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An Easy Method to Introduce MIRR into Introductory Finance Classes

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An Easy Method to Introduce MIRR into Introductory Finance Classes

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In this paper, the modified internal rate of return (MIRR) is demonstrated to be a holding period return calculation that is not dependent on knowing a project's internal rate of return (IRR) nor the process for finding the IRR. Further, the MIRR calculation can be directly connected to the calculation of the profitability index (PI) and the net present value (NPV) if project cash flows are discounted using a firm's weighted average cost of capital. This connection to the PI and NPV allows for an intuitively appealing presentation of the MIRR calculation.

INTRODUCTION

Modified internal rate of return (MIRR) is a capital budgeting technique that tends to be inapproachable because it is difficult to see the issue it is intended to resolve and many believe that one needs to understand internal rate of return (IRR) to produce the MIRR. MIRR is actually more related to calculating a holding period return and (assuming discounting with a firm's weighted average cost of capital) can be generated from the profitability index (PI) which relies on the same calculations required for the net present value (NPV).

In the next section, the "reinvestment issue" that MIRR addresses is demonstrated with the holding period return calculated for a bond. In the following section, MIRR is calculated using the PI (and NPV). The last section concludes the paper.

THE "REINVESTMENT ISSUE"

The "reinvestment issue" concerning a project occurs as the project receives cash flows through time. Assuming a risk-adjusted discount rate of "k", the project in its entirety will not generate a holding period return of "k" unless cash flows are reinvested at "k" throughout the life of the project. This can be seen very easily with a bond.

A \$1,000.00 ten-year bond with 10% annual coupons (i.e. coupons = \$100.00 = 10% × \$1,000.00) can be bought for \$1,064.18. The yield to maturity for the bond based on its price is 9% APR and is equivalent to the discount rate for a project.

Compute the holding return (HPR) for this bond assuming it is held to maturity assuming no reinvestment of the coupon payments.

$$\text{HPR} = 6.51\% = \{[\ \$1{,}000.00 + 10 \times \$100.00] \div \$1{,}064.18\}^{1/10} - 1 \qquad (1)$$

Notice, the HPR does not equal the yield to maturity because proceeds received during the life of the bond are not reinvested. More generally, if the proceeds received through the life of a project are not reinvested, the HPR for the project will be less than "k".

If the coupons from the bond are reinvested at 9% APR, then the contribution of the coupons to the investment in the bond becomes $1,519.29. The HPR for the bond investment is now equal to the yield to maturity:

$$\text{HPR} = 9.00\% = \{[\ \$1{,}000.00 + \$1{,}519.29] \div \$1{,}064.18\}^{1/10} - 1 \qquad (2)$$

Similarly, the HPR for a project will only be equal to the discount rate (k) for the project if the proceeds received throughout the life of the project are reinvested at a rate equal to "k". This is the reinvestment issue relative to net present value (NPV) or profitability index (PI). Relative to IRR, the reinvestment issue is that the HPR for a project will only be equal to the IRR for the project if the proceeds received throughout the life of the project are reinvested at a rate equal to the IRR.

MIRR is a calculation that adjusts for this issue of project proceeds being reinvested throughout the life of the project. In general, the reinvestment of these proceeds is assumed to be at the weighted average cost of capital (WACC)[1] for the firm. Cash inflows are appreciated at the WACC (i.e. FV(Cash inflows)) until the project terminates at period "N" and cash outflows are discounted to the beginning of the project using the WACC (i.e. PV(Cash outflows)) as well. The MIRR is actually an HPR calculation[2] based on the appreciation and discounting of the cash flows at the WACC:

$$\text{MIRR} = \{\text{FV(Cash inflows)} \div \text{PV(Cash outflows)}\}^{1/N} - 1 \qquad (3)$$

In a sense, the MIRR tells you what return a firm actually earns on the project when considering what is actually done with the interim cash flows.

Another analogy may be useful here. Upon winning the lottery and receiving the prize through installment payments over time, the MIRR calculates your actual return by assuming you put the money into a savings account rather than into the purchase of more lottery tickets.

PI-BASED MIRR CALCULATION

In the previous section it was demonstrated that MIRR is actually an HPR calculation and not directly related to an IRR calculation. If a project's PI (and correspondingly, its NPV) is calculated using the WACC, the MIRR can be easily calculated through the PI.

Looking back at equation (3), the FV(Cash inflows) is equivalent to the discounted cash inflows (call this PV(Cash inflows)) multiplied by one plus the weighted average cost of capital raised to the "Nth" power (i.e. $(1 + WACC)^N$):

$$\text{FV(Cash inflows)} = \text{PV(Cash inflows)} \times (1 + \text{WACC})^N \qquad (4)$$

Substituting equation (4) into equation (3), the MIRR identity becomes:

$$\text{MIRR}=\{\text{PV(Cash inflows)}\times(1+\text{WACC})^N\div\text{PV(Cash outflows)}\}^{1/N}-1 \qquad (5)$$

Notice, the PI equals PV(Cash inflows) divided by the PV(Cash outflows) assuming the weighted average cost of capital (WACC) is the discount rate (note: this is a common assumption for the PI and NPV metrics). Inserting PI into equation (5) produces:

$$\text{MIRR} = \{\text{PI}*(1 + \text{WACC})^N\}^{1/N} - 1 \qquad (6)$$

This is the Arnold and Nixon (2011) version of the equation. However, equation (6) can be simplified further to produce equation (7).

$$\text{MIRR} = \{\text{PI}^{1/N}\}*(1 + \text{WACC}) - 1 \qquad (7)$$

Equation (7) is intuitively appealing because the MIRR calculation is now directly connected to the very popular NPV metric through the PI calculation. Assuming NPV is calculated using the WACC as the discount rate then:

- $NPV > 0$ implies: $PI > 1$ and $MIRR > WACC$
- $NPV = 0$ implies: $PI = 1$ and $MIRR = WACC$
- $NPV < 0$ implies: $PI < 1$ and $MIRR < WACC$

A numerical demonstration can be easily created from any standard NPV type of problem. Let a given project have an initial of cost of $500.00 and beginning next year, produces annual cash flows of $200.00, $300.00, and $400.00 respectively. Assuming a WACC of 10%, the NPV becomes:

$$\text{NPV} = \$200 \div (1 + 10\%)^1 + \$300 \div (1 + 10\%)^2 + \$400 \div (1 + 10\%)^3 - \$500$$

$$\text{NPV} = \$730.28 - \$500 = \$230.28 \qquad (8)$$

The corresponding PI becomes:

$$PI = \$730.28 \div \$500 = 1.4606 \quad (9)$$

The corresponding MIRR becomes:

$$MIRR = \{(1.4606)^{1/3}\}*(1 + 10\%) - 1 = 24.81\% \quad (10)$$

Although equation (3) may not be as intuitively appealing as equation (7) for calculating MIRR, equation (3) is the more general form for MIRR and is not dependent on the discount rate being set at the WACC. Consequently, it is instrumental to demonstrate that equation (3) produces the same answer:

$$FV(\text{Cash inflows}) = \$200*(1+10\%)^2 + \$300*(1+10\%)^1 + \$400 = \$972 \quad (11)$$

$$PV(\text{Cash outflows}) = \$500 \quad (12)$$

$$MIRR = \{\$972 \div \$500\}^{1/3} - 1 = 24.81\% \quad (13)$$

CONCLUSION

In this paper, the MIRR is demonstrated to be an HPR calculation that can be directly linked to the PI and NPV calculations assuming the discount rate is set to the firm's WACC. By demonstrating the HPR calculation structure of MIRR, it dispels the notion that one needs to understand and calculate the IRR before attempting the calculation of the MIRR.

Because there are benefits to calculating the MIRR (see Chang and Swales, 1999), it is the authors' hope that MIRR become more prevalent in the introductory corporate finance curriculum. Perhaps, using the techniques demonstrated in this paper will allow for an easier understanding of MIRR due to the connection that can be made between the MIRR and the PI (and NPV).

ENDNOTES

[1]Technically, MIRR does not require reinvestment at the WACC. Brigham and Houston (2012) assumes reinvestment at the WACC, but notes that another explicit rate can be used if is a more reasonable assumption. Ross, Westerfield, and Jordan (2013) describe the rate more generally as the required rate of return. The firm's WACC will be used as the reinvestment rate in this paper as it seems reasonable that, on average, a firm should be able to reinvest at is overall cost of capital.

[2] This calculation is consistent with Lin (1976), but is not universally accepted (see McDaniel, McCarty, and Jessell (1988) or Shull (1994)). Some advocate that all cash flows after the project's inception, including negative cash flows, should be compounded at the WACC until the project's termination.

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