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Sundials and Their Shady Past

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Mathematical Connections - 2003

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Introduction

Throughout history, patterned approaches to determining and subdividing time have been considered among the greatest natural mathematical connections ever conceived by mankind. Over the years, this mathematical quest for time has certainly been shrouded in utility (as evidenced by atomic clocks that are accurate to within trillionths of a second by using the mathematically predictable resonance frequencies of elements such as Cesium), but the more *romantic* and aesthetic virtues of historical time pieces lie in the interplay between pure mathematics and architectural beauty. Although the movement of celestial bodies relative to one another has acted as a basis for the historical development of time telling devices, it is the clocks themselves that provide clues to human insights, ingenuity, and romance. And so it follows that perhaps history really is written in the stars.

Historical Background

It is safe to assume that early in human history, men must have recognized the passage of time, whether it were in the form of night following day, or perhaps by the sequencing of seasons. Subdivisions of daylight hours were perhaps most clearly observable through the apparent movement of shadows, which changed slowly in their direction and length as the day grew old. The measuring of shadows to tell time is even illustrated in Chaucer's Canterbury Tales. In the opening lines of his *Parson's Prologue* he writes:

> It was four o'clock according to my guess,
> Since eleven feet, a little more or less,
> My shadow at the time did fall,
> Considering that I myself am six feet tall.

This simple use of ratios was probably the precursor to the sundial, but we do not know exactly when men first began to use instruments similar to modern sundials. Some biblical historians believe that the Old Testament refers to the sundial of Ahaz, about 700 B.C. (see Isaiah 38:8 and II Kings 20:11). About a century later, the Greek philosopher and astronomer Anaximander is said to have introduced the sundial into Greece. From there, sundials grew in popularity and were the primary timekeeping medium for centuries. They even remained in use long after the invention of the clock, since early mechanical clocks tended to be a bit erratic and needed frequent correction by the sundial. Today, their purpose is primarily aesthetic, yet they remain a source of fascination and testimony of man's early ingenuity.

Sundial Mechanics

Most sundials are heavily dependent on trigonometric ratios and either use tables or some other type of geometric procedures to mark the hour lines on a “dial plate.” There are many different kinds of sundials, so the mathematical models that govern their construction are as varied as the sundials themselves. Three of the more common sundials in particular will be discussed here: the Polar Dial, the Horizontal Dial, and the Perforated Ring Dial. These three types of sundials have been chosen because they illustrate increasingly sophisticated mathematical models.

The mechanics of sundials have nothing to do with moving parts, but rather predictable patterns of the apparent movement of the sun over the surface of the earth. This apparent movement causes shadows to be cast from vertical references. Nearly all types of horizontal sundials require a meridian mark of some sort, and the simplest sundials have North-South references to this mark. One possible method for finding the meridian mark without the use of special instruments is to create concentric circles around a vertical object such as a nail. These concentric circles can be used to measure the length and direction of the shadows in both the morning and the afternoon. When the path of the shadow is measured in the morning at a given direction and length, and again in the afternoon at a reflective direction and length, the result is an angle having its vertex at the base of the nail. The resulting angle bisector provides a fairly accurate representation of the meridian line. Since the meridian mark represents the center of a sundial, shadows cast from a vertical object resting on the meridian line are very predictable. The sun has an apparent movement of 15 degrees East to West each passing hour of the day. This is true at any latitude since the 360 degrees of the sun’s circular path around the earth is divided evenly into 24 parts, each part being one hour. Because congruence can be established by vertical angles at the tip of the shadow source, it follows then that a 15 degree angle from the tip of the vertical object to the dial plate would allow one to calculate the distance of successive hour marks from the meridian mark using tangents for 15, 30, 45 degrees and so on if the height of the vertical object is known. An even simpler process yet would be to use a protractor and a string to simulate a shadow’s path from the tip of the vertical object to the dial plate.

The Polar Dial

The Polar Dial is possibly the simplest type of sundial, both architecturally and mathematically. A Polar Dial typically uses a rectangular dial plate upon which shadows are cast, a vertical gnomon, and a style (which rests atop the gnomon and determines the shadow line on the dial plate). The dial plate itself, although roughly horizontal, is not necessarily level, but rather lies parallel to the earth’s axis and has a center line which runs North-South representing local apparent noon. Local apparent noon is simply the meridian mark upon which a vertical, North-South running Gnomon rests where no shadow is cast. Each successive hour line runs parallel to the noon mark and is spaced according to the length of shadows at a given time of day. The hour marks are symmetric about the noon mark with the morning hours indicated at the left of the noon mark as cast by westerly shadows and the afternoon hours to the right of the noon mark as cast by easterly shadows. The vertical plate extending from the center of the dial plate is a rectangular Gnomon (derive from the a Greek word meaning “one who knows”) and rests perpendicular with respect to the dial plate. This type of sundial is certainly limited in that the sun can never throw a shadow onto the dial plate of a polar dial until sometime after 6:00 A.M., and since the shadow leaves the dial sometime before 6:00 P.M., there is no need to mark the dial plate past those hours. Although the polar dial is not limited by season, the completed dial plate is set up for use with the 12:00 hour and the gnomon both in the meridian and with the entire dial plate tipped up to make an angle with the horizon equal to the latitude where the dial rests.

The Horizontal Dial

The horizontal sundial is by far the most common of the sundials. Their popularity is probably due in part to the fact that they tell the time whenever the sun is shining, where other types of dials are restricted to certain daylight hours (i.e. the polar dial). They are also comparatively easy to construct and place for use. The components of a horizontal sundial are basically the same as for the polar dial (dial plate, gnomon, and style), but the horizontal sundial is different from the polar dial in that the shape of the gnomon restricts the latitude in which the dial can be used. For horizontal dials, the gnomon is a roughly triangular plate that sticks vertically out of the dial plate like a shark fin. In order to work properly, the base angle of the gnomon must be congruent to the latitude where the dial rests. It is for this reason that most commercially produced sundials, sold in garden shops and other stores, work only at one given latitude. For instance, a horizontal sundial used in North Dakota would have a gnomon angle much different than one designed for southern Texas. There are several ways to plot the hour lines on the dial plate of a horizontal sundial, including a graphic method using a unit square and requiring no mathematical calculations other than measuring angles with a protractor. The dial plate may also be marked by table values or by computation. The computation method takes a basic understanding of trigonometry, which in and of itself makes it fascinating from a historical standpoint.

To mark the dial plate by computation one must know the latitude where the dial is being placed. The hour lines on the dial plate can be marked by drawing a line from the base of a predetermined North/South meridian line at an angle calculated by a special formula. Suppose we start with a North/South line representing 12:00 noon. The 11:00 hour and the 1:00 hour (15 degrees of apparent solar movement to the right and left of the meridian) can then be drawn using the following formula: $\tan H = \tan 15(\sin L)$ where H is the angle measure between the meridian line and the first hour line on either side of the noon mark, and L is the latitude where the sundial rests. Say for instance that the sundial is located at 30 degrees North latitude.

$$\tan H = \tan 15(\sin 30)$$

$$\tan H = (.268)(.5)$$

$$\tan H = .134$$

$$H = 7.63 \text{ degrees}$$

Therefore, the 11:00 hour line would lie at an angle of 7.63 degrees to the left of the meridian mark. Also, since the hour marks are symmetrical with respect to the noon mark, the 1:00 hour lies 7.63 degrees to the right of the meridian. One would continue in this fashion using 30, 45, 60, 75, and 90 degrees respectively to mark additional daylight hours on the dial plate.

The Perforated Ring Dial

Perforated ring dials were occasionally fashioned in the form of a wide circular band centered with a very small hole in the middle of one side. Through this hole, the sun could shine and project a small point of light onto the interior of the dial where the hour lines were marked. This particular dial used a point of light rather than a shadow to indicate the time of day and contained hour lines derived from a mathematical model that is a bit more sophisticated. In use, the ring was hung from a vertical position with the aperture turned toward the sun. The altitude of the sun and its relative path through the hole of the ring dial determined the time of day. The hour marks drawn on the inside of the ring were generally wavy because unlike the polar and horizontal dials, the ring dial tracked the apparent movement of the sun in a North/South fashion as well as East and West. The hole in a ring dial could be placed anywhere on the upper half of the circle, but was most effective where the angle from the horizontal to the center of the ring to the hole was equal to the colatitude. Also, because of their reliance on apparent North/South movement of the sun, ring dials are particularly sensitive to changes in polar angles with respect to the sun as happens slightly each day, month, and season. Without proper markings for each month, ring dials are probably inaccurate.

Conclusion

Although only three types of sundials were discussed here, there are many more kinds of sundials, each with its own unique artistic flavor and mathematical model governing how it works. But it is perhaps most ironic to consider that in some form or another, sundials have stood the test of *time*, the very time they have been measuring for centuries and perhaps will continue to measure for centuries to come.

O God! Methinks it were a happy life,
To be no better than a homely swain;
To sit upon a hill, as I do now,
To carve out dials quaintly, point by point,
Thereby to see the minutes how they run,
How many make the hour full complete;
How many hours bring about the day;
How many days will finish up the year;
How many years a mortal man may live.

Shakespeare
King Henry VI, Part III, 2, v.

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