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γ-ray Spectroscopy of ^{166}Hf: X(5) in $N > 90$?

E. A. McCutchan

N. V. Zamfir

R. F. Casten

M. A. Caprio

H. Ai

See next page for additional authors

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E. A. McCutchan,[1] N. V. Zamfir,[1,2,3] R. F. Casten,[1] M. A. Caprio,[1] H. Ai,[1] H. Amro,[1] C. W. Beausang,[1] A. A. Hecht,[1] D. A. Meyer,[1] and J. J. Ressler[1]

[1]*Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520, USA*

[2]*Clark University, Worcester, Massachusetts 01610, USA*

[3]*National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania*

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Excited states in ^{166}Hf were populated in the β^+/ϵ decay of ^{166}Ta and studied through off-beam γ-ray spectroscopy at the Yale moving tape collector. New coincidence data found no support for two previously reported excited 0^+ states and led to a substantially revised level scheme. Similarities between the revised level scheme of ^{166}Hf and the X(5) critical point symmetry are discussed, and the extent of X(5) behavior in this mass region is explored through the W and Os isotopes. Among X(5) candidates with $N > 90$, good agreement is observed for most energies and interband $B(E2)$ strengths, while all exhibit similar disagreements with other key observables, in particular, yrast $B(E2)$ values and spacing in the excited $K = 0^+$ sequence.

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I. INTRODUCTION

The X(5) critical point symmetry, introduced by Iachello [1] as an analytic solution to describe the transition from a spherical harmonic vibrator to an axially deformed rotor, has sparked considerable interest both experimentally and theoretically. Since the development of X(5), numerous experiments (see for example [2–4]), discussions [5–7], and modifications [8–10] have followed. Within the mass 150 region, experimental studies have established X(5) behavior in the $N = 90$ isotones of Sm [2], Nd [3], Gd [11], and Dy [12].

One proposed signature [13,14] of phase transitional behavior is a sharp rise in the $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ value as a function of neutron number as nuclei evolve from vibrational ($R_{4/2} = 2.0$) to rotational ($R_{4/2} = 3.33$). The X(5) solution has an $R_{4/2}$ value between that of the vibrator and rotor, with $R_{4/2} = 2.91$. The evolution of $R_{4/2}$ values in even-even nuclei as a function of N for the Nd through Os isotopic chains [15] is given in Fig. 1. The dashed lines indicate a range of $R_{4/2}$ values close to the X(5) predictions ($R_{4/2} = 2.91 \pm 0.10$). An abrupt change in $R_{4/2}$ as a function of N is clearly exhibited by the Nd, Sm, Gd, and Dy nuclei. Each of these chains intersects the X(5) region only at $N = 90$. As Z increases, the same overall trend is observed, but, the evolution becomes much less dramatic. Nevertheless, one nucleus in each of the Yb, Hf, W, and Os chains exhibits an $R_{4/2}$ value very close to the X(5) predictions, specifically, ^{162}Yb, ^{166}Hf, ^{170}W, and ^{176}Os. New measurements on ^{162}Yb, with $N = 92$, have found a structure similar to the X(5) predictions in terms of energies [16]. The extent of X(5) behavior in this mass region, whether the behavior is particular to $N \sim 90$ or also present in Hf, W, and Os nuclei with $N > 90$, requires a more detailed analysis of their structure. This is particularly true since there is a family of structures with $R_{4/2} = 2.9$ ranging from the X(5) description to γ-soft rotors. Thus, $R_{4/2}$ values of 2.9 do not necessarily involve phase transitional behavior and therefore serve only as a guide to possible X(5) candidates.

The nucleus ^{166}Hf, with $N = 94$, has an $R_{4/2}$ value similar to the X(5) model yet it differs with the X(5) predictions for some low-lying states, most importantly, the location of the excited 0_2^+ state. To better determine the structure of ^{166}Hf, and subsequently the extent of X(5) behavior in this mass region, accurate knowledge of the properties of non-yrast states in this nucleus is required. The present work makes use of high-statistics γ-ray coincidence data, leading to a substantial revision of the previous ^{166}Hf level scheme and improved measurements of intensities of low-lying transitions.

II. EXPERIMENT

Low-lying states of ^{166}Hf were populated in the β^+/ϵ decay of ^{166}Ta and studied through off-beam γ-ray spectroscopy at the Yale University moving tape collector [17,18]. The parent ^{166}Ta nuclei were produced through the ^{159}Tb(^{16}O, 9n) reaction. A 3-pnA, 155-MeV ^{16}O beam provided by the Yale ESTU tandem accelerator bombarded a 4-mg/cm^2 natural Tb target. The recoil products were collected onto a 16-mm-wide aluminized Kapton tape and transported periodically to a low-background counting area. To prevent the primary beam from reaching the tape, a 3-mm-diameter gold plug was placed 7 cm downstream of the target and 1.5 cm in front of the tape. While most of the unreacted primary beam particles were stopped by the plug, most fusion evaporation products, which recoil downstream in a larger recoil cone of angles, bypassed the plug and reached the tape with $\sim$75% acceptance. The decay of ^{166}Ta (with the production calculated by the code PACE [19] as 22% of the 1.5 b total reaction cross section) was optimized by advancing the tape at 1-min intervals [given the 32-s half-life of the $(2)^-$ ground state of ^{166}Ta].

The γ rays were detected by an array of three Compton-suppressed segmented YRAST ball clover high-purity Ge detectors [20] and one low-energy photon spectrometer (LEPS) in close, coplanar geometry, with an array photopeak efficiency of 1.0% at 1.3 MeV. Both γ-ray singles and γ-γ coincidence data were simultaneously acquired in event mode with both a Ge single and doubles trigger. In the 105-h experiment, 1.3×10^8 γ-γ coincidence events were acquired. Although the

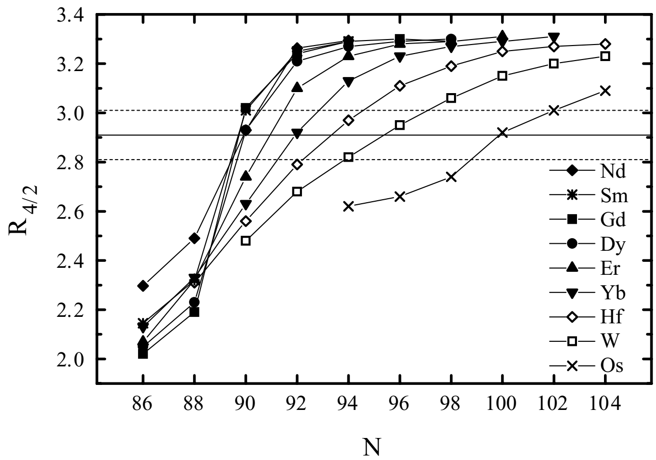

FIG. 1. Empirical $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$ ratios for even-even nuclei for the Nd to Os nuclei with $N \leqslant 104$ [15]. The solid line represents the X(5) value of $R_{4/2} = 2.91$; the dashed line indicates the region of possible X(5) candidates, $R_{4/2} = 2.91 \pm 0.10$.

array was shielded with a ~15-cm thickness of boron-loaded paraffin to thermalize and absorb reaction neutrons, the $9n$ reaction channel yielded considerable background neutron flux. Neutron background dominates the combined clover singles spectrum as shown in Fig. 2 (top). Despite this, the γ-γ spectrum is extremely clean as shown in Fig. 2 (bottom) where a gate is placed on the 159-keV, $2_1^+ \to 0_1^+$ transition. The energy range of the spectrum was limited to $E_\gamma^{\max} \sim$ 2.4 MeV.

Table I summarizes the γ rays assigned to ^{166}Hf based on γ-γ coincidences from the present experiment, including their placements, intensities, and the most useful coincidence relations. Table II lists the levels populated in ^{166}Hf and their γ decay. The level scheme deduced in the present work is given in Fig. 3.

A. Previously reported levels from β decay for which no evidence is found

Several levels assigned to ^{166}Hf [21] were identified in a single β-decay study [22], based on singles γ-ray data and limited coincidence data. The reaction mechanism chosen in this and the previous work produced contamination not only from the large flux of evaporated neutrons but also from the numerous additional reaction channels. The complicated nature of the resulting singles spectrum made the accurate identification of levels and γ rays belonging to ^{166}Hf quite challenging. Several of the previously reported low-lying levels in ^{166}Hf are found to be unsubstantiated by the present, high-statistics γ-ray coincidence data. Evidence used in dismissing these levels is summarized below. The previous level scheme is given in Fig. 4 for reference. Intensities given in the following discussion are normalized to the intensity of the 159-keV, $2_1^+ \to 0_1^+$ γ-ray transition ($I_{159} \equiv 100$). Spin and parity assignments are taken from the literature except where noted.

(0_2^+) *at 695 keV*: The evidence [22] for the proposed 0^+ level at 695 keV is based on one depopulating transition of 536.0(4) keV with intensity 4.5(10) to the 2_1^+ level. The previous study [22] finds no γ rays populating the 695-keV level and indicates that it is one of only two levels not directly fed in the decay of ^{166}Ta. In addition, the 536-keV transition is doubly placed, also as a transition from the 3_1^+ level to the 4_1^+ level, with intensity 3.0(9). The two previous placements are illustrated in Fig. 5, labeled A and B for reference. From spectra gated on the 312-keV transition, the intensity of transition B is found to be 2.0(2) with an energy of 536.81(7) keV. From spectra gated on the 159-keV transition, which should yield the sum of intensities from transitions A and B, a 536.9(2)-keV transition of intensity 1.9(2) is observed. The intensity of transition B thus accounts for the entire combined measured intensity, leaving no residual intensity for transition A. The measured intensity of 2.0(2) in the present study does not account for the total reported intensity of 7.5(10) [21]. A significant amount (~50%) of the singles

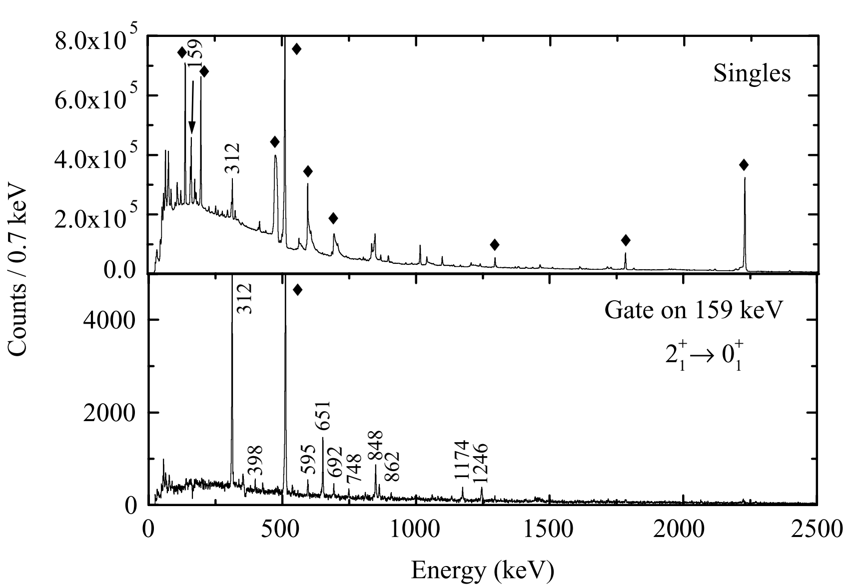

FIG. 2. (top) Clover singles spectrum. The most intense γ transitions assigned to ^{166}Hf are marked with their energies. Contaminant and background lines are indicated by a diamond. (bottom) Coincidence spectrum gated on the 159-keV, $2_1^+ \to 0_1^+$ transition.

TABLE I. Observed γ-ray transitions in ^{166}Hf, arranged in order of increasing transition energy. Intensities (normalized to $I_{159} \equiv 100$) and the most useful coincidence relations[a] are given.

E_γ (keV)	E_i (keV)	E_f (keV)	I_γ	Coincidences[a]
158.64(4)	158.64	0.00	100(3)	312, 398, 537, 651, 748, 848, 1174
311.87(5)	470.51	158.64	44.7(9)	159, 537, 692, 748, 862, 1081
397.6(1)[b]	1404.6	1007.1	2.9(3)	159, 848
426.7(1)[c]	897.2	470.51	1.21(17)	159, 312
536.81(7)	1007.1	470.51	2.0(2)	159, 312
544.27(10)[b]	1551.4	1007.1	0.94(18)	159, 312, 848
594.65(10)[d]	1404.6	809.9	5.7(9)	159, 651, 810
651.26(5)	809.9	158.64	18.9(4)	159, 595
692.23(6)	1162.74	470.51	4.4(3)	159, 312
748.25(7)[d]	1218.79	470.51	3.0(2)	159,312
810.0(3)	809.9	0.00	20.2(18)	595
848.41(6)	1007.1	158.64	12.7(9)	159, 398, 544
861.97(7)	1332.42	470.51	5.4(3)	159, 312
906.35(9)	1065.0	158.64	2.1(3)	159
1060.2(1)[b]	1218.79	158.64	2.3(3)	159
1080.86(12)[c]	1551.4	470.51	1.6(2)	159, 312
1132.75(11)[b]	1603.2	470.51	2.5(3)	159, 312
1173.74(7)	1332.42	158.64	5.9(4)	159
1218.8(3)[b]	1218.79	0.00	1.0(4)	
1246.37(7)[b]	1404.6	158.64	5.4(3)	159
1444.4(2)[b]	1603.2	158.64	2.3(1)	159

[a]Only those coincident transitions most relevant to the placement of the tabulated transition or to the measurement of its intensity are listed. For low-lying transitions coincident with a large number of feeding transitions, the weaker feeding transitions are omitted.
[b]γ-ray line was not previously reported [21].
[c]Transition was not previously reported in β decay [21].
[d]γ-ray line was not reported in this placement [21].

intensity measured in the present work at this energy comes from a 537.64(4)-keV contaminant transition in ^{166}Yb [21], identified by its coincidences with the 228-, 368-, and 998-keV transitions in that nucleus. Since the location of the first excited 0^+ state is central to the structural interpretation of ^{166}Hf, a further inspection of the existence of transition A can be performed by examining, in a gate on the 537-keV transition, the ratios of intensities of the 159-keV (I_{159}) and 312-keV (I_{312}) transitions. From the intensities given in the literature [21], the ratio I_{159}/I_{312} in a gate on the 537-keV transition should yield 2.5(10). From spectra gated on the 537-keV transition, the ratio $I_{159}/I_{312} = 1.0(1)$ is obtained. This provides further evidence that the 537-keV γ ray decays only through transition B, depopulating the 3_1^+ level at 1007 keV. Therefore, having found no support for the reported [21] transition depopulating the 0_2^+ level at 695 keV and with no observation of any direct or indirect population of the level, there is no evidence for the existence of a 0^+ level at 695 keV.

(2^+) *at 852 keV*: The proposed level [22] at 852 keV is based on two depopulation transitions, a 693.2(5)-keV transition to the 2_1^+ level and a 851.7(6)-keV transition to the 0^+ ground state. The prior study gives the 693.2(5)-keV transition a double placement, as a transition from the 852-keV level to the 2_1^+ level with intensity 2.0(8) and as a transition from a level at 1164-keV to the 4_1^+ level with intensity 1.2(5). From the reported intensities, the ratio I_{159}/I_{312}, in a gate on the 693-keV transition, should yield a value of 2.7(15). From spectra gated on the 693-keV transition, the ratio $I_{159}/I_{312} = 1.0(1)$ is obtained. This confirms the latter placement, with a 692.23(6)-keV transition depopulating the 1164-keV level with intensity 4.4(3). No evidence was found in singles for the reported 852-keV transition. A limit on the intensity of a 852-keV transition of <0.3 was obtained compared with the reported intensity of 3.4(14).

(0_3^+) *at 909 keV*: The evidence for the proposed 0^+ level at 909 keV is based on one depopulating transition of 750.0(5)-keV to the 2_1^+ level with intensity 10.4(18). As shown in Fig. 6(a), the present coincidence data show a 748.25(7)-keV transition in coincidence with the 159-keV, $2_1^+ \to 0_1^+$ transition, with intensity 3.0(2), which is close in energy to the adopted energy [21] of 750.0(5) keV. However, the present data demonstrate the 748-keV transition to be in coincidence with both the 159-keV, $2_1^+ \to 0_1^+$ transition and the 312-keV, $4_1^+ \to 2_1^+$ transition, as shown in Fig. 6(b). Intensity measurements show that the 748-keV transition populates only the 4_1^+ state. Since the sum energy is in good agreement with a newly observed level at 1219 keV (supported by additional coincidence relations, see Table I), the 748-keV line is placed as a transition from the 1219-keV level to the 4_1^+ state with intensity 3.0(2). The discrepancy between the

TABLE II. Levels populated in ^{166}Hf and their γ decay. Relative (in β decay) intensities are given for γ-ray transitions depopulating the levels and compared with literature values [21] where available. Intensity limits are given for spin-allowed but unobserved transitions between low-lying levels relevant to the structural interpretations of the nucleus. For these limits, the approximate transition energy expected from the level energy difference is shown in brackets.

$J_i^{\pi\,a}$	E_i (keV)	J_f^π	E_f (keV)	E_γ (keV)	I_γ	I_{lit}^b
2^+	158.64(5)	0^+	0.00	158.64(4)	100(3)	100.0(35)
4^+	470.51(6)	2^+	158.64	311.87(5)	44.7(9)	53.6(21)
2^+	809.9(1)	0^+	0.00	810.0(3)	20.2(18)	18.6(11)
		2^+	158.64	651.26(5)	18.9(4)	16.1(11)
		4^+	470.51	[340]	<0.27	
6^+	897.2(2)^c	4^+	470.51	426.7(1)^d	1.21(17)	
3^+	1007.1(1)	2^+	158.64	848.41(6)	12.7(9)	13.6(27)
		4^+	470.51	536.81(7)	2.0(2)	3.0(9)
		2^+	809.9	[197]	<0.40	
$(0)^{+e}$	1065.0(1)	2^+	158.64	906.35(9)	2.1(3)	11.5(15)
		2^+	809.9	[255]	<0.19	
(?)	1162.74(8)	4^+	470.51	692.23(6)	4.4(3)	1.2(5)
		2^+	158.64	[1004]	<0.15	
		2^+	809.9	[353]	<0.51	
2^{+e}	1218.79(8)^f	0^+	0.00	1218.8(3)^g	1.0(4)	
		2^+	158.64	1060.2(1)^g	2.3(3)	
		4^+	470.51	748.25(7)^h	3.0(2)	10.4(18)
		2^+	809.9	[409]	<0.69	
		3^+	1007.1	[212]	<0.13	
(?)	1332.42(8)	2^+	158.64	1173.74(7)	5.9(4)	9.7(4.7)
		4^+	470.51	861.97(7)	5.4(3)	7.1(20)
		2^+	809.9	[523]	<0.65	
		3^+	1007.1	[325]	<0.33	
(?)	1404.6(1)^f	2^+	158.64	1246.37(7)^g	5.4(3)	
		2^+	809.9	594.65(10)^h	5.7(9)	6.7(9)
		3^+	1007.1	397.6(1)^g	2.9(3)	
		4^+	470.51	[935]	<0.17	
$(5^+)^e$	1551.4(2)^c	4^+	470.51	1080.86(12)^d	1.6(2)	
		3^+	1007.1	544.27(10)^g	0.94(18)	
(?)	1603.2(2)^f	2^+	158.64	1444.4(2)^g	2.3(1)	
		4^+	470.51	1133.75(11)^g	2.5(3)	
		2^+	809.9	[793]	<0.33	

[a]Level spin assignments are nominal assignments from evaluation [21] except as noted.
[b]Literature values for relative intensities are from the evaluated ^{166}Ta β^+/ϵ decay data of [21].
[c]Level was not previously reported in β decay [21].
[d]γ-ray line was not previously reported in β decay [21].
[e]Probable spin assignment is given on the basis of observed transitions to levels of known spin.
[f]Level was not previously reported [21].
[g]γ-ray line was not previously reported [21].
[h]γ-ray line was not reported in this placement [21].

measured intensity in the present work and the total reported intensity [21] is most likely due to the large contribution to the singles intensity from a 747.82(4)-keV contaminant transition in ^{164}Yb [23], identified by its coincidences with the 123-, 741-, and 864-keV transitions in that nucleus. Therefore, with no observation of any direct population of the level at 909 keV and no support for the reported [21] transition depopulating the level, there is no evidence for the existence of a 0^+ level at 909 keV.

(3,4) *at 1023 keV*: This level was proposed [22] in β^+/ϵ decay to account for two depopulating transitions, of 552.4(4) and 864.1(5) keV, placed as branches to the 4_1^+ and 2_1^+ states,

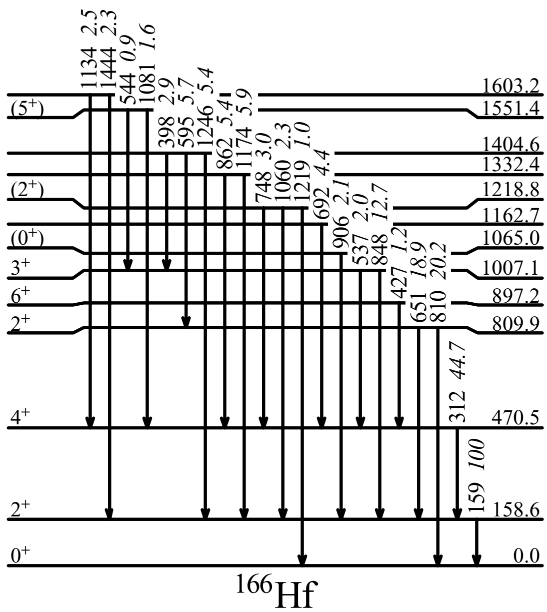

FIG. 3. Complete level scheme of ^{166}Hf populated in ^{166}Ta β^+/ϵ decay. Levels are marked with their energy in keV. Transitions are labeled by their energy in keV and relative intensities (Table I).

respectively. Neither γ ray was found to be coincident with any known transitions in ^{166}Hf. Coincidence data established both lines as corresponding to the decay of ^{164}Lu to ^{164}Yb, produced in a competing reaction channel. The intensities observed in the present experiment originate entirely from 552.01(3)- and 863.89(3)-keV contaminant transitions in ^{164}Yb [23].

(2^+) *at 1213 keV*: This level was identified [22] in β^+/ϵ decay on the basis of two depopulating transitions, of 742.8(4) and 1054.4(10) keV, placed as branches to the 4_1^+ state and 2_1^+ state, respectively. Neither γ ray was found to be coincident with any known transitions in ^{166}Hf. Coincidences observed between the 743-keV line and a 123-keV transition identify the 743-keV transition as the $2_2^+ \to 2_1^+$ transition in ^{164}Yb [23]. The observed 1054-keV transition is associated with a 1054.7(6)-keV contaminant transition in ^{166}Yb [21], identified by its coincidences with the 102- and 228-keV transitions in that nucleus.

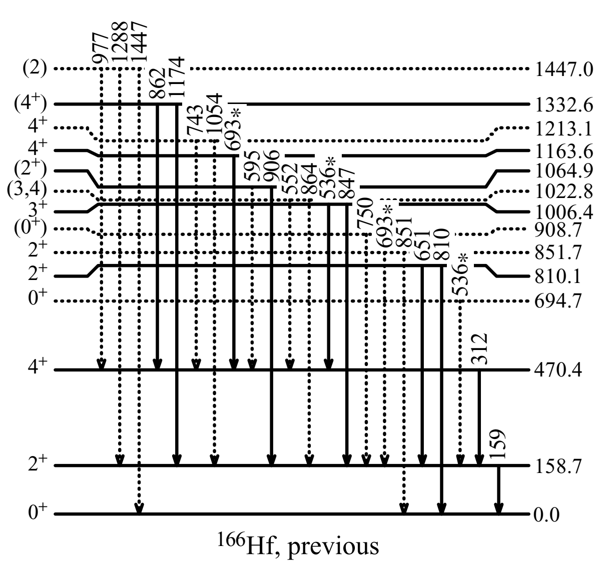

FIG. 4. Previous β^+/ϵ decay level scheme of ^{166}Hf. Levels and transitions for which the present study finds no evidence are indicated by a dashed line. Double placements are indicated by a $*$.

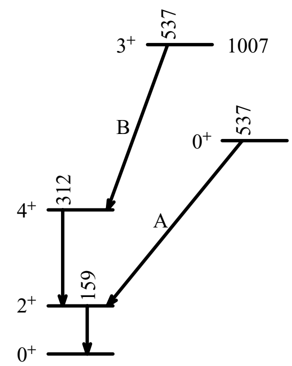

FIG. 5. Partial level scheme of ^{166}Hf illustrating previous double placement of the 537-keV transition.

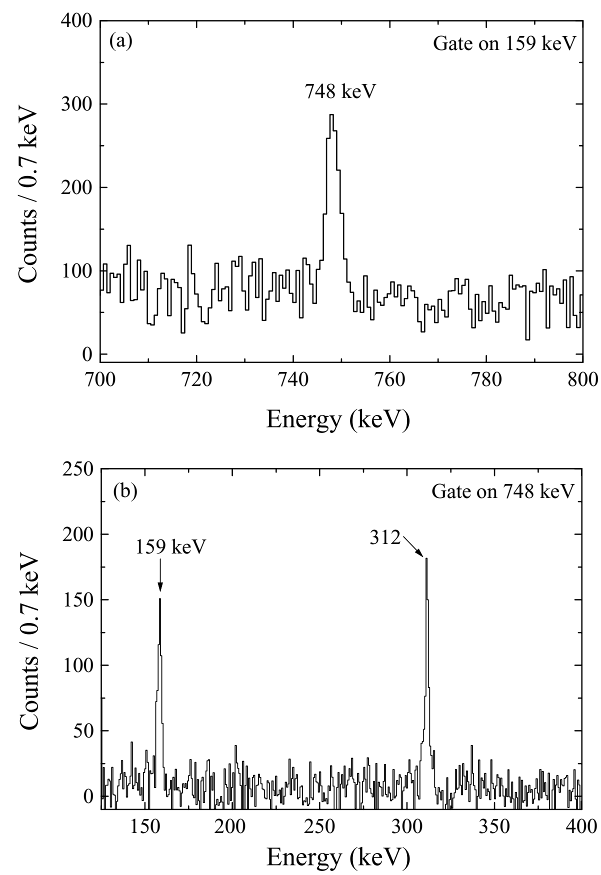

FIG. 6. Gated coincidence spectra giving evidence for the new placement of the 748-keV transition. Spectra gated on the (a) 159-keV, $2_1^+ \to 0_1^+$ transition focusing on the energy range 700–800 keV and the (b) 748-keV γ ray.

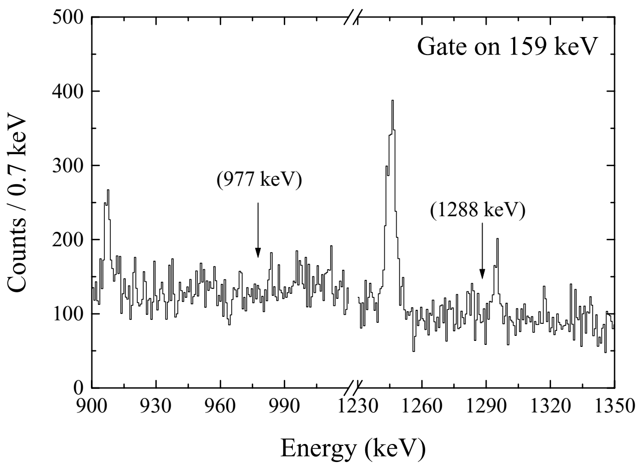

FIG. 7. Spectrum gated on the 159-keV, $2_1^+ \rightarrow 0_1^+$ transition, highlighting the energy range 900–1350 keV and demonstrating the unobserved coincidences with the 977- and 1288-keV transitions.

(2) *at 1447 keV*: The evidence for the proposed level at 1447 keV is based on three depopulating transitions, a 977.0(8)-keV line with intensity 4.7(11), a 1288.3(12)-keV line with intensity 5.8(21), and a 1447.0(20)-keV line with intensity 6.3(16), placed as branches to the 4_1^+, 2_1^+, and 0_1^+ level, respectively. From spectra gated on the 159-keV, $2_1^+ \rightarrow 0_1^+$ transition, no coincidences are observed with either 977- or 1288-keV γ rays, as shown in Fig. 7. (From the present data, the coincident intensity of the 159-keV transition with a possible 977-keV transition is found to be <0.15, and that with a possible 1288-keV transition is <0.12.) No evidence was found in singles for a 1447-keV transition.

B. Transitions depopulating some low-lying levels

As indicated in Table II, several revisions have also been made to other reported levels [21]. Here we discuss the experimental results for levels that are central to the low-lying structure of ^{166}Hf.

Level at 1065 keV: The state at 1065 keV was previously assigned [21] to decay to the 4_1^+ and 2_1^+ levels through transitions of 595 and 906 keV, respectively, and assigned a J^π of 2^+ [22]. Here, a 906.35(9)-keV transition is observed with an intensity 2.1(3), which is significantly reduced compared with the literature value [21] of 11.5(15). The present data show the 595-keV transition to be noncoincident with the 312-keV, $4_1^+ \rightarrow 2_1^+$ transition, as illustrated in Fig. 8(a). Rather, the 595-keV γ ray is in coincidence with both the 810-keV, $2_2^+ \rightarrow 0_1^+$ transition, and the 651, $2_2^+ \rightarrow 2_1^+$ transition, as shown in Fig. 8(b). The 595-keV line, with intensity 5.7(9), is now placed as a transition populating the 2_2^+ state at 810 keV.

The level at 1065 keV is therefore supported now by a single depopulating transition of 906 keV to the 2^+ ground state, making it a candidate for an excited 0^+ state in ^{166}Hf. Angular correlation measurements were performed for detectors separated by 75° and 15°. The ratio of coincidences of the 159- and 906-keV γ rays was measured as $W(75°)/W(15°) = 0.50(10)$. This is consistent with the theoretical, unattenuated value of 0.49 for a spin $0^+ \rightarrow 2^+ \rightarrow 0^+$ cascade. In comparision, the ratio of coincidences for a known $2^+ \rightarrow 2^+ \rightarrow 0^+$ cascade between the 651- and 159-keV transitions gives a value of $W(75°)/W(15°) = 0.99(12)$. This suggests a tentative spin assignment of 0^+ for the level at 1065 keV.

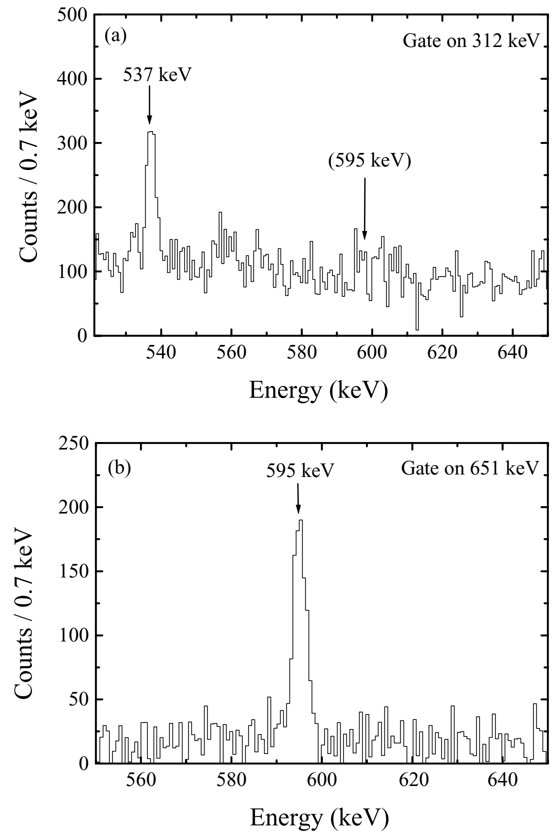

FIG. 8. Gated coincidence spectra giving evidence for the new placement of the 595-keV transition. Spectra gated on the (a) 312-keV, $4_1^+ \rightarrow 2_1^+$ transition and the (b) 651-keV, $2_2^+ \rightarrow 2_1^+$ transition.

Level at 1219 keV: A level at 1218.8(3) keV is identified on the basis of newly observed transitions to the 4_1^+, 2_1^+, and 0_1^+ states. These transitions suggest a spin assignment of 2^+, although a spin of 3^- is possible if $E3$ multipolarity is considered.

Level at 1552 keV: The level at 1552 keV and the transition of 1082 keV were not previously observed in a β-decay experiment but were identified in a heavy-ion-induced in-beam study of this nucleus [24]. The 1552-keV level was given a tentative spin assignment of 5^- although spins of 6^+ and 5^+ could not be excluded from the previously observed transitions [24]. In the present experiment, the 1082-keV transition to the 4_1^+ level is observed along with an additional depopulating transition, a 544.27(10)-keV line to the 3_1^+ state. This new transition is inconsistent with the previous 5^- spin assignment and excludes a spin of 6^+, suggesting a spin assignment of 5^+ is more probable.

III. DISCUSSION

The X(5) model [1] provides a simple geometric description of nuclei at the critical point of a first-order phase transition

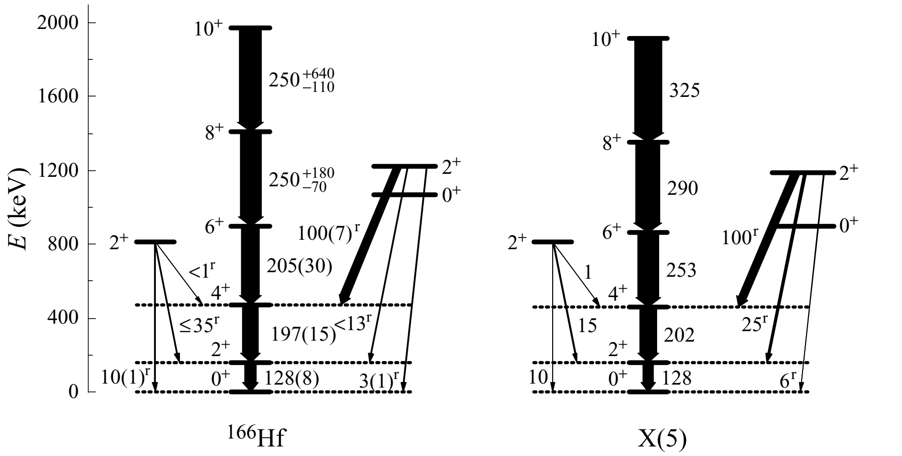

FIG. 9. Comparison of energy levels and $B(E2)$ transitions in ^{166}Hf (left) and X(5) (right). Transitions are labeled by the corresponding absolute or relative $B(E2)$ value. Theoretical energies are normalized to the 2_1^+ energy of ^{166}Hf, and absolute $B(E2)$ values in the ground state band are normalized to the $2_1^+ \rightarrow 0_1^+$ transition in ^{166}Hf. Transitions from the $K = 0_2^+$ band and quasi-γ band are relative $B(E2)$ strengths (indicated by "r").

from a spherical harmonic vibrator to an axially symmetric rotor. The X(5) solution is obtained when the potential of the Bohr Hamiltonian is taken as an infinite square well in the β deformation. All resulting predictions for energies and $E2$ transition strengths for the $K^\pi = 0^+$ bands are parameter free except for scale. In this section, the newly revised level scheme of ^{166}Hf is compared with the X(5) predictions as well as with structurally similar nuclei in the W and Os isotopes.

As previously mentioned, the energy spacing of the yrast band levels in X(5) are intermediate between a vibrator and a rotor with $R_{4/2} = 2.91$. The energy spacing of the yrast band levels in ^{166}Hf closely matches the X(5) predictions as shown in Fig. 9. An accurate comparison with the intraband $B(E2)$ strengths turns out to be more difficult due to lack of precise knowledge of the absolute values. The data [25] generally indicate that the spin-dependence of the yrast band $B(E2)$ values is consistent with that of an ideal rotor; however, the large uncertainties in the higher spin transitions make a meaningful distinction between the rotor and X(5) difficult.

Another essential signature of X(5) is the prediction of the position of the first excited collective 0^+ state at 5.67 times that of the energy of the 2_1^+ state. The present study finds no evidence for the previously reported low-lying 0^+ state at 695 keV or for another 0^+ state at 909 keV. The tentatively assigned new 0^+ state at 1065 keV, based on decay properties and angular correlation measurements, is located slightly higher than the X(5) predictions, with $E(0_2^+)/E(2_1^+) = 6.7$, as shown in Fig. 9.

The most likely candidate for the 2^+ member of the 0_2^+-band sequence is the level at 1219 keV. Assuming this is the correct assignment, the experimental energy spacing between the 1219-keV level and the 0_2^+ state, 154 keV, is considerably compressed compared with the X(5) prediction of 285 keV, as shown in Fig. 9. This feature is also observed in the $N = 90$ isotones (e.g., ^{152}Sm [2],^{150}Nd [3]). The branching properties of the (2^+) state at 1219 keV are compared with the X(5) predictions in Fig. 9, illustrating reasonable overall agreement. The only discrepancy is in the decay to the 2_1^+ state, which is larger by a factor of 2 in the X(5) predictions. The other possible spin assignment for the level at 1219 keV is a 3^- state. From the observed depopulating intensities, however, this assignment would result in an anomalously large $B(E3)$ value for the ground-state transition, making this assignment unlikely.

The other possible candidate levels for the 2^+ member of the 0_2^+ band, from energy and spin considerations, are the 1163- and 1332-keV levels, both with unknown spins. The 1163-keV level is a questionable candidate since a decay is seen only to the 4_1^+ state. The energy of this level and its observed decays suggest a better assignment would be the 4^+ member of the quasi-γ band. The 1332-keV level is observed to decay to the 4_1^+ and 2_1^+ states. If this level is a member of the 0_2^+ sequence, the experimental energy spacing to the 0_2^+ state, 267 keV, is quite close to the X(5) predictions. An alternative assignment for the 1332-keV level could be as a 3^- state. The energy as well as the observed decays are consistent with negative parity excitations in this mass region [15].

The previous study [22] identified the 2^+ and 3^+ states of the quasi-γ band at 810 and 1007 keV, respectively. The transitions from these states and their intensities were confirmed in the present work. The energy of the quasi-γ bandhead is not fixed in the X(5) predictions [26]; however, there are parameter-free predictions for its decay. These are also compared with the branching properties of ^{166}Hf in Fig. 9, again showing consistency with X(5). An alternative fit to ^{166}Hf described in Ref. [27] used an interacting boson model (IBA) Hamiltonian with a somewhat more γ-soft potential and obtained a similar level of agreement.

Extending the X(5) comparison to the W and Os region has only recently become possible thanks to relatively new experiments on the light W and Os nuclei by Kibédi *et al.* [28,29] providing extensive data on previously unobserved $K = 0_2^+$ and quasi-γ bands. The two nuclei closest to the X(5) predictions in terms of the $R_{4/2}$ ratio (see Fig. 1) are ^{170}W and ^{176}Os. The exact location of the 0_2^+ state in ^{170}W has not been observed experimentally;however, higher spin members of the first excited $K = 0^+$ band have been identified on the basis of strong $E0$ transitions to the ground state band. Estimating the location of the first excited 0^+ state from the higher spin members of the band with a variety of energy formulas—ranging from the anharmonic vibrator (AHV) formula, which describes a wide range of nuclei from spherical to deformed [30], to the soft rotor formula [31], an expansion in $J(J + 1)$, or the Harris formula [32]—, consistently gives the first excited

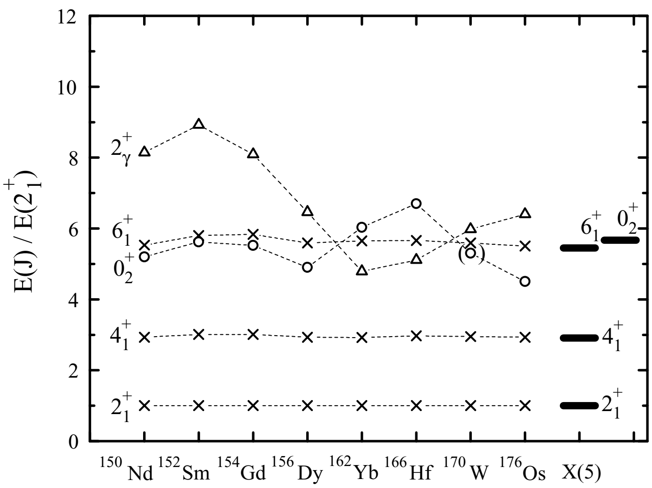

FIG. 10. Normalized (to the 2_1^+ energy of each nucleus) energy of low-lying states, 2_1^+, 4_1^+, 6_1^+, 0_2^+, and 2_γ^+, in several X(5) candidate nuclei compared with the X(5) predictions. The excited 0^+ state in ^{170}W is approximated from higher spin members of the band (see text) and given in parenthesis.

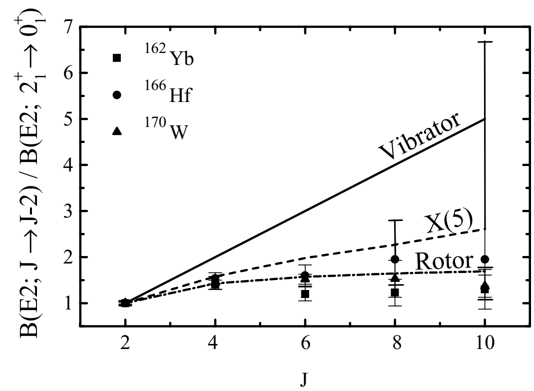

FIG. 11. Yrast band $B(E2)$ values [normalized to $B(E2;\, 2_1^+ \to 0_1^+)$] for ^{162}Yb, ^{166}Hf, and ^{170}W. Rotor, vibrator, and X(5) predictions are shown for comparison.

0^+ state, within 20 keV, at an average of 840 keV. This gives a ratio $E(0_2^+)/E(2_1^+)$ around 5.3. For ^{176}Os, the ratio is calculated to be $E(0_2^+)/E(2_1^+) = 4.5$. A comparison of the low-lying states of candidates for X(5) nuclei in the rare-earth region is given in Fig. 10. Evident from Fig. 10 is that regardless of the N or Z value, these nuclei display a close similarity to the X(5) predictions in terms of $R_{4/2}$ and $E(0_2^+)/E(2_1^+)$ ratios. This is in contrast to the location of the quasi-γ bandhead, which varies widely throughout these nuclei. This is perhaps expected since the energy of the quasi-γ band is not fixed in the X(5) model [26].

Looking at only the energy ratios $R_{4/2}$ and $E(0_2^+)/E(2_1^+)$ across the entire rare-earth region, there are no significant differences between the $N=90$ isotones of Nd, Sm, Gd, and Dy and the candidates in the Yb, Hf, W, and Os isotopes with $N > 90$. A more detailed comparison can be performed by looking at both intraband and interband $B(E2)$ strengths. As mentioned previously, the yrast $B(E2)$ values of ^{166}Hf are not in obvious agreement with the X(5) predictions. This same discrepancy is found in all the $N \geqslant 92$, X(5) candidates with known yrast $B(E2)$ values as shown in Fig. 11 for ^{162}Yb [33], ^{166}Hf [21], and ^{170}W [34]. While the yrast energies for each of these nuclei agree very closely with the X(5) predictions, the yrast $B(E2)$ values evolve much closer to the rotor predictions. Perhaps a remeasurement of these yrast $B(E2)$ values, making use of the gated coincident method [35], could yield more reliable values and help to clarify this picture. If, on the other hand, this discrepancy remains, new models are now being developed [8,36] which can account for the $R_{4/2}$ values intermediate between the vibrator and rotor limits and the yrast $B(E2)$ values which vary with spin similar to the rotor.

The known relative $B(E2)$ values from the $K = 0_2^+$ sequence of ^{170}W and ^{176}Os are given in Fig. 12 along with the respective predictions for the X(5) model. The relative branching ratios from both the 2^+ and 4^+ states exhibit excellent agreement with the X(5) predictions. This is in contrast to the $N=90$ isotones of Sm, Gd, and Dy where the strength of the interband transitions is much weaker relative to the intraband transitions when compared to the X(5) model [12,37,38]. While these heavier nuclei exhibit better agreement with

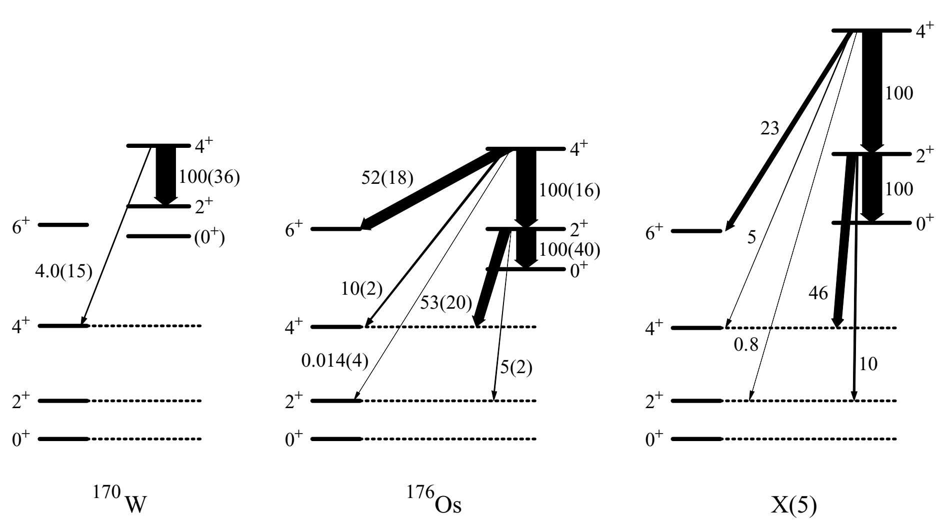

FIG. 12. Comparison of energy levels and relative $B(E2)$ transitions in ^{170}W, ^{176}Os, and X(5). Theoretical energies are normalized to the 2_1^+ energy of each nucleus.

the X(5) predictions in terms of interband $B(E2)$ strengths, Fig. 12 also highlights a significant discrepancy. The X(5) model predicts a somewhat deformed ground state structure ($R_{4/2} = 2.90$) and a less deformed excited $K = 0^+$ sequence ($R_{4/2} = 2.70$). These coexisting structures have been observed in the Nd, Sm, Gd, and Dy isotopes with $N = 90$. Figure 12 illustrates that the picture is reversed for ^{170}W and ^{176}Os, with the excited $K = 0_2^+$ sequence just slightly more deformed ($R_{4/2} = 3.0$) than the ground state band in both nuclei. In addition to the discrepancy in the energy ratios in the excited 0^+ sequence, the absolute energy spacing is significantly more compressed in the $N > 90$ nuclei than the X(5) predictions. This feature is also observed in the $N = 90$ isotones. Once again, newly developed perturbation schemes [8–10] to the idealized X(5) model may be able to account for this disagreement and provide more insight into the structure of these nuclei.

IV. CONCLUSION

Off-beam, γ-ray spectroscopy was performed on the nucleus ^{166}Hf populated in β^+/ϵ decay. New coincidence data provided evidence for the dismissal of two previously proposed excited 0^+ states and a substantial revision to the previous level scheme. A comparison of ^{166}Hf along with other nuclei that are candidates for X(5), with $N > 90$, finds a similarity in the level of agreement of certain key observables. ^{162}Yb, ^{166}Hf, ^{170}W, and ^{176}Os agree well with the X(5) predictions for most energies and interband $B(E2)$ strengths. However, they all show trends for yrast $B(E2)$ values as well as energy spacings in the excited $K = 0^+$ band that disagree with the X(5) predictions. Clearly, it is a challenge for these simple models to consistently reproduce the wide variety of spectroscopic data (energies, $E2$ branching ratios, and absolute transition rates) in transitional nuclei.

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