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Articles

THE IMPORTANCE OF THE TAX SYSTEM IN DETERMINING THE MARGINAL COST OF FUNDS*

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This paper examines the effect on the marginal cost of public funds of two alternative ways in which the tax schedule can be altered: one that maintains the progressivity of the tax schedule and another that rotates the tax schedule. We calculate values of these marginal-cost-of-funds concepts for plausible ranges of key parameters. Our results point to the considerable importance of specifying the exact manner in which the tax schedule is altered when calculating the marginal cost of public funds.

I. INTRODUCTION

In evaluating the costs and benefits of public expenditure, the costs are calculated using the concept of the marginal cost of funds *(MCF)*. Many recent studies, including Wildasin [1984], Browning [1987], and Stuart [1984], have provided numerical calculations of the *MCF* for balanced-budget changes in taxes under a variety of assumptions. Stuart's paper is typical of general-equilibrium models in this area in that *simulation* methods are used in calculations of the *MCF*. More recently, Mayshar [1991] has derived *analytic* expressions for the *MCF* in a general-equilibrium model of the type that Stuart used, in order to isolate which parameters are crucial and which parameters wash away.

* The views expressed in this paper are those of the authors and do not nesessarily reflect those of the Federal Reserve Bank of Philadelphia or of the Board of Governors of the Federal Reserve System or any other members of its staff.

Generally, the literature has considered tax experiments that either assume proportional taxes (so that the marginal and average tax rates are one and the same) or maintain the degree of progressivity of the tax system (defined as the ratio of the marginal tax rate to the average tax rate).[1] The purpose of this note is to show that this assumption is crucial. We show the different ways in which the tax schedule can affect the *MCF*. We also provide some new results that have not been noted in the previous literature.

Our general result is that the *MCF* depends more on how the tax schedule is altered than it does on changes in the values of other parameters (such as the initial levels of the marginal and average tax rates). This is important because the literature has often focused on these other parameters, without discussing how the tax schedule is altered. Specifically, we illustrate this point by considering a rotation of the tax schedule (a rise in the marginal tax rate without changing the intercept of the tax schedule), as an alternative to maintaining progressivity.

Our main specific result is that if the uncompensated labor-supply elasticity is zero and if the tax schedule is rotated as noted above, then our formula shows clearly that the *MCF must* be unity, irrespective of the values of other parameters of the model. We consider this result important because previous studies of the *MCF* have considered the situation in which there is a zero uncompensated labor-supply elasticity as a useful benchmark case. Therefore, the sensitivity analyses in those studies depend heavily on the assumption that the progressivity of the tax system is maintained.

II. ANALYTICAL FRAMEWORK

The government's total tax revenue, R, is given by:

$$R = T + mwN \tag{1}$$

so that only labor earnings (the product of the wage rate, w, and the labor input, N) are taxed, at marginal tax rate m. The parameter T is the lump-sum tax amount implicit in the tax system.[2] For example, suppose the tax system is set up with some amount of deductions from income (D) and some tax exemptions (E), so that $R = m(wN - D) - E$. Then $T = -(mD + E)$ puts this tax system in the form of eq. (1). The average tax rate is $t = R/wN$.

We focus on two alternative balanced-budget tax experiments below:

(I) A change in the marginal tax rate that maintains progressivity, implying that dT is whatever is necessary to make $d(m/t) = 0$.

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1 Exceptions are Wildasin [1984] and Browning [1987], who discuss the sensitivity of calculations of the *MCF* to different ways that the tax schedule can be altered.

2 As our model looks only at the short run in which capital doesn't vary, we can't examine the incidence of taxation as in Latham and Naisbitt [1986].

(II) A change in the marginal tax rate that rotates the tax schedule with no change in T.

While some general tax changes may be thought of as maintaining progressivity, others may reduce or raise progressivity. For a single-bracket income tax, a combination of experiments (I) and (II) can provide any potential change in the tax system. Tax experiment (I) alone, which has typically been used in this literature, is not sufficient to characterize tax changes like that in the U.S. in 1986, when the marginal tax rate was reduced without changing the average tax rate significantly.

Consider a representative agent who allocates her given total endowment of time between leisure (L), and labor (N).[3] Her utility function is given by:

$$U = U(C, L, G) \tag{2}$$

where C is consumption and G is government purchases of goods and services. She maximizes utility subject to a budget constraint:

$$wN + I + S = C + R \tag{3}$$

where I is non-labor income, and S is a lump-sum transfer from the government. The agent's virtual income, Z, is the component of her after-tax income that is unaffected by her labor-supply choice, so $Z = I + S - T$. The government must balance its budget, so $G + S = R$.

If the general-equilibrium response of labor following a tax change is dN, the marginal cost of funds is given by:

$$MCF \equiv 1 - mw\frac{dN}{dR}. \tag{4}$$

As Fullerton [1991] notes, the *MCF* is the change in welfare, evaluated in consumption units, per dollar of revenue raised.[4]

To calculate *MCF,* we assume that the entire marginal revenue raised from a change in the tax rate is spent on government goods and services, so that $dR = dG$. We also make the simplifying assumption shared by the bulk of the literature that G does not affect the tradeoff between consumption and leisure.[5]

3 The model we use is quite similar to that of Mayshar [1991]. Because the model is one of a representative consumer, we can't deal with the marginal cost of funds for redistribution; for an analysis of that, see Ballard [1991].

4 Because our analysis takes place entirely at the margin, we needn't be concerned with the differences between compensating and equivalent variation, as discussed by Pauwels [1986].

5 If G affects the leisure-consumption choice, numerical calculations of the *MCF* may change substantially; see Ahmed and Croushore [1996].

III. FORMULAS FOR THE MARGINAL COST OF FUNDS

Let *MCFM* denote the marginal cost of funds when the tax schedule is shifted so as to "Maintain" progressivity and *MCFR* denote it when "Rotating" the schedule. Our analytical expressions for the *MCF* for the two cases are obtained by totally differentiating the first-order conditions of the representative agents' utility maximization problem, then solving for dN and substituting the results in eq. (4):

$$MCFM = 1 + \frac{\eta + \left(\frac{m}{t} - 1\right)\eta^c}{\frac{1 + \gamma\eta^c}{m} - (1 + \eta)} \tag{5}$$

$$MCFR = 1 + \frac{\eta}{\frac{1 + \gamma\eta^c}{m} - (1 + \eta)} \tag{6}$$

where η and η^c are, respectively, the uncompensated and compensated labor-supply elasticities with respect to the wage rate, and γ is minus the elasticity of the wage rate with respect to labor.[6]

Mayshar [1991] derived a related expression for the *MCF* that is not restricted to particular types of shifts in the tax schedule; however, his *MCF* is expressed in terms of dm/dt. Except in the case where progressivity is maintained (so $d(m/t) = 0$), his formula is less useful than ours because dm/dt is an *endogenous* variable yet to be solved for in Mayshar's formula. Since $m = t - T/wN$, calculating dm/dt depends on general-equilibrium effects on w and N. So dm/dt itself is a function of η, η^c, and other parameters that affect the tax base.[7]

III. A. Comparison of MCFM and MCFR

Our main point can be illustrated by considering eq. (6). *When the tax schedule is rotated, then a zero uncompensated labor-supply elasticity* ($\eta = 0$) *implies that the MCF is 1, irrespective of the values of the other parameters of the model.* This is not the case when the tax schedule is shifted up so as to maintain the initial level of progressivity. For example, taking Mayshar's illustrative parameter values (which in turn are taken from Stuart) of $\eta = 0$, $\eta^c = 0.20$, $m = 0.427$, $t = 0.273$, $\alpha = 0.72$, and constant returns to scale, which implies $\gamma = 1 - \alpha$, the *MCF* calculated from eq. (5) is

6 These formulas differ from those derived by Usher [1984] because we allow taxation to be progressive. The marginal cost of funds depends heavily on the labor-supply elasticities in general equilibrium, as Georgakopoulos [1991] suggests.

7 When progressivity is maintained, our eq. (5) is identical to Mayshar's eq. (3).

MCFM = 1.077, as in Mayshar.[8] With a lower marginal tax rate, $m = 0.33$, and other parameters unchanged, the *MCFR* is still unity, of course, but *MCFM* drops down to 1.019. Thus, with $\eta = 0$, *MCFM* is sensitive to the initial level of the marginal tax rate, while *MCFR* is not. Also, *MCFR* becomes closer to *MCFM* as the tax system becomes less progressive.

The intuition behind the result that *MCFR* is unity when $\eta = 0$ is as follows. If $\eta = 0$, then keeping fixed virtual income $(Z = I - T + S)$, the substitution and wealth effects of a change in the marginal tax rate (m) on labor supply exactly cancel out. This means that labor supply will not change, and hence from Fullerton [1991], which can also be seen from eq. (4) here, *MCF* will differ from unity only if virtual income changes. If the tax schedule is rotated (so $dT = 0$), there are no additional wealth effects arising from a change in virtual income following the change in taxes. Hence there is no effect on the equilibrium quantity of labor, so *MCF* = 1. On the other hand, with progressivity maintained, a change in the marginal tax rate necessitates a change in T, and hence labor supply, and the *MCF* is not equal to unity even with $\eta = 0$.

There are two important practical implications of our main result. First, the results of earlier general-equilibrium studies on sensitivity analysis with respect to how the *MCF* is affected by various parameters (as in Stuart) may depend significantly on the manner in which the tax schedule is shifted. Second, the *MCF* is likely to differ substantially for different types of observed tax rate changes, say the 1986 tax reform compared to the 1982-83 tax cuts in the United States.

Note that our argument does not imply that the marginal cost of funds is unresponsive to labor-supply elasticities. The uncompensated and compensated labor-supply elasticities are crucial determinants of how much labor supply changes, and hence of the *MCF,* following a tax change. For example, if we consider the alternative values of $\eta = 0.2$ and $\eta^c = 0.4$ (vs. $\eta = 0$ and $\eta^c = 0.2$ earlier) considered by Stuart, the marginal costs of funds are *MCFM* = 1.303 (vs. 1.077 earlier) and *MCFR* = 1.142 (vs. 1.000 earlier). So the *MCF* remains sensitive to the labor-supply elasticity. But the point is that the degree to which the *MCF* is affected by changes in any parameters, including tax rates, elasticities, and income shares, depends on the manner in which taxes are changed.

Eqs. (5) and (6) also imply that if we have a progressive tax system $(m > t)$, then *MCFM* > *MCFR*. This result is also quite intuitive. When the tax system is progressive, a shift in the tax schedule that maintains the initial level of progressivity implies that a greater amount of income is subject to distortionary taxation than is the case if the tax schedule is shifted up proportionately.

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8 Mayshar reports a value of 1.076; the difference between this and our 1.077 is due to rounding.

IV. CONCLUSIONS

This paper has developed analytical formulas for the *MCF* for two types of changes in the tax system: a rise in the tax schedule maintaining the initial level of progressivity, which has been standard in most of the literature, and a rotation of the tax schedule. The results indicate that the *MCF* is quite sensitive to the manner in which the tax system is changed. In particular, when we consider the same benchmark case used by many previous studies in which the uncompensated labor-supply elasticity is zero, but rotate the tax schedule, the *MCF must* be unity because, under these conditions, there are exactly offsetting substitution and wealth effects on labor supply.

Our results also indicate that the *MCF* is higher when progressivity is maintained than when the tax schedule is rotated. These results point to the considerable importance of specifying precisely how the tax system is being changed when calculating the marginal cost of funds numerically (or investigating its sensitivity to variations in other parameters), especially for the purpose of subsequent use by researchers and policymakers in benefit-cost analyses. Clearly the marginal cost of funds will differ for different types of tax reforms, say the 1986 tax reform compared to the 1982-1983 tax cuts in the United States.

APPENDIX

This Appendix derives eqs. (4), (5), and (6) in the text.

1. Derivation of eq. (4).

Since all output (Y) is either consumed (C) or spent by government (G), we have:

(A1) $$dC = dY - dG.$$

If the production function is $Y = F(N)$, then $dY = F' dN$. Since the wage rate equals the marginal product of labor, this gives:

(A2) $$dY = w\ dN.$$

(A1) and (A2) imply:

(A3) $$dC = w\ dN - dG.$$

Now, since a change in labor is offset by a change in leisure, we get:

(A4) $$dN = -dL.$$

Define the compensating surplus (following Stuart) as the amount of consumption needed to be given to the consumer such that the change in utility is zero, so:

(A5) $$dU = 0 = U_1(dC + CS) + U_2 dL.$$

The first-order conditions of the agent's maximization problem gives the usual marginal utility relationship:

(A6) $$\frac{U_2}{U_1} = (1 - m)w.$$

Since the government budget must balance, $dR = dG$. Using this in (A3) gives:

(A7) $$dC = w\ dN - dR.$$

From (A5), using (A4), we get:

(A8) $$CS = \frac{U_2}{U_1} dN - dC.$$

Using (A6) and (A7) in (A8) gives:

(A9) $$\frac{CS}{dR} = 1 - mw\frac{dN}{dR}.$$

With *MCF* defined as CS/dR, this is eq. (4) in the text.

2. Derivation of eqs. (5) and (6).

First, use the Slutsky equation, following Mayshar (1991) to get:

(A10) $$(1 - m)(1 + \gamma\eta)dN = -\eta N\ dm - (\eta^c - \eta)dZ/w$$

where $Z = I + S - T$ and $\gamma = -(dw/w)/(dN/N)$, so $d(wN) = (1 - \gamma)wdN$.

[To derive this, start with the Slutsky equation applied to labor supply: where $N = N[w(1 - m), Z]$; the Slutsky equation is $\eta^c = \eta - w(1 - m)N_2$. Totally differentiating the expression for N gives $dN = N_1 d[w(1-m)] + N_2 dZ$; where $d[w(1-m)] = (1-m)dw - wdm$. Multiplying this expression through by $w(1-m)$ and using the definitions of η^c and η gives: $w(1-m)dN = \eta N(1-m)dw - \eta Nwdm - (\eta^c - \eta)$ $N_2 dZ$. Now use the definition of γ to simplify this expression to (A10).]

From the production function $Y = f(\bar{K}, N)$, with capital ($\bar{K}$) fixed in the short run, and assuming competitive labor markets so that $w = f_N(\bar{K}, N)$, we get $dY = wdN$. From differentiating the expression for total income ($Y = wN + I$), $dY = d(wN) + dI$, so $dI = \gamma wdN$.

Totally differentiating the expression for virtual income ($Z = I + S - T$), rearranging terms, and using $dI = \gamma wdN$ gives:

(A11) $$dZ = \gamma w\ dN - dT.$$

Rotating the Tax Schedule

Differentiating eq. (1) in the text gives $dR = dT + m(1 - \gamma)wdN + wNdm$. Totally differentiating the definition of the average tax rate ($t = R/wN$) gives:

(A12) $$dR = t(1 - \gamma)w\ dN + wN\ dt.$$

Substituting (A12) into the total derivative of (1) gives $dT = (t - m)(1 - \gamma)wdN + wN(dt - dm)$. When the tax schedule rotates, $dT = 0$, so that:

$$dm - dt = -(m - t)(1 - \gamma)/N\ dN. \tag{A13}$$

Since $dT = 0$, (A11) simplifies to:

$$dZ = \gamma w\ dN. \tag{A14}$$

Solving (A13) for dm, and plugging this and (A14) into (A10), gives:

$$dt = -[1 - (1 + \eta)m + (1 - \gamma)\eta t + \eta^c\gamma]/\eta N\ dN \tag{A15}$$

if $\eta \neq 0$. If $\eta = 0$, then $dN = 0$, so $MCFR = 1$.

Using (A15) in (A12) gives $dN/dR = -\eta/w[1 - (1 + \eta)m + \eta^c\gamma]$. Using this in eq. (4) in the text gives the following result, which is identical to eq. (6):

$$MCFR = 1 + \frac{m\eta}{[1 - (1 + \eta)m + \eta^c\gamma]} \tag{A16}$$

Maintaining Progressivity

When progressivity is maintained, $d(m/t) = 0$, so $dm = m/t\ dt$, and:

$$dm - dt = \left(\frac{m}{t} - 1\right) dt. \tag{A17}$$

Using (A17) in the expression for dT and plugging this into (A11) gives:

$$dZ = [\gamma + (m - t)(1 - \gamma)]w\ dN + \left(\frac{m}{t} - 1\right) wN\ dt. \tag{A18}$$

Using (A17) and (A18) in (A10) allows us to solve for dt in terms of dN:

$$dt = -\frac{1 - (1 + \eta)m + (1 - \gamma)\eta t + \eta^c[\gamma + (m - t)(1 - \gamma)]}{[\eta + \left(\frac{m}{t} - 1\right) \eta^c]N} dN \tag{A19}$$

if $\eta + (m/t - 1)\eta^c \neq 0$. If $\eta + (m/t - 1)\eta^c = 0$, then $dN = 0$, so $MCFM = 1$.

Using (A12) in (A19) gives $dN/dR = -[\eta + \eta^c(m/t - 1)]/[1 - (1 + \eta)m + \gamma\eta^c]w$. This can be used in eq. (4) to get eq. (5).

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