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# The Marginal Cost of Funds with Nonseparable Public Spending

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## The marginal cost of funds with nonseparable public spending

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How do we measure the welfare effects of additional government spending when that spending must be financed by distortionary taxation? In empirical studies, a common approach is to treat the costs and benefits separately. In this approach, the costs of raising the funds to finance a project are calculated independently of the project being financed. In deciding whether a particular project is worthwhile, one must evaluate the project-specific benefits and then subtract the cost of raising funds (which is the same for all projects) to find the project's net benefits. If these net benefits are positive, the project should be undertaken.

The cost side in these calculations is obtained using the concept of the marginal cost of funds (MCF), defined as the fall in welfare per dollar of revenue raised. The MCF is equal to $1 (for taxes paid per dollar of revenue raised) plus an amount that represents an additional effect on economic welfare that arises because the non-lump-sum nature of taxation distorts economic decisions.

However, if government spending substitutes for private consumption or private investment, the costs of a particular project cannot be calculated independently of its benefits. In such situations, government spending is said to be nonseparable in utility or production because the marginal utility of private consumption, or the marginal product of private inputs, is not independent of the publicly provided goods and services. These nonseparabilities in utility and production imply that the benefits associated with the public project in turn influence marginal decisions of economic agents, thereby affecting variables such as the equilibrium quantity of the labor input and hence the MCF. Although the theoretical literature in public finance has recognized the importance of nonseparabilities, empirical calculations of MCF are based on the assumption, often made implicitly, that either publicly provided goods are separable in utility and production (so that the benefits of the public spending are independent of the costs) or the change in government spending consists of only transfer payments that substitute one to one for private consumption.(1)

In this article, we calculate the quantitative impact that the nonseparabilities associated with public spending have on agents, economic decisions and welfare. Specifically, we highlight a measure labeled the net marginal cost of funds (NMCF), which represents the costs of raising public funds net of those benefits of the government spending that are not independent of agents, economic decisions. The NMCF can be interpreted as the magnitude of the additional separable benefits that a particular project will have to satisfy for it to be a worthwhile undertaking from an economic welfare perspective.(2) When public spending affects private agents' economic choices, the relevant concept in welfare analysis is the NMCF.

We provide numerical calculations of the NMCF for aggregate government purchases of goods and services in the U.S. economy and examine the policy implications of our results.(3) Our empirical work thus partly satisfies the recommendation of Fullerton (1989), who suggests, "The best procedure is to specify the particular project in the utility of consumers, including any complementarity to labor or leisure, and then calculate whether the tax and spending package increases welfare" (p. 15).

## WELFARE EFFECTS OF GOVERNMENT

## SPENDING AND TAXATION

Suppose the economic welfare of the typical agent in the economy is given by the utility function U(C, [L.sub.2], G), where C is private consumption, [L.sub.2] is leisure, and G is publicly provided goods.(4) A typical economic agent earns labor income by working [L.sub.1] hours at a wage rate of w and paying taxes on labor income at the marginal tax rate t'. Government spending on publicly provided goods increases at the margin by an amount dG, financed by a higher tax rate on labor income. Letting T stand for total tax collections, dG = dT. In response to this policy change, the agent makes marginal changes in consumption (by an amount dC) and labor supply (by an amount [dL.sub.1]). The change in labor supply implies a change of [dL.sub.2] = [-dL.sub.1] in leisure given the fixed endowment of time. Given the changes dG, dC, and [dL.sub.2], one can calculate the effect of the government spending and tax change on the welfare of the agent.

To compare benefits to costs, we use the MCF. The MCF is defined as the fall in welfare per dollar of revenue raised.(5) It has been shown (e.g., see Fullerton 1991 and our appendix) that the MCF can be written as

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This equation shows the MCF to consist of taxes paid by the individual per dollar of revenue raised (which by definition is 1 for a balanced-budget change) plus an amount, [MATHEMATICAL EXPRESSION NOT REPRODUCIBLE IN ASCII], that depends on the change in labor supply, which can be thought of as the fall in welfare associated with the reduction in labor supply brought about by the increase in distortionary taxation.

## SEPARABLE GOVERNMENT SPENDING

To see why nonseparable effects are important, it is useful first to consider the case in which government spending is strongly separable. Suppose the utility function is

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In this equation, u represents utility obtained from consumption and leisure and v is utility obtained from the publicly provided good. The assumption of strong separability means that G does not affect the utility enjoyed from private consumption and leisure.(6) The studies mentioned earlier, such as Stuart (1984), have calculated MCF numbers based on equation (1) with [dL.sub.1] calculated from a general equilibrium model in which G is separable in utility, as in equation (2).

## NONSEPARABLE GOVERNMENT SPENDING

How are these results on welfare effects and the MCF modified when we allow for the substitutability of publicly provided goods for private consumption and private inputs in production?(7) Suppose there are three types of government spending. One type, [G.sub.c], substitutes for private consumption goods and is not separable in the utility function. Another portion of government spending, [G.sub.s], is separable in utility and does not affect production. A third portion represents government's investment in capital goods, [G.sub.i].

Now the utility function is of the form

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where [C.sup.*] is effective consumption. Barro (1981) suggested that effective consumption is a function of private consumption and government services. For example, the value of transportation by private car depends on the quality and availability of publicly provided roads. The value of a privately purchased lunchtime apple is enhanced by eating it in a publicly provided park. Following Barro, we postulate that effective consumption, [C.sub.*], is a linear function of consumption goods purchased by households, C, and publicly provided goods that substitute for private consumption goods, [G.sub.c]. Thus

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The parameter [[theta].sup.*] represents the degree of substitutability of publicly provided consumption goods and services for private consumption. Theoretically, publicly provided goods can be either a substitute for ([[theta].sup.*] [is greater than] 0) or a complement to ([[theta].sup.*] [is less than] 0) private consumption. Empirical estimates in Aschauer (1985) and Kormendi (1983) suggest that they are substitutes. Equation (4) implies that indifference curves between C and [G.sub.c] are linear with slope equal to [-[theta].sup.*].(8)

To allow for nonseparability in production, suppose the economy's production function is shifted by changes in government spending on investment goods ([G.sub.i]). For example, increased infrastructure spending may reduce the costs of production, leading firms to increase output. The marginal productivity of government spending, [MPG.sup.*], defined as the marginal increase in output due to the marginal increase in government spending on infrastructure ([G.sub.i]), which increases the public capital stock, is a direct benefit of the government spending. In addition, because the increased output may affect both nonlabor income and labor income in the economy, it will affect labor-supply decisions. The nonseparability of government spending in utility and production thus affects both the benefits of the public spending and the MCF.

The marginal benefits (MB) of an extra dollar of public spending now consist of three components. The separable benefits are measured by the derivative of the term [v.sup.*]([G.sub.s]) in equation (3) divided by [u.sub.1] to put the change in utility in units of the consumption good ([v.sup.*]'/[u.sub.1]. The benefits of [G.sub.c] are measured by [[theta].sup.*]. And benefits from the increase in private production due to higher government capital can be measured by the marginal product of the government spending, $MPG^*$.

Suppose an initial fraction $f_c$ of total government spending is nonseparable in utility so that $G_c = f_c G$, an initial fraction $f_s$ of total government spending is separable in utility so that $G_c = f_s G$, and an initial fraction $f_i$ of total government spending is devoted to investment so that $G_i = f_i G$. Now imagine a marginal increase in G divided in the same proportion between these different kinds of spending. The marginal benefits are [MATHEMATICAL EXPRESSION NOT REPRODUCIBLE IN ASCII], which we write as [MATHEMATICAL EXPRESSION NOT REPRODUCIBLE IN ASCII], and $MPG = f_i MPG^* G$. Henceforth, we work just with $\theta$ v', and MPG rather than with the variables with asterisks; this simplifies our notation.

The separable benefits ($v/u_1$) are likely to include redistributional effects that are difficult to measure in an environment focusing on the average agent, as our setup does. For this reason, we define (and provide estimates of for the U.S. economy) a new variable, the nonseparable marginal benefits of government spending (NSMB), as the sum of the direct impact on effective consumption and the marginal product of government spending:

NSMB x $\theta$ MPG.

The calculation of MCF is still based on equation (1). However, the typical agent's labor-supply decision is affected by the benefits of government spending, and so the term $t'w(dL_1/dT)$ in equation (1)(and hence the MCF) differs depending on the magnitude of the nonseparability parameters $\theta$ and MPG.

The NMCF consists of the MCF less those benefits of the public spending that arise because the publicly provided good is nonseparable in utility and production (NSMB). Thus NMCF = MCF - NSMB. We believe that, in the presence of nonseparabilities, the NMCF is a more relevant concept than the MCF. This is because, as Mayshar (1991) has argued, when the public spending provides services that substitute for private consumption, it is ambiguous what part of the fall in welfare arising from the change in labor should be added to costs and what part should be subtracted from benefits.

## THE MODEL

Previous empirical work, which treats government spending on goods and services as separable in utility and production, is exemplified by the general equilibrium model of Stuart (1984). Because we wish to compare our results with nonseparable government spending to those with the standard approach, we take Stuart's model and extend it to the nonseparable case. Our model encompasses Stuart's; if the nonseparability parameters are set to zero, the model is exactly the same as Stuart's.

In the model, households maximize utility, taking lump-sum transfers from the government and nonlabor income as given. Firms maximize profits subject to a technology that allows publicly provided goods to be a productive input. The model is a one-period model of resource allocation with no uncertainty, a fixed capital stock that may be used in either the government sector or the private sector, and a fixed endowment of time allocated between market and home production. Alternatively, it can be interpreted as the steady state of a dynamic model. Government spending is exogenously given, and the economy is always at a full-employment equilibrium.

## PREFERENCES AND THE HOUSEHOLD'S OPTIMIZATION PROBLEM

The utility function of the representative household is assumed to take the form

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with the functional form for u given by

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where [C.sup.*] is effective consumption as defined in equation (4), [Y.sub.2] is home production, G is government spending on goods and services, and the functions u(??) and v(??) are such that U(C, [Y.sub.2], G) is continuously differentiable, is strictly quasi-concave, and yields interior optima given positive income and prices.

We follow Stuart (1984) in modeling home production as

[Y.sub.2] = [BL.sub.2.sup.b],

where [L.sub.2] is time devoted by the household to home activities.

The representative household in the economy chooses consumption, C, and labor supply (hours worked), [L.sub.1], to maximize utility (6), given the functional forms (4), (7), and (8), subject to the following resource and time constraints, respectively.

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In these equations, I is nonlabor income (exogenously given to the household), [wL.sub.1] is labor earnings with w being the real wage, R is lump-sum transfers from the government, t is the average tax rate, and L is the endowment of time. All variables are expressed in units of consumption, the numeraire good.

The first-order necessary conditions of the household's maximization problem are the constraints (9) and (10), together with

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where t' is the marginal tax rate applying to labor income. Equation (11) states that the marginal rate of substitution of the untaxed use of time (leisure), [L.sub.2], for effective consumption is equal to the after-tax real wage.(9) Note that because G is exogenous, the term v(G) in equation (6) has no bearing on the agent's decisions although it does have welfare implications.

## MARKET PRODUCTION AND THE FIRM'S OPTIMIZATION PROBLEM

Market output, [Y.sub.1], is assumed to be produced by a Cobb-Douglas technology. Specifically,

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where K is the fixed private capital stock, [K.sub.G] is the government capital stock, [L.sub.1] is labor demand (measured in hours), and [A.sup.*] is a scale factor, changes in which can be used to capture technological progress.(10) The parameter [beta], if positive, allows publicly provided capital to directly enhance private production possibilities, as in Barro (1981), for example.(11) A positive [beta] implies that, unlike the assumptions made in previous studies, the effect of [K.sub.G] on private production is not independent of private capital and labor. In other words, government expenditures that increase [K.sub.g] can be thought of as leading to proportional shifts in the production function, affecting the marginal products of capital and labor. Note that equation (12a) implies that the technology displays constant returns to scale with respect to the private inputs, with a and 1 -- a being the labor and capital shares, respectively, of total income.(12)

Now suppose the economy is in a stationary state in which government investment spending is proportional to the government capital stock; then [K.sub.G] = [mG.sub.i] [mf.sub.1]G.(13) The proportionality implies that we can rewrite the production function as

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where A = [A.sup.*]([mf.sub.i])[beta] and D(G) = [AG.sup.[beta]].

The representative firm maximizes profits, given by

I = [Y.sub.1] = [wL.sub.1],

subject to the production function (12). Although households take nonlabor income as given, it reflects the payments to capital in a general equilibrium. At the firm's optimum choice of labor demand, the real wage is equated to the marginal product of labor so that

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## GOVERNMENT BUDGET CONSTRAINT

We consider only balanced-budget changes in government expenditure, and so without loss of generality we assume that the government's budget is balanced to begin with. Thus the government's budget constraint is given by

R + G = T,

where total tax collections (T) equal [twL.sub.1]. When taxes are changed to finance a change in government spending (dG = dT), we assume that the ratio of the marginal tax rate (t') to the average tax rate (t) remains unchanged at a constant [tau].(14)

## EQUILIBRIUM

Two aggregate consistency conditions are now used to close the model.

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Equation (16) is the goods market-clearing condition, whereas equation (17) states that total output is exhausted in payments to capital and labor.

## RESULTS

## DATA AND PARAMETERS

The parameters of the model are the utility function parameters ([alpha], [delta], and [rho]), the parameters measuring the nonseparable effects of publicly provided goods in utility and production ([theta] and [beta]), the production function parameters (a, A, b, and B), the endowment of time (L), and the parameter representing die constant ratio of the marginal tax rate to the average tax rate ([tau]). Data values are needed for the variables C, [Y.sub.1], [L.sub.1], [L.sub.2], N, G, K, w, I, t, and t'.

We report results for 1976 data to make them comparable to those of Stuart (1984), who focused on that year. The following figures are taken directly from Stuart @dollar figures are in billions of 1976 dollars): labor supply [L.sub.1] = 1,008.14, hours available L = 3,660, output [Y.sub.1] = $1,527.40, labor's share a = .720, marginal tax rate t' = .427, average tax rate t = .273, and the ratio of the marginal tax rate to the average tax rate [tau] = 1.564. Also following Stuart, we set the coefficient of leisure in the home production function equal to labor's share of aggregate income (i.e., b = a = .720), although the results are not sensitive to that choice. Utility function parameter B is set so that [u.sub.1] = [u.sub.2], which sets the marginal rate of substitution of the home-produced good for the market good to unity. This just amounts to a normalization of the units in which home production is measured.

The utility function parameters [alpha], [rho], and [delta] are set by pinning down the uncompensated and compensated labor-supply elasticities ([e.sub.u] and [e.sub.c], respectively). Once again, we use Stuart's (1984) benchmark values of [e.sub.u] = 0.0 and [e.sub.c] = 0.2. The exact values of [alpha], [rho], and [delta] that these elasticities imply depend on the value of [theta]. The values of [theta] and [beta] used in our benchmark case are [theta] = 0.284 and [beta] = 0.182. Our procedure for obtaining these estimates is described later.

We take the capital stock, K, to be the current-dollar net stock of capital, reported in Musgrave (1986), which has a value of $3,242,20. Finally, government purchases are G = $361.40.(15)

NONSEPARABLE MARGINAL BENEFITS

To estimate the marginal product of government spending, we use a method similar to that of Ram (1986). We use time-series data to estimate a Cobb-Douglas production function for the U.S. economy with output as a function of total hours worked, capital, government purchases of goods and services, and a deterministic time trend to account for factors such as technical progress (see the appendix for details). For 1976 values of the variables, our estimation procedure yields an estimate of the marginal product of aggregate government spending given by MPG = 0.769; that is, $1 of extra aggregate government spending (divided in fixed proportions among [G.sub.c], [G.sub.i], and [G.sub.s]) increases output by $0.769; by shifting out the production function. This value is quite large but consistent with the significant productivity effects of infrastructure spending found in Aschauer (1989).

To find the value of O, the extent to which the publicly provided good substitutes for private consumption, we follow a procedure analogous to that of Aschauer (1985). Aschauer estimated [theta] by using the condition that economic agents will lend or borrow optimally to smooth effective consumption over time. We extend Aschauer's method to allow for work effort being a choice variable, thereby also incorporating the condition that individuals allocate time efficiently between labor and leisure (see details in the appendix). Our estimation gives [theta] = 0.284, which is close to the value of 0.23 obtained by Aschauer.(16)

These results indicate that the nonseparable benefits associated with a $1 increase in publicly provided goods are substantial, as [theta] + MPG = $1.053. The next question is whether they outweigh the costs involved in raising this $1 of revenue using distortionary taxation.

MCF AND NMCF WITH SEPARABLE AND NONSEPARABLE SPENDING

We begin by replicating Stuart's (1984) results with separable government spending. To do this, we set [theta] = 0 and MPG = 0, calculate the change in labor (d[L.sub.1]) and plug this into equation (1) to get MCF. Stuart found MCF = $1.072; our replication gives MCF = $1.077.(17) The small difference between these results is entirely computational.(18)

Next, we look at the impact of nonseparability of government spending on the MCF measure. Table 1 shows estimates of the MCF when the parameters [theta] and MPG take on alternative values of [theta] = 0 or 0.284 and MPG = 0 or 0.769. When [theta] = 0.284 and MPG = 0.769, the MCF is $1.144. Nonseparable marginal benefits are NSMB = $1.053, and so the NMCF = $0.091; that is, when taxes and government spending are raised by $1, consumers get back (in utility terms) $1.053 worth of direct benefits ($0.289 of effective consumption plus $0.769 of additional output). But it costs $1.144 to finance the spending, and so there is a net welfare loss (not counting any separable benefits) of $0.091.

TABLE 1: The Marginal Cost of Funds When Government Spending Is Nonseparable

| | NSMB | MCF | NMCF |
|---|---|---|---|
| [theta] = 0, MPG = 0 | 0.000 | 1.077 | 1.077 |
| [theta] = 0.284,MPG = 0.769 | 1.053 | 1.114 | 0.091 |
| [theta] = 0.284,MPG = 0 | 0.284 | 1.115 | 0.831 |
| [theta] = 0, MPG = 0.769 | 0.769 | 1.106 | 0.337 |

NOTE: NSMB = nonseparable marginal benefits; MCF = marginal cost of funds; NMCF = net marginal cost of funds; MPG = marginal productivity of government spending.

If publicly provided goods are completely nonseparable in utility, so that v'/[u.sub.1], is zero, then the fact that the NMCF is positive means that the costs of additional government spending outweigh the benefits. Whenever the separable benefits, measured by v'/[u.sub.1], are less than $0.091, additional spending is not justified and in fact government spending should be reduced. If the separable MB are valued at exactly $0.091, then government spending is at its optimal level. If the separable MB are valued at more than $0.091, then a rise in aggregate government spending beyond the current value would increase welfare.

Another way to look at the results is to compare our NMCF of $0.091 to Stuart's (1984) MCF of $1.077. We have captured many of the benefits of government spending in the NSMB term, so that separable benefits must exceed just $0.091 for additional spending to be justified. In Stuart's case, all the benefits of government spending are assumed to be separable, so that the benefits must exceed $1.077 for additional spending to be justified. In Stuart's case, suppose there are two types of benefits, Type 1 and Type 2. Suppose further that we were able to measure the Type 1 benefits at the margin, and they came to $1.053 (a number equal to our measure of NSMB). Then, according to Stuart's measure, Type 2 benefits would have to be $0.024, which equals $1.077 (MCF) - $1.053 (Type 1 benefits). However, our measure of NMCF ($0.091) is larger than this because we have a higher MCF due to our assumption that the Type 1 benefits are nonseparable.

SENSITIVITY ANALYSIS

Because our values for the parameters [theta] and MPG are estimated values, it is important to test the sensitivity of the results to alternative values of these parameters. Our estimate of [theta] is 0.284 with a standard error of 0.056. Our estimate of MPG is 0.769 with a standard error of 0.099. Table 2 shows the MCF, NSMB, and NMCF for high, benchmark, and low values of [theta] and MPG where the high value is the benchmark value plus 2 standard errors and the low value is the benchmark value minus 2 standard errors.

TABLE 2: Ranges for NSMB, MCF, and NMCF

| | MPG | | |
|---|---|---|---|
| | 0.571 | 0.769 | 0.968 |
| [theta] = 0.171 | | | |
| NSMB | 0.742 | 0.940 | 1.139 |
| MCF | 1.121 | 1.129 | 1.137 |
| NMCF | 0.379 | 0.189 | -0.002 |
| [theta] = 0.284 | | | |
| NSMB | 0.855 | 1.053 | 1.252 |
| MCF | 1.137 | 1.144 | 1.152 |
| NMCF | 0.282 | 0.091 | -0.100 |
| [theta] = 0.397 | | | |
| NSMB | 0.968 | 1.166 | 1.365 |
| MCF | 1.152 | 1.160 | 1.167 |
| NMCF | 0.184 | -0.006 | -0.198 |

NOTE: MPG = marginal productivity of government spending; NSMB = nonseparable marginal benefits; MCF = marginal cost of funds; NMCF = net marginal cost of funds.

Table 2 shows that, for the parameter values in this model, increases in [theta] or MPG increase the MCF. However, at the same time that the MCF is rising, the NSMB is rising by more, and so the NMCF decreases as [theta] and MPG rise. The lowest value of MCF comes when [theta] and MPG are at their lowest values, with MCF = $1.121; the highest MCF occurs when [theta] and MPG are at their highest values, with MCF = $1.167. Similarly, the NMCF ranges from $0.379 when [theta] and MPG are low, falling to -$0.198 when [theta] and MPG are high. The results in Table 2 also suggest that the NSMB is much more sensitive to changes in MPG than it is to changes in [theta].

These results, the qualitative properties of which are all consistent with theory, imply that variations in the parameters capturing the nonseparable benefits of publicly provided goods have significant effects on the MCF. This means that the MCF is project specific. Our benchmark results can be thought of as having calculated the MCF for a particular government project that we believe to be an interesting one: a rise in publicly provided goods keeping fixed the existing composition between (a) publicly provided goods that affect the separable term v(G) in the utility function, (b) publicly provided goods that have an impact on effective consumption C*, and (c) publicly provided goods that affect productivity.

## SUMMARY AND CONCLUSIONS

In this article, we have provided empirical estimates of the welfare effects of changes in distortionary taxation and government spending, allowing publicly provided goods to substitute partially for private consumption and to act as inputs into private production. The MCF is not independent of the marginal benefit of public spending. The MCF associated with publicly provided goods is in the range of $1.121 to $1.167 (Table 2). Thus the MCF is higher than the comparable measure of Stuart (1984), which sets the nonseparable effects equal to zero, and lower than what Stuart finds with transfer payments, which substitute one to one for private consumption. Accounting for the nonseparable benefits of government spending yields an NMCF in the range of -$0.198 to $0.379 (Table 2). To justify the current level of government expenditure, the marginal $1 of government spending must provide as much as $0.379 in separable benefits when [theta] and MPG are low. Only in certain ranges for [theta] and MPG (involving relatively high values for [theta] and MPG) is the NMCF negative. This suggests that, in general, the nonseparable benefits by themselves may not be sufficient to make additional government spending valuable to society.

Our results illustrate the empirical importance of accounting for the nonseparability of publicly provided goods in calculating the MCF. The nonseparabilities have effects on economic agents' decisions that are likely to be very different depending on the nature of the project. Thus the MCF is project specific and cannot be calculated independently of the intended use of the funds.

## APPENDIX

### The MCF With Nonseparable Public Spending

### DERIVATION OF EQUATION (1)

Substitute for C from equation (16) ([Y.sub.1] = C + G) into (4) (C* = C + [theta]* [G.sub.c) in the text, use the equations [theta] = [f.sub.c] [theta]* and [G.sub.c] = [f.sub.c] G, and totally differentiate to yield

dC* = d[Y.sub.1] - (1 - [theta])dG. (A1)

Then totally differentiate (12b) ([Y.sub.1] = D(G)[K.sup.1-a][L.sub.1.sup.a] and use (14) (w = aD(G) [K.sup.1-a][L.sub.1.sup.1]) and the definition of MPG (MPG = ?[Y.sub.1]/?G = D'(G)[K.sup.1-a] [L.sub.1.sup.a]) to give

d[Y.sub.1] = w d[L.sub.1] + MPG x dG. (A2)

(A1) and (A2) imply

dC* = w d[L.sub.1] - (1 - [theta] - MPG)dG. (A3)

Now totally differentiate (8) ([Y.sub.2] = B[L.sub.2.sup.b] and (10) ([L.sub.1] + [L.sub.2] = L) and combine these to eliminate d[L.sub.2] to get

d[Y.sub.2] = -bB[L.sub.2.sup.b-1]d[L.sub.1]. (A4)

Define the compensating surplus (following Stuart 1984) as the amount of consumption needed to be given to the consumer such that the change in utility is zero, ignoring separable effects (i.e., assuming $v'/u_1 = 0$):$du = 0 = u_1(dC^* + CS) + u_2 dY_2$, where CS is the compensating surplus. Substituting (A3) and (A4) into this expression and plugging in $u_2 = u_1 (1 - t')w/bBL_2^{b-1}$, which is implied by (11), it follows that

$$CS = (1 - \theta - MPG)dG - t'w\, dL_1. \quad (A5)$$

Using $dG = dT$ and the definitions $NMCF = CS/dT$, $NSMB = \theta + MPG$, and $NMCF = MCF - NSMB$ gives

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This is equation (1) in the text.

## ESTIMATES OF $\theta$ AND MPG

### Marginal Product of Government Spending

Using annual data on K, $L_1$, G, and $Y_1$, from 1950 to 1989, equation (12b) in the text yields a time series on $D(G) = AG^{\beta}$. The data used are output ($Y_1$), capital (K), hours worked ($L_1$), and government purchases (G) per employed worker. Denoting the time period by the subscript t to capture technical progress, we postulate $\log(A_t) = d_0 + d_1 t + \epsilon_t$, where $\epsilon_t$, is a first-order autoregressive process. The parameter $\beta$ is then estimated by applying maximum likelihood to the regression

$$\log D(G_t) = d_0 + d_1 t + \beta \log (G_t) + \epsilon_t. \quad (A7)$$

The marginal product of government spending is then obtained from the estimate of $\beta$ and data on $Y_1$ and G by using $MPG = \beta Y_1/G$. Our estimated value of $\beta$ is given by $\beta = 0.182$ with a t statistic of 7.80. Given the values of $Y_1$ and G for 1976, this implies an MPG of 0.769.

### Substitutability of Government Spending for Private Consumption

The parameter $\theta$ is obtained by using the efficient labor-leisure choice condition given by equation (11) in the text and the following condition, which implies efficient allocation of consumption over time:

$$MUC_t/MUC_{t+1} = \gamma_1, \quad (A8)$$

where $MUC_t$ is the marginal utility of effective consumption at time t (i.e., $u_1$ evaluated with time t values of the variables) and $\gamma_1$ is the intertemporal relative price.

To obtain an estimate of $\theta$, we first transform our variables to ones that have a constant steady state by dividing by the scale factor $A_t$ of the production function. Then we linearize equation (11) in the text and equation (A8) around the steady-estate values of the transformed variables. Estimation of these linearized equations yields an estimate of $\theta$. Denoting the percentage deviation of a transformed variable from its steady-state value by putting a hat over the variable, these linearized equations take the form

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where b and f with subscripts $\phi$, 1, 2, 3, are constants, g and c are the steady-estate values of $G_t/A_t$, and $C_t/A_t$, respectively, $E_t$ is the rational expectations operator conditional on all information available at time t, and $\epsilon_{Ct}$ and $\epsilon_{Lt}$ are error terms. To solve for the expectations terms in these equations, we make a forecast of the transformed government spending variable by regressing that variable on two lags of itself and one lag of the transformed real government budget deficit. Taking the mean values of the transformed variables over the sample period to be steady-estate values, we estimate equations (A9) and (A10) simultaneously, subject to the implied cross-equation restrictions, to yield an estimate of $\theta$. Our estimate is $\theta = 0.284$ with a t statistic of 5.04.

NOTES

(1.) Atkinson and Stern (1974) show that the MCF can differ from the marginal rate of transformation of the publicly provided good for the private good (typically assumed to be unity) for several reasons. These include the nonseparability effect arising from changed consumption of the taxed good due to the effect on agents' decisions of the nonseparability in utility and production of the public and private goods. The nonseparability effect has also attracted attention in the theoretical literature on labor supply including Gwartney and Stroup (1983), Wildasin (1984), Bohanon and Van Cott (1986) Gahvari (1986) and Snow and Warren (1989) The only empirical studies of the nonseparability effect are those of Hansson (1989), who considers government expenditure on infrastructure with empirical estimates for Sweden, and Conway (1992), who uses state government expenditure data with micro-labor-supply data and finds a very large MCF.

(2.) For example, suppose a project that costs $1 has nonseparable benefits of $1.05 and an MCF of $1.15; then the NMCF is $0.10 The project is worth undertaking if the separable benefits of the project (see equation [3] later in the text) are valued at 0.10 or more.

(3.) Previous empirical work, assuming separability of the public good and the private good in utility, obtains estimates of the MCF that cover a wide range. Browning (1987) estimates an MCF ranging from $1.32 to $1.47 for his preferred parameter values, Stuart (1984) estimates it in the range of $1.07 to $1.71, add Ballard Shoven, and Whalley (1985) estimate the MCF for labor-income taxes to be between $1.16 and $1.31. Studies with capital distortions have obtained even bigger numbers, as Ballard et al. (1985) show. According to Fullerton (1991), many of the differences in these results are explained by the use of different welfare cost measures.

(4.) The model and notation used throughout the article are designed to match those of Stuart (1984) for comparison purposes.

(5.) In calculating the amount by which utility falls, measured in units of consumption, we use the measure of compensating surplus, following Stuart (1984) (see our appendix). However, alternative measures, such as the compensating variation used by Mayshar (1991), yield similar results.

(6.) Our point really depends only on weak separability, which requires that G not affect the marginal rate of substitution between private consumption and leisure. However, the results are easier to show assuming strong separability.

(7.) Recently, Mayshar (1991) and Snow and Warren (1989,1992) have derived analytical expressions for the marginal cost of funds for both separable G and nonseparable G in the utility function. However, their methods do not deal with the case in which G affects the production function.

(8.) The assumption of linear indifference curves seems restrictive. However, we have also investigated two cases where these indifference curves are not linear and where the value of [theta]* is used to pin down the slope of the indifference curve only at the initial equilibrium. The two cases are C* = ln(C) + [theta]* ln([G.sub.c]) and C* = [[CG.sub.c.sup.[theta]*. The results are not very sensitive to these alternative specifications.

(9.) Following much of the literature in this area, we assume that the labor income tax is the only margin of distortion created by the government. For a model with additional distortions, see, for example, Ballard et al. (1985).

(10.) Note that we are assuming the same production function for the government and private sectors, as is typical in this literature.

(11.) Of course, a negative value of [beta] would that government spending was harmful. Empirical work by Ram (1986)and Aschauer (1989) suggests that [beta] is positive.

(12.) The assumption of constant returns to scale with respect to private inputs and a positive [beta] imply that private production exhibits increasing return over all inputs inclusive of the public capital stock, [K.sub.G]. The usual argument for economies of scale behind the public provision of some of the inputs to private production makes this a plausible assumption (see, e.g., the discussion in Aschauer (1989,180). On the other hand, for those types of government expenditures where congestion effects are severe, the assumption of increasing returns to scale with respect to all inputs can be questioned.

(13.) Note that this must be the case along the steady-estate path under the assumption that, along such a path, the government capital stock grows at a constant rate. To see this, consider the following equation describing the evolution of the public capital stock over time: [K.sub.G,t+1] = (1 - [[delta].sub.G] [K.sub.G,t] + [G.sub.i,t], where [[delta.sub.G] is the depreciation rate on the public capital stock. Divide both sides by [K.sub.G,t] and note that along a steady-estate path [K.sub.G,t+1]/[K.sub.G,t] = 1 + [[gamma].sub.k], where [[gamma].sub.k] is the growth rate of the public capital stock. Then it follows that 1 + [[gamma].sub.k] = (1 - [[delta].sub.G] + [G.sub.i]/[K.sub.G], or [G.sub.i]/[K.sub.G] = [[gamma].sub.k] + [[delta].sub.G] [equivalent to] [.sub.1/m], where [G.sub.i] [K.sub.G] represents the steady-estate value of [G.sub.i,t][K.sub.G,t].

(14.) This assumption about maintaining the ratio of the marginal tax rate to the average tax rate follows that of Stuart (1984) Ahmed and Croushore (1993) show how the MCF is affected by making alternative assumptions about how the tax structure is shifted. Because we are looking at the MCF for government spending, not transfers, we set dR = 0. In earlier versions of this article, we investigated the case where dR [is not equal to] 0; however, the fact that this is a representative agent model makes an evaluation of transfer payments less useful.

(15.) This is the level of total government purchases in 1976, using Stuart's (1984) data source. Stuart actually used a different value (G = 227.7) as he tried to separate different types of government spending into government consumption and transfers. But because of the way the model is parameterized, the choice of the initial level of G does not affect the marginal cost of funds when [theta] = 0 and MPG = 0.

(16.) However, Cushing (1992) finds [theta] to be statistically insignificantly different from zero in his study of borrowing constraints.

(17.) The result that the MCF is $1.077 is based on the same parameter values as Stuart (1984) used and on separable government spending. In particular, the uncompensated labor supply elasticity (e.sub.u) is set to zero. This raises an interesting question. If [e.sub.u] = 0, why do we get any change in the labor input from raising distortionary taxes and, therefore, why is the MCF anything different from $1, the amount of taxes paid? Fullerton (1991) provides an answer to this question (see also Ballard and Fullerton 1992 and Ballard 1990). Fullerton's article demonstrate that the equilibrium quantity of the labor input can change, even with [e.sub.u] = 0, if the change in taxes causes a change in virtual income. Virtual income consists of nonlabor income plus the lump-sum portion of the tax system that arises from having deductions or credits. For example, if total tax collections can be written as T = T + t'w[L.sub.1], then T is the implicit lump-sum portion of the tax system that enters virtual income. With [e.sub.u] = 0, the substitution and income effects on labor supply from a change in (1 - t')w cancel out, but there is an additional effect through virtual income if T changes. See Ahmed and Croushore (1993) for further details.

(18.) Stuart (1984) examines a one-percentage-point increase in the tax rate and calculates the discrete changes in the variables. We prefer the method of analysis at the margin, which comes from totally differentiating the model's system of equations. Mayshar (1991) also performs marginal analysis, finding MCF = $1.076, which is the same as our result except for rounding.

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## Abstract

Mathematical models have been used to quantify the beneficial effects of public spending funded by disproportionate taxation separate from the benefits of the project being financed. However, if public spending replaces private investment or private use, it is considered nonseparable in applicability or production. A mathematical model has been created to quantify the numerical effects that nonseparabilities linked to government spending have on economic agents' marginal decisions. Results indicate that the nonseparabilities markedly change the marginal cost of funds previous models have quantified.