8-2009

Search for correlated fluctuations in the B+ decay of Na-22

Mark P. Silverman
Trinity College, mark.silverman@trincoll.edu

Wayne Strange

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

August 2009

EPL, **87** (2009) 32001
www.epljournal.org
doi: 10.1209/0295-5075/87/32001

Search for correlated fluctuations in the β^+ decay of Na-22

M. P. SILVERMAN[(a)] and W. STRANGE

Department of Physics, Trinity College - Hartford, CT 06106, USA

received 15 June 2009; accepted in final form 21 July 2009
published online 27 August 2009

PACS 24.80.+y – Nuclear tests of fundamental interactions and symmetries
PACS 23.40.-s – β decay; double β decay; electron and muon capture
PACS 02.50.Ey – Stochastic processes

Abstract – Claims for a "cosmogenic" force that correlates otherwise independent stochastic processes have been made for at least 10 years, based on visual inspection of histograms whose shapes were interpreted as suggestive of recurrent patterns. Building on our earlier work to test nuclear alpha, beta, and electron-capture decay processes for non-randomness, we searched for correlations in the time series of e^+e^- annihilations deriving from the β^+ decay of ^{22}Na. Coincident gamma photons were counted within narrow time and energy windows over a period of 167 hours leading to a time series of more than 1 million events. Statistical tests for correlated fluctuations in the time series and its histograms were in all cases consistent with statistical control, giving no evidence of a "cosmogenic" force.

Introduction. – One of the most extraordinary claims in the scientific literature is that of the observation of correlated fluctuations between ostensibly independent stochastic processes. The claims, which have been made for more than a decade [1,2] and which, to our knowledge, have not been refuted, retracted, or independently confirmed, were based on visual inspection of the shapes of the histograms of stochastic processes like disintegration of radioactive nuclei.

The observation of at least two kinds of histogram patterns has been the basis for speculation of a "cosmogenic force". The first purportedly manifested what the authors in ref. [1] believed to be evidence of "discrete states during macroscopic fluctuations". The histogram was constructed of layers in which the first recorded frequencies of events $i=1\dots I$, the second recorded frequencies of events $i=1\dots 2I$, and so on, the j-th layer recording frequencies of events $i=1\dots jI$ for some integer I. A striking pattern of well-defined articulations in the layers signified "discrete states". The second kind of reported evidence was a perceived recurrence in time of histograms of similar shapes (*e.g.* for α-decay of ^{239}Pu), suggesting a "cosmogenic force" with daily and monthly periods.

We stress here that the "shape" of a histogram is an ill-defined geometrical feature and not an invariant characteristic. It can take widely differing forms for a given set of events depending on the number and widths of the arbitrary classes (*i.e.* categories) into which events are assigned. Moreover, the branch of mathematics known as Ramsey theory [3] virtually guarantees that any sought-for pattern can be found in the distribution of a sufficiently large set of points. Only rigorous statistical analysis can reveal whether time series and frequency distributions actually manifest correlated fluctuations. This is the motivation underlying the present article, a brief report of which was made at the *Fall 2008 Meeting of the American Physical Society* [4].

We have chosen to examine the β^+ decay of ^{22}Na for several reasons. First, the process should be governed by Poisson statistics; thus the parent probability function is known and all other pertinent statistical quantities can be determined analytically. Second, this transmutation is an example of a weak nuclear interaction with long half-life, so the time series of decays over the period of our experiment is very nearly stationary. Third, the decay yields a stable nuclide of neon and a single outgoing positron, which immediately interacts with an ambient electron leading to e^+e^- annihilation to produce two counter-propagating 511 keV γ-photons. The simplicity of the final state together with spatial correlation and narrow energy uncertainty of the γ's permits us to make coincident measurements with very low background and high signal-to-noise ratio.

Experiment and data structure. –

Test of stationary decay distributions. An initial 0.079 μCi ^{22}Na source (half-life $T_{1/2}=2.6027\pm0.0010$ y

[(a)] E-mail: mark.silverman@trincoll.edu

Table 1: Distributions of nuclear decay statistics.

Statistic	Distribution	Symbol	Probability density
Counts	Poisson $\sim$ Normal	$\mathbf{X} = \mathbf{P}(\mu) \sim \mathbf{N}(\mu, \mu)$	$f_{\mathrm{P}}(x;\mu) = e^{-\mu}\frac{\mu^x}{x!}$
Amplitude	Normal	$\{\boldsymbol{\alpha}, \boldsymbol{\beta}\} = \mathbf{N}(0, \frac{1}{2}\mu)$	$f_{\mathrm{N}}(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$
Squared Amplitude	Gamma	$\{\lvert\boldsymbol{\alpha}\rvert^2, \lvert\boldsymbol{\beta}\rvert^2\} = \mathbf{G}(\frac{1}{2}, \mu^{-1})$	$f_{\mathrm{G}}(x;r,s) = \frac{s^r}{\Gamma(r)} x^{r-1} e^{-sx}$
Power	Exponential	$\lvert\boldsymbol{\alpha}\rvert^2 + \lvert\boldsymbol{\beta}\rvert^2 = \mathbf{E}(\mu)$	$f_{\mathrm{E}}(x;\mu) = \frac{1}{\mu} e^{-x/\mu}$
Modulus	Rayleigh	$(\lvert\boldsymbol{\alpha}\rvert^2 + \lvert\boldsymbol{\beta}\rvert^2)^{1/2} = \mathbf{R}(\mu)$	$f_{\mathrm{R}}(x;\mu) = \frac{2}{\mu} x e^{-x^2/\mu}$
Amplitude Ratio	Cauchy	$\boldsymbol{\beta}/\boldsymbol{\alpha} = \mathbf{C}(0,1)$	$f_{\mathrm{C}}(x;r,s) = \frac{1}{\pi s(1+(\frac{x-r}{s})^2)}$
Autocorrelation	Normal	$\mathbf{R}(\tau > 0) = \mathbf{N}(0, \mu^2/N)$	$f_{\mathrm{N}}(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$

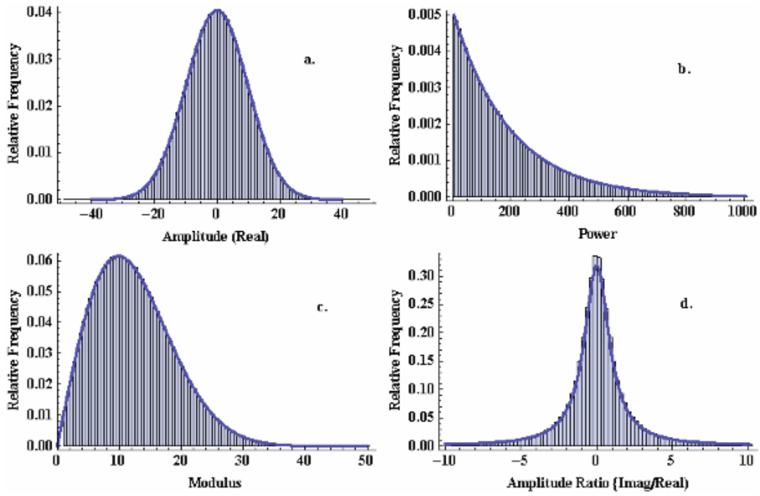

Fig. 1: (Color online) Observed and theoretical (solid line) distributions of Fourier amplitudes $\{\phi_j = \alpha_j + i\beta_j\}$ of the ^{22}Na decay time series: (a) real part $\{\alpha_j\}$, (b) power $\{\alpha_j^2 + \beta_j^2\}$, (c) modulus $\{(\alpha_j^2 + \beta_j^2)^{1/2}\}$, (d) ratio $\{\beta_j/\alpha_j\}$.

[5]) gave rise to correlated γ's, which were detected in coincidence by means of NaI(Tl) scintillation detectors within a coincidence interval of 50 ns and within a 375 keV energy range from 345 keV to 720 keV. The number of coincidences were recorded sequentially within sampling windows (bins) of $\Delta t = 0.439$ s for a total time of 167 hours, resulting in "bags" of data of 8192 bins to the hour (1 bag), with a mean coincidence count rate of approximately 441 s^{-1} and background 0.021 s^{-1}.

The time series of counts $\mathbf{X} = \{x_t\}(1 \leqslant t \leqslant N = 167 \times 8192)$ was partitioned into a temporal sequence $\{x_{a,b}\}$ of $1 \leqslant a \leqslant 167$ bags of $1 \leqslant b \leqslant 8192$ bins/bag. A maximum-likelihood line of regression was fit to the scatter plot ($\ln \mu_a$ *vs.* a) of bag means $\{\mu_a\}$ to obtain the stationary mean ($\mu_X = 193.8$), variance ($\sigma^2_{\mu_X} = 0.024$), and negative trend of magnitude $\hat{\lambda} = (8.27 \pm 0.57) \times 10^{-9}$ s^{-1}. The full data set was then transformed to a series $\mathbf{Y} = \{y_t\} = \{x_t - \mu_X + \hat{\lambda} t\}$ of 0 mean and 0 trend whose Fourier amplitudes $\{\phi_j = \alpha_j + i\beta_j\}(0 \leqslant j \leqslant \frac{1}{2}N)$ were calculated by a discrete Fourier transform (DFT) algorithm. The autocorrelation, $r_\tau = R_\tau / R_0$, in which $R_\tau = N^{-1} \sum_{t=1}^{N-\tau} y_t y_{t+\tau}$ is the autocovariance at lag τ, was calculated by inverse DFT of the power spectrum, $\{S_j = \alpha_j^2 + \beta_j^2\}$ according to the Wiener-Khinchin theorem.

The null hypothesis throughout this analysis is that the probability of a single nuclear decay within short sampling interval Δt is $p = \hat{\lambda}\Delta t$, independent of outcomes in previous and subsequent time intervals. The stationary mean is $\mu_X = N_0 \hat{\lambda} \Delta t$, where N_0 is the initial number of nuclei. Under the condition $\mu_X \gg 1$ pertinent to our experiments, there then follow all the statistical distributions summarized in table 1. (The order of parameters in the density functions is the same as in the random variable symbols.)

Figure 1 shows observed distributions with theoretical densities superposed. Table 2 gives results χ^2_{obs} of corresponding χ^2 tests of d degrees of freedom with P-values defined as the probability $P(\chi^2_d \geqslant \chi^2_{\mathrm{obs}})$ and $d =$ (number of classes) − (number of estimated parameters) − 1. Each of the distributions, determined exclusively by the single experimental parameter μ_X, tests different facets of the time series and Fourier amplitudes. The figure and table support the null hypothesis of independent decays.

Histograms and discrete structures. Histograms $\{H_a\}$ $(a = 1, \ldots, 167)$ with $K = 91$ classes of unit width

Table 2: χ^2 Test of distributions of Fourier amplitudes.

Distribution	χ^2_{obs}	d	P
Real part	45.6	45	0.45
Imaginary part	50.7	40	0.12
Square of the real part	13.3	13	0.43
Square of the imag. part	28.3	26	0.34
Power	38.8	40	0.48
Modulus	44.7	40	0.28
Ratio: imag./real	46.9	40	0.21

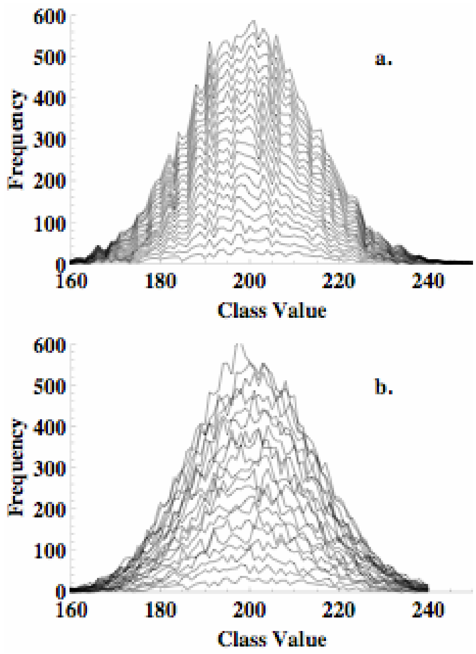

Fig. 2: 20-layered histograms (class width $=1$) generated by a Poisson RNG ($N=1000$ samples). (a) Correlated layers showing discrete structure. (b) Uncorrelated layers showing no discrete structure.

were made of the bag frequencies and subjected to χ^2 tests for fits by Poisson probability functions of corresponding means $\{\mu_a\}$. The distribution of resulting values $\{\chi^2_a\}$ was fit by the theoretical density for χ^2_{89}. A test of this fit with 14 classes yielded $P=0.26$ for $\chi^2_{13}=15.76$, which supports the null hypothesis.

The assembly of a layered histogram manifested "discrete structures" when each layer L_I was constructed as previously described, $L_I=\sum_{a=1}^{I} H_a$, but showed *no* such structure when each layer comprised a *non*-overlapping sequence of basis histograms: $L_1=H_1$, $L_2=H_2+H_3$, $L_3=H_4+H_5+H_6$, etc. The discrete structures reflect the increasing (with I) degree of correlation of layers and can be reproduced, as shown in fig. 2, by a Poisson random number generator (RNG). As such, they are an artifact of the mode of data presentation and have nothing whatever to do with correlated fluctuations arising from any physical force.

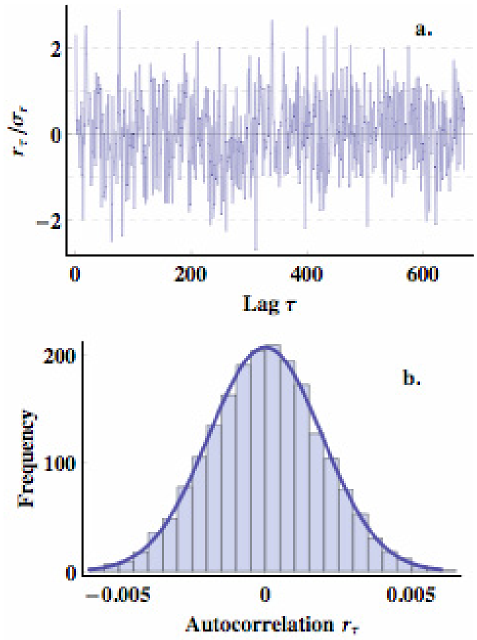

Fig. 3: (Color online) (a) Autocorrelation coefficients $r_\tau(0\leqslant\tau\leqslant 671)$ of ^{22}Na decay time series ($N=2^{18}$); lag interval $\Delta\tau=512$ bins ~ 224.77 s. (b) Distribution of r_τ fit by Gaussian density for $\mathbf{N}(0,N^{-1})$ (solid line); $P=0.81$ for $\chi^2_{19}=13.5$.

Recurrence, autocorrelation and periodicity. A histogram is a graphical representation of a multinomial distribution $\mathbf{M}(\{n_k\},\{p_k\})\equiv\mathbf{M}(\mathbf{n},\mathbf{p})$ of outcomes $k=1\dots K$ with frequencies $\{n_k\}$ and probabilities $\{p_k\}$ governed by the (discrete) probability function

$$f_M(\mathbf{n};\mathbf{p})=n!\prod_k \frac{p_k^{n_k}}{n_k!}, \tag{1}$$

subject to the constraint $\sum_{k=1}^{K} n_k=n$. If the null hypothesis is valid, the probability p_k of an outcome in the k-th class is the Poisson probability

$$p_k\equiv f_{\mathrm{P}}(x_k;\mu)=e^{-\mu}\frac{\mu^{x_k}}{x_k!}\underset{\mu\gg 1}{\longrightarrow}\frac{e^{-(x_k-\mu)^2/2\mu}}{\sqrt{2\pi\mu}} \tag{2}$$

for occurrence of x_k decays. The mean frequency and variance of the k-th class are, respectively, $\bar{n}_k=np_k$, $\text{var}(n_k)=np_k(1-p_k)$, and the covariance of two frequency classes is $\text{cov}(n_j,n_k)=-np_jp_k$. (The correlation is negative because the sum of all events is a constant.)

Since the frequency distribution (1) depends on the single parameter μ in (2), a periodicity in the chronological sequence of histograms can occur only if the population mean is periodic in time. Such periodicity could be revealed in the power spectrum and/or autocorrelation of the coincident count time series.

Figure 3a shows the autocorrelation r_τ of the time series $\{y_t\}$ as a function of lag τ up to a maximum lag of 671 units, corresponding to about 41 hours. The coefficients $\{r_k\}$ $(k\neq 0)$ are distributed normally as $\mathbf{N}(0,N^{-1})$ to an excellent approximation as shown in fig. 3b. The vertical

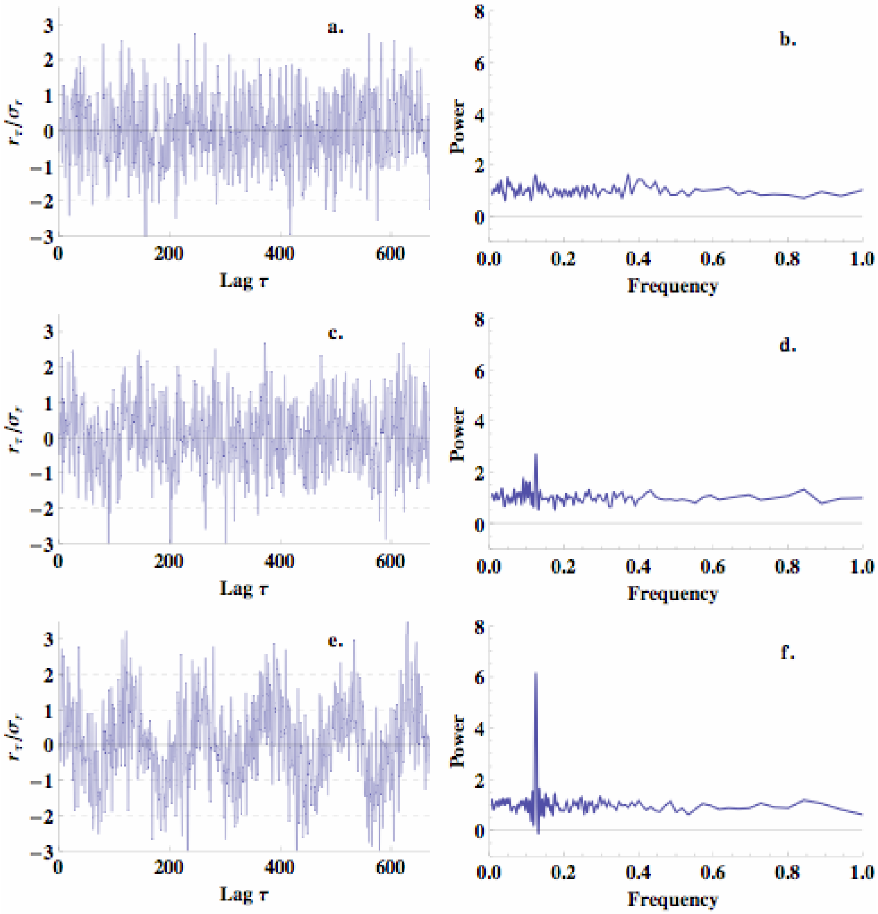

Fig. 4: (Color online) Autocorrelation and power spectrum for Poisson RNG simulated time series with periodic mean of harmonic amplitude $\beta =$ (a, b) 0, (c,d) 0.3%, (e,f) 0.5%.

scale of fig. 3a is in units of standard deviation $\sigma_r \approx N^{-1/2}$. A test of Gaussian fit led to $P = 0.81$ for $\chi^2_{19} = 13.5$. Figure 3a is indicative of white noise; there is no evidence of statistically significant correlations.

If the null hypothesis is valid, the ordinates $\{S_j\}$ of the power spectrum of $\{y_t\}$ are distributed exponentially (see table 1) with $\sigma_S = \mu_S = \sigma_X^2 = \mu_X$. The statistical significance of the *largest* ordinate can be tested by the Walker-Fisher (W-F) harmonic test [6], whereby the probability that at least one element of the set $\{S_j\}$ comprising J elements exceeds the largest observed value $S_{\max}$ is $1-(1-e^{-S_{\max}/\mu_S})^J$. By the Shannon sampling theorem, $J = \frac{1}{2}N = 687{,}847$ harmonics (corresponding to maximum period of 83.5 h). In our test the largest ordinate was $S_{\max} = 2894$ and $\mu_X = 193.8$, which yielded the probability $P(S \geqslant S_{\max}) = 0.20$, consistent with pure chance. Note that one ignores harmonic $j = 1$ because it corresponds in every DFT spectrum to the length of the time series. Thus, neither the power spectrum nor the autocorrelation spectrum gave evidence of a statistically significant component of period $T \leqslant 83.5$ h in the time series of coincident counts.

We can place an approximate limit on the sensitivity of the data to reveal a periodic component by simulating a decay time series with a Poisson RNG of time-varying mean: $\mu_X(t) = \mu_{X0}(1+\beta\cos(2\pi t/T))$. Figures 4(a)–(f) show the progressive change in the power spectrum (right panels) and autocorrelation (left panels) as the harmonic amplitude β takes on the sequential values 0 (a,b), 0.003 (c,d), 0.005 (e,f) for a period T less than the duration of the time series T_{exp}. At a threshold value $\beta \lessapprox 0.3\%$, the power ordinate $S_{\max}$ passes the W-F test for statistical control and the harmonic variation in r_τ merges with the noise. Thus, if a harmonic with larger amplitude β were present in the time series $\{y_t\}$, it would have been revealed by analysis even though visual inspection of the sequence of 167 histograms would show no statistically significant recurrences.

A time series of duration T_{exp} does not permit measurement of a period $T > T_{\text{exp}}$. However, a partial-period component, if present, would be equivalent to a trend and thereby lead to low-frequency oscillations in the power spectrum $\{G_l\}$ of the autocorrelation $\{r_\tau\}$ [7],

$$G_l = 1 + 2\sum_{\tau=1}^{m-1} r_s \cos\left(\frac{\pi\tau l}{m}\right) + (-1)^l r_m \, (l = 0, 1, \ldots, m), \tag{3}$$

Table 3: Runs up/down of descending sorted intervals of the mean (194).

Counts category C_k	Length of sequence n	Number of runs (observed)	Number of runs (theory)	Normalized residual z_R	Probability $P(z \geqslant \lvert z_R \rvert)$
190	37977	25223	25318	−1.15	0.25
191	38606	25800	25737	0.76	0.45
192	39050	26031	26033	−0.02	0.98
193	39464	26366	26309	0.68	0.50
195	39029	26099	26019	0.96	0.34
196	38839	25928	25892	0.43	0.67
197	38280	25466	25520	−0.65	0.52
198	37430	25015	24953	0.76	0.45
199	36728	24467	24485	−0.22	0.82

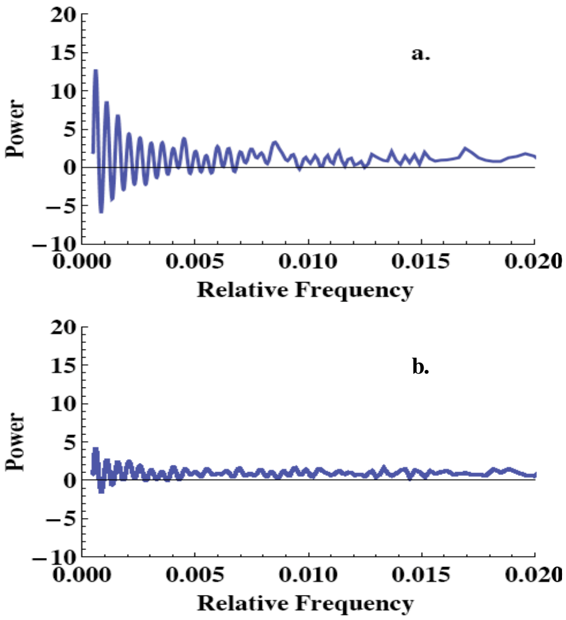

Fig. 5: (Color online) Power spectrum of autocorrelation of coincident-count time series: (a) unadjusted for negative trend due to natural lifetime; (b) Poisson RNG simulated spectrum for 1/4-cycle variation in mean with $\beta = 2.0\%$. Lag interval $\Delta\tau = 83$ bins ~ 36.44 s; relative frequency 1.0 corresponds to $(\Delta\tau)^{-1}$.

as shown in fig. 5a for the unadjusted time series $\{x_t\}$; $m = 2048$ is the maximum lag in units of sampling time $\Delta t = 36.44$ s. Transforming to the detrended series $\{y_t\}$ removes the oscillations. From computer simulations of $\{G_l\}$ for partial-period components of various amplitudes, as shown in fig. 5b, together with the fact that no linear trend other than that attributable to natural lifetime was manifested by $\{x_t\}$ to within the precision stated earlier, we conclude that our experiment would have revealed a trend resulting from an external influence of period up to $5T_{\mathrm{exp}}$, *i.e.* 835 h or about 35 days.

Recurrence, intervals and runs. A time series generated by a random process must display certain recurrent patterns, otherwise the process would be *non*random. This may seem surprising at first, since the concept of randomness ordinarily implies the *absence* of regularity. However, it is the basis for runs tests, which we applied to various nuclear decay processes in our earlier work [8–10]. In the final set of tests reported here, we searched for recurrent histogram shapes by examining the ^{22}Na decays for correlations in the intervals of different count frequencies. Recurrence in a time series of histograms requires (by definition of how a histogram is constructed) a regularity in recurrence of the frequencies of classes $C_k (k = 1 \ldots K)$, otherwise it is meaningless to say that two histograms have the same shape.

If the null hypothesis is correct, then the probability that a decay count takes the value x_k is governed by a Poisson distribution, and the interval between recurrence of two identical counts x_k in the time series of counts follows a geometric distribution with mean time $\bar{t} = f_{\mathrm{P}}(x_k;\mu)^{-1}$ and variance $\sigma_t^2 = f_{\mathrm{P}}(x_k;\mu)^{-1}[1 - f_{\mathrm{P}}(x_k;\mu)]$. In the first part of our test, comparison of the observed and predicted mean intervals for a range of count values (190–199) about the central value (194) of histograms $\{H_a\}$ showed statistical control in all cases.

The second part of our test, employing runs, was designed to test for correlations among the intervals of different classes. In a sequence of n observations, $x_1, x_2, \ldots x_n$, the $n-1$ differences $x_{i+1} - x_i$ give rise to a sequence of $n-1$ signs, "+" (run up), "−" (run down). Under statistical control, the mean number of cumulative runs up and down of length k or longer is given by [9]

$$R_k = \frac{2}{(k+2)!}\left[n(k+1) - (k^2 + k - 1)\right] \quad k \leqslant n-1. \quad (4a)$$

The theoretical total mean number of runs up and down and its variance are then

$$R_1 = \frac{1}{3}(2n - 1), \quad (4b)$$

$$\mathrm{var}(R_1) = \sigma_{R_1}^2 = \frac{1}{90}(16n - 29). \quad (4c)$$

For a sufficiently long series ($n > \sim 20$), the residual $z_R = (R - R_1)/\sigma_{R_1}$, where R is the observed total, is normally distributed as $\mathbf{N}(0,1)$.

If there is a causal periodicity to the recurrence of histograms $\{H_a\}$, then intervals for different classes must be correlated, or there is again no meaning to the idea of equivalent shapes. We defined the central class $C_0 = 194$, corresponding closest to the mean count per bin μ_X. Let C_k signify class $194 + k$, where $5 \geqslant k \geqslant -4$. Runs up/down tests were made of the intervals of C_k to establish that the results were all under statistical control. The intervals of C_0 were then arranged in descending order, and the intervals of the other classes were sorted in the corresponding order. Runs up/down tests were again performed on the intervals of C_k ($k \neq 0$) to test whether the sequences of intervals were still under statistical control or whether they were correlated with the now highly improbable rank ordering of the intervals of C_0. The results, reported for total number of runs in table 3, show that the re-ordered intervals of C_k are still under statistical control, signifying no correlation with the intervals of C_0 or with each other.

The runs-of-intervals test showed no evidence whatever that the histograms of a long time series of ^{22}Na decays gave rise to recurrent shapes.

Conclusions. – We have performed a comprehensive set of statistical tests on a time series of e^+e^- annihilations arising from β^+ disintegrations of ^{22}Na recorded continuously over a period of 167 hours. The set of tests were designed to reveal correlations in the time series of counts or in the frequencies of counts such as could lead to recurrence of histogram shapes improbably accounted for by chance alone. In all cases the time series and count frequencies passed the statistical tests, revealing no evidence of correlations or pattern formation outside of statistical control for recurrence periods $\lessapprox 35$ days.

From computer simulations of Poisson time series with amplitude-modulated means, we determined that evidence of a periodic influence would have been manifest in the autocorrelation and power spectrum at a threshold ratio of harmonic to dc term of $\sim 0.3\%$. The authors of ref. [1] claimed to have visually observed shape recurrences in the histograms of nuclear decay. Had the count frequencies of our experiment been correlated, our statistical tests would have revealed this feature even under conditions where visual inspection of histograms could not.

We note that quantum theory predicts the non-exponential decay of quasi-stationary states [11] for times short compared to a coherence time of the system (about 10^{-18} s for ^{22}Na) or long compared to a mean lifetime (about 2.5 y for ^{22}Na). The duration of our experiment (167 h) was well outside both time domains and therefore consistent with our null hypothesis, which leads to exponential decay.

The question of whether it is possible within the framework of the known laws of physics for ostensibly-independent nuclear processes to be correlated is an interesting one. The Standard Model of particles and forces predicts an ever-present background field (Higgs field) pervading all space. Similarly, the Standard Model of cosmology ("big bang" + inflation) requires an all-pervasive field (dark matter) to account for cosmic distribution of mass. Whether such fields could lead to correlated fluctuations in nuclear decay is highly dubious. Nevertheless, recent reports [12] of variable nuclear decay rates correlated with the Earth's orbital position, although contested [13,14], could conceivably lead, if true, to an annual periodicity in the histograms of nuclear decay and would call for novel explanations that may lie outside the current standard models. To examine this possibility further, we are presently modifying our experiment to observe simultaneously and for a longer duration two independent nuclear decay processes.

REFERENCES

[1] Schnoll S., Kolombet V., Pozharskii E., Zenchenko T., Zvereva I. and Konradov A., *Phys. Usp.*, **41** (1998) 1025.

[2] Schnoll S., Panchelyuga V. and Shnoll A., *Prog. Phys.*, **2** (2008) 151, and list of other references to Schnoll therein.

[3] Graham R., Rothschild B. and Spencer J. H., *Ramsey Theory* (John Wiley and Sons, NY) 1990.

[4] Silverman M. P. and Strange W., *Search for correlated fluctuations in the β^+ decay of Na-22*, in *Fall 2008 Joint Meeting of the American Physical Society and the American Association of Physics Teachers, Boston, MA, 2008.*

[5] National Nuclear Data Center at Brookhaven National Laboratory, information extracted from the Chart of Nuclides database, `http://www.nndc.bnl.gov/chart/`.

[6] Kendall M. G., Stuart A. and Ord J. K., *The Advanced Theory of Statistics, Design and Analysis and Time-Series*, Vol. **3** (Macmillan, New York) 1983, pp. 589–590.

[7] Bendat J. S. and Piersol A. G., *Measurement and Analysis of Random Data* (Wiley, New York) 1966, pp. 291–293.

[8] Silverman M. P., Strange W., Silverman C. R. and Lipscombe T. C., *Phys. Lett. A*, **262** (1999) 265.

[9] Silverman M. P., Strange W., Silverman C. R. and Lipscombe T. C., *Phys. Rev. A*, **61** (2000) 0421061.

[10] Silverman M. P. and Strange W., *Phys. Lett. A*, **272** (2000) 1.

[11] Winter R. G., *Phys. Rev.*, **123** (1961) 1503.

[12] Jenkins J. H. *et al.*, *Evidence for Correlations Between Nuclear Decay Rates and Earth-Sun Distance*, arXiv:0808.3283v1 [astro-ph] (25 August 2008).

[13] Cooper P. S., *Astropart. Phys.*, **31** (2009) 267, arXiv:0809.4248v1 [astro-ph] (24 September 2008).

[14] Norman E. B. *et al.*, *Astropart. Phys.*, **31** (2009) 135, arXiv:0810.3265v1 [astro-ph] (17 October 2008).