions using precise written language. It is therefore inaccurate to call mathematics a substrate of natural language given that any natural language must exist within a mathematical structure and which also requires the capacity to express mathematical constructs to be complete.

Relating Mathematical and English Syntax

Recall that when we refer to syntax in a communication system, we are talking about the structural rules of the system and not the meaning (semantics) that emerges from those rules. It is comforting to recognize that the basic syntax of both systems can be learned relatively early in our educational lives. Having said that, however, it is also important to remember that the rules exist to generate meaning in a systematic and consistent way, and that a very basic rule structure can generate semantic expressions of tremendous complexity. This statement is true for both written English and for mathematical expressions.

There are currently approximately 171,000 words in the English language not including slang, obsolete words, special contemporary jargon, contextual and dialectic additions. These words are the substance of things, actions, ideas, modifiers, connectors, qualifiers, and quantifiers. They allow for semantic expression in an infinite number of ways using only a few structural protocols. This is also true in mathematics. The syntax of mathematics may appear to be very different from written English on the surface, but under closer examination, it can be determined that using a very similar set of rules results in a very similar type of communication, the difference is really an issue of precision. After all, there are fewer than 100 symbols in all of mathematics, but there are still an infinite number of ways to express mathematical meaning using only basic syntax. This is because mathematical symbols more consistently represent a single function than English words, which often have multiple meanings.

Perhaps the most effective way to illustrate the previous point is to begin with the most predominant rules for written English (the parts of speech) and directly compare each to a similar mathematical categorization. Contemporary grammar sources typically categorize English into anywhere from eight to ten basic parts of speech. The eight most common parts of speech will be used for our purposes: 1) Nouns, 2) Verbs, 3) Adjectives, 4) Adverbs, 5) Conjunctions, 6) Prepositions, 7) Pronouns, and 8) Determiners (Articles). Presumably, knowing the parts of speech and how they must be mutually arranged illustrates a basic understanding of grammar. Likewise, algebraic expressions can be categorized into the following parts: 1) variables, 2) constants, 3) coefficients, 4) exponents, 5) operators, 6) and groupings.

In the English rules list, we have deliberately omitted Interjections because they are more closely related to semantics than syntax. Similarly, in the mathematics rules list, we have omitted Comparators (equality or inequality signs) because they are focused on restatement or substitution and do not represent an application of syntax within an expression. The following table illustrates how the basic syntactic structures relate to one another.

English Category	Mathematical Category	Purpose
Noun/Pronoun	Variable/Constant	Antecedent object or idea
Verb	Operator	Action, function, or operation
Adjective/Adverb	Coefficient/Exponent	modifier or descriptor
Conjunction	Groupings	connects/joins clauses or phrases
Preposition	Differentials (Not discussed here)	links noun to another word/phrase to show relationship
Determiner	(Not discussed here)	expresses a reference in context

Based on the table, we might decide that the two systems have similar structural components, but that does not necessarily mean that there is automatically a congruence in the other operational factors of the system. To verify some additional syntactic nuances, we will continue with a basic structural example for each system that illustrates an increasing complexity of expression within each step. This illustration is designed to begin with the simplest expression of an object and build description and detail into the expression.

English	Mathematics
The dog…	x
The large dog…	5x
The large dog and the small cat…	5x + 2y
The large dog and the small cat both slept.	S(5x + 2y)

In this example we can see that an antecedent object, the dog, can be represented in a mathematical expression as a variable. We are no doubt accustomed to assuming a numeric value when we encounter a variable in an algebraic expression, but this does not have to be the case. A variable certainly represents an unknown, but because a variable is simply a name for the unknown we wish to represent, it can take on many forms. For now, it will not matter if we choose a value or object. In the next step, we add a level of complexity to our expression by modifying it with the adjective "large."

The exact same process is used to modify our expression "x" when we place a coefficient in front of the variable. The number (5) in the algebraic expression used to represent "large" was chosen at random but could have just as easily applied another variable to the subjective value of "large" and then used a comparator (an inequality sign) to justify the relative size between the dog and the cat. Specifically, we could have stated $nx + ky$ where $n>k$, which would have still assigned x as the dog, y as the cat, and n and k as subjective (variable) sizes that satisfy the condition that n is larger than k (since we know the dog is bigger than the cat). In the third iteration, we add another level of complexity by including another variable y (the cat) and the coefficient descriptor of 2 (also meaning "small") to indicate the cat's size relative to the dog's size because 2 represents a magnitude that is less than 5. Finally, the last step of the problem includes the complex idea of grouping an action that is applied to both the dog and the cat. Recall that the large dog (5x) and the small cat (2y) both slept. This step of the algebraic representation uses a variable S to represent the object of sleep with the action of sleeping by using the *distributive property of multiplication*, which carries the S through the grouping symbols, effectively applying the object S to both 5x and 2y. We can demonstrate this mathematics language congruence by suggesting that the following sentences have the same basic meaning:

The large dog and the small cat both slept = the large dog slept and the small cat slept

$$(5x \quad + \quad 2y)\ S \quad = \quad 5xS \quad + \quad 2yS$$

Using English Syntax to Teach Mathematics

While it is not uncommon for educators to suggest "math is a language," it is less likely educators recognize language as having an operational foundation in math. Certainly there are formulaic behaviors in language, English or otherwise, that demonstrate patterned approaches and create frameworks for understanding ideas in context, just as in math. Some of these patterns and practices have already been illustrated in this exploration.

Tangential to linguistics, and beyond typical instructional practice (such as suggesting learners use words to tell math stories), however, are opportunities for expanding understanding through transferrable skills between the two pedagogies such as demonstrating sets of familiar conditioned responses and logical reasoning. By overlapping the instructional practices, learners use to put content into context across a host of experiences. Such is the case with suggesting English syntax can be leveraged to teach mathematics. Just as "rules" (agreed-upon patterns from which deviation causes misunderstanding, dissonance or disruption) apply in math, so, too, do they hold true in language. Pursuant to this point, consider the sentence, "Juan and Jorge are here." By reordering the sentence to "Jorge and Juan are here," the elements and the meaning of the communication are unchanged (at least in this context).
*English Language Example***:** Juan and Jorge are here. Jorge and Juan are here.
*Mathematical Example***:** 3 and 5 are 8. 5 and 3 are 8. a+b=b+a

Combinations of words in written language can be denoted as quantities, as well, just as in math. For example, "John and Mary are here," is understandable as representing two separate people, both of whom are in the same place. While there are many alternative approaches to expressing the group "John and Mary" (such as "he and she," "the two of them," "a man and a woman"), a simplified version of the same (using the mathematical property of addition) might be, "They are here." In other words, replacing "John and Mary" with a plural pronoun ("they") is not dissimilar from adding "3" to "5" to achieve "8," where "and" represents "=" and "they" is an outcome—a pronoun representing a new, simplified way of seeing a group.
English Language Example: John and Mary are they. They are John and Mary. They are here.
Mathematical Example: 3+5 = 8. 5+3=8. 8 is the total here.

Although math and language appear to share a symbiotic, syntactic origin, the similarities between the two systems, and the interdependence of each to the other, do not stop here. Neither do the opportunities to use English syntax to teach mathematics cease with the categories, properties and postulates outlined to this point in this article.

On the contrary, there are additional, obvious mathematical properties educators might consider introducing to primary-grade students, and could continue reinforcing throughout youths' academic careers. After all, English teachers are, unwittingly, math teachers.

Consider the very basic example of the *reflexive property* of algebra, where a=a; or, as an example, 3=3. The quantity "3," of course, is always equal to itself. This is true in English language expression, as well, and can be illustrated by examining proper nouns. In the earlier example of Juan and Jorge, Juan is always Juan and Jorge is always Jorge. It is impossible for Juan, although grouped with Jorge, to *become* Jorge (and vice versa).
English Language Example: Juan is Juan. Jorge is Jorge.
Mathematical Example: 3=3. a=a.

Additionally, Kenney (2005) suggests instructors consider the similarities between math and language by examining nouns and verbs, thinking of mathematical actions as verbs, and problem-solving as a process, a strategy she notes grew out of The Harvard Graduate School of Education's Balanced Assessment Program (Schwartz & Kenney, 1995).

Conclusion

While teachers of both math and language might acknowledge their disciplines, taken in aggregate, have structural parallels, there is a paucity of evidence suggesting researchers agree on the level of interdependence between math and language, and whether the potential interplay between the two has an impact on the relevance of this relationship relative to teacher perception, student outcomes and instruction.

Moreover, while few scholars acknowledge a systemic math/language interdependence, fewer still articulate the potential ramifications of one discipline serving as a framework for the other. For this reason, using mathematical postulates to frame the interchangeability of mathematical language with written English syntax offers new opportunities for instruction. Examining how mathematics postulates frame sentence structure, incorporate delimiters, use symbols, exercise substitution and function systematically, suggests mathematical concepts can rightly be applied to deciphering and understanding language. Moreover, it is a way to consider the question, are English teachers actually the first math teachers?

Although students of a young age might agree math exists extraneous of language (for example, while a tool of measurement has been invented to gauge the distance of the earth from the sun, this distance exists, even in the absence of a framework to explain it), strategies for examining the syntax of language—relative to math—are few. In addition, research in math and language, seldom, if ever, takes the brave step of suggesting language might be an *outcome* of math.

As such, the suggestion of conversely using English syntax to teach mathematics opens the door to a more pragmatic approach of considering math as a framework for language, and postulates as elemental to this structure. To evaluate the potential for mathematic postulates to impact math and language instruction, additional study is warranted.

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