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Lyapunov Exponent and Surrogation Analysis of Patterns of Variability: Profiles in New Walkers With and Without Down Syndrome

Beth A. Smith
University of Michigan - Ann Arbor

Nicholas Stergiou
University of Nebraska at Omaha, nstergiou@unomaha.edu

Beverly D. Ulrich
University of Michigan - Ann Arbor

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Beth A. Smith, Nick Stergiou, and Beverly D. Ulrich

In previous studies we found that preadolescents with Down syndrome (DS) produce higher amounts of variability (Smith et al., 2007) and larger Lyapunov exponent (LyE) values (indicating more instability) during walking than their peers with typical development (TD) (Buzzi & Ulrich, 2004). Here we use nonlinear methods to examine the patterns that characterize gait variability as it emerges, in toddlers with TD and with DS, rather than after years of practice. We calculated Lyapunov exponent (LyE) values to assess stability of leg trajectories. We also tested the use of 3 algorithms for surrogation analysis to investigate mathematical periodicity of toddlers' strides. Results show that toddlers' LyE values were not different between groups or with practice and strides of both groups become more periodic with practice. The underlying control strategies are not different between groups at this point in developmental time, although control strategies do diverge between the groups by preadolescence.

Keywords: gait, nonlinear analysis, developmental disabilities

Movement scientists have a long history of using linear methods to analyze movement variability. Linear tools focus on the magnitude of variability and assume each repetition of a behavior, such as a step, is independent from those preceding and following. Nonlinear methods, in contrast, focus on the structure of variability by examining patterns in the variability across time and are designed to reveal how one movement influences the next. Both the magnitude and structure of variability during movement can differ between persons, and each reflects a different characteristic of the performance (Stergiou, Harbourne & Cavanaugh, 2006; Sosnoff & Newell, 2006). Within a specific magnitude of variability, differences in structure may exist. This is important, as the structure of variability has been linked to the health of biological systems. Healthy systems are those that are stable yet adaptable. Either too much consistency across repetitions (e.g., gait cycles) with

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Smith and Ulrich are with the Developmental Neuromotor Control Laboratory, School of Kinesiology, University of Michigan, Ann Arbor, MI. Stergiou is with the Nebraska Core Biomechanics Facility, University of Nebraska at Omaha, NE, and the Dept. of Environmental, Agricultural, and Occupational Health Sciences, College of Public Health, University of Nebraska Medical Center, Omaha, NE.

extremely periodic organization of variability, or lack of consistency with random organization of variability has been linked to poor health in cardiac, respiratory, and neurologic disease (Goldberger et al., 2002; Peng, Havlin, Stanley, & Goldberger, 1995; Seely & Macklem, 2004). These variability states can be thought of as opposite ends of a continuum. In between these two ends exists a deterministic yet nonperiodic pattern that provides a balance between flexibility and stability of behavior (Stergiou, Harbourne, & Cavanaugh, 2006). This state is associated with maximum complexity, which is defined as the highly variable fluctuations in physiological processes resembling mathematical chaos.

Nonlinear analysis tools offer a way to measure the patterns of variability displayed by a system and researchers have begun to apply them to the study of walking in humans. The Lyapunov Exponent (LyE) is one specific nonlinear tool that has been used to explore the variability of walking kinematics. LyE measures divergence within the trajectories of movement trajectories by quantifying their exponential separation in state space (see Figure 1). Previous work has shown that compared with their peers with typical development (TD), preadolescents with Down syndrome (DS) displayed a larger magnitude of variability (Smith, Kubo, Black, Holt & Ulrich, 2007) and higher LyE values reflecting more divergence of thigh, shank and foot segmental angles (Buzzi & Ulrich, 2004) from one walking stride to the next. The LyE results showed that there are changes in the structure, in addition to the known increases in magnitude, of gait variability occur during walking in preadolescents with DS.

Although an increase in the magnitude of variability is often associated with a decrease in stability, this relationship can also be an inverse one. This is one reason the concept of stability is difficult to define. England and Granata (2007) found a differential effect for walking speed on magnitude and structure of gait variability. They calculated LyE values for the ankle, knee and hip angles of healthy adults as they walked on a treadmill. Slow walking speeds were associated with an increase in magnitude of variability and smaller LyE values (less divergence, more order), while at faster walking speeds both variables increased (England & Granata, 2007).

The uncontrolled manifold analysis (UCM) is another tool that researchers have used to investigate the relationship between variability and stability. In regard to preadolescents with DS, use of the UCM revealed that although preadolescents with DS demonstrated a larger magnitude of variability with respect to the position of both the center of mass and the head at heel contact during gait, they also partition more variance along the manifold ($UCM_{//}$) than preadolescents with TD. Variance along the manifold is that which does not compromise performance of the task (Black, Smith, Wu, & Ulrich, 2007). This is another example where a larger magnitude of variability is not necessarily related to an increase in instability.

The studies cited above and others show that the nonlinear method of LyE is one tool that can be successfully applied to increase our understanding of the patterns of variability in adult or preadolescent gait and how its structure differs in response to injury or disease. By using both linear measures of magnitude and nonlinear measures of structure of variability, we are able to understand adult and preadolescent control strategies for movement in a more complete manner, rather than using either one in isolation. Our goal here is to use LyE to examine control strategies in the early stages of walking in toddlers with DS, before years
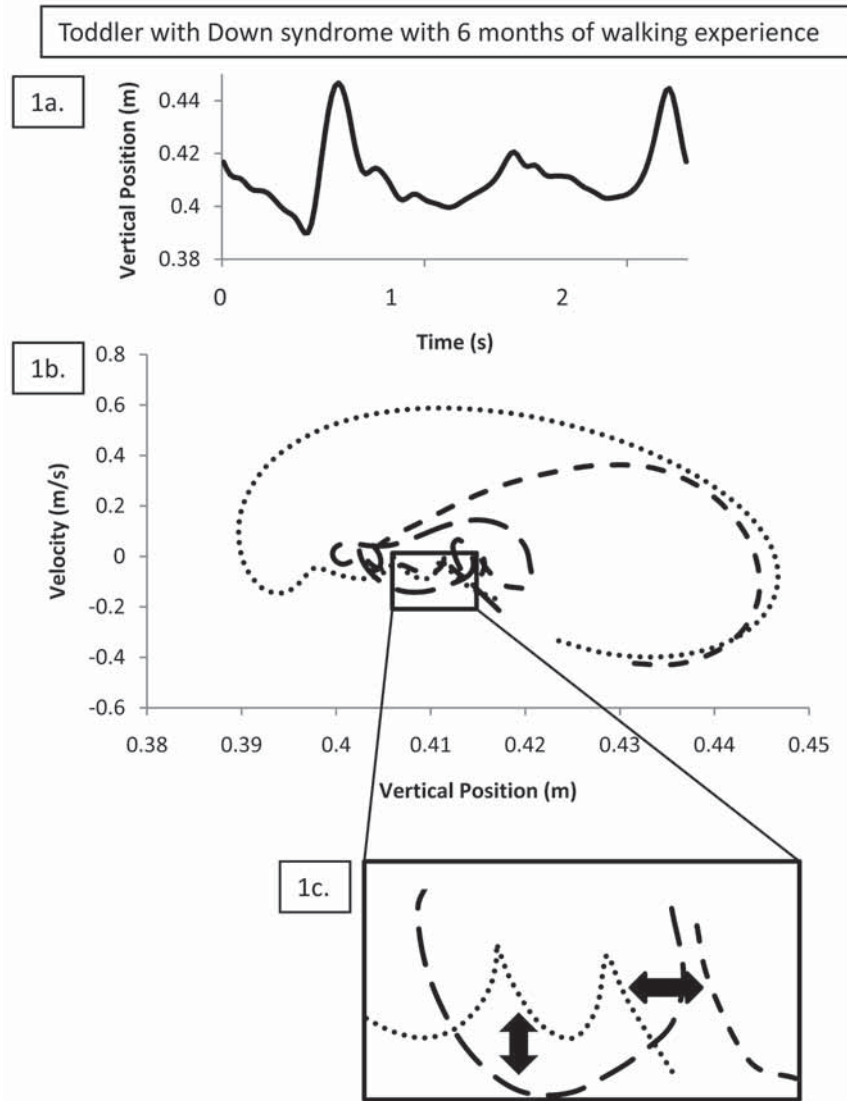
Figure 1 — Visual analogy of what Lyapunov Exponent (LyE) calculates, using the variability across cycles of the knee marker of a toddler with Down syndrome (DS). 1a is the knee marker vertical position time series, 1b shows 3 strides extracted from the time series (1a) and overlaid as position vs. corresponding velocity and 1c demonstrates a magnified version of an isolated segment of the state space to show the divergence between neighboring trajectories. One important point to note is that LyE values for data in this manuscript were calculated in an 8-dimensional space, not the 2-dimensional space pictured here.

of practice have influenced their control strategies. By preadolescence, they have figured out a way to allow a larger magnitude of variability in their performance without detracting from its success, yet we do not know if control strategies are different than their peers with TD at the earliest stages of walking.

Researchers have used linear methods to describe many aspects of variability in toddlers' walking patterns. However, they often demonstrate a limited ability to identify differences between typically and atypically developing groups, often due to the high amount of variability inherent in toddler data. Throughout the first six months of walking experience, for example, DS and TD toddlers produce a large amount of variability in EMG data (Chang, Kubo, Buzzi, & Ulrich, 2006; Chang, Kubo, & Ulrich, in press), step lengths and widths (Looper, Wu, Angulo Barroso, Ulrich, & Ulrich, 2006), and stiffness and impulse values (Black et al., 2009). As practice increases, variability in many of their step parameters decreases quantitatively. Step length variability decreases (Looper et al., 2006) as walking experience increases as does variability in interlimb (Clark, Whitall, & Phillips, 1988) and intralimb lower extremity phasing (Clark & Phillips, 1993). But there is more to the story; toddlers also show an increase over these first six months in variability of some parameters, such as step width (Looper et al., 2006), walking speed (Bril & Brenière, 1992) and stiffness and impulse values (Black et al., 2009) before settling down to lower levels of variability. In theory, nonlinear measures, by analyzing the temporal organization of variability, have the potential to extend our understanding beyond linear measures for these periods of seeming contradictions in emergent skill. They can discover if distinctions in underlying control exist between toddlers with DS and their peers with TD during early walking, despite both groups having high amounts of variability.

Our primary goal in this study was to use nonlinear analysis, specifically LyE, to compare the patterns of variability produced by toddlers with DS and TD at the onset of walking. We chose LyE because it allows us to assess the patterns of variability of the position in space of the lower extremity from one stride to the next, as opposed to linear measures of variability such as the standard deviation of stride length or knee displacement, which summarize performance but ignore temporal relationships. Understanding how variability is structured at the onset of walking will help us understand if toddlers with DS are following a similar or unique developmental trajectory in relation to their peers with TD. We must know what early patterns look like to understand the impact of and relationship between inherent neural and physiological characteristics and practice of the task itself. This knowledge may be used, ultimately, to guide intervention.

In addition to LyE, we also employed surrogation analysis to test for determinism (i.e., order) in the time series. Our secondary goal was to determine when walking becomes sufficiently mathematically periodic to be recognizable as deterministic, when its pendular flexion-extension motion (the intracycle periodic structure) develops. We explored different surrogation techniques to address the inherently noisy characteristics of toddler data.

In summary, we are asking both methodological and theoretical questions about typical and atypical development. Fundamentally, we want to understand differences in control strategies as they relate to the emergence of typical or atypical walking patterns, across the first months of independent walking. We believe

nonlinear tools, in combination with linear measures, give a more complete answer to this question than linear tools alone. However, there are methodological concerns about the application of nonlinear tools to inherently noisy, short toddler data sets. Ultimately we hope to help other researchers understand how and when these tools can be applied to human movement successfully and when they are limited.

Methods

Data Collection

Participants whose data we analyzed here were part of a larger longitudinal study approved by the University of Michigan Institutional Review Board. Consent was obtained from parents before their child's participation in the study. Parents brought their toddlers to the laboratory at the onset of walking (defined as 3–6 consecutive independent steps) and at 1 month of walking experience. In addition, participants with DS were tested at 3, 4, 6, and 8 months of walking experience and participants with TD at 2, 3, 4, 5, and 6 months of walking experience. For this analysis, we included the treadmill-walking data of 9 toddlers with DS and 9 toddlers with TD at 3, 4 and 6 months of walking experience. We evaluated treadmill walking rather than overground data because it increased the number of continuous strides we had available for analysis. Due to space constraints of our 3D motion analysis calibrated volume we were often able to collect only 5 or 6 consecutive overground strides.

When participants arrived in the laboratory, we allowed them time to play and get comfortable with the setting and staff. We removed clothing except for diapers and attached 2-cm-diameter reflective markers bilaterally at bony landmarks of their temperomandibular joints, shoulders, elbows, greater trochanters, knees, midshanks, heels, and third metatarsophalangeal joints. We also collected EMG data for the tibialis anterior, gastrocnemius, rectus femoris, biceps femoris, erector spinae and rectus abdominus muscles. To minimize wire movement and toddlers' attention to electrode wires, participants wore a pair of dark tights with holes cut out to expose their feet and the reflective markers. Results for EMG and resultant center of mass data (which required multiple reflective markers) will not be discussed further here.

Participants walked overground to their parents at a self-selected speed, over a GAITRite mat (CIR Systems, Havertown, PA) at visits 1 and 2. They performed walking trials until we collected 4 passes of 3–6 usable steps (Visit 1) or 10–15 steps (Visit 2). At subsequent visits, toddlers walked overground as in Visit 2 and then walked on a motorized treadmill (Parker brand, LET Medical Systems Corp., Miami Lakes, FL) with close supervision. We used GAITRite software to calculate the average over ground walking speed of each participant, which we used to adjust the belt speed for the treadmill phase of testing, during which participants walked on the treadmill for 30 s trials at 40%, 58%, 75%, 92%, and 110% of their self-selected over ground speed. We operationalized comfortable treadmill speed as 75% of self-selected over ground speed based on previous work from our laboratory (Ulrich, Haehl, Buzzi, Kubo, & Holt, 2004), as well as subjective reports that comfortable speeds on a treadmill are slower than over ground (Alton, Baldey, Caplan, & Morrissey, 1998).

As participants walked over the gait mat and on the treadmill, we collected 3-dimensional joint marker position at 60 Hz using a six-camera Peak Motus real-time system (Vicon Peak, Centennial, CO). At the end of each walking collection we assessed developmental milestones using the motor component of the Bayley Scales of Infant Development (The Psychological Corporation, San Antonio, TX) and measured body segment lengths, height and weight.

Data Analysis

Appling nonlinear methods to toddlers' data presents unique challenges. In theory, the mathematical approach of LyE is based on an infinite amount of data (Wolf, Swift, Swinney, & Vastano, 1985). In practice, researchers often employ techniques such as LyE and surrogation, which tests for determinism in a time series, using data from approximately 40 s (Buzzi et al., 2003) to 9 min of continuous walking (Hausdorff et al., 2001). To understand how new skills are being acquired, we need to start as early as possible, yet we are limited by the ability of the toddlers to take only a few consecutive strides at walking onset. A second challenge is the fact that toddler data are "noisy". Although variability is the focus of these measurement techniques, too much noise renders them ineffective. With surrogation analysis, for example, extreme noise makes it difficult to detect inherent patterns and distinguish them from randomly generated surrogate equivalents. Figure 2 illustrates that new walkers clearly produce continuous, alternating strides, yet the data are noisy making it more difficult, mathematically, to identify their periodicity as compared with more skilled behaviors. We will, therefore, investigate three available algorithms (as described below) to address these concerns.

After extensive pilot work, we selected the left knee marker as representative of the pendular motion of the lower extremity during walking. We wanted to assess the relationship of each walking stride to the next, beyond what the linear measures of the standard deviation of the stride length and knee displacement could show us. Pilot work consisted of examining the time series of all lower extremity markers for 4 participants (2 in each group, across time). We concluded that there was not enough displacement of the greater trochanter marker, and too much extraneous motion (noise) at the foot markers (See Figure 3). For each participant at each visit, we selected the longest segments of continuous strides for treadmill walking at the 75% speed. As length of data sets must be equivalent for this type of analysis, we shortened longer segments to 276 data points (7–8 strides) of the left knee marker time series for all three directions of the three-dimensional data. We used the Tools for Dynamics software (Applied Nonlinear Sciences, LLC and Randle, Inc, Del Mar, CA) to identify the embedding dimension of our data using the Global False Nearest Neighbor algorithm (Abarbanel, 1996). The embedding dimension represents the number of dimensions needed to unfold the structure of a given dynamical system in space (Mitra, Riley, & Turvey, 1997). Our calculations indicated that 8 embedding dimensions were necessary to form a valid state space from the toddlers' knee time series, as compared with typical calculations of 5 for adult gait data (Buzzi et al., 2003; Dingwell, Cusumano, Cavanagh, & Sternad, 2001). This finding reflects the increased noise present in toddler data. In addition, we chose the appropriate time delay by calculating the first local minimum of an average mutual information algorithm (Abarbanel, 1996). We then used Chaos Data

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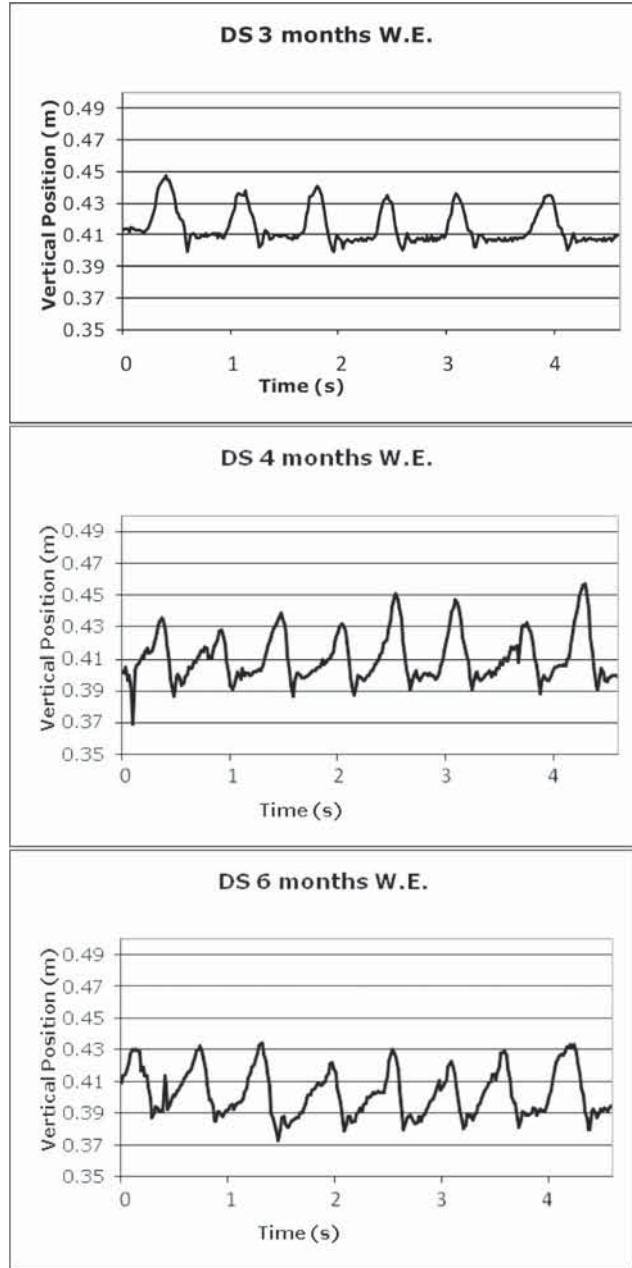
Figure 2 — Exemplar time series of knee marker data in the vertical axis across months of walking experience. Although the behavior is clearly periodic by observation, there is enough variability across strides in time, amplitude, and shape that it is difficult to identify the mathematical rules for defining each period (TD = typical development, DS = Down syndrome).
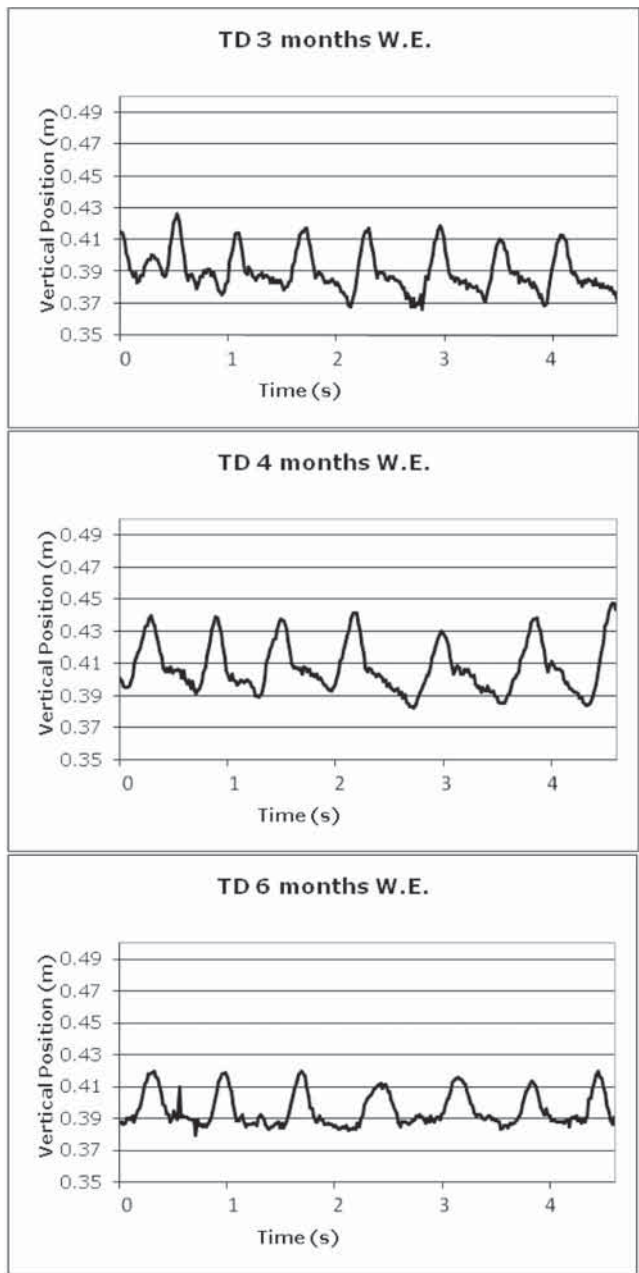
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b

Figure 2 — continued.

Analyzer Professional Version software (Physics Academic Software, Raleigh, NC) to calculate the LyE values for each direction of the three-dimensional left knee data. LyE measures divergence within the trajectories of movement patterns by quantifying their exponential separation in state space (see Figure 1). Larger values indicate more variability in the system, more divergence and possibly randomness. Shifts toward smaller values indicate less variability, less divergence and possibly rigidity. In the case of our data, a higher LyE value indicates more divergence in the trajectory of the knee movement from one stride to the next, and can reflect the use of different underlying control strategies for the production of successive strides.

We used the procedure of surrogation to further explore the structure of the data. Surrogate datasets were generated for all original knee time series using MATLab software (The MathWorks, Inc., Natick, MA) with the algorithms described in the next section. We then computed LyE values for all surrogate time series and compared them to the value of the original time series to test for differences between the original time series and its surrogate counterparts. The calculation of the LyE values from the surrogate data sets was performed as described above.

Surrogate Data

The process of surrogation removes any deterministic structure from the original data set by generating a random equivalent with the same mean and variance as the original data. We created surrogate data for the kinematic time series data using three distinct methods: the Small pseudoperiodic algorithm (Small, Yu, & Harrison, 2001) and Theiler's algorithms 0 and 1 (Theiler, Eubank, Longtin, Galdrikian, & Doyne Farmer, 1992).

The Small algorithm looks for determinism on top of inherently periodic data by preserving the intracycle dynamics and shuffling the intercycle dynamics (Miller, Stergiou, & Kurz, 2006). It tests pseudoperiodic time series data against the null hypothesis of a periodic orbit with uncorrelated noise. Theiler's Algorithm 1 generates phase-randomized surrogates of the time series by computing Fast Fourier transforms (FFT) of the original data, randomizing the phase spectra, and computing the inverse FFTs. The power spectrum and correlation function are preserved while the probability distribution is different. When we compare this surrogate data to its original form, the null hypothesis being tested is that original time series is linearly filtered noise (Theiler et al., 1992).

With Theiler's Algorithm 0, the data are simply shuffled. Here we preserve the probability distribution; however there may be a different power spectrum and a different correlation function. Comparing this surrogate data to its original form tests the null hypothesis that the original time series is an independent and identically distributed noise (Theiler et al., 1992).

Statistical Analysis

For each direction, LyE values of the surrogate data were compared with the LyE values of the original data using a paired *t* test. For data sets with significant differences between original and surrogate data sets, we used a linear mixed model to examine fixed effects of group, time (walking experience) and a group by time interaction for differences in LyE values. We used an unstructured variance-covariance structure for the repeated measures across time (for more
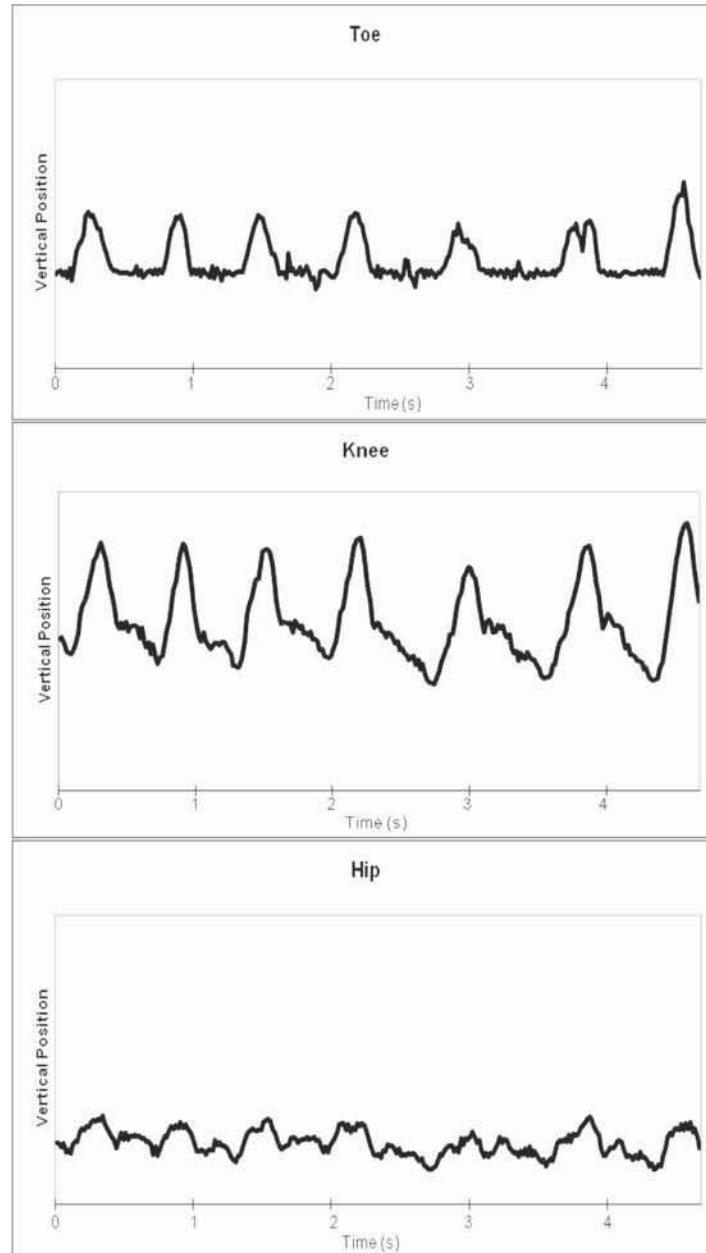
Figure 3 — Exemplar vertical direction time series from treadmill walking for one toddler with typical development with 4 months walking experience (W.E.). For the hip marker, the cyclic motion of the leg is not clear. The toe displays cyclic motion; but noise in the time data, particularly in the troughs, would decrease accuracy of results. The knee marker displays cyclic motion with the least amount of noise.

information see the study of Bagiella, Sloan, & Heitjan (2000). Missing data points were defined as missing at random (technical reasons or child/family related missing data) and did not exclude the toddler's other data points from analysis. We used SPSS Version 15.0 (SPSS, Chicago, IL) with an alpha level of significance set at 0.05 to calculate all statistical tests.

Results

When using the Small algorithm for surrogation, all surrogate data sets (across ages and groups) produced lower LyE values than their original counterparts. This indicates the surrogation technique was not successful. Surrogate data, as purposely-generated random equivalents of the original data, should have higher LyE values. The Small algorithm was unable to detect determinism in the data as they are not periodic to begin with. The assumption that this algorithm makes that the data have inherent intracycle periodic dynamics is not valid as the toddlers have not yet developed clear pendular walking cycles. We then tested the Theiler algorithms and their underlying assumptions about the structure of the original time series.

Using Algorithm 1, which assumes the original time series are linearly filtered noise, we found significant differences between the original and surrogate data sets for the medial-lateral (mean pair difference [original—surrogate] = 0.05, t = 9.55, df = 64, p < .01) and vertical (mean pair difference [original—surrogate] = 0.01, t = 2.88, df = 64, p = .01) directions. However, in both cases, the LyE values of the surrogate data were lower than those of the original data, again indicating surrogation was not successful as surrogate data, as purposely-generated random equivalents of the original data, should have higher LyE values.

For Algorithm 0, which assumes the original time series data are an independent and identically distributed noise, significant differences were found between the original data and surrogate data in all directions. Again, the LyE values of the surrogate data were lower than those of the original data for the anterior-posterior (mean pair difference [original—surrogate] = 0.09, t = 14.00, df = 64, p < .01) and medial-lateral (mean pair difference [original—surrogate] = 0.06, t = 6.63, df = 64, p < .01) directions. Results for the vertical direction (mean pair difference [original—surrogate] = -0.03, t = -3.66, df = 64, p = .01), however, indicated the fluctuations observed in the original time series were more periodic than their randomly derived counterparts. Consistent with its deterministic origin, the order and periodicity of the knee marker data in the vertical direction is recognized by the surrogation algorithm. Defining success by its ability to identify differences between surrogate and original data, this surrogation technique is more successful as age increases (see Figure 4).

We then calculated the LyE of the knee vertical direction data. The LyE values showed no significant differences between the groups (F[1,12.6] = 0.23, p = .64). There was not a time effect (F[2,11.5] = 0.81, p = .47) or a group by time interaction (F[2,11.5] = 0.02, p = .98) see Figure 5).

Successful Surrogation
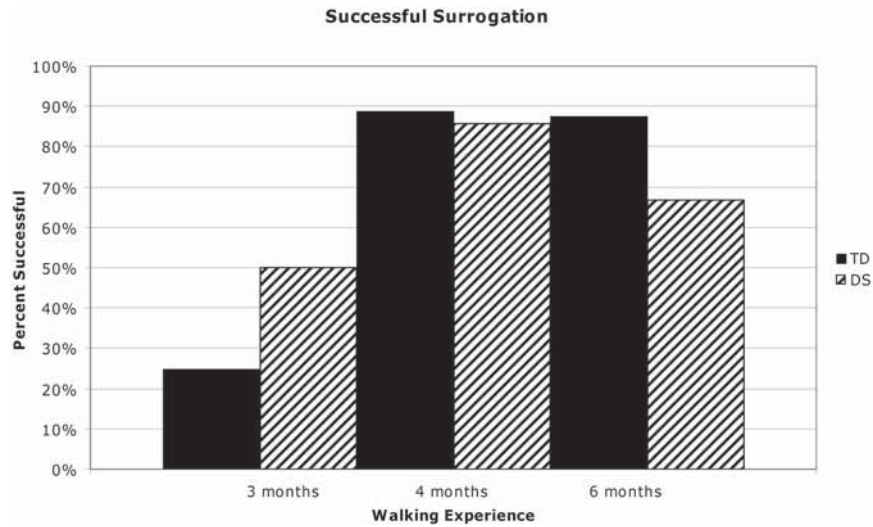
Figure 4 — Success rates of surrogation increase across time using Theiler's Algorithm 0 to create surrogate data. Success is defined as identifying more periodicity in knee vertical direction time series than in their randomly-generated surrogate counterparts (TD = typical development, DS = Down syndrome).

Knee, vertical direction
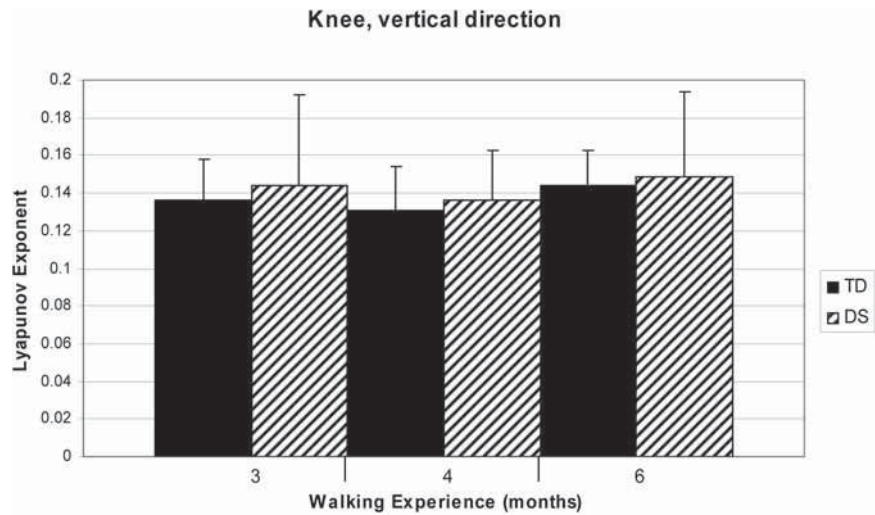
Figure 5 — Mean LyE values for the left knee vertical direction by group across time. Error bars represent 1 standard deviation (TD = typical development, DS = Down syndrome).

Discussion

Our efforts to apply LyE analysis to the emergent patterns of variability in toddlers' gait produced important insights into the existing limitations of this mathematical tool but also revealed some new information about the level of control underlying their early walking. Developmentalists consider walking to have emerged when infants "toddle" for three consecutive strides before falling. By one month of experience toddlers can walk across the room, stop and start, and change directions, yet even when they have been practicing this skill for six months many of their joint motions are not sufficiently "smooth" to be mathematically defined as periodic using existing mathematical algorithms. This reveals a unique contrast between the control issues underlying the emergence of truly novel behaviors that are extremely complex and difficult to acquire (as we present here) and behaviors that have been practiced for many years (e.g., gait in adolescents or adults), the latter of which have been used in most previous applications of these tools in the human motor control literature.

In addition, this type of analysis works best when copious amounts of data- or minutes worth of continuous walking cycles are analyzed. This is quite difficult to procure for voluntary movements during infancy and early childhood, thus the capacity to observe optimal performance in a comfortable and relaxed context for young performers is limited. Here we chose to elicit walking by placing toddlers on a motorized treadmill. While it allowed us to monitor more continuous strides per toddler than we could have for overground walking, we still had comparatively small numbers of strides to analyze and the treadmill itself added a cognitive challenge. Toddlers do not understand walking in place as easily as they understand walking overground to retrieve toys. The treadmill also provides the challenge of adapting to an external pace rather than driving the pace. However, we believe we minimized this challenge by engaging these toddlers in the game of walking in place with our encouragement and the pace was tailored to their individual self-selected overground gait velocity.

By three months of walking experience, toddlers were able to produce 7–8 consecutive strides of treadmill walking. This allowed us a small number of strides for LyE calculation. Our results suggest that LyE values in the *vertical* direction were not different between groups, nor did they change significantly through additional months of practice. It appears that after three months of practice, toddlers have achieved a functionally stable, although noisy, walking pattern with a minimum level of quantifiable stability to be able to produce continuous strides. The fact that it required three months or less to achieve and did not change appreciably in the subsequent three months suggests one of two things: 1) this is the necessary and sufficient "region" of stability, or 2) changing beyond this necessary "region" will require considerable additional practice, perhaps years.

Although LyE analysis was not able to reveal changes in the stability of the knee trajectory from one stride to the next as walking experience increased during this period of development, surrogation analysis enabled us to quantify periodicity. Overall, toddlers' strides become more periodic between 3 and 6 months of practice. Surrogation analysis was more successful as experience increased, indicating the vertical direction variability is becoming more periodic (see Figure 4).

While movement in the vertical direction displayed variability that is stable and becoming more periodic, by 6 months of practice motion in the anterior-posterior and medial-lateral directions did not meet the criteria for successful surrogation analysis. This failure indicates that movement in these axes is not yet mathematically defined as periodic. In other words, the data contain too much variability for successful use of existing algorithms. This was a disappointing finding. In a similar experience, Polk and colleagues (2008) examined toddler thigh and shank gait phase portraits. Using linear analysis to quantify variability, the authors found their results to be limited by too much intra and interindividual variability (Polk et al., 2008). We anticipated that nonlinear analysis could succeed where linear analyses have not. For the moment, this is not the case.

LyE analysis does allow us to investigate further patterns of variability by measuring the stability of trajectories, or quality of the movement, across time. For example, both of our groups of new walkers showed high quantities of variability. We predicted that toddlers with DS, due to their inherent ligamentous laxity, hypotonia and balance difficulties, would show less stable knee motion from one stride to the next than toddlers with TD. Although our prediction was not supported, a significant difference does emerge by preadolescence. Eight to ten-year olds with DS show significantly higher quantity of variability and less structure/more adaptability (higher LyE values) for hip, knee and ankle segmental angles as compared with their peers with TD (Buzzi & Ulrich, 2004). This is a similar to the developmental pattern of stiffness and impulse values. There is not a group difference in toddlers' stiffness or impulse values (Black et al., 2009), however a group difference does emerge by preadolescence, when 8–10 year-olds with DS demonstrate higher stiffness and no significant difference in impulse when walking overground and higher stiffness and impulse values when walking on a treadmill as compared with their peers with TD (Ulrich et al., 2004).

It is also possible that the high inter and intraindividual variability merely masks a group difference, as it often does when comparisons are performed between typical and atypical toddler groups. As toddlers become more skilled in their behavior, group differences in emergent control strategies, as measured by LyE, become apparent and statistically significant by preadolescence (Buzzi & Ulrich, 2004). The overall question of whether a lack of difference in LyE values between the groups reflects their use of a similar control strategy, and thus similar values, or is due to high variability and a lack of power, is a difficult one to answer. We performed a power analysis on our data, which indicated that we would need 37 participants per group to find a difference in LyE values. In addition, an average effect size of 0.2 indicates a small effect of the group difference. These data suggest that if a statistically significant population difference exists, it is very small and not likely to be of practical or interpretable significance.

Based on several published studies, we know that control strategies used by preadolescents and adults with DS are different from and more variable than that of their peers with TD. We also know that their energy cost for preferred movement patterns is greater (Ulrich et al., 2004), but also modifiable to some extent (Smith et al., 2007). Understanding as much as we can about their unique patterns of movement and control strategies allows us to better decide when and how to intervene to help them learn the most efficient and functional movement patterns possible. Measuring structure of variability, in addition to quantity of variability,

gives us further insight into the emergence of their unique patterns. Our findings here indicate that, although some of the endpoint parameters of gait (e.g., step width and stride length) show differences between toddlers with DS and TD due to inherent differences in their systems, the underlying control strategies are not different between groups at this point in developmental time. Control strategies do, however, appear to diverge between the groups across years of walking practice and experience as the walkers "settle in" on preferred patterns.

Conclusion

Although the focus of nonlinear analysis is variability, current algorithms are very limited in their ability to analyze short sets of inherently noisy and highly variable new walker data. Especially in the case of toddler data, researchers should select variables that consist of longer, less noisy continuous movement time series that are believed to reflect the outcome of the toddlers' selected control strategies. Although challenging to apply, especially to toddler data, we believe nonlinear analysis techniques have the potential to help, in a unique way, to answer questions about how variability is expressed in different systems and how organizational control emerges and changes with practice. We successfully applied nonlinear analyses to show that toddlers' LyE values were not different between groups or with practice and strides of both groups become more periodic with practice. Preadolescents with DS produce larger LyE values during walking than peers with TD, a difference in control strategy that emerges over years of practice.

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