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I. Introduction

Economists have long been concerned about the best way to finance government deficits. Finding the proper fiscal policy and monetary policy mix is a crucial decision. When government debt grows too fast, interest rates rise and capital is crowded out. If the money growth rate is excessive, inflation occurs.

The study of this issue at the theoretical level requires a model which incorporates the following features: (1) modeling money and bonds as endogenous financial assets, whose rates of return are determined in general equilibrium, (2) examination of the utility maximization decisions of individuals, so that welfare analysis of alternative policies may be made, (3) modeling the government's optimization problem and its budget constraint, and (4) modeling capital investment, showing how the returns to financial assets affect investment decisions. In such a model, the government's financing decisions affect the rates of return on money and bonds, which affect the welfare of individuals. Standard models in the economic literature do not satisfy all these features. The purpose of this paper is to derive such a model.

Macroeconomic models often assume that it is optimal to maximize consumption or output of the economy. This paper shows clearly that such an assumption is false in a dynamic context. Output can be maximized by government policy which drives the interest rate to a low level. But the interest rate affects intertemporal consumption possibilities, and a low interest rate may reduce welfare.

The goal of this paper is thus to examine the optimal growth rates of money and government debt, considering their implications for welfare. Changes in the rates of return on money and debt affect welfare in three ways: (1) they affect the capital stock and thus output, (2) they affect consumption directly because people take out consumption loans, and (3) they affect the quantity of resources devoted to transactions costs.

We develop a model of the financial-asset holding decision. Money and bonds are the two financial assets. Bonds are an explicit contract between two agents in the economy, a contract which is costly to write. Money is issued by the government by fiat, and represents a costless, implicit contract with a zero nominal rate of return.

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Capital in the model is not a store of value, merely a nondurable factor of production.[1] Individuals differ in capital productivity.[2] When people are young, they borrow to buy capital, which increases their productivity in middle age.

In section II, we describe the individual's maximization problem. Individuals face two major decisions—how much capital to purchase and whether to hold money or bonds as a store of value. These decisions depend upon the interest rate, the inflation rate, and the productivity of capital. The rates of return on money and bonds are determined endogenously, depending upon individuals' decisions. Section III describes the aggregate equilibria. In section IV we compare government intervention in the market to laissez-faire, and suggest how government may increase social welfare. A numerical example illustrates the results. The results are interpreted and related to the existing macroeconomic literature in section V.

II. The Individual's Maximization Problem

Capital goods and consumption goods are assumed to be physically identical, merely put to different uses. Capital goods are completely used up in production in one period. The production function is $F(k)$, where k is the amount of capital which a person uses. It is assumed that $F(0) = \alpha > 0$, $F'(0) \geqq 0$, $F''(k) < 0$ for all k, and $\lim_{k\to\infty} F'(k) \leqq 0$.

People live three periods. They borrow when young to buy capital goods and consumption goods. In middle age they produce output using their capital goods and labor. They use either money or bonds as a store of value to finance old-age consumption. When bonds are used as a store of value, transactions costs in the fixed real amount γ are incurred.[3] This cost represents the expense of explicitly writing a debt agreement, which may be interpreted as including expenses such as transportation and time costs of travel to a bank, costs of investigating credit-worthiness, search costs, and the like.

The maximization problem facing the individual is:

$$\max_{c_y, c_m, c_o, k, s, b, l} \quad U = \ln c_y + \delta \ln c_m + \delta^2 \ln c_o$$

subject to: $c_y, c_m, c_o, k, s, b, l \geqq 0$,

$$b/p_y = c_y + k, \tag{1}$$

$$F(k) = (1 + i_y)\, b/p_y + c_m + s/p_m + l/p_m + \gamma \quad \text{if } s > 0, \text{ or} \tag{2a}$$

$$F(k) = (1 + i_y)\, b/p_y + c_m + l/p_m \quad \text{if } s = 0, \tag{2b}$$

$$[1/(1 + \pi_m)]\, l/p_m + (1 + i_m)\, s/p_m = c_o, \tag{3}$$

where ln is the natural logarithm, c_y, c_m, and c_o are consumption while young, middle-aged, and old, k is the size of the capital stock; s and l are the nominal amounts of bonds and money held as stores of value, b is the nominal amount of borrowing, p_y, p_m, and p_o are the

1. It would be simple to incorporate durable capital into the model, but it contributes little and adds additional terms to already-complicated equations.

2. This setup could well describe human capital.

3. The transactions-cost setup is identical to that of Baumol [2] and many others who followed.

price levels facing the individual in successive periods. The gross real return on bonds is $1 + i_t (t=y,m)$ while the gross real return on money holdings is $1/(1 + \pi_m)$, where π_m is the rate of inflation. In general, $1 + \pi_t = p_{t+1}/p_t$, where t is a time index. Equations (1), (2), and (3) are simply the budget equations facing the individual in each period.

Equation (1) shows that a young person borrows amount b to spend on consumption goods (c_y) and capital goods (k). A middle-aged person chooses how much money (l) and bonds (s) to hold as a store of value for old age, incurring the transactions costs γ in equation (2a) if bonds are held. Equation (2) shows that real income (output) is $F(k)$, and that the principal and interest on borrowing when young are repaid. The real returns to money holdings and bond holdings are used to finance consumption in old age, as shown in equation (3).

The shape of the utility function requires that we get $c_y > 0$, $c_m > 0$, and $c_o > 0$, if utility is not to be driven infinitely negative. Using budget equations (1) and (3), nonnegativity of consumption requires $b > 0$ and either $s > 0$ or $l > 0$. Because of the nonconvexity of transactions costs, the choice between holding bonds and money must be made by comparing utility levels under each. No marginal decision rule can be obtained. Because of the nature of the (nonconvex) budget constraint, money and bonds are never used by the same person. If the marginal conditions on money and bonds are the same, money is used because bonds have a positive fixed-cost component. First, we examine the case where bonds are used.

Utility Maximization Using Bonds

When an individual uses bonds, equation (2a) is used in setting up the Lagrangian. The Kuhn-Tucker condition on k yields:

$$F'(k) \leq 1 + i_y, \tag{4}$$

with equality if $k > 0$. Thus if capital is used, we get the normal result that the marginal product of capital equals the rate of interest. It is easy to show that a person will choose $k = 0$ only if $F'(0) \leq 1 + i_y$. If $F'(0) \leq 1 + i_y$, then since $F''(k) < 0$, we must have $F'(k) < 1 + i_y$ for all $k > 0$. When $F'(0) > 1 + i_y$, the optimal value of k is finite since $\lim_{k\to\infty} F'(k) \leq 0$ and $1 + i_y > 0$.[4]

Figure 1 illustrates the optimal choice of capital. This diagram shows two production functions, F_a and F_b. The marginal product of capital at $k=0$ is greater than the real marginal cost of borrowing ($F'_a(0) > 1 + i_y$), so the optimal capital stock is positive, and occurs at level k^*_a, for the production function F_a. For production function F_b, the marginal product of capital is always less than the real marginal cost of borrowing ($F'_b(0) < 1 + i_y$), so it is not optimal to purchase capital.

A lifetime-budget constraint is derived in present-value terms by combining equations (1), (2a), and (3), eliminating b and s. We define this in terms of the present value at middle age:

$$F(k) - (1 + i_y)k - \gamma = (1 + i_y)c_y + c_m + [1/(1 + i_m)]c_o. \tag{5}$$

The left-hand side of (5) is the present value at middle age of income minus nonconsump-

4. $1 + i_y$ must be positive to have any economic meaning.

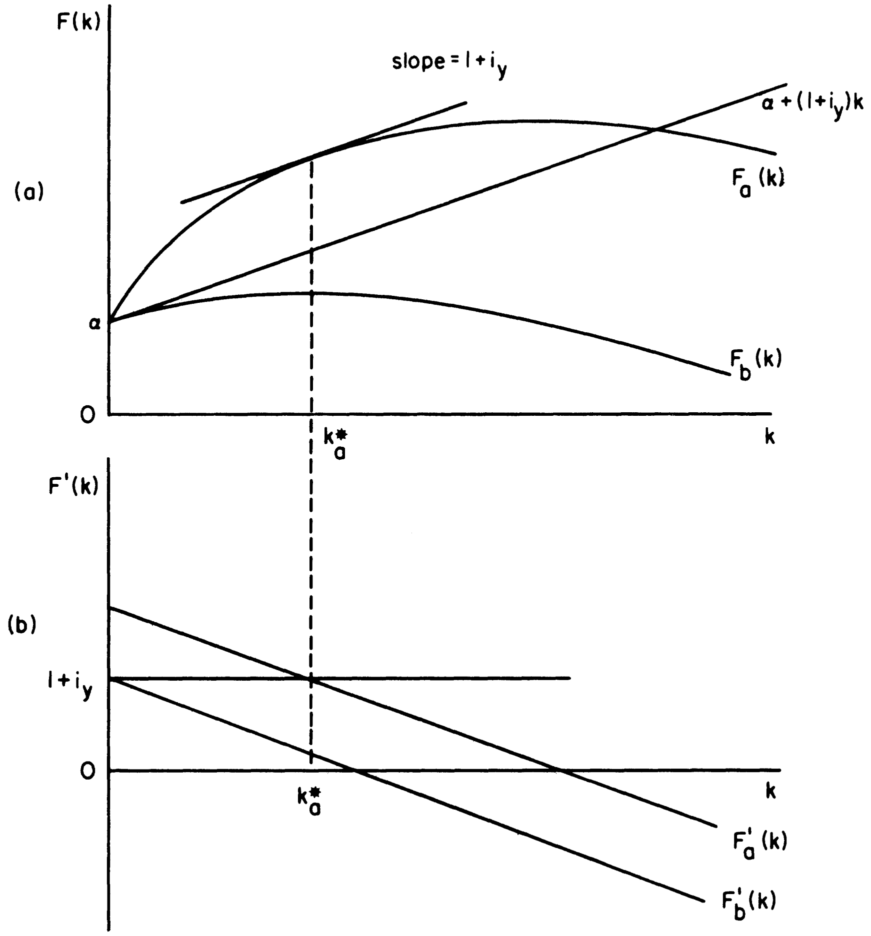

Figure 1

tion expenditures, while the right-hand side shows consumption expenditures. The optimal choice of k maximizes the left-hand side of (5). As long as $F'(k) > 1 + i_y$, an increase in k causes $F(k)$ to rise relative to $(1 + i_y)k$.

Figure 2 illustrates the returns to capital. k^* is the optimal level of capital. Gross returns to capital are $F(k^*) - \alpha$, in units of the consumption good at middle age. The interest cost of capital is $A - \alpha = (1 + i_y)k^*$. Net returns to capital are $F(k^*) - A = F(k^*) - \alpha - (1 + i_y)k^*$.

We now consider the production function given by:

$$F(k) = \alpha + \psi k - (\beta/2)k^2, \tag{6}$$

where $\alpha > 0$, $\psi \geq 0$, $\beta > 0$.

This function has the desired properties: (a) $F(0) = \alpha > 0$; (b) $F'(0) = \psi \geqq 0$; (c) $F''(k) = -\beta < 0$; (d) $\lim_{k\to\infty} F'(k) = \psi - \beta k \leq 0$. The marginal product of capital is:

$$F'(k) = \psi - \beta k. \tag{7}$$

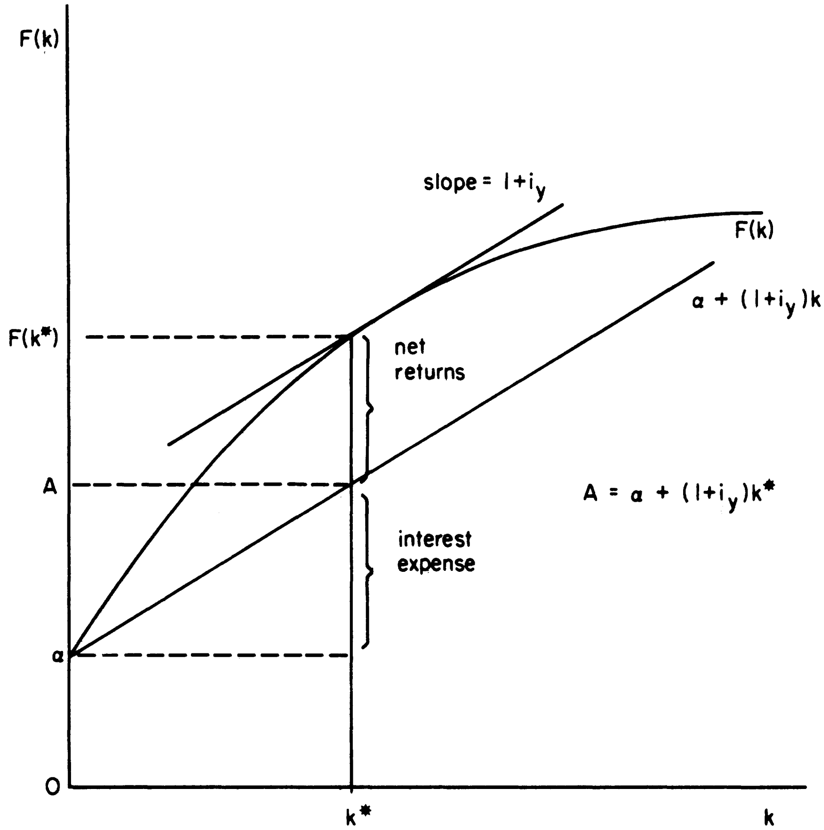

Figure 2

Since $F'(0) = \psi$, then $k > 0$ if and only if $\psi > 1 + i_y$. Using (7) in (4), the optimal capital purchase is

$$k = [\psi - (1 + i_y)]/\beta \text{ for } \psi \geq 1 + i_y. \tag{8}$$

As expected, $\partial k/\partial(1 + i_y) < 0$, so the capital stock falls as the interest rate rises. Given this capital purchase, output is:

$$F(k) = \alpha + (1/2\beta)\,[\psi^2 - (1 + i_y)^2]. \tag{9}$$

The net return to capital, as shown in Figure 2, is given by:

$$F(k) - \alpha - (1 + i_y)k = (1/2\beta)\,[\psi - (1 + i_y)]^2. \tag{10}$$

We now summarize the solution to the individual's choice problem:

a. When $\psi > 1 + i_y$, then $k > 0$, and:

$$c_y = [1/(1 + \delta + \delta^2)][1/(1 + i_y)]\,\{\alpha + (1/2\beta)[\psi - (1 + i_y)]^2 - \gamma\}, \tag{11}$$

$$\begin{aligned} b/p_y = {} & [1/(1 + \delta + \delta^2)]\,\{[1/(1 + i_y)]\,(\alpha + (1/2\beta)\psi^2 - \gamma) \\ & - (1/2\beta)(1 + i_y) + (\delta + \delta^2)(1/\beta)[\psi - (1 + i_y)]\}, \end{aligned} \tag{12}$$

$$s/p_m = [\delta^2/(1+\delta+\delta^2)]\,\{\alpha + (1/2\beta)\,[\psi - (1+i_y)]^2 - \gamma\}. \tag{13}$$

b. When $\psi \leq 1 + i_y$, then $k = 0$ and $F(k) = \alpha$, so:

$$c_y = b/p_y = [1/(1+\delta+\delta^2)](1/(1+i_y)]\,(\alpha - \gamma), \tag{14}$$

$$s/p_m = [\delta^2/(1+\delta+\delta^2)]\,(\alpha - \gamma). \tag{15}$$

In both cases (when $k > 0$ and when $k = 0$) consumption in middle age and old age is:

$$c_m = \delta(1+i_y)c_y, \tag{16}$$

$$c_o = \delta(1+i_m)c_m. \tag{17}$$

Utility Maximization Using Money

When money is used as a store of value, equation (2b) is used in setting up the Lagrangian. Conditions for optimal k are unaffected by the choice between money and bonds. The solution to the individual's choice problem is now modified:

a. When $\psi > 1 + i_y$, then $k > 0$, and:

$$c_y = [1/(1+\delta+\delta^2)][1/(1+i_y)]\,\{\alpha + (1/2\beta)\,[\psi - (1+i_y)]^2\}, \tag{18}$$

$$b/p_y = [1/(1+\delta+\delta^2)]\,\{[1/(1+i_y)]\,[\alpha + (1/2\beta)\psi^2] - (1/2\beta)(1+i_y) + (\delta+\delta^2)(1/\beta)\,[\psi - (1+i_y)]\}, \tag{19}$$

$$l/p_m = [\delta^2/(1+\delta+\delta^2)]\,\{\alpha + (1/2\beta)[\psi - (1+i_y)]^2\}. \tag{20}$$

b. When $\psi \leq 1 + i_y$, then $k = 0$ and $F(k) = \alpha$, so:

$$c_y = b/p_y = [1/(1+\delta+\delta^2)][1/(1+i_y)]\,\alpha, \tag{21}$$

$$l/p_m = [\delta^2/(1+\delta+\delta^2)]\,\alpha. \tag{22}$$

In both cases, consumption in middle age and old age is:

$$c_m = \delta(1+i_y)c_y, \tag{23}$$

$$c_o = \delta[1/(1+\pi_m)]\,c_m. \tag{24}$$

The effect on the lifetime consumption stream of the choice between money and bonds can be seen by comparing equations (11) to (18), (16) to (23), and (17) to (24). Consumption while young and middle-aged is lower when bonds are used as a store of value, due to the added term $-\gamma$ in equation (11) which does not appear in (18). However, this is offset by higher consumption in old age, when the return to bonds $(1 + i_m)$ in equation (17) exceeds the return to money $[1/(1 + \pi_m)]$ in equation (24). The same effect can be seen when capital isn't used by comparing equation (14) to equation (21).

The impact of the use of capital can be seen by comparing equations (11) to (14) and (18) to (21). Consumption throughout the lifetime is clearly higher when capital is used.

We have completely characterized the individual's choice problem in this economy. Our next step is to examine the macroeconomy.

III. Aggregate Equilibrium

We now partition individuals into four classes, based on their choices of capital purchases and whether they use bonds or money as a store of value. These classes are denoted:

I. Capital is purchased and bonds are used
$$(k > 0, s > 0, l = 0),$$
II. Capital is purchased and money is used
$$(k > 0, s = 0, l > 0),$$
III. Capital is not purchased and bonds are used
$$(k = 0, s > 0, l = 0),$$
IV. Capital is not purchased and money is used
$$(k = 0, s = 0, l > 0).$$

The aggregate equilibrium depends upon the proportion of the population in each class (I. $k > 0$, $s > 0$; II. $k > 0$, $s = 0$; III. $k = 0$, $s > 0$; IV. $k = 0$, $s = 0$). The equilibrium interest-rate and inflation-rate sequences both affect and are affected by the number of people in each class. Consequently, the equilibrium values of the endogenous variables are difficult to find.

To keep things fairly simple, we assume that there are two types of people, rich and poor, who are distinguished by the term ψ in the production function (6).[5] This term is the marginal productivity of capital at $k = 0$. The well-endowed rich have ψ_r and the poor have ψ_p, where $\psi_r > \psi_p$.

Three patterns of the bond-money choice are possible: (1) the all-bond equilibrium in which both rich and poor use bonds; (2) the mixed-bond equilibrium in which the rich use bonds and the poor use money; and (3) the all-money equilibrium in which everyone uses money as a store of value.[6] Similarly, three possibilities for capital exist: (1) an all-capital equilibrium in which everyone uses some capital; (2) a mixed-capital equilibrium in which the rich use capital while the poor do not; and (3) a no-capital equilibrium. Thus there are nine possible equilibria, depending on the bond-money choice and the capital choice. By changing the parameters of the model $(\alpha, \psi_r, \psi_p, \gamma)$ and depending on the government's role in the economy (to be described later), any one of the nine equilibria can be reached, except for the mixed-bond, no-capital equilibrium.[7]

We denote the population of each generation born at time t of the rich and poor as $N_r(t)$ and $N_p(t)$. We assume that both groups grow at rate n, so that we have an overlapping-generations model with a growing population.

$$N_r(t) = (1 + n)N_r(t - 1), \qquad N_p(t) = (1 + n)N_p(t - 1). \tag{25}$$

5. Sargent and Wallace [7] examine a similar rich-poor setup. This allows people who have differing endowments to demand different types of financial assets.

6. The solutions to the individual's maximization problem preclude the possibility of a reverse-mixed equilibrium in which the rich hold money while the poor hold bonds.

7. The mixed-bond, no-capital equilibrium can't be reached because if neither rich nor poor use capital, both types of people produce the same output, so both groups use the same store of value. We assume that the parameters are such that we never reach an "indifference equilibrium" in which some group is just indifferent to holding either bonds or money, so that some fraction of the group holds each asset.

We assume that the government's role is to maximize the social welfare of each generation while treating each member of each type (rich, poor) equally across generations. Thus the government seeks a stationary solution with $U^r(t) = U^r$ and $U^p(t) = U^p$ for all t. We assume further that government policy is distribution-neutral, so that social-welfare weights are assigned proportional to the output a person produces. Thus social welfare at time t is a monotonic transformation of the stationary function:

$$W = \phi U^r + (1 - \phi)(N_p/N_r)\, U^p, \tag{26}$$

where $\phi = F^r(k^*)/[F^r(k^*) + F^p(k^*)]$, and $N_p/N_r = N_p(t)/N_r(t)$ for all t, from equation (25). The term k^* is the level of capital chosen in the absence of all frictions in the model at a Pareto-optimal equilibrium.[8]

The government maximizes social welfare subject to its budget constraint, given by:

$$\begin{aligned} G(t) + [1 + R(t-1)]\, \bar{S}^g_b(t-1) + \bar{D}^g_b(t) = \\ \bar{S}^g_b(t) + [1 + R(t-1)]\bar{D}^g_b(t-1) + X(t) + M(t) - M(t-1), \end{aligned} \tag{27}$$

where $1 + R(t-1) = [1 + i(t-1)][1 + \pi(t-1)]$. Government spending $(G(t))$ plus interest and principal payments on bonds sold last period $(\bar{S}^g_b(t-1))$ plus new lending $(\bar{D}^g_b(t))$ is equal to total government outflows of funds. Inflows of funds come from selling bonds $(\bar{S}^g_b(t))$, collecting interest and principal payments on outstanding loans $(\bar{D}^g_b(t-1))$, collecting taxes $(X(t))$, and printing new money $(M(t) - M(t-1))$. For simplicity, we assume that no taxes are collected and that the sole government spending item is transactions costs on bond purchases and sales (described below). Since the government can borrow or lend, and these are symmetric in the government budget constraint, we define the variable $D^g_b \equiv \bar{D}^g_b - \bar{S}^g_b$.

We now suppose that the government incurs transactions costs whenever it exchanges bonds and money in the market. The transactions costs are a marginal cost of $\theta(|\bar{D}^g_b(t)| + |\bar{S}^g_b(t)|)$, where $\theta \geq 0$. Under this cost setup, the government either borrows or lends, not both. With the structure of the model described above, it is always optimal for the government to set $D^g_b(t) > 0$. Thus $G(t) = \theta D^g_b(t)$.

A stationary solution in the money and bond markets is one for which $1 + i_t = 1 + i$ and $1 + \pi_t = 1 + \pi$ for all t. These solutions can only be reached in this model when real government debt and monetization each grow at the rate of population growth, so that $D^g_b(t)/N_r(t)p(t) = D^g_b/N_r p$ and $M(t)/N_r(t)p(t) = M/N_r p$ for all t. Using these expressions, equation (27) can be rewritten as:

$$[(1 + \theta)(1 + n) - (1 + i)]\, D^g_b/N_r p = [(1 + n) - (1/(1 + \pi))]\, M/N_r p. \tag{28}$$

The aggregate variables are as follows:

a. Aggregate use of goods in transactions:

$$L(t) = Z_b^r N_r(t-1)\gamma p(t) + Z_b^p N_p(t-1)\gamma p(t) + \theta D^g_b(t), \tag{29}$$

where $Z_b^r\,(Z_b^p)$ equals 1 if the rich (poor) use bonds and equals 0 if they use money as a store of value.

8. Of course, the results of this paper vary quantitatively with the weights chosen. We examine here only one possible set of weights—that set which is "distribution-neutral." The weight ϕ could be adjusted easily to examine the implications of redistributional efforts. Such an investigation is beyond the scope of this paper.

b. Aggregate demand for bonds:

$$D_b(t) = Z_b^r N_r(t-1)s^r(t) + Z_b^p N_p(t-1)s^p(t) + D_b^g(t). \tag{30}$$

c. Aggregate supply of bonds:

$$S_b(t) = N_r(t)b^r(t) + N_p(t)b^p(t). \tag{31}$$

d. Aggregate demand for money:

$$D_m(t) = (1 - Z_b^r)N_r(t-1)l^r(t) + (1 - Z_b^p)N_p(t-1)l^p(t). \tag{32}$$

e. Aggregate supply of money:

$$S_m(t) = D_m(t-1) + [M(t) - M(t-1)]. \tag{33}$$

The real interest rate which clears the bond market can be found by setting $D_b(t) = S_b(t)$, substituting equations to eliminate all choice variables, and solving algebraically. The result is the following cubic expression:

$$\begin{aligned}
&\{[1/(1+n)]\,\delta^2(1/2\beta)\,[Z_b^r\,Z_k^r + Z_b^p\,Z_k^p\,(N_p/N_r)]\}(1+i)^3\\
&+ \{(1/\beta)(1/2+\delta+\delta^2)[Z_k^r + Z_k^p\,N_p/N_r] - [1/(1+n)]\,\delta^2(1/\beta)[Z_k^r\,Z_b^r\,\psi_r\\
&+ Z_k^p\,Z_b^p\,(N_p/N_r)\psi_p]\}(1+i)^2 + \{[1/(1+n)]\delta^2[(Z_b^r + Z_b^p\,N_p/N_r)(\alpha-\gamma)\\
&+ (1/2\beta)(Z_k^r\,Z_b^r\,\psi_r^2 + Z_k^p\,Z_b^p\,(N_p/N_r)\psi_p^2)] + (1+\delta+\delta^2)(1 - Z_b^r\,Z_b^p)D_b^g/N_r p\\
&- (\delta+\delta^2)(1/\beta)(Z_k^r\psi_r + Z_k^p\,(N_p/N_r)\psi_p)\}(1+i) + \{\gamma[Z_b^r + Z_b^p\,N_p/N_r]\\
&- (1 + N_p/N_r)\alpha - (1/2\beta)[Z_k^r\psi_r^2 + Z_k^p\,(N_p/N_r)\psi_p^2]\} = 0,
\end{aligned} \tag{34}$$

where Z_k^r (Z_k^p) equals 1 if the rich (poor) have a positive capital stock, and equals 0 if their capital stock is zero. Equation (34) is not really as complex as it first appears. The Z terms allow us to pack all eight possible equilibria into one expression. For some equilibria, this equation simplifies considerably.

The real return to money which clears the money market is given by:

$$1/(1+\pi) = (1+n)\{1 - [[(1+\theta)(1+n) - (1+i)]/DENOM]\,D_b^g/N_r p\}, \tag{35}$$

$$\begin{aligned}
DENOM = {}&[\delta^2/(1+\delta+\delta)^2]\,\{\alpha[(1-Z_b^r) + (1-Z_b^p)N_p/N_r]\\
&+ (1/2\beta)[Z_k^r(1-Z_b^r)(\psi_r - (1+i))^2\\
&+ Z_k^p(1-Z_b^p)(\psi_p - (1+i))^2]\}.
\end{aligned} \tag{36}$$

Again, this expression represents the general case for all eight possible equilibria, but simplifies considerably for any particular equilibrium. The inflation rate is undefined if no group uses money as a store of value.

IV. The Government's Optimization Problem

In this section we compare optimal government intervention in the bond market to laissez-faire.[9] First, we examine the laissez-faire case when the government does not intervene in

9. By "intervention" we mean a case in which the government exchanges bonds for money or money for bonds in

Table I. Equilibrium Values of Variables

	a	b	c	d
θ	—	0	.05	.1
Equilibrium $(Z_b^r, Z_b^g, Z_k^r, Z_k^p)$	1,0,1,0	0,0,1,1	0,0,1,1	1,0,1,1
$D_b^g/N_r p$	0	7.87	7.52	2.06
$1+i$	2.42	1.50	1.57	1.62
$1/(1+\pi)$	1.50	1.50	1.50	1.27
L/N_r	.333	0	.376	.540
W	5.03	5.27	5.21	5.19
k^r	.952	.970	.969	.968
F^r	27.9	28.0	28.0	28.0
s^r/p	7.51	0	0	7.74
l^r/p	0	7.93	7.91	0
U^r	5.60	5.84	5.78	5.76
k^p	0	.0100	.00850	.00767
F^p	3	3.02	3.02	3.01
s^p/p	0	0	0	0
l^p/p	.897	.897	.897	.897
U^p	−.547	−.0648	−.114	−.275
$Y = F^r + (N_p/N_r)\,F^p$	29.441	29.486	29.483	29.481

the bond market. Next, we show that welfare can be improved by government bond purchases which reduce the interest rate, when the government faces no transactions costs.[10] However, as the transactions costs facing the government rise, the welfare gains from intervention fall. Finally, we compare the above results to a situation in which the government intervenes in the market not to maximize social welfare, but to maximize the total output of the economy.

Equilibrium When Government Does Not Intervene in the Bond Market

If the government does not intervene in the bond market, the nominal money supply remains constant, and there occurs a deflation at the rate of population growth:

$$1/(1+\pi) = 1 + n. \tag{37}$$

The real interest rate is determined by equation (34) with $D_b^g/N_r p = 0$. Because of the complications of examining all eight possible equilibria analytically, we instead look at a numerical example to illustrate the results of the model. We let the parameters take the following values: $\delta = .9$, $\gamma = .5$, $n = .5$, $N_p/N_r = .5$, $\alpha = 3$, $\beta = 50$, $\psi_r = 50$, $\psi_p = 2$.

Given these parameters, the laissez-faire equilibrium is shown in Table I, column a. The economy is in a mixed-bond, mixed-capital equilibrium, the rich using bonds and holding a positive capital stock, the poor using money as a store of value and not using any capital.

the market. This is termed "active" government policy. If the government does not intervene in the market, then it neither changes the money supply nor exchanges any money for bonds.

10. It should be noted that the welfare comparisons being made here are comparisons of steady states. They are not valid for evaluating a change in policy from an existing policy to a new one, because the non-steady state welfare is not analyzed. The welfare comparisons in the paper may be thought of in the following sense: given a known economic structure, what is the optimal government policy which should be followed from the beginning of time?

In this equilibrium, the real interest rate is over 100%. The demand for borrowing is very high, since people wish to borrow both for consumption and for capital formation. A high real interest rate is necessary to equate the demand and supply for bonds.

The rich find it profitable to utilize much more capital than the poor. A considerable difference in output is apparent, with the rich producing 27.9 units and the poor producing only 3 units.

We now show that government intervention may be optimal by changing the interest rate and inflation rate to maximize social welfare.

Equilibrium When Government Intervenes to Maximize Social Welfare

When the government tries to maximize social welfare, it must satisfy its budget constraint and achieve a stationary equilibrium which is stable. This can be done by finding the optimal money supply and government bond level for each of the eight possible equilibria. The equilibrium with the highest social-welfare value is then chosen. Finally, the stability of the equilibrium must be verified.

In the numerical example, the optimal government intervention is for the government to lend enough that it eliminates the use of bonds as a store of value, when there exist no transactions costs ($\theta = 0$). The all-money, all-capital equilibrium can be reached with $D_b^g / N_r p = 7.87$, as shown in Table I, column b.

A Pareto-improvement occurs relative to the situation in which the government does not intervene (compare columns a and b of Table I). Both the rich and the poor are made better off. The real interest rate falls, to equal the economy's (population) growth rate, with no change in the inflation rate.[11] Output is increased because both rich and poor people purchase more capital than before. Consumption increases because of higher output and because of a reduction in resources used up in transactions (L / N_r).

The results are due primarily to an intergenerational free lunch. Government intervention provides funds to the bond market, lowering the interest rate significantly. Everyone is made better off due to the perpetual nature of the government loan, which need never be repaid. In a sense, this is a reverse Ponzi scheme, where the government keeps lending more (in real terms) as the population grows. The Pareto improvement occurs because the scheme effectively allows trading between generations, as in many models of overlapping generations.

Equilibrium under Optimal Government Intervention with Transactions Costs

When the government incurs transactions costs, the scope of its optimal intervention in the bond and money markets is reduced. Because the θ term in the government's budget constraint becomes positive, resources must be obtained by the government to pay these transactions costs.

In the numerical example, with $\theta = .05$, the all-money, all-capital equilibrium is still sustainable, but now $D_b^g / N_r p$ is reduced to 7.52. The results are shown in Table I, column c.

11. This result is identical to those in the literature on optimal growth with money—a golden "financial asset" rule is followed as in Tobin [8], Friedman [5] and Bewley [3]. The inflation rate does not change in this case, because while the money stock has risen, so has money demand. Rich people switch from holding bonds as a store of value to holding money.

Compared with the case of no transactions costs (column b), this equilibrium has a higher interest rate, hence less capital and less output. Everyone is worse off.

Notice that in comparison with the laissez-faire equilibrium (column a), everyone is better off. Output is higher because the reduced real interest rate has led to increased capital use. But transactions costs eat up a lot of output. In fact, transactions costs (L/N_r) are higher here than in the laissez-faire equilibrium, so that aggregate consumption is reduced. The reduction in consumption is overcome by the better intertemporal tradeoff of consumption possibilities, and a Pareto improvement occurs.

As the transactions costs faced by the government rise, intervention becomes less valuable. As these costs rise, optimal government policy moves closer to laissez-faire, and the benefits from the intergenerational free lunch decline.

This is illustrated in column d of Table I, where $\theta = .1$ (i.e., 10% of the funds government lends must go to transactions costs). The equilibrium becomes one in which the rich switch to using bonds as a store of value, so that we have a mixed-bond, all-capital equilibrium. Optimal government intervention is reduced substantially to $D_b^g/N_r p = 2.06$. This causes the real interest rate to rise while the real return to money falls.

Maximization of Output

Governments sometimes follow a policy of maximizing their nation's gross national product (GNP) or some other measure of economic output. An increase in output is obtainable in this model, but only at the cost of decreased social welfare.

For example, when there are no transactions costs on government intervention, output (Y) can be increased from 29.486, as in column b of Table I, to 29.493 in the all-money, all-capital equilibrium. This is accomplished by an increase in government lending from 7.87 to 8.69, causing the real interest rate to fall from .50 to .35. Capital usage is increased by both rich and poor.

The amount of goods available for consumption increases more than the transactions costs of generating additional output. Despite the increase in consumption, social welfare falls. The changes in the real interest rate and the inflation rate change people's intertemporal consumption possibilities, making them worse off.

The implication of this result is clear—the government should *not* drive the interest rate as low as possible in order to increase investment, national product, and consumption. It must trade off the gains from additional output with the loss in terms of intertemporal tradeoffs available in consumption.

V. Interpretation and Conclusions

The major result of this paper is to demonstrate theoretically that the government's entry into capital markets affects social welfare in three ways: (1) by changing the amount of resources used up in transactions; (2) by modifying the equilibrium interest rate and inflation rate, transforming people's intertemporal consumption paths (potentially, as in this paper, yielding a Pareto-improvement); and (3) by affecting the utilization of capital in the economy, thus affecting output. Further, we see that it is not optimal to maximize total output.

These results are obtained in a fairly simple model in which there is neither government spending on goods and services nor taxation. The optima found in the paper are generally second-best, given the existence of transactions costs. If lump-sum taxation and transfers are available to the government, first-best optima are obtainable. Also, the result that it is optimal for the government to lend to, rather than borrow from, the private sector, could be reversed if the government has some spending to finance. Thus the results of this paper are only suggestive of some of the important considerations in a more complete model which incorporates government spending and taxation. Such a model is a logical extension of the model of this paper. It seems likely to yield the result that government spending can be financed partially by taxation, partly by borrowing, and partly by printing money, with the relative shares of each method of finance being determined by (1) transactions costs, (2) intertemporal consumption tradeoffs, and (3) the optimal capital stock.

It is important to realize that in this model, the government does far more than just reducing aggregate transactions costs. In fact, as the numerical examples illustrate, it may be optimal for government intervention to cause resources spent on transactions costs to increase. This result is in contrast to that of Bryant and Wallace [4] in which bond finance is inefficient relative to money finance because it raises transactions costs. Similarly, some economists argue that government debt should be increased whenever people face liquidity constraints (including cases as in this paper, where the rates of return on borrowing and lending differ, due to transactions costs). The results of this paper show that while this should be done, there is a limit to the amount of government intervention which is optimal.

The focus of this paper is strictly long run. An interesting extension would be to incorporate a business cycle into the model, examine the role of liquidity constraints in such a model, and see how optimal government policy would vary over the cycle. Also, for both long-run and short-run analyses, it may be interesting to examine the role of technological change (changes in the production function over time) and change in the transactions technology γ (for example, due to computerization in the financial-services industry).

The results of this paper are relevant to the empirical testing of the effects of government debt and monetization on interest rates. The costs of trading debt for money and the costs of servicing debt are likely to be important. This suggests that different types of debt ought to be treated differently, for example Treasury debt versus the debt of the Social-Security system.

Finally, the Ricardian Equivalence Theorem does not hold in this model. We are examining a non-Ricardian regime, one in which there are no future tax liabilities implied by the existence of current government debt.[12] Consequently, changes in government debt affect interest rates.

12. The importance of the distinction between Ricardian and non-Ricardian regimes has been made by Sargent [6] and Aiyagari and Gertler [1].

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