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# Movement Variability and the Use of Nonlinear Tools: Principles to Guide Physical Therapist Practice

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1 TITLE: Perspective on Movement Variability and the Use of Nonlinear Tools: Principles to  
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## Abstract

Fields studying movement generation, including robotics, psychology, cognitive science and neuroscience utilize concepts and tools related to the pervasiveness of variability in biological systems. The concepts of variability and complexity, and the nonlinear tools used to measure these concepts open new vistas for physical therapy practice and research in movement dysfunction of all types. Because mounting evidence supports the necessity of variability for health and functional movement, we argue in this perspective for changes in the way therapists view variability both in theory and in action. By providing clinical examples, as well as applying existing knowledge about complex systems, we hope to create a springboard for new directions in physical therapy research and practice.

Keywords: Variability, nonlinear tools, complexity

## Introduction

1  
2           Variability in human performance, and the nonlinear manner in which skills and  
3 characteristics of movement change over time, reflects the complexity of the movement system.  
4 As Bernstein<sup>1</sup> described, multiple degrees of freedom of the body including joints, muscles, and  
5 the nervous system, combine with external forces during movement to produce countless  
6 patterns, forms, and strategies. The redundancy of the system allows for the use of multiple  
7 strategies to accomplish any given task. Logically, there are multiple performance variants for  
8 each movement, depending on the constraints of each individual's system. How do therapists  
9 utilize these redundancies in practice? Although many movement science and neuroscience  
10 students are now well-versed in the importance of variability, this information has not been  
11 integrated into many physical therapy interventions and is far from being embedded in general  
12 practice.<sup>2</sup>

13           Think about the last time you encouraged a patient to acquire a new or more efficient  
14 movement. How did you go about it? Did you demonstrate the movement and ask the patient to  
15 copy your movement pattern? Or did you ask the patient to experiment with various movement  
16 strategies to find success? Perhaps you asked the patient to think about the movement, the goal, a  
17 similar task, or a mental image of performance. Was some feature of the movement a focus for  
18 measurement to determine progress or change? Regardless of the practice setting, we ask patients  
19 to move, and we generally ask them to move differently than they moved before they came to us.  
20 Although our goals often do not explicitly target variability in movement, our implied  
21 expectation is that the functional movement that emerges will be adaptive and flexible enough to  
22 meet the everyday goals of our patients. To achieve this flexibility, our patients need adequate

1 variability of the motor system. It follows that adequate variability should be a focal point of  
2 examination and intervention in order to achieve optimal function for the individual.

3 Clinicians trained in medical fields use linear models for prediction and problem  
4 solving.<sup>3</sup> However, it is becoming increasingly clear that linear models are limited in many  
5 cases, and are certainly not the optimal model for function. Several areas of healthcare, including  
6 epidemiology, infectious disease processes, and biomedicine, are turning to nonlinear models for  
7 solutions to difficult problems.<sup>4</sup> For example, drug dosages are nonlinear; one cannot increase  
8 the dose for more effect, because there is generally a threshold value at which the desired effect  
9 occurs, and beyond which negative effects occur.<sup>5</sup> In our field, therapists know that a given  
10 amount of practice cannot assure the learning of a skill in a linear manner. Our patients usually  
11 learn sporadically, progressing in a nonlinear manner over time. In addition, rates and paths of  
12 progress vary between individuals with the same diagnoses who are similar in characteristics.  
13 Motor learning progresses nonlinearly, exhibiting nonlinear learning curves depending on the  
14 task, conditions, and characteristics of the learner.<sup>6</sup> Linear can be defined as pertaining to a  
15 straight line, or consisting of only one dimension\*. Nonlinear, usually used with the term  
16 dynamics, as in nonlinear dynamics, can be defined as “not in a straight line”, or as a system  
17 whose output is not proportional to its input. Nonlinear systems are more complex than linear  
18 systems, necessitating the use of sets of equations producing unpredictable outcomes that exhibit  
19 chaotic features. In general, biological systems, including humans, are complex, nonlinear  
20 systems with inherent variability in all healthy organisms.<sup>7</sup>

21 In this perspective, we will provide an overview of the constructs of variability,  
22 complexity in human movement, and nonlinear analysis. This overview focuses on clinical

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\* See Appendix A for definitions

1 relevance, and the definitions and explanations herein are therefore more conceptual than  
2 mathematical. We first describe variability of movement and its relationship to complexity, and  
3 the changing view of the importance of variability for successful function. Next, we review  
4 nonlinear measures and the concept of complexity in movement. Finally, we propose principles  
5 and examples of the use of variability, complexity and nonlinearity in examination, intervention,  
6 and research components of physical therapy practice.

### 7 The Importance of Variability and the Concepts of Nonlinearity

#### 8 *Variability: For better or worse?*

9 Before discussing the changing perspective on variability as it pertains to clinical  
10 application, consistent interpretation requires definitions of terminology. First, the definition of  
11 variability occurs in behavioral, biological, and statistical forms. Behavioral variability describes  
12 differences in observed behavior when an entity is placed in the exact same situation.<sup>8</sup> Webster's  
13 dictionary defines variability in a biological sense as "the power possessed by living organisms,  
14 both animal and vegetable, of adapting themselves to modifications or changes in their  
15 environment, thus possibly giving rise to ultimate variation of structure or function."<sup>9</sup> Statistical  
16 variability refers to measures of centrality around a mean or an average and includes measures  
17 such the standard deviation, the range of possible values, or the variance. All of these definitions  
18 contribute to our understanding of human variability.

19 Human movement variability encompasses the normal variations that occur in motor  
20 performance across multiple repetitions of a task over time.<sup>10</sup> Variability is inherent within all  
21 biological systems and reflects variation in both space and time, which can be easily illustrated in  
22 human movement. As a person walks through sand or snow, their footprints never repeat exactly,  
23 reflecting variability from step to step in a continuous cycle of movement. During quiet standing,

1 we sway around a central equilibrium point without ever remaining exactly still as we maintain  
2 orientation to the world. Are these examples of variability in movement should be considered  
3 errors in motor performance, or are they the normal output of a healthy motor system?

4 Traditionally, random error or noise within the system was deemed responsible for the  
5 variability measured during repetitions of a skill.<sup>11</sup> Motor learning textbooks usually describe  
6 movement variability as error and skilled movement as movement with decreased variability.<sup>12</sup>  
7 Generalized Motor Program theory (GMPT)\* considered variation in a given movement pattern  
8 to be the result of errors in the ability to predict the necessary parameters for employing the  
9 underlying motor program.<sup>13</sup> However, there is mounting evidence of the importance of  
10 variability in normal movement, which reveals variation not as error, but as a necessary  
11 condition for function. Variability reflects multiple options for movement providing for flexible,  
12 adaptive strategies that are not reliant on rigid programs for each task or for each changing  
13 condition encountered. Optimal variability as a central feature of normal movement is consistent  
14 with a nonlinear approach.

15 Counter to a therapeutic assumption that equilibrium is an indicator of health, nonlinear  
16 theories emphasize disequilibrium\* as healthy. This means that the system never quite settles  
17 into a stable state, and constant fluctuations characterize the healthy variability that allows  
18 adaptation to environmental change.<sup>4</sup> A complex dynamic system is in slight disequilibrium with  
19 the environment and maintains this disequilibrium over time.<sup>14</sup> Goldberger describes complete  
20 equilibrium as equivalent to the death of the organism because it implies a static, non-dynamic  
21 state.<sup>15</sup> Therefore, health is indicated by a dynamic equilibrium that is not a static state.

22 Rather than thinking of variability as a negative feature, variability reflects important  
23 information for the maintenance of the health of the system. Reduced variability is known to

1 cause repetitive stress injury in a mechanical sense. Although outwardly this appears to be a  
2 mechanical problem, the underlying story describes an information problem. The lack of  
3 variability in movement leads to abnormal mapping of the sensory cortex, which subsequently  
4 disturbs motor function. These neural maps (both sensory and motor) are more complex when  
5 movement variability is present, and less complex when variability is reduced.<sup>15-18</sup> Movements  
6 with an optimal amount of variability avoid this abnormal mapping, and essentially contribute to  
7 the neuroplasticity needed for maintaining or achieving functional skill. Thus, variability of the  
8 movements used for the task contributes information to the nervous system, which then serves to  
9 prevent injury.

10       Too much variability can also be a problem, such as in an individual with an ataxic  
11 movement disorder. Movements that usually fall within a specific range of variability to  
12 accomplish a task such as gait, unexpectedly fall both in and outside the acceptable range in an  
13 ataxic individual. When one movement falls outside the expected range, the next movement is  
14 perturbed. Gait is an example of a continuous, cyclic task, within which the steps cannot be  
15 random, but also are not completely repeatable or robotic. Thus, we propose that optimal  
16 movement variability lies between too much variability and complete repeatability.<sup>10</sup> Scientists  
17 define this optimal range of variability using mathematical chaos. Furthermore, techniques from  
18 mathematical chaos describe important features of variability.<sup>19, 20</sup>

### 19 *How nonlinear tools advance understanding of variability*

20       We have established that variability is not a negative feature of functional movement or  
21 of biological systems in general. However, exactly when is variability good? Moreover, how do  
22 we determine if the variability we see is good variability or bad variability? The answer to these  
23 questions lies in understanding dynamic models of movement.



1           The conversation about variability began with systems theory\*, a departure from linear  
2 models. In 1990 at the II Step conference, dynamic systems theory (DST\*) was promoted as a  
3 model for the progression of movement skills and as a way to model change in movement skill  
4 for physical therapists.<sup>21,22</sup> DST introduced the notion of stability\* and nonlinearity to explain  
5 variability. Based on environmental, biomechanical, and morphological constraints, any  
6 biological system will self-organize to find the most stable solution. An increased amount of  
7 variability indicates less cooperative behavior between the components of the system, which  
8 eventually can drive the system to new attractor states\* or behaviorally stable solutions. This is a  
9 nonlinear system because the input does not lead to a linear change in output. Input changes the  
10 variability of the system, which may drive the emergence of a new behavior. Importantly, the  
11 measure of the variability provides a means to classify the stability of the system. Thus, in DST  
12 small amounts of variability will indicate a highly stable behavior.

13           The GMPT and DST perspectives are similar in that both recognize that decreased  
14 variability results from the efficient execution of a given movement pattern. DST differs,  
15 however, because it proposes that at a specific critical point, variability increases and dominates  
16 the system as new movement emerges. The system becomes highly unstable and switches to a  
17 new stable behavioral state. An example of this process occurs when an individual learns to ride  
18 a bike. If a child has been using training wheels, pedaling and controlling the direction of the  
19 bike is a stable behavior. However, when the training wheels are removed, the system is  
20 perturbed. Pedaling and directional control is very erratic and unstable as the individual learns  
21 the interaction of balance with the speed of the bike and controlling movement through space. At  
22 the point where the individual understands how the parameters of speed, balance and directional  
23 control interact, the behavioral state (independent bike riding) becomes stable. The system must

1 cross a critical point, where the speed of the bike makes balance easier for the behavior to  
2 become less variable in its new state. In DST, increased variability in the system reveals growing  
3 instability, which may lead to a shift to a new attractor, or a new behavior. Conversely, a lack of  
4 variability traps a behavior in a specific state or attractor. Thus, DST advances our understanding  
5 of transitions between behavioral states, with variability not considered error, but rather as a  
6 source of behavioral change. Through DST, the importance of variability was brought to the  
7 attention of developmentalists, and is recognized as a theoretical starting point for studying the  
8 emergence of developmental actions, perceptions, and cognitive skills.<sup>23</sup> However, the variability  
9 within an existing state (as opposed to a developing state), behavior, or established movement  
10 function has not been appreciated as important to skillful movement in adults or in describing  
11 pathological conditions. The use of linear tools, such as the standard deviation, limits our  
12 understanding of variability as a window to view the nature of adaptation in functional skills.  
13 Consequently, the ideas from DST of “stable state” and “attractor” are not intrinsically accepted  
14 as part of our therapeutic world. This is partially because we have lacked the tools to see the  
15 “hidden” information in the variability of movement progressing over time. Variability and its  
16 underlying characteristics are not completely described or quantified in either GMPT or DST,  
17 even though variability has an important role in both theories.

18 Over time, it was recognized that neither GMPT nor DST account for the observation that  
19 some behaviors, which appear to be stable, paradoxically occur in quite variable ways. This is  
20 especially evident when we observe elite sports players or musicians performing skillfully. Even  
21 though they perform the same skill as others, they seem to have developed an infinite number of  
22 ways to perform it. Thus, it seems that a very stable behavioral state is supported by a very “rich”  
23 behavioral repertoire. If we consider fundamental motor skills (i.e. gait, posture) and not the

1 skills of an elite athlete, we are all skillful in our ability to walk through crowds or on diverse  
2 and challenging terrains. Therefore, it seems that variability does not decrease when we develop  
3 and refine a stable behavioral state but actually increases. Therefore is important to describe the  
4 structure of variability and not just the amount. This is feasible with nonlinear tools. Nonlinear  
5 tools best capture variation in how a motor behavior emerges in time, for which the temporal  
6 organization in the distribution of values is of interest. Temporal organization, or “structure” is  
7 quantified by the degree to which values emerge in an orderly (i.e., predictable) manner, often  
8 across a range of time scales. Nonlinear tools quantify the nature or the structure of variability,  
9 and provide the missing ability to quantify the concept of “stability” from DST.

10 We illustrate the above concepts of variability using Figure 1. The figure pictures four  
11 different time series, with the linear measure of range and the nonlinear measure of Approximate  
12 Entropy (ApEn) listed beside each signal. The first and third rows picture signals that look  
13 messy, seeming to be random, with one signal larger in amplitude than the other. Beside these  
14 rows, you can see that the range values reflect the varying amount of the signal between the two  
15 traces, with the larger number by the larger signal; however, the ApEn values are equivalent for  
16 these two signals. This reflects the fact that only the amplitude varies, but not the structure of the  
17 time series. The second and fourth rows picture time series that are very regular, a sine wave.  
18 Again, comparing the range one can see they are different in amplitude, but the same in structure  
19 as reflected in the ApEn value. However, if we compare the first to the second signal (and the  
20 third to the fourth), we can see that the amplitude, quantified by the range, is the same (and the  
21 standard deviation, a linear measure of variability, would also be the same), but the structure of  
22 the series, described by the nonlinear ApEn value, is different. Therefore, the amount of  
23 variability measured by the standard deviation (linear) and the structure of variability measured



1 is because measuring each part of the movement separately will not give us an overall measure  
2 of the complexity required for success in the function. Complexity is something that is “hidden”  
3 within the time series of a movement sequence or strategy as it emerges over time. Movements  
4 which occur at one moment affect and are affected by movements that occur either before or  
5 after the movement in the moment. Tools for measuring complexity come from nonlinear  
6 dynamics, and mathematical models incorporate time to describe this complexity.

7 *Linear measures; traditional measures of dispersion*

8 Linear tools to measure variability provide information about the quantity of a signal, but do not  
9 tell us about the time-evolving nature of the signal. Linear tools include the statistics of range,  
10 standard deviation, and the coefficient of variation, and are limited in their explanation of human  
11 movement variability for several reasons. One reason is that data from several trials are  
12 averaged to generate a “mean” picture of the subject’s movement pattern. This approach  
13 removes the temporal variations of the movement and masks the true structure of variability  
14 present in the movement pattern. In addition, the valid use of linear tools to study variability  
15 assumes that variations between repetitions of a task are random and independent (of past and  
16 future repetitions), which has been shown to be false.<sup>24-26</sup> Finally, linear tools provide different  
17 answers when compared with nonlinear tools regarding the way that they evaluate variability.  
18 For example, traditional linear measures of postural sway quantify only the magnitude of sway  
19 and not the temporally evolving dynamics (or disequilibrium) of postural control. In spite of  
20 long-term use in many studies, researchers are beginning to realize that linear measures, such as  
21 the range and standard deviation of the center of pressure (COP) do not quantify stability of the  
22 postural control system, because it is possible to have a large area of the COP path while having  
23 a stable posture or an unstable posture.<sup>27, 28</sup> Therefore, linear measures of variability do not

1 accurately define constructs important in movement, such as stability because they only provide  
2 insight into the amount of variability (Figure 1). Nonlinear tools give us essential additional  
3 information about the structure of variability, which completely describes the evolution of the  
4 movement over time. This is possibly the reason why previous theoretical accounts of variability  
5 in human movement (i.e. GMPT, DST) supported the notion that small amounts of variability  
6 characterize a very stable behavioral state and that improved stability links directly to decreasing  
7 variability. If we measure improved performance linearly, this conclusion is very reasonable.  
8 However, nonlinear measures provide additional information, and allow understanding of  
9 complexity.

10 In Figure 2, one can visually note that the time series signals on the left are different from  
11 each other; the first is very regular, the second seems to have some type of pattern that is hard to  
12 describe verbally, and the third seems to have no pattern. However, when these signals are  
13 plotted versus their velocities (phase plane), it becomes clear that the first signal is completely  
14 regular, with no variation from the first cycle to the last cycle. The second signal forms a  
15 complex yet organized pattern, with similar paths for each cycle, but not repeating the same path,  
16 with each path dependent on a previous path and influencing the next path. The third is a random  
17 signal, where the paths are not similar, and not dependent on each other. We propose that  
18 functional and healthy movement resembles chaos, the complex center picture. Nonlinear tools  
19 can determine whether a chaotic structure, or complexity, is present in movement.

20 *Why is variability inherent in biological systems?*

21 Variability is inherent in biological systems because it ensures survival. Gerald Edelman,  
22 a Nobel laureate, describes this pervasive rule as “population thinking”, and he used it to  
23 describe the complexities of the immune system, and then expanded the principles to

1 neuroscience and the way the mind works.<sup>29</sup> The basic idea is that variability allows an organism  
2 or group of organisms to be more successful. This pertains to antibodies and viral agents, animal  
3 species, the nervous system, and the evolutionary progress of plants and animals. Variability  
4 allows choices among options, selection of strategies, and flexibility to adapt to variations in the  
5 environment. If an animal rigidly performs limited behaviors, or functions only within a  
6 restricted environment, it is challenged for survival by a more adaptive animal. This Darwinian  
7 principle describing the advantage of variability lies within many levels of organisms, and is  
8 explicitly described for the growth of genetic complexity in an information-rich environment.<sup>30</sup>  
9 Variability exists at many levels and within interacting components of a system, often operating  
10 at different time scales. Thus, variability may not be obvious at one level, but can be revealed at  
11 another level. The variability inherent in biological systems from genes to behavior cannot be  
12 considered error if it is pervasive from one species to another and is linked to survival.

13 *Why isn't movement variability just error?*

14         If movement variability is equivalent to error, we can reasonably assume that more  
15 skillful individuals would have less error (or variability) at the outset of learning, and then  
16 quickly drop to zero error. In fact, the opposite is true. Individuals who use a high degree of  
17 variability in cognitive strategies at the beginning of task development have greater learning and  
18 eventual success in performing the task.<sup>31</sup> Movement researchers have started to understand the  
19 importance of variability in motor skill learning, and examine performance differently. For  
20 example, a study of coordination variability in jumpers revealed a U-shaped curve in the  
21 progression of variability.<sup>32</sup> Initial high variability occurs as different strategies are attempted.  
22 Subsequently the performer moves toward a reduction in variability as the learner performs more  
23 successfully. But then, surprisingly, as the learner becomes an expert, the variability increases

1 again. This skillful variability indicates increasing flexibility of skill to allow adaptation to  
2 perturbations. Thus, the variability at the beginning of task learning may seem like error because  
3 the task is not performed efficiently or accurately. However, this initial variability can also be  
4 seen as necessary to map the possibilities of movement for the task, which then is refined into a  
5 different type of variability when the performer is skillful. Although variability is typically  
6 known to decrease as a skill is acquired, think about how our notions of the mechanisms of skill  
7 acquisition change if we consider the *role of variability*. If therapists consider variability to be  
8 error, the variability is seen as an impairment. However, if therapists consider variability  
9 necessary for skill acquisition, they will examine the structure of variability to help build skill.  
10 Nonlinear measures can unmask the hidden structure inherent in variability so that intervention  
11 can successfully address different features of variability as necessary during skill development.  
12 While clinicians can easily understand the behavioral variability we have been discussing, such  
13 as the number of strategies for accomplishing a functional task, it is more difficult to understand  
14 what underlies that variability.

15         Motor skills researchers increasingly find nonlinear tools useful in revealing information  
16 through time series analysis. The ApEn measure revealed significant differences between  
17 athletes who had a concussion and healthy controls by analysis of the COP time series, even after  
18 other standard linear measures indicated that the athletes had fully recovered from the  
19 concussion.<sup>42-43</sup> Moraiti et al used the LyE measure to show that a group of patients with anterior  
20 cruciate ligament deficient knees exhibited significantly more rigid and predictable walking  
21 patterns than the healthy controls, suggesting a decrease in system complexity and narrowed  
22 functional responsiveness.<sup>47</sup> Kurz and Stergiou used an entropy measure to show that  
23 neurophysiological changes associated with aging may result in less certainty of the



1 neuromuscular system in selecting a stable gait.<sup>67</sup> Therefore, utilizing a nonlinear perspective to  
2 examine variability can assist in differentiating between health and non-health.

3 *Nonlinear measures: new ways to describe the nature of variability*

4         We have established that the amount and structure of variability are two different things  
5 (Figure 1). As a result, changes in measures of the amount of variability may be in a completely  
6 different direction than changes in measures that evaluate structure of variability. Similarly, in  
7 studies of postural control, gait, and force production, researchers found that as measures of the  
8 amount of variability increased, measures of the structure or organization of variability  
9 decreased.<sup>33-35</sup> Let us reflect on an example of postural sway in standing. As a person's range of  
10 sway increases, the standard deviation increases, indicating a greater amount of variability.  
11 However, if we use a nonlinear tool to examine the features of the variability in postural sway,  
12 we may note that sway has become more regular (with more repeatable movement patterns).  
13 This makes sense, because the individual must have some specific strategy of control to make the  
14 appropriate adjustments for balance maintenance when the range of movement is large;  
15 otherwise, they would fall. The relationship between linear and nonlinear tools as described  
16 above can further our understanding of the emergence of functional, adaptive movement.

17         Nonlinear measures always describe a time series, or a series of measurements taken at  
18 specific intervals over uninterrupted time. For example, the range of a joint during each step as a  
19 person walks, taken with an electrogoniometer or motion analysis equipment sampled at 30 times  
20 per second over a 2-minute period of time, can be presented as a time series. The importance of  
21 looking at a measure in the context of time is so that we understand the ability of the system for  
22 adaptations as conditions change. The period of time may vary from seconds to days, but the  
23 important concept is that a behavior emerging from the complex system can be described over

1 time with specific mathematical nonlinear tools that are used to quantify order, predictability,  
2 regularity and complexity. A time series is also valuable because information that is important to  
3 understand the health or function of the individual may be revealed at different time scales.

4 Characterizing the nature of the complexity present in a time series is of great interest in  
5 many scientific domains, including biology. Healthy systems, whether referring to heart rate or  
6 the center of pressure time series, correspond to a rich behavioral state with high complexity.<sup>10</sup>  
7 This state is defined as highly variable fluctuations in physiological processes resembling  
8 mathematical chaos. This allows the system to have a relatively predictable course, which can  
9 adapt if a change in the environment occurs. Low levels of complexity correspond to states that  
10 resemble random, noisy and erratic behavior or rigid, periodic and regular behavior. Therefore,  
11 with low complexity, adaptability suffers. Nonlinear measures allow us to extract information  
12 hidden in a time series and evaluate complexity. Some examples of possible tools follow.

13 Approximate entropy (ApEn) is a measure that can quantify the regularity of a time series  
14 or predictability of a time series (Figure 1).<sup>36-40</sup> Increasing ApEn values reveal greater  
15 irregularity. Conversely, lower values reveal a more regular or periodic behavior. ApEn has been  
16 useful in the identification of differences between young and old in the COP time series in  
17 standing,<sup>41</sup> in revealing deficits in athletes with a concussion when compared to healthy  
18 athletes,<sup>42, 43</sup> and in detecting developmental changes in sitting postural control.<sup>25</sup> ApEn  
19 measures the probability that the configuration of one segment of data in a time series will allow  
20 the prediction of the configuration of the another segment of the time series a certain distance  
21 apart.<sup>37</sup> Data segments with a greater likelihood of having the same configuration or pattern  
22 upon comparison will result in lower approximate entropy values, while data segments with a  
23 low likelihood of similarity between segments will result in higher values. Values closer to 0 are

1 consistent with maximum regularity, and values nearing 2 represent maximum irregularity.  
2 Clinically, ApEn is useful for understanding the predictability of a movement. For example,  
3 Cavanaugh et al utilized ApEn to determine the predictability of everyday walking activities,  
4 using time series data from a pedometer.<sup>44</sup> They found that inactive elderly individuals walked  
5 less and had more predictable walking activity than inactive elderly. This allows a more  
6 complete picture of the differences in walking between active and inactive individuals, and  
7 allows the clinician to understand why the elderly may have a harder time responding to  
8 fluctuations in routine or adaptations to different walking demands. This insight suggests  
9 intervention that can elicit greater complexity in the activity, rather than just simply increasing  
10 the amount of activity.

11         The largest Lyapunov Exponent (LyE) is a nonlinear measure that can measure the  
12 divergence of the movement trajectories (Figure 2). The LyE describing purely sinusoidal\* and  
13 completely repeatable data with no divergence in the trajectories is zero because the trajectories  
14 overlap rather than diverging (Figure 2A). This shows minimal change in the structure of the  
15 variability over time in the data. The LyE for random noisy data indicates greater divergence in  
16 the data trajectories (Figure 2C). The LyE values for the random data is larger, with values  
17 above 0.4.<sup>19</sup> The LyE values from data that are described by mathematical chaos (i.e. Lorenz  
18 attractor; Figure 2B) are in between these two extremes. Complexity is defined by such values  
19 since it is described as highly variable fluctuations in physiological processes resembling  
20 mathematical chaos. The LyE has been used with gait time series data to characterize the  
21 underlying complexity during movement.<sup>19, 45-48</sup> Yamada reported body sway that resembles  
22 mathematical chaos from COP data during standing in normal adults using the LyE, thus  
23 revealing inherent complexity.<sup>49</sup> Use of the LyE in an ongoing intervention study with infants

1 with cerebral palsy (CP) was important as a fine-grained measure to detect advancing postural  
2 control in sitting, when linear measures of the COP and clinical tools did not detect change.<sup>50</sup> A  
3 standardized test, the sitting section of the Gross Motor Function Measure<sup>51</sup> did not consistently  
4 detect change. However, a variety of features of the child's movement (more attempts to stay  
5 vertical, the ability to turn the head without falling, more attempts to reach while sitting with  
6 support) were noted as changes in behavior by the parents and therapist. These small changes in  
7 movement, or attempts at new strategies, were quantified by the nonlinear measures of ApEn and  
8 LyE, but were not indicated in the linear measures. This is an example of a nonlinear tool  
9 providing "hidden" information that would not be easily measured or documented otherwise.

10 Surrogation is a technique used to determine if the source of a given time series is  
11 deterministic (has order) in nature.<sup>19, 52</sup> The technique compares the actual data and a random  
12 data set that has a similar structure to the original data set in question. In other words, the  
13 deterministic structure from the original data set is removed by generating a random equivalent  
14 with the same mean, variance, and power spectra as the original. Subsequently, the LyE (or  
15 another nonlinear measure) is compared from the surrogate data to the LyE value of the original  
16 data. Significant differences between the LyE in the surrogate data as compared to the original  
17 indicate that the original data is not randomly derived and therefore, may be deterministic and  
18 possibly complex in nature. Harbourne and Stergiou<sup>25</sup> and Boker et al<sup>26</sup> have used this technique  
19 to show that variability in the center of pressure time series from infants during the development  
20 of independent sitting is not just noise, but it has a deterministic origin. This means that infants  
21 learning to sit are not just randomly "wiggling". For the clinician, it is important to recognize  
22 that within these outwardly unorganized and noisy-looking movements are orderly patterns and  
23 the beginnings of strategies for postural control. The implication for therapists is that a

1 movement that is just beginning to emerge will be unorganized, but necessarily so. The  
2 variability inherent in this disorganization may be necessary for ultimate successful selection of  
3 movement and postural strategies. This may reflect the system “mapping” the territory around  
4 the skill region allowing the individual to “get back” to the successful region when perturbed.

5 Nonlinear analysis includes several additional tools such as the Detrended Fluctuation  
6 Analysis, Correlation Dimension, Mutual Information, Hurst Exponent, Symbolic Entropy,  
7 Recurrence Quantification Analysis, and others. These methods have a common goal, to evaluate  
8 the structure or organization of variability and uncover the underlying complexity. However,  
9 they differ in the mathematic manipulation of the available time series. Here we do not to  
10 provide a complete list of all nonlinear tools, but describe only a few tools to provide the basic  
11 concepts of nonlinear analysis. A more comprehensive review on the topic is available for the  
12 interested reader.<sup>19</sup>

### 13 Application of Nonlinear Concepts in Practice

#### 14 *Complexity in health*

15 Goldberger describes the use of complexity at the bedside for physicians by providing  
16 examples of periodic behavior of pathologic systems.<sup>56</sup> Disease brings about a loss of complexity  
17 with resulting increased rigidity, such as Cheyne-Stokes breathing in heart failure, tremors in  
18 neurologic disease, and a sinusoidal appearance of heart rate variability in patients with  
19 congestive heart failure.<sup>58,59</sup> The medical field is beginning to recognize the need for a nonlinear  
20 view toward complexity, particularly for problems that affect multiple systems. Ahn describes a  
21 traditional reductionist approach as the antithesis of a complexity-oriented approach, with the  
22 reductionist approach being appropriate for use with acute, single system problems, such as an  
23 acute infection.<sup>60</sup> However, a disease such as diabetes requires management as a problem,

1 affecting many systems that interact in various ways. Extending the nonlinear view, one could  
 2 argue that very few problems are truly single system problems and solvable by linear reasoning,  
 3 because each system interacts with other systems for optimal function. As in medicine, many  
 4 clinical problems seen in physical therapy need a nonlinear approach.

5 Clinical uses for nonlinear analysis appear in a variety of disciplines including  
 6 cardiology, neurology, and psychiatry. Heart rate analysis using ApEn has been used to evaluate  
 7 risk factors for sudden infant death syndrome.<sup>37, 38</sup> Nonlinear analysis has been useful in  
 8 verification of implantable cardiac defibrillator interventions by using entropy analysis of heart  
 9 rate variability.<sup>61</sup> In addition, postural control analysis using stabilometry has been improved by  
 10 the addition of nonlinear analysis, which can serve to more accurately identify features of  
 11 postural control indicating subtle problems in infants,<sup>50</sup> developmental differences between the  
 12 young and elderly,<sup>41</sup> or changes that accompany a disease state such as Parkinsons.<sup>62, 63</sup> Gait  
 13 variability has also been studied and modeled using nonlinear tools.<sup>64</sup> Cavanaugh et al utilized  
 14 measures of variability including nonlinear measures in gait during daily community activity to  
 15 characterize functional limitations.<sup>44</sup> Applications in the clinic for physical therapy are now a  
 16 realistic possibility.

17 *Clinical application of nonlinear principles and use of complexity in physical therapy*

18 Table 1 lists some proposed principles for physical therapy intervention emerging from  
 19 the theory and applications in other fields.

Table 1. Proposed Principles of Nonlinearity in the Acquisition and Maintenance of Motor Skill
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1. An optimal amount of variability is necessary for movement to be functional and efficient; normal, efficient movement includes both deterministic and random characteristics, which can fluctuate within an optimal range.
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2. Healthy motor control has characteristics of nonlinearity including the spontaneous generation of new patterns of movement, movement possibilities that are sensitive to initial conditions, and a limited ability to precisely predict future movement based on current status.
3. If variability increases in a system without enough variability, new movement options can emerge spontaneously.
4. Because motor function is sensitive to initial conditions, each person brings a slightly different set of conditions to a motor problem, and the optimal solution to that problem may be unique to that person. Therefore, therapists cannot “prescribe” the best motor pattern or strategy that is common to all patients.
5. Input into the system can drive the system into other possibilities for movement that are not predictable. The input can come from more than one system, i.e., not only motor but also sensory, cognitive, emotional, social.
6. Measures of complexity can help predict the emergence of a new behavior, or direct appropriate intervention to allow variability changes to affect function.
7. Traditional measures of variability are not equivalent to measures of complexity. For example, as the measure of standard deviation increases, the measure of approximate entropy can decrease. Measures of complexity describe the structure of variability in new ways that can help quantify subtle movement changes or characteristics.
8. Complexity is necessary for systems to adapt to changing conditions; loss of complexity means decreased ability for adaptation.

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3 *An example of clinical application in intervention follows:*

1           Two clinicians must perform an initial evaluation on a elderly man who has had a stroke.  
2   One therapist will utilize a traditional, linear approach, while the other therapist utilizes an  
3   approach based on principles of nonlinearity. As a general rule, the therapist using a linear  
4   approach assumes that decreasing the variability of movement is equivalent to improving  
5   functional skill. Therefore, the therapist has in mind the “correct” movement pattern for various  
6   functional skills, which she will guide her patient toward during intervention. Using principles of  
7   motor learning, the therapist first gives 100% feedback, and then fades subsequently to 50%  
8   feedback for various skills.<sup>65</sup> Because the therapist wants measurable outcomes, she uses a  
9   standard walking course that has 25-foot increments up to 200 feet. To determine decreasing  
10   variability and increasing accuracy, the therapist counts errors in the set of procedures to sit-  
11   stand-walk a standardized course. This principle of fading feedback applies as she guides the  
12   patient in transferring out of the wheelchair. She first locks the brakes of the wheelchair of the  
13   patient, and tells the patient to scoot to the edge of the chair. The patient receives assistance to  
14   lean forward and place his feet under his center of mass both with physical guidance and with  
15   verbal guidance. As the patient starts to reach for the walker, the therapist tells the patient to  
16   push up from the arms of the chair. The clinician then has the patient practice the sit-to-stand  
17   activity 5 times, giving feedback as described each time. The next day the patient makes the  
18   same errors, and the therapist provides less cueing, hoping to fade cueing over the next 2 weeks.  
19   In addition to counting errors during the sit-stand-walk practice, the PT notices that the patient’s  
20   steps are of unequal length, and the affected side shuffles forward rather than exhibiting a heel-  
21   toe pattern of stepping. The PT uses the same approach of using verbal and physical guidance to  
22   point out the errors in the patient’s gait pattern, fading the feedback over time and counting  
23   errors within the measured distance of the walking track.



1           In contrast, the therapist using a nonlinear approach assumes that the general rule for this  
2 patient is to enhance complexity of movement in order to improve gait and functional mobility  
3 skills. This will include the concept of disequilibrium, or keeping the patient in a state of  
4 dynamic equilibrium (as described earlier in this paper) during therapy sessions. Additionally,  
5 the PT uses the strategy of only providing information for the patient on how to do a task if the  
6 patient does not have a way to get the information; the rationale is that variability is encouraged  
7 if the patient seeks information independently, and the patient is kept in a dynamic state. The PT  
8 first asks the patient if he would like to go sit on a chair 10 feet away next to his wife. He agrees,  
9 and the therapist invites the patient to begin the task, assuring the patient that the task is safe  
10 while the therapist is present. The patient pushes back in the unlocked wheelchair, and the chair  
11 rolls back, putting the patient further away from the targeted goal. The therapist notes that this is  
12 a point where the patient is not gathering enough information for the task, and addresses this  
13 problem by having the patient do some guided exploration within the task to increase flexibility  
14 in terms of availability of options. The therapist tells the patient that he can roll the chair in many  
15 different directions just using his feet, and challenges him to find 10 different directions of chair  
16 movement by pushing with his feet. During this exercise, the patient maps the way that foot force  
17 effects the chair movement. The therapist asks if there is a way to keep the chair from rolling,  
18 and the patient remembers the brakes. The patient then makes multiple errors in his attempts to  
19 stand, including incorrect foot placement, reaching for the walker instead of the chair arms, and  
20 leaning back and to one side instead of leaning forward to get up. However, the therapist does  
21 not provide guidance at this point because the patient is not making the same error, but rather is  
22 exhibiting a variety of strategies, which at this stage of skill development is desirable. At several  
23 points, the therapist asks if the strategy just used was successful, and when answered in the

1 negative, the therapist reminds the patient to try some different strategies, just as they did with  
2 the pushing the chair in different directions with the feet. Occasionally the therapist gives light  
3 touch cues to suggest an effective strategy. At the end of the trial and error session, the patient  
4 stands and walks over to his wife to sit in the chair. After resting and conversing, the therapist  
5 asks the patient to walk back to the wheelchair, without giving any verbal instructions. The  
6 patient makes a few errors, self-corrects, and visibly thinks through the process of coming to  
7 stand, but is markedly faster than the first try. The next day the patient makes only one error,  
8 moves from sit to stand with guarding, and elects to sit on a different chair, a couch, for some  
9 social contact with another patient. During walking, the therapist noted that the patient had short  
10 steps and shuffled with the affected leg. The therapist sees this as a possible problem. The PT  
11 then uses barefoot tasks, having the patient identify different textures/objects under the feet,  
12 place pressure on different parts of his feet during walking and standing, and walk with a variety  
13 of patterns through different paths and obstacle courses. The PT is more concerned about  
14 increasing the adaptive capacity of walking by increasing variability at a functional speed than in  
15 producing a consistent heel-toe pattern.

16         What differs in these approaches? The basic difference is that the linear therapist seeks to  
17 reduce variability of responses within the intervention and the measurement of the task goals,  
18 and come to a state of complete equilibrium; the nonlinear therapist seeks to enhance complexity  
19 by encouraging slight disequilibrium, particularly at the initial stage of task learning. The  
20 nonlinear therapist builds complexity into the task, with the use of multiple systems: cognitive,  
21 social, motor and sensory. The practice space is strategically varied and multiple movement  
22 approaches are encouraged, as well as an expanded environmental context in which the practice  
23 takes place. In addition, the functional task proceeds in such a way that the series of movements

1 that lie within the task are related to each other, and dependent on each other. In the linear  
2 therapist's approach, each subtask is a task unto itself, and separated from the other parts of the  
3 task due to the interjection of the therapist's instructions. Therefore, the series of movements and  
4 postural adjustments in the task of standing from the wheelchair are unrelated to each other, and  
5 unrelated to the overriding environmental context and underlying values of the patient. And most  
6 importantly, the linear therapist prevents response flexibility from being part of the learning  
7 process by insisting that the patient avoid errors, precisely because the therapist considers them  
8 undesirable. Thus, the linear therapist focuses on the absolute "correct" pattern of movement,  
9 allowing little complexity in the intervention, and preventing the emergence of a flexible strategy  
10 that works for this individual patient. If these therapists had the benefit of examining the  
11 patient's initial gait by linear and nonlinear analysis, they may have noted high variability of the  
12 step length (by looking at the standard deviation) but low values on a nonlinear measure  
13 (Lyapunov Exponent), somewhat like series in the last row of Figure 1. If one wanted to decrease  
14 variability, one would work toward a time series that looks like the second series (linear  
15 approach). However, if the goal is to increase the structure of the variability for greater  
16 complexity, one would work in a nonlinear fashion toward a time series that looks like the third  
17 row of Figure 1. You can see that looking at variability from these differing perspectives could  
18 lead toward different types of intervention, as we have just described.

19 Another example of the use of the principles of nonlinearity to acquire or maintain motor  
20 skill is a treatment approach for the movement problems of Parkinson's Disease (PD). The BIG  
21 training program focuses on a single attentional parameter to drive changes in the motor  
22 system.<sup>66</sup> This parameter is "Think BIG", emphasizing attention to large amplitude movements.  
23 The principles of treatment include high intensity, multiple repetitions, saliency and complexity,

1 leading to neuroplastic changes and functional improvement. Although patients with PD have  
2 many movement problems, including speed, smoothness, accuracy and quality of movement  
3 patterns such as step length during gait, this approach ignores these other movement deficiencies.  
4 The focus on increasing amplitude changes the initial conditions driving the movement, and  
5 shifts the system into a new state space where the movement is more skillful and complex. In this  
6 way, the BIG approach addresses complexity because amplitude serves as the avenue to provide  
7 enhanced adaptability and increased responsiveness. Several principles of nonlinearity are  
8 inherent in this approach. First, the variability of movement behavior overall for these patients is  
9 increased as a fallout of the increase in amplitude. The individuals with PD can now make large  
10 movements as well as small, and all the increments in between, whereas they had previously  
11 been restricted to only small movements. Secondly, another principle of nonlinearity is apparent  
12 because a change in just one movement parameter, amplitude, causes a change in other  
13 movement parameters that are difficult to predict precisely. Third, the input needed for a motor  
14 system change can come from a different system, such as cognitive/attentional/perceptual  
15 systems. The focus on attending to making a “bigger” movement, in this case, re-calibrates the  
16 perceptual system to recognize when a movement is actually big, vs. the small movements  
17 common to individuals with PD. The therapist would not teach a particular movement form or  
18 strategy, but rather let the patient discover that increased complexity of various movements have  
19 an inherent value in producing success during daily tasks.

#### 20 *Use of nonlinear tools in motor skill research*

21           Setting up methodology for nonlinear analysis within a research project may seem  
22 to be a daunting task. Here are suggestions to keep in mind when designing such a project:

- 1 1. Carefully design your experimental set-up incorporating a healthy matched control group to  
2 provide reference points for observed changes in the nonlinear parameters.
- 3 2. Seek partnerships with mathematicians, neuroscientists, and biomechanists who are  
4 knowledgeable in nonlinear tools. However, this is easier nowadays with the use of the internet  
5 and it possible from a distance. Technical expertise is needed from start to finish on the project,  
6 including sampling frequency, the length of the time series needed, and examination of the data  
7 with appropriate nonlinear tools for your questions. Knowledge in movement measurement is a  
8 benefit in interpretation of the results. Remember that you will not “speak the same language” as  
9 your collaborator, and be very patient so that ideas can be exchanged comfortably.
- 10 3. Measure a task, skill, or variable that may show emergence of a new level of function.
- 11 4. Variable and task selection must incorporate a time series; you should take as long a time  
12 series as possible within the constraints of your target population and considering the tools you  
13 are using. Many nonlinear tools need thousands of data points for accurate use.
- 14 5. Take pilot data, and examine them to get an idea of the nature of the fluctuations in the  
15 variable or in the behavior. Plotting position and velocity against each other and even three-  
16 dimensionally by incorporating acceleration can help examine the organization in the data.
- 17 6. a A replication of a previously reported project, but adding nonlinear tools, can help with  
18 planning the methods and interpreting your results. This is a new area, and consequently difficult  
19 to know the “standard” approach to common issues.
- 20 7. If you are considering an intervention study, keep in mind that a focus on increasing or  
21 decreasing variability will need to be determined carefully, using a variety of tools to measure  
22 both linear and nonlinear factors. The two approaches are complementary and they do not negate  
23 each other.

1 8. Measure more than one variable; different variables may reflect component skills or constructs  
2 differently.

### 3 *Limitations of nonlinear measures*

4           Because nonlinear measurement tools require the use of mathematical equations and  
5 software to evaluate time series data, nonlinear analysis is primarily done in the research setting.  
6 However, the burgeoning interest in nonlinear tools in many scientific fields bodes well for  
7 clinicians. In the future, there will likely be devices that have embedded software to calculate  
8 important measures of variability using nonlinear tools. Another limitation is the understanding  
9 of variability and complexity in the field of physical therapy. Therapists are taught to use a  
10 reductionist approach, as in most medical fields. This lack of introduction to nonlinear principles  
11 early in the education of therapists biases the field against the productive use of variability and  
12 complexity.

13           Additional limitations of the technique itself create challenges for clinical use.

14 Translation of nonlinear measures to clinical problems requires concurrent use of linear tools to  
15 appreciate both amount and structure of variability and to make associations and determine  
16 clinical meaning. The lengthy time series required for analysis prohibits the use of nonlinear  
17 tools for movement that is extremely limited. Lastly, to evaluate correctly variability measures  
18 need to be taken over multiple repetitions of a movement and not with only few observations of a  
19 short duration.

### 20 *Conclusions*

21           Optimal variability in human movement is a characteristic of healthy functioning.

22 Nonlinear tools reveal complexity inherent in normal variability indicating features of motor  
23 control that are important for physical therapists to measure and implement in intervention. The

- 1 application of principles based on nonlinear dynamics and use of nonlinear tools for analysis can
- 2 provide innovations to guide physical therapy practice and research.
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## Appendix A

The following are working definitions for terms needed to understand nonlinear concepts.

Attractor; attractor state – behaviorally is a preferred state (i.e. walk, run). Mathematically is a set to which a dynamical system evolves after time. Points that get close enough to the attractor remain close even if slightly perturbed. Geometrically, an attractor can be a point, a curve, or even a fractal structure known as a strange attractor. Describing the attractors of chaotic dynamical systems the focus of the mathematical theory of chaos.

Chaos-is one subject in the field of nonlinear dynamics, which is part of the broader field of dynamical systems.

Complexity - highly variable fluctuations in physiological processes resembling mathematical chaos.

Constraints – Variability is introduced into the system from the constraints that dictate the system's behavior. The constraints are morphological, biomechanical, environmental and task specific.

Deterministic – for a given starting condition, the future state of the system is determined; randomness is not present.

Dynamic system – a system that evolves over time.

Linear measures - measures that describe the central tendency or dispersion of the values within a set of numbers, such as the mean, the range, and the standard deviation.

Long-range power law correlations – fluctuations at any given moment are statistically related to fluctuations occurring at other times, both short term and long term.



- 1 Mathematical Chaos - the behavior of several related systems many times seems to be erratic
- 2 with no order (i.e. random). However, nonlinear measures demonstrate that such variations are
- 3 not random but have a deterministic pattern, meaning that their future dynamics are fully defined
- 4 by their initial conditions. This behavior is known as mathematical or deterministic chaos, or
- 5 simply chaos.
- 6 Noise - that part of a system description which is not deterministic. For simplicity, it is usually
- 7 assumed to have a simple form such as white noise.
- 8 Nonlinear measures - measures that quantify the relationship, or dependency, of the numbers
- 9 throughout the time series. Nonlinear measures describe the patterns or structure within the time
- 10 series, not simply the quantity.
- 11 Periodic - Happening or appearing at regular intervals like a sine wave.
- 12 Phase Space (or phase plane) - a representation of the behavior of the dynamic system in state
- 13 space. Typically, it takes the form of a two-dimensional plot of the position  $X$  of the time series
- 14 (on the horizontal-axis) versus the first derivative  $X'$  (on the vertical axis).
- 15 Random - lack of pattern or order; lack of a relationship between points in a time series or parts
- 16 of a system.
- 17 Regularity - the repeatability of a pattern.
- 18 Self-organization – the formation of moving patterns is a function of the cooperation of all of the
- 19 sub-systems and their interaction with the environment; it is not centrally coded or commanded.
- 20 Self-similarity- the statistical properties of the pieces of the system are proportional to the
- 21 statistical properties of the whole system.
- 22 Stability, stable state-is a rich behavioral state characterized by high complexity.

1 State space - the set of all possible states of a dynamical system. When modeling a dynamic  
2 system, the number of variables needed to describe the system is called the dimension of the  
3 state space.

4 Stochastic – synonymous with random; the current state does not determine the next state.

5 Time series – Time series data is a specific example of an ordered list of numbers, where time is  
6 the parameter that gives order to the list.

7 Variability- is inherent within all biological systems and can be described as the normal  
8 variations that occur in motor performance across multiple repetitions of a task.

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2 **FIGURE CAPTIONS**

3 Figure 1. Comparison of linear and nonlinear measures of several signals. Four signals are  
4 displayed with the respective values for range and ApEn (Aproximate Entropy).

5 Figure 2. Time series and corresponding phase plane plot. Top panel: periodic function Middle  
6 panel: time series from a chaotic system, the Lorenz attractor. Bottom panel: random time series.

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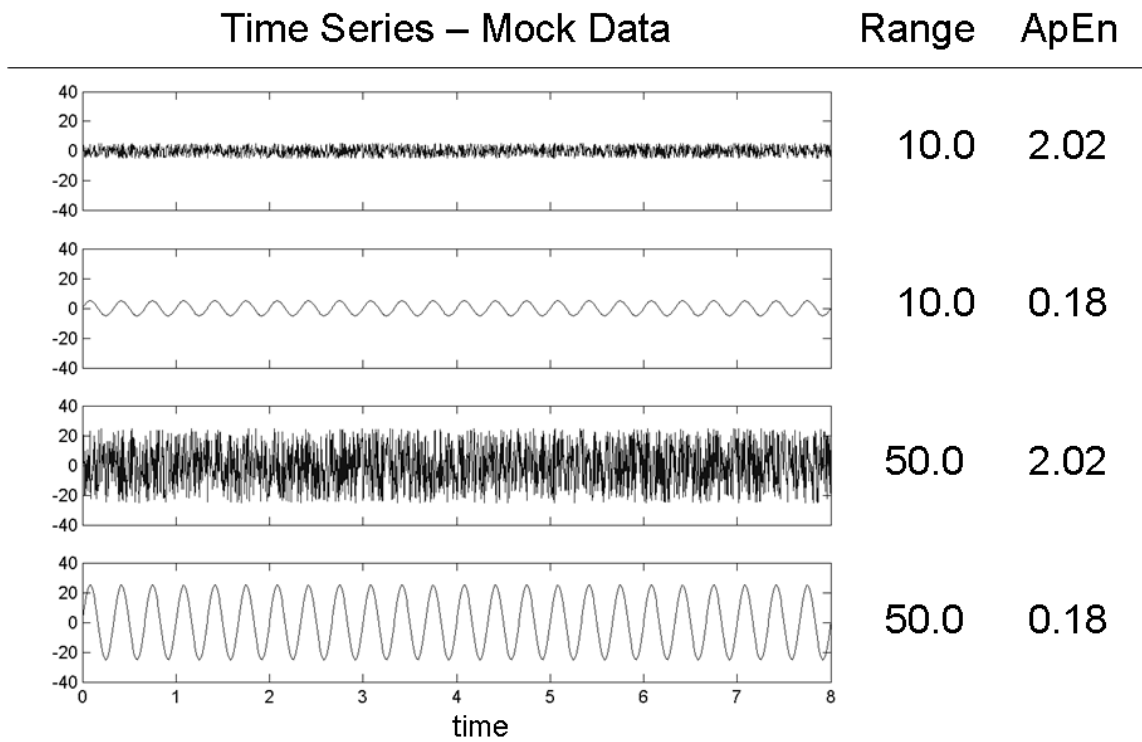
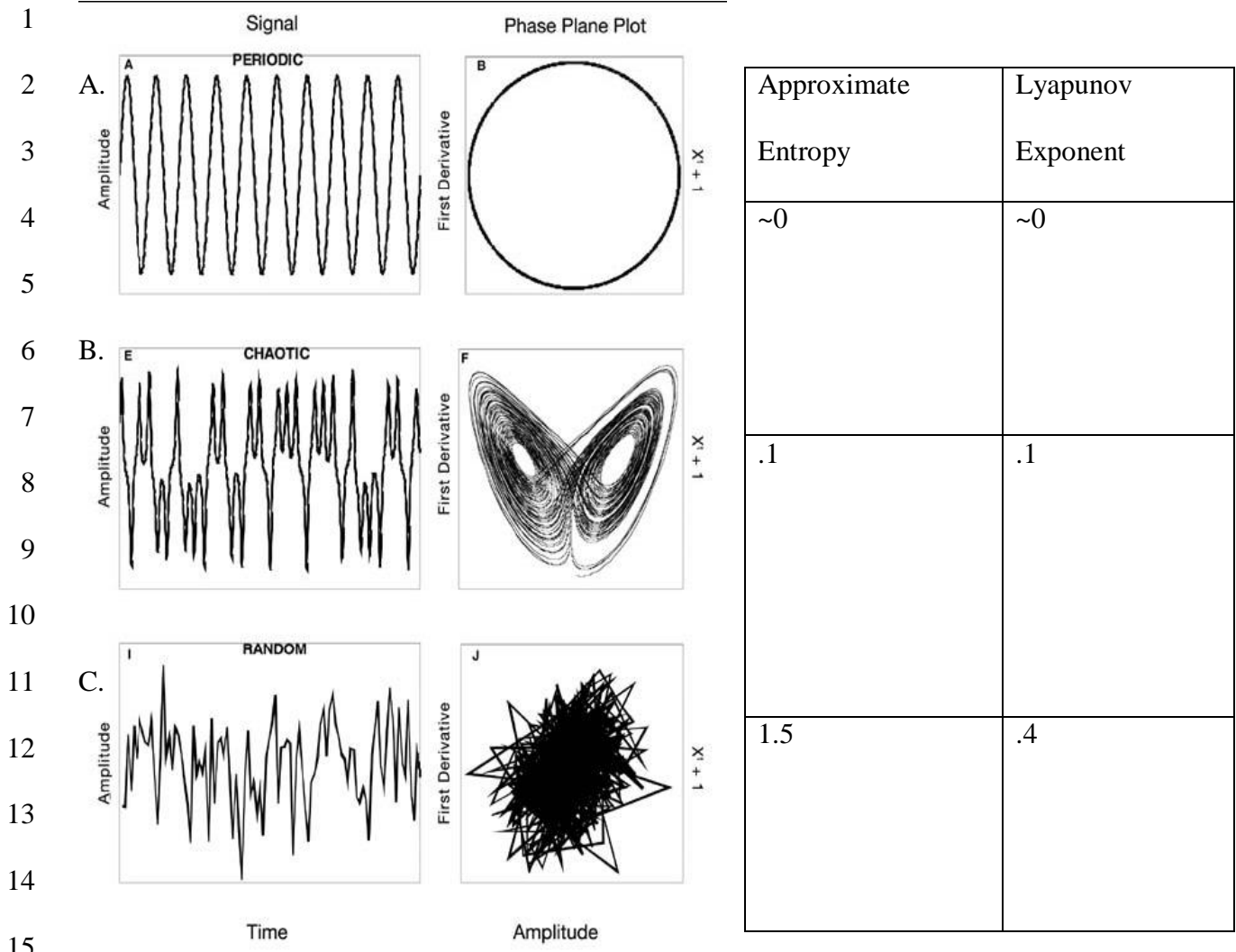


Figure 1



17 Figure 2

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