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Use of Pen-Based Technology in Calculus Courses

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1. Abstract

The author and his students used Tablet computers in Calculus I and Calculus II classes, providing students with dynamic digital transcripts that they could replay at their convenience. He and his students agreed that these graphic replays provide an effective alternative to the static explanations found in textbooks and in traditional course notes. Two specific examples are given in this paper.

2. Problem Statement and Context

Several fundamental concepts in the standard calculus course are best understood by means of a temporal sequence of graphic images. Textbooks are, by their static nature, unable to convey that kind of dynamic *progression of events. Calculus instructors can do so on a chalkboard in* the classroom. But traditionally, the calculus student has had no way to preserve that dynamic presentation for later study. The dynamic process cannot be recalled from the textbook or from the students' notes.

3. Solution Employed

Pen-based technology now allows the student to retain a permanent digital transcript of dynamic derivations. The author has employed this technology in his calculus courses. This paper describes these efforts and their evaluation.

In the fall of 2004, the author used a pen-based Tablet computer running the DyKnow Vision software to present all of his lectures in his Calculus II course. His students used the DyKnow client to access and review the lectures after class.

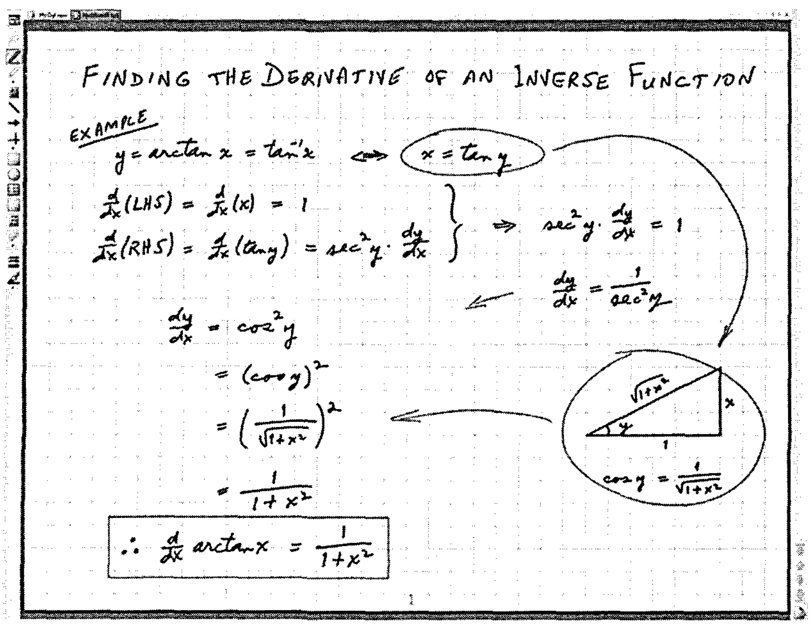
In the fall of 2005, he used the same technology again in one section of his Calculus I course. In this case, class met in a lab where students had their own pen-based Tablet computer by which DyKnow provided interactive access to the class sessions.

The DyKnow software has many features that facilitate better understanding and retention of topics in the calculus sequence. The particular feature that is relevant to this paper is its replay feature. This allows the instructor or the student to replay the sequence of pen strokes recorded in a particular panel during the class presentation. Several examples are provided below to illustrate the effectiveness of this feature.

3.1. The Derivative of the Arctangent

The formula for the derivative of the arctangent function is one of the more important derivative formulas in Calculus I. Moreover, the technique used to derive it is the standard method for deriving derivatives of other inverse functions.

Figure 1



The panel shown in the screen capture in Figure 1 shows the usual derivation of the formula for the derivative of the arctangent function. It is similar to that found in most calculus textbooks. The key steps are:

1. Express the function with the equation $y=\arctan x$.
2. Invert the function with the equation $x=\tan y$.
3. Employ implicit differentiation to obtain the differential equation

$$\sec^2 y \frac{dy}{dx} = 1$$

4. Solve the equation for

$$\frac{dy}{dx} = \cos^2 y$$

5. Obtain the formula for $\cos y$ in terms of x:

$$\cos y = \frac{1}{\sqrt{1+x^2}}$$

6. Substitute that into the previous formula to obtain the final result:

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Steps 3 and 5 are the places where students have the most difficulty. The sequence of substeps for each of those two steps are not transparent from the static image that appears in the textbook or on the panel in Figure 1. The substeps in step 5 are particularly tricky because they depend upon deriving relations on the right triangle in an order that is not apparent from the static image. However, when the DyKnow replay feature is employed, the progress of substeps becomes clear. And each time the student replays it, at various speeds and with pauses whenever needed, it becomes even more clear.

3.2. The Alternating Series Test

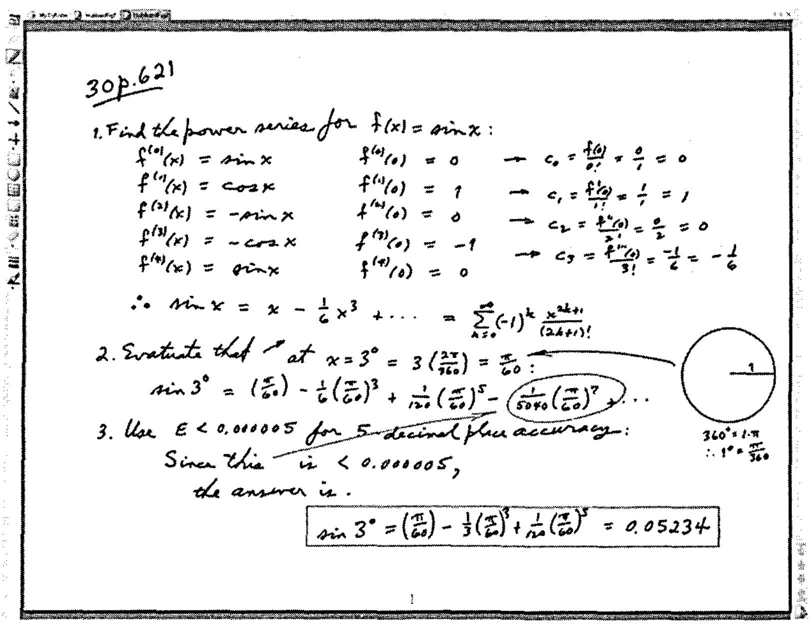
In Calculus II, there are several theorems that provide means to testing the convergence of infinite series. One of these theorems is called the alternating series test. It asserts that a series of decreasing terms must converge to a finite sum if the sequence terms converges to zero and if their signs alternate. For example, the harmonic series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ must converge to a finite sum because the sequence $\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ converges to zero and the signs in the series alternate $(-, +, -, +, \ldots)$.

The proof of this theorem is based on a fairly simple idea that is easily explained dynamically, in terms of motion along the x-axis. Moreover, this motion can be illustrated quite simply with a sequence of graphic motions on the x-axis. When viewed in real time, the idea becomes transparent and easy to remember.

The trouble with the dynamic graphic explanation is that when viewed on the printed page, the sequence of motions is hard to describe and loses its appeal. This explains why this explanation is not included in most calculus textbooks.

The author's pen-based Tablet computer demonstration is shown in Figure 2 below. The key feature is the spiral diagram that depicts the motion from one point to the next on the x-axis. The distance of each movement is the value of the corresponding term in the infinite series. The destination points on the x-axis are the corresponding partial sums of the series. The requirement that the individual terms decrease in value translates into decreasing displacements back and forth on the x-axis. Their alternating signs cause each displacement to be in the opposite direction of its preceding displacement. And the requirement that the terms converge to zero guarantees that the spiral

Figure 2



closes in on a single limit point, which is then seen to be the actual sum of the series.

The preceding descriptive argument is not easy for students to grasp. However, when they can replay the progress of steps in their DyKnow transcript, at various rates and with pauses where needed, the concept becomes transparent. The geometry of the argument comes to life!

The alternating series test has an important corollary that is used for actual computations of alternating series. The corollary states that the limit of the series can be approximated by any partial sum and the error in that approximation will be less than the magnitude of the next term in the series. The analytic proof of this fact requires an argument with inequalities that always seems to leave half the class asleep and the other half gazing out the window. Calculus-level mathematics students are generally uncomfortable with algebraic inequalities. But the dynamic sequence of geometric steps that the students can replay with their DyKnow transcript makes the argument transparent without any explicit use of inequalities.

4. Evaluation

Student evaluations were conducted after the semester was over. Ten of the 22 enrolled students responded to an online questionnaire. All of these students were first-semester freshmen.

Here are the responses to questions about the DyKnow Replay feature:

1. I used the DyKnow replay feature on my own computer:
   A. Several times per week. 20%
   B. About once per week. 10%
   C. Only a few times.. . . . . 30%
   D. Not at all. 40%
2. I used the DyKnow replay feature on my computer in class:
   A. Nearly every class.. . . . . 10%
   B. About once per week. 0%
   C. Only a few times.. . . . . 30%
   D. Not at all. 60%
3. I found the DyKnow replay:
   A. Very helpful. 10%
   B. Somewhat helpful. 50%

C. Not helpful. 10%
D. N/A. 30%

The questionnaire also asked for free response on opinions on Dyknow. These three were received:

- "Dyknow gave me the opportunity to be more attentive during class all throughout the semester. As Professor Hubbard spoke throughout the lesson, I was able to give 100 percent of my attention to his methods of teaching since I did not have to write down the examples that were provided."
- "It was nice to be able to go through the class notes when prepping for an exam. On the down side, while in class I felt as though it was not a necessity to pay attention as I would have the notes to review at a later time."
- "DyKnow was a very useful tool outside of class when going over my homework to prepare for the tests. However, I did not find the replay feature useful. Also, I felt that by using DyKnow in class, I was given an excuse to not pay attention because I knew I would have the assignments on my computer at home. This is no fault to the professor, but I think it would have been more useful if we had to take notes throughout class, rather than have the notes handed to us by the professor."

It is clear from these responses that a greater effort has to be made by this instructor, at least in first-semester freshman courses, to persuade his students how take best advantage of this powerful technology.

5. Future Work

Second only to one-on-one development work between student and instructor, the replay feature of DyKnow Vision provides a simple mechanism for students to see the logic of complex derivations, step-by-step. I now look forward to revising my class presentations so that these derivations are more central to the main course content.

I also plan to find more effective ways to persuade my students to use the various features of DyKnow to their best advantage. Toward that end, I expect to develop specific exercises to which the replay feature will facilitate solutions.