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Supply Chain Network Sustainability Under Competition
and
Frequencies of Activities from Production to Distribution

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Abstract: In this paper, we develop a competitive supply chain network model with multiple firms, each of which produces a differentiated product by brand and weights the emissions that it generates through its supply chain network activities in an individual way. The supply chain network activities of production, transport and distribution, and storage have associated with them distinct capacities and the firms seek to determine their optimal product flows and frequencies of operation so that their utilities are maximized where the utilities consist of profits and weighted emissions. Multiple production, storage, and transport mode options are allowed. The governing equilibrium concept is that of Cournot-Nash equilibrium. We provide both path and link flow variational inequality formulations of the equilibrium conditions and then propose an algorithm, which, at each iteration, yields closed form expressions for the underlying variables. Numerical examples illustrate the generality of the model and the information provided to managerial decision-makers and policy-makers.

This paper adds to the growing literature on sustainable supply chains through the development of a computable general competitive supply chain network game theory model, which brings a greater realism to the evaluation of profit and emission trade-offs through the incorporation of frequencies.

Keywords: game theory, supply chains, networks, Nash equilibrium, emissions, sustainability, variational inequalities, activity frequencies, production, distribution, storage

1. Introduction

Supply chains have revolutionized the manner in which goods are produced, stored, and distributed around the globe and serve as critical infrastructure networks for economic activities. Products may be manufactured on one continent, transported over thousands of miles over land and/or sea to storage facilities on yet another continent, and then further distributed to demanding customers.

Consumers have come to expect fresh produce in any season, new high tech products, as soon as they become available, stylish fashions on a regular basis, as well as medicines and pharmaceuticals, whenever needed. Each supply chain network activity, however, may have associated with it some environmental cost – from production to transportation and distribution to the storage of the products in terms of emissions generated, in addition to the energy and natural resources, such as water, that are utilized. Hence, the quantification of environmental impacts associated with supply chains, in their myriad network forms, through the minimization of emissions, is essential to sustainability.

Today, there are over 7 billion people on our planet with more than half of the world's population residing in cities. In order to quantify the impacts of supply chains on the environment it is imperative to view them holistically, in a system-wide manner. In addition, it is critical to be able to assess how different production technologies, transport modes, storage facilities, along with the frequency of the associated activities of production, transportation, and storage affect the environmental emissions (notably greenhouse gasses (GHG), especially carbon, which plays the major role today in climate change) (cf. Nagurney (1999a)). For example, in urban areas alone, the transport of freight may account (cf. Arvidsson (2013)) for 20-30% of the total vehicle distance traveled and for 16-50% of the emissions from transportation (see also Dablanc (2007)). In North America, freight transportation CO_2 emissions represented 7.8% of total US emissions in 2008 and 8% of total Canadian CO_2 emissions in 2007. In the European Union 27, according to EUROSTAT (2012), transport emissions (freight and other) comprised 19.7% of the GHG emissions for 2010.

It has been emphasized that collaboration on supply-chain carbon accounting and reporting should be developed for different modes of freight transport in order to help the freight sector lower fuel use and GHG emissions, thus reducing costs across the supply chain and improving competitiveness (see Commission for Environmental Cooperation (2011)). The transportation sector in North America is second only to electricity generation in terms of CO_2 emissions generated.

Clearly, when it comes to transport and distribution in the context of supply chain networks, the freight mode, the load on the freight mode as well as the frequency of operation will affect the emissions generated (see, e.g., Aronsson and Huge-Brodin (2006), Cullinane and Khanna (2008), VTI (2008), Arvidsson (2013)). The same can be said with respect to production activities as well as storage. The Intergovernmental Panel on Climate Change (IPCC) in its revised 1996 report has also emphasized the impact of industrial processes, including manufacturing, on various GHG emissions, in industrial sectors ranging from cement production to the food industry. According to Hadhazy (2009), manufacturing and industrial processes release GHGs, which the Environmental Protection Agency (EPA) estimates are equivalent to approximately 350 million metric tons of carbon dioxide emissions – 5% of the total US greenhouse gas emissions. According to EUROSTAT (2012), industrial processes, including manufacturing, accounted for 7.3% of the GHG emissions in 2010 in the European Union 27.

Due to both theoretical relevance and practical significance, the topic of sustainable supply chains, from modeling, analysis, to design, has garnered growing research activity. Researchers (cf. Beamon (1999), Sarkis (2003), Corbett and Kleindorfer (2003), Nagurney and Toyasaki (2005), Sheu, Chou, and Hu (2005), Kleindorfer, Singhal, and van Wassenhove (2005), Nagurney, Liu, and Woolley (2007), Seuring and Muller (2008), Linton, Klassen, and Jayaraman (2007), Nagurney and Woolley (2010), Boone, Jayaraman, and Ganeshan (2012)) have noted that sustainable supply chains are essential for both operations and the environment. In addition, Nagurney and Woolley (2010) emphasized that customers and suppliers will punish polluters in the marketplace that violate environmental rules, with the consequence that polluters may face lower profits, also called a "reputational penalty," which will reveal itself in a lower stock price for the company (Klein and Leffler (1981), Klassen and McLaughlin (1996)). Roper Starch Worldwide (1997) noted that more than 75% of the public would switch to a brand associated positively with the environment when price and quality are equal; and nearly 60% of the public favors companies that support the environment. Furthermore, sound environmental practices may reduce a firm's risk (Feldman, Soyka, and Ameer (1997)).

Certain firms, from the automobile manufacturer, Ford Motor Co. (Trudell (2013)), to the Swedish clothing retailer and manufacturer, Hennes & Mauritz, commonly known as H&M, are taking active efforts to reduce emissions. For example, Ford, the Dearborn, Michigan-based company, is targeting a 30% reduction in carbon dioxide emissions per vehicle from its factories by 2025 after a 37% cut from 2000 to 2010. H&M identified that 51% of its carbon imprint in 2009 was due to transportation. In order to reduce the associated emissions, it began more direct shipments that avoided intermediate warehouses, decreased the volumes shipped by ocean and air by 40% and increased the volume of products shipped by rail, resulting in an over 700 ton decrease in the amount of carbon dioxide emitted (see also Nagurney and Yu (2012)). In 2011, H&M achieved its target of a 5% year-on-year reduction in its carbon emissions, according to the company's 2011 corporate sustainability report (see Environmental Leader (2012)). The company's CO_2-equivalent emissions per million SEK ($148,500) of sales were 3.16 metric tons, down from 3.33 in 2010. H&M says the reduction was achieved through reducing the transportation of goods via air by 32%, improving energy efficiency in its stores, and offsetting using Gold Standard-verified carbon reduction projects.

Procter & Gamble (P&G) realizes that sustainability drives efficiency and that this is particularly the case in supply chain transport and logistics and notes that a company need not sacrifice profits to achieve sustainability (see Waters (2013)). Since 2002, according to Waters (2013), P&G has more than halved the impact it has on the environment through energy usage, CO_2 emissions, water usage, and waste disposal and has redesigned its network through the location of its distribution centers in Europe as well as its use (and loadage) of transport modes. Another illuminating example is that of ICA, a Swedish grocery chain. It reduced emissions by centralizing its distribution network with the basic idea that, instead of each supplier sending one small truck directly to each store, they are all routed to a single central warehouse from which ICA then sends one large consolidated truck to the store (cf. ICA (2008)). Thus, more tonne-kms but fewer vehicle-kms, which has resulted in lower emissions, an estimated reduction of 20%. This demonstrates the importance of the selection of the links in the network and also their frequencies as each store gets deliveries from fewer but larger trucks (lower cost, lower emissions). The frequency in the number of trucks in ICA is reduced, but (in most cases) not the frequency of delivery of each product group as they share vehicles.

In this paper, we develop a competitive supply chain network model consisting of a finite number of firms competing in an oligopolistic manner. Each firm produces a product, which is associated with its brand, and seeks to maximize its utility, with a firm's utility function consisting of its profits minus its weighted emissions generated. We note that, in many industries, from pharmaceuticals to high technology to fast fashion and even certain food products, products may be distinguished by the producer or the brand (cf. Nagurney et al. (2013)). Such products are, nevertheless, substitutes. In our competitive supply chain network model, each firm may have, at its disposal, multiple production technologies, with different associated emissions generated, multiple transportation modes for shipping the product to storage facilities and for ultimate distribution to the demand markets, also with different associated emissions, as well as multiple storage options, if feasible. In addition, each firm's supply chain network activities have associated with them capacities (capacity of the manufacturing facility for production, capacity of a transport mode (truck, ship, airplane, etc.), capacity of the warehouse or distribution center). Moreover, the frequencies of operation of each link activity, which provide information as to the number of shipments, the number of manufacturing runs needed, the number of warehouse content replacements, etc., are variables in our model.

Our new model builds upon a growing literature on the sustainability of supply chains and extends those that have appeared in the literature in several ways as discussed below.

1. It considers elastic demand and multiple competing firms, along with profit maximization, whereas Nagurney (2013) focused on a single firm, cost minimization, and the case of fixed demands. The latter paper was also concerned with supply chain network design as was the paper by Nagurney and Nagurney (2011), which also considered only a single supply chain firm in the network and had no frequencies included.

Our new model, unlike the above-noted ones, handles total operational cost functions on the links that are nonseparable, and that may depend not only on the particular link's flow but on the flows on the links in the particular firm's supply chain network, as well as on those on the other firms' supply chain links, in order to capture competition for resources. In addition, we include link cost functions associated with the frequency of operation. Moreover, since the firms' utility functions include profits, the demand price functions associated with the firms' brands at the demand markets are also nonseparable and can depend, in general, not only on the demands for the specific firm's product at that and the other demand markets, but also on the other firms' product demands at all the markets. This generality provides flexibility in modeling and in capturing different competitive environments.

2. It extends the model of Nagurney and Yu (2012) to include link capacities and frequencies as well as multiple production technology options and multiple storage technology options, with associated emissions. Hence, in our new model, in order to better reflect reality, firms have their own capacities associated with their supply chain network activities and select the frequency of their link operations, from production through storage, transportation, and ultimate distribution, which, in turn, will affect emissions. The emission functions now depend on both the link flow and on the frequency, which is a generalization of the emission functions of Nagurney and Yu (2012). Hence, a firm can evaluate its impact on the environment and on its profits by varying the weight that it imposes on its environmental emissions. In addition, the weights can play the role of environmental taxes and, thus, our model is also useful to policy-makers who may wish to assess the impact on emissions by

imposing such taxes on firms.

3. Our new game theory model is broader in scope than several existing models since it does not focus on a single industrial sector or application as does the work of Nagurney, Masoumi, and Yu (2012) for blood supply chains and that of Nagurney and Nagurney (2012) in medical nuclear ones, both of which used a generalized network framework to capture product perishability. See also Yu and Nagurney (2013) for a game theory network model for food supply chains. Moreover, the application-based models were concerned primarily with waste minimization in sustainable supply chains, whereas, in this paper, we emphasize the environmental emissions generated. Also, much of the previous healthcare applications assumed cost minimization, whereas here we consider profit maximization for each firm, as well as the minimization of emissions generated, with an appropriate weight for each firm.

4. Furthermore, the proposed algorithmic scheme, which yields closed form expressions in flows and frequencies for each of the competing firms, can be interpreted as a discrete-time adjustment process until the equilibrium state is achieved. It proceeds from iteration (time period) to iteration and reveals the type of information needed, which consists of, for a given firm, its flows and frequencies from the preceding period and the other firms' flows and frequencies, which can be observed or estimated in practice. Since the model is computable, a firm may evaluate different forms for its functions, explore the addition or deletion of competitors as well as the addition and deletion of demand markets, modes of transportation, production technologies, and changes in link capacities, etc., to name just a few scenarios. The model, with the accompanying algorithm, also allows for various sensitivity analysis exercises to be conduced as we demonstrate in the numerical example section.

This paper is organized as follows. In Section 2, we present the competitive supply chain network model with brand differentiation, and with supply chain activity frequencies. We derive the governing equilibrium conditions for the noncooperative game theory model and also present two equivalent variational inequality formulations. We also describe the information provided from the solution of the game theory model, which is of value to managerial decision-makers and to policy-makers. In Section 3, we provide a computational procedure that yields closed form expressions, at each iteration, for the variables, that is, the path flows, the link frequencies, and the Lagrange multipliers associated with the link capacity constraints. In Section 4, we present our numerical examples. We summarize and conclude in Section 5.

2. The Sustainable Supply Chain Network Model Under Competition and Frequencies

In the model, there are I firms, with a typical firm denoted by i, who are involved in the production, transport/shipment, storage, and distribution of a product and who compete noncooperatively in an oligopolistic manner. Each firm corresponds to an individual *brand* representing the product that it produces. The products are, hence, substitutable but are not homogeneous. The economic activities of each firm are represented but its supply chain network, as depicted in Figure 1.

Each firm i; $i = 1, \ldots, I$ is considering n^i_M manufacturing facilities/plants; n^i_D distribution centers, and serves the same n_R demand markets. Let L^i denote the set of directed links representing the supply chain network economic activities associated with firm i; $i = 1, \ldots, I$. Let $G = [N, L]$ denote the graph consisting of the set of nodes N and the set of links L in Figure 1, where $L \equiv \cup_{i=1,\ldots,I} L^i$.

We emphasize that the network topology in Figure 1 is only representative, for definiteness. In fact, the model can handle any prospective supply chain network topology provided that there is a top-tiered node to represent each firm and bottom-tiered nodes to represent the demand markets with a sequence of directed links, corresponding to at least one path, joining each top-tiered node with each bottom-tiered node. Hence, different supply chain network topologies to that depicted in Figure 1 correspond to distinct supply chain network problems.

The links from the top-tiered nodes i; $i = 1, \ldots, I$, representing the respective firm, in Figure 1 are connected to the manufacturing nodes of the respective firm i, which are denoted, respectively, by: $M^i_1, \ldots, M^i_{n^i_M}$, and these links represent the manufacturing links. The multiple links represent different manufacturing technologies and have associated with them distinct emissions.

The links from the manufacturing nodes, in turn, are connected to the distribution center nodes of each firm i; $i = 1, \ldots, I$, which are denoted by $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$. These links correspond to the transportation/shipment links between the manufacturing plants and the distribution centers where the product is stored. Observe that there are alternative shipment links to denote different possible modes of transportation (which would also have associated with them different levels of emissions). Different modes of transportation may include: rail, air, truck, ship, as appropriate. A shipment link may also represent the option of intermodal transport (see Floden (2007)). Both the manufacturing links and the shipment links have distinct capacities associated with them. Capacities, in the case of a transport mode, rep-

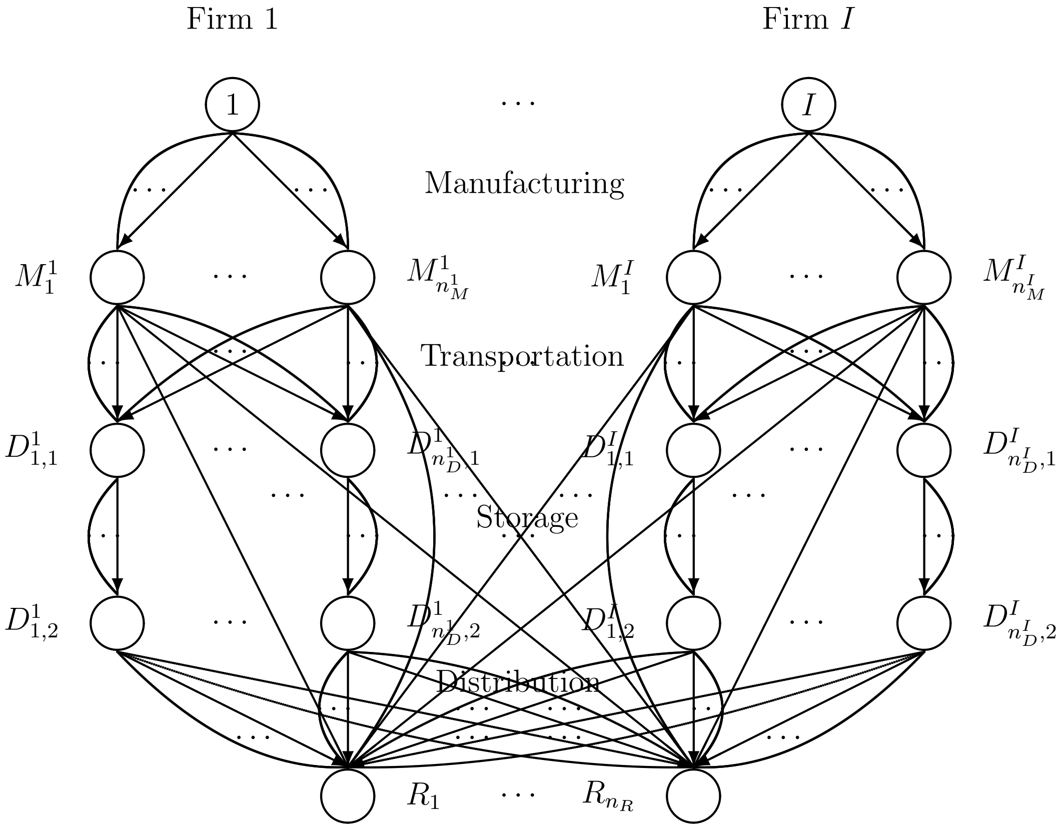

Figure 1: The Sustainable Supply Chain Network Topology

resent the volumes (flows) of the product that the mode can transport. In the case of a manufacturing link, the capacity denotes the amount of flow (volume) of the product that can be produced in a single manufacturing run.

The links joining nodes $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$ with nodes $D^i_{1,2}, \ldots, D^i_{n^i_D,2}$ for $i = 1, \ldots, I$ correspond to the storage links. The multiple storage links represent the available storage options and have associated with them different capacities, representing the maximum volume of the product (flow) that can be stored at the warehouse / distribution center.

Finally, there are possible shipment/distribution links joining the nodes $D^i_{1,2}, \ldots, D^i_{n^i_D,2}$ for $i = 1, \ldots, I$ with the demand market nodes: $R_1, \ldots, R_{n_R}$. Here, we also allow for multiple modes of transportation, as depicted by multiple arcs in Figure 1. For the sake of generality, we refer to the bottom-tiered nodes in Figure 1 as demand markets. Of course, they may correspond to retailers.

In addition, we allow for the possibility that a firm may wish to have the product transported directly from a manufacturing plant to a demand market, and avail itself of one or more transportation shipment modes. Having such an option may also be attractive to consumers and also, possibly, for the environment.

We assume that the firms, as decision-makers, are rational, which is a common assumption in game theory models, as well as in economics. In addition, we assume that the firms possess perfect information on their underlying functions, which is not unreasonable. In the discussion of the algorithm in the next Section, we also provide an interpretation of the computational scheme as a discrete-time adjustment process and the information needed.

Let d_{ik} denote the demand for firm i's product; $i = 1, \ldots, I$, at demand market R_k; $k = 1, \ldots, n_R$. The demands are variables and are not fixed. Let x_p denote the nonnegative flow on path p joining (origin) node i; $i = 1, \ldots, I$ with a (destination) demand market node. Then the following conservation of flow equations must hold:

$$\sum_{p \in P^i_k} x_p = d_{ik}, \quad i = 1, \ldots, I; \; k = 1, \ldots, n_R, \tag{1}$$

where P^i_k denotes the set of all paths joining the origin node i; $i = 1, \ldots, I$ with destination node R_k, and $P \equiv \cup_{i=1,I} \cup_{k=1,n_R} P^i_k$, denotes the set of all paths in Figure 1. According to (1), the demand for firm i's product at demand point R_k is satisfied by the product flows from firm i to that demand market. We group the demands d_{ik}; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$ into the $I \times n_R$-dimensional vector d, and the path flows x_p; $p \in P$ into the n_p-dimensional vector x, where n_p is the number of all the paths in Figure 1.

We denote the demand price of firm i's product at demand market R_k by ρ_{ik} and we assume, as given, the demand price functions:

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; \; k = 1, \ldots, n_R, \tag{2a}$$

that is, the price for firm i's product at a particular demand market may depend upon not only the demands for this product at the other demand markets, but also on the demands for the other substitutable products at all the demand points. Hence, (2a) captures competition on the demand side of the competitive supply chain network.

In view of (1), we can define the demand price functions $\hat{\rho}_{ik}$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$, in product flows, that is

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) = \rho_{ik}(d). \tag{2b}$$

We assume that the demand price functions are continuous, continuously differentiable, and monotone decreasing.

In addition, let f_a denote the flow on link a. We must have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{3}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise. In other words, the flow on a link is equal to the sum of flows on paths that contain that link. Observe that, since the firms share no links, we do not need to distinguish with superscripts the individual firm path and link flows.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P. \tag{4}$$

Let γ_a denote the activity frequency of link a. With the existing link capacities, denoted by $\bar{u}_a$; $a \in L$, which are assumed to be positive, the following constraints must hold:

$$f_a \leq \bar{u}_a \gamma_a, \quad \forall a \in L, \tag{5}$$

that is, the product flow on a link does not exceed that link's capacity times the activity frequency of that link. We group the link flows and the activity frequencies into the respective n_L-dimensional vectors f and γ. We assume that all vectors are column vectors.

The total operational cost on a link, be it a manufacturing/production link, a shipment/distribution link, or a storage link is assumed, in general, to be a function of the product flows on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \tag{6}$$

The above total cost expressions capture competition among the firms for resources used in the manufacture, transport, and storage of their products. We assume that the total cost on each link is convex and is continuously differentiable.

The total cost of operating link a at a frequency γ_a is assumed to be a function of the activity frequency of that link, that is,

$$\hat{g}_a = \hat{g}_a(\gamma_a), \quad \forall a \in L. \tag{7}$$

These frequency operational cost functions are also assumed to be convex and continuously differentiable.

In addition, all the firms are concerned with their environmental impacts along their supply chains, but, possibly, to different degrees. As done in Nagurney (2013), we denote the emission-generation function associated with link a by $\hat{e}_a$, and assume that

$$\hat{e}_a = \hat{e}_a(f_a, \gamma_a), \quad \forall a \in L. \tag{8}$$

These functions are also assumed to be convex and continuously differentiable. Here, for definiteness, we assume that the emission functions correspond to GHG emissions as in carbon emissions. However, the model and (8) are also relevant to other emissions, including particulate matter (PM), which has a large negative impact on air quality and human health (see World Health Organization (2006, 2013)). PM is generated in transport and in manufacturing, among other human activities.

Let X_i denote the vector of path flows associated with firm i, that is, $X_i \equiv \{\{x_p\}|p \in P^i\} \in R_+^{n_{P^i}}$, where $P^i \equiv \cup_{k=1,\dots,n_R} P_k^i$, and let n_{P^i} denote the number of paths from firm i to the demand markets. Γ_i is the vector of activity frequencies associated with firm i, that is, $\Gamma_i \equiv \{\{\gamma_a\}|a \in L^i\} \in R_+^{n_{L^i}}$, where n_{L^i} denotes the number of links associated with firm i. The strategy variables, then, associated with firm i are its product flows and its activity frequencies, denoted by Y_i, where $Y_i \equiv (X_i, \Gamma_i)$. Y is then the vector of all the firms' strategies, that is, $Y \equiv \{\{Y_i\}|i = 1, \dots, I\}$.

The profit of firm i; $i = 1, \dots, I$, is the difference between the firm's revenue and its total costs, and each firm i seeks to maximize its profit, that is,

$$\text{Maximize} \quad \sum_{k=1}^{n_R} \hat{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{g}_a(\gamma_a). \tag{9}$$

In addition, each firm seeks to minimize its entire environmental impact, in terms of emissions generated, that is,

$$\text{Minimize} \quad \sum_{a \in L^i} \hat{e}_a(f_a, \gamma_a). \tag{10}$$

We can now construct a weighted utility function associated with the two criteria faced by each firm. The term ω_i is assumed to be the price that firm i would be willing to pay for each unit of emission on each of its links and it is nonnegative. This term, hence, represents the environmental concern of firm i, with a higher ω_i denoting a greater concern for the environment. Consequently, the multicriteria decision-making problem faced by firm i; $i = 1, \dots, I$, is:

$$U_i = \sum_{k=1}^{n_R} \hat{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{g}_a(\gamma_a) - \omega_i \sum_{a \in L^i} \hat{e}_a(f_a, \gamma_a). \tag{11}$$

Note that, in the case of governmental regulations, the ω_is would correspond to a tax on emissions (carbon or related).

In view of (1)-(11), we may write:

$$\hat{U} = \hat{U}(Y), \tag{12}$$

where $\hat{U}$ is the I-dimensional vector of all the firms' utilities.

According to the oligopolistic market mechanism, the I firms select their product path flows and their activity frequencies in a noncooperative manner, each one trying to maximize its own utility.

Definition 1: Supply Chain Network Cournot-Nash Equilibrium

A path flow and link frequency pattern $Y^* \in K = \prod_{i=1}^{I} K_i$ is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\hat{U}_i(Y_i^*, \hat{Y}_i^*) \geq \hat{U}_i(Y_i, \hat{Y}_i^*), \quad \forall Y_i \in K_i, \tag{13}$$

where $\hat{Y}_i^* \equiv (Y_1^*, \ldots, Y_{i-1}^*, Y_{i+1}^*, \ldots, Y_I^*)$ and $K_i \equiv \{Y_i | Y_i \in R_+^{n_{P^i}+n_{L^i}}\}$.

Note that, according to (13), an equilibrium is established if no firm can individually improve its utility, by changing its production path flows and its activity frequencies, given the decisions of the other firms.

The λ_a; $a \in L$ are the Lagrange multipliers associated with constraint (5). We group the Lagrange multipliers into the n_L-dimensional vector λ.

The variational inequality formulations, in path flows and in link flows, respectively, of the Cournot-Nash (Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980)) sustainable supply chain network problem satisfying Definition 1 are given in the following theorem.

Theorem 1

Assume that for each firm i; $i = 1, \ldots, I$, the utility function $\hat{U}_i(Y)$ is concave with respect to the variables in Y_i, and is continuously differentiable. Then $Y^* \in K$ is a sustainable supply chain network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{Y_i} \hat{U}_i(Y^*), Y_i - Y_i^* \rangle \geq 0, \quad \forall Y \in K, \tag{14}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{Y_i} \hat{U}_i(Y)$ denotes the gradient of $\hat{U}_i(Y)$ with respect to Y_i. The solution of variational inequality (14), *in turn, is equivalent to the solution of the variational inequality: determine* $(x^*, \gamma^*, \lambda^*) \in K^1$ *satisfying:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}+\omega_i\frac{\partial \hat{E}_p(x^*,\gamma^*)}{\partial x_p}+\sum_{a\in L^i}\lambda_a^*\delta_{ap}-\hat{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*]$$

$$+\sum_{i=1}^{I}\sum_{a\in L^i}\left[\frac{\partial \hat{g}_a(\gamma_a^*)}{\partial \gamma_a}+\omega_i\frac{\partial \hat{E}_p(x^*,\gamma^*)}{\partial \gamma_a}-\bar{u}_a\lambda_a^*\right]\times[\gamma_a-\gamma_a^*]$$

$$+\sum_{i=1}^{I}\sum_{a\in L^i}\left[\bar{u}_a\gamma_a^*-\sum_{q\in P}x_q^*\delta_{aq}\right]\times[\lambda_a-\lambda_a^*]\geq 0,\quad \forall(x,\gamma,\lambda)\in K^1, \tag{15}$$

where $K^1 \equiv \{(x,\gamma,\lambda) | x \in R_+^{n_P}, \gamma \in R_+^{n_L}, \lambda \in R_+^{n_L}\}$ *and for each path* p*;* $p \in P_k^i$*;* $i = 1,\ldots,I$*;* $k = 1,\ldots,n_R$,

$$\frac{\partial \hat{C}_p(x)}{\partial x_p}\equiv\sum_{b\in L^i}\sum_{a\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}, \tag{16a}$$

$$\frac{\partial \hat{E}_p(x,\gamma)}{\partial x_p}\equiv\sum_{a\in L^i}\frac{\partial \hat{e}_a(f_a,\gamma_a)}{\partial f_a}\delta_{ap}, \tag{16b}$$

$$\frac{\partial \hat{E}_p(x,\gamma)}{\partial \gamma_a}\equiv\frac{\partial \hat{e}_a(f_a,\gamma_a)}{\partial \gamma_a}, \tag{16c}$$

$$\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p}\equiv\frac{\partial \rho_{il}(d)}{\partial d_{ik}}. \tag{16d}$$

In addition, (15) can be re-expressed in terms of link flows as: determine $(f^*, d^*, \gamma^*, \lambda^*) \in K^2$*, such that:*

$$\sum_{i=1}^{I}\sum_{a\in L^i}\left[\sum_{b\in L^i}\frac{\partial \hat{c}_b(f^*)}{\partial f_a}+\omega_i\frac{\partial \hat{e}_a(f_a^*,\gamma_a^*)}{\partial f_a}+\lambda_a^*\right]\times[f_a-f_a^*]$$

$$+\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\rho_{ik}(d^*)-\sum_{l=1}^{n_R}\frac{\partial \rho_{il}(d^*)}{\partial d_{ik}}d_{il}^*\right]\times[d_{ik}-d_{ik}^*]$$

$$+\sum_{i=1}^{I}\sum_{a\in L^i}\left[\frac{\partial \hat{g}_a(\gamma_a^*)}{\partial \gamma_a}+\omega_i\frac{\partial \hat{e}_a(f_a^*,\gamma_a^*)}{\partial \gamma_a}-\bar{u}_a\lambda_a^*\right]\times[\gamma_a-\gamma_a^*]$$

$$+\sum_{i=1}^{I}\sum_{a\in L^i}[\bar{u}_a\gamma_a^*-f_a^*]\times[\lambda_a-\lambda_a^*]\geq 0,\quad \forall(f,d,\gamma,\lambda)\in K^2, \tag{17}$$

where $K^2 \equiv \{(f,d,\gamma,\lambda) | \exists x \geq 0,\ and\,(1)\ and\,(3)\ hold,\ and\,\gamma \geq 0,\ \lambda \geq 0\}$.

Proof: (14) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987).

In order to obtain variational inequality (15), we note that, for a given firm i, under the imposed assumptions, (14) holds if and only if (see, e.g., Bertsekas and Tsitsiklis (1989)) the following holds:

$$\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[-\frac{\partial \hat{U}_i}{\partial x_p}+\sum_{a\in L^i}\lambda_a^*\delta_{ap}\right]\times[x_p-x_p^*]+\sum_{a\in L^i}\left[-\frac{\partial \hat{U}_i}{\partial \gamma_a}-\bar{u}_a\lambda_a^*\right]\times[\gamma_a-\gamma_a^*]$$

$$+\sum_{a\in L^i}\left[\bar{u}_a\gamma_a^*-\sum_{q\in P}x_q^*\delta_{aq}\right]\times[\lambda_a-\lambda_a^*]\geq 0,\ \forall(x,\gamma,\lambda)\in K_i^1, \qquad (18)$$

where $K_i^1\equiv\{(x,\gamma,\lambda)|x\in X_i,\ \gamma\in\Gamma_i,\ \lambda\in\Lambda_i,\}$, and $\Lambda_i\equiv\{\{\lambda_a\}|a\in L^i\}\in R_+^{n_{L^i}}$. For each path p; $p\in P_k^i$,

$$\begin{aligned}
\frac{\partial \hat{U}_i}{\partial x_p}&=\frac{\partial\left[\sum_{l=1}^{n_R}\hat{\rho}_{il}(x)\sum_{q\in P_l^i}x_q-\sum_{b\in L^i}\hat{c}_b(f)-\sum_{b\in L^i}\hat{g}_b(\gamma_b)-\omega_i\sum_{b\in L^i}\hat{e}_b(f_b,\gamma_b)\right]}{\partial x_p}\\
&=\sum_{l=1}^{n_R}\frac{\partial\left[\hat{\rho}_{il}(x)\sum_{q\in P_l^i}x_q\right]}{\partial x_p}-\frac{\partial\left[\sum_{b\in L^i}\hat{c}_b(f)\right]}{\partial x_p}-\frac{\partial\left[\sum_{b\in L^i}\hat{g}_b(\gamma_b)\right]}{\partial x_p}-\omega_i\frac{\partial\left[\sum_{b\in L^i}\hat{e}_b(f_b,\gamma_b)\right]}{\partial x_p}\\
&=\hat{\rho}_{ik}(x)+\sum_{l=1}^{n_R}\frac{\partial\rho_{il}(d)}{\partial d_{ik}}\frac{\partial d_{ik}}{\partial x_p}\sum_{q\in P_l^i}x_q-\sum_{a\in L^i}\frac{\partial\left[\sum_{b\in L^i}\hat{c}_b(f)\right]}{\partial f_a}\frac{\partial f_a}{\partial x_p}-\omega_i\sum_{a\in L^i}\frac{\partial\left[\sum_{b\in L^i}\hat{e}_b(f_b,\gamma_b)\right]}{\partial f_a}\frac{\partial f_a}{\partial x_p}\\
&=\hat{\rho}_{ik}(x)+\sum_{l=1}^{n_R}\frac{\partial\rho_{il}(d)}{\partial d_{ik}}\sum_{q\in P_l^i}x_q-\sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial\hat{c}_b(f)}{\partial f_a}\delta_{ap}-\omega_i\sum_{a\in L^i}\frac{\partial\hat{e}_a(f_a,\gamma_a)}{\partial f_a}\delta_{ap} \qquad (19)
\end{aligned}$$

and for each link a; $a\in L^i$,

$$\begin{aligned}
\frac{\partial \hat{U}_i}{\partial \gamma_a}&=\frac{\partial\left[\sum_{l=1}^{n_R}\hat{\rho}_{il}(x)\sum_{q\in P_l^i}x_q-\sum_{b\in L^i}\hat{c}_b(f)-\sum_{b\in L^i}\hat{g}_b(\gamma_b)-\omega_i\sum_{b\in L^i}\hat{e}_b(f_b,\gamma_b)\right]}{\partial \gamma_a}\\
&=\sum_{l=1}^{n_R}\frac{\partial\left[\hat{\rho}_{il}(x)\sum_{q\in P_l^i}x_q\right]}{\partial \gamma_a}-\frac{\partial\left[\sum_{b\in L^i}\hat{c}_b(f)\right]}{\partial \gamma_a}-\frac{\partial\left[\sum_{b\in L^i}\hat{g}_b(\gamma_b)\right]}{\partial \gamma_a}-\omega_i\frac{\partial\left[\sum_{b\in L^i}\hat{e}_b(f_b,\gamma_b)\right]}{\partial \gamma_a}\\
&=-\frac{\partial\hat{g}_a(\gamma_a)}{\partial\gamma_a}-\omega_i\frac{\partial\hat{e}_a(f_a,\gamma_a)}{\partial\gamma_a}. \qquad (20)
\end{aligned}$$

By making use of the definitions in (16a)-(16d), variational inequality (15) is immediate. In addition, the equivalence between variational inequalities (15) and (17) can be proved with (1) and (3). □

Variational inequalities (15) and (17) can be put into standard form (see Nagurney (1999b)): determine $X^*\in\mathcal{K}$ such that:

$$\langle F(X^*),X-X^*\rangle\geq 0,\quad \forall X\in\mathcal{K}, \qquad (21)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. Indeed, if we define the column vectors: $X \equiv (x, \gamma, \lambda)$ and $F \equiv (F_1(X), F_2(X), F_3(X))$, where

$$F_1(X) = \Big[\frac{\partial \hat{C}_p(x)}{\partial x_p} + \omega_i \frac{\partial \hat{E}_p(x,\gamma)}{\partial x_p} + \sum_{a \in L^i} \lambda_a \delta_{ap} - \hat{\rho}_{ik}(x) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x)}{\partial x_p} \sum_{q \in P^i_l} x_q;$$
$$p \in P^i_k;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R \Big], \tag{22a}$$

$$F_2(X) = \Big[\frac{\partial \hat{g}_a(\gamma_a)}{\partial \gamma_a} + \omega_i \frac{\partial \hat{E}_p(x,\gamma)}{\partial \gamma_a} - \bar{u}_a \lambda_a;\ a \in L^i;\ i = 1, \ldots, I \Big], \tag{22b}$$

$$F_3(X) = \Big[\bar{u}_a \gamma_a - \sum_{q \in P} x_q \delta_{aq};\ a \in L^i;\ i = 1, \ldots, I \Big], \tag{22c}$$

and $\mathcal{K} \equiv K^1$ then (15) can be re-expressed as (21). If we define the column vectors: $X \equiv (f, d, \gamma, \lambda)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X), F_4(X))$, where

$$F_1(X) = \Big[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} + \omega_i \frac{\partial \hat{e}_a(f_a, \gamma_a)}{\partial f_a} + \lambda_a;\ a \in L^i;\ i = 1, \ldots, I \Big], \tag{23a}$$

$$F_2(X) = \Big[-\rho_{ik}(d) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d)}{\partial d_{ik}} d_{il};\ i = 1, \ldots, I;\ k = 1, \ldots, n_R \Big], \tag{23b}$$

$$F_3(X) = \Big[\frac{\partial \hat{g}_a(\gamma_a)}{\partial \gamma_a} + \omega_i \frac{\partial \hat{e}_a(f_a, \gamma_a)}{\partial \gamma_a} - \bar{u}_a \lambda_a;\ a \in L^i;\ i = 1, \ldots, I \Big], \tag{23c}$$

$$F_4(X) = \Big[\bar{u}_a \gamma_a - f_a;\ a \in L^i;\ i = 1, \ldots, I \Big], \tag{23d}$$

and $\mathcal{K} \equiv K^2$ then (17) can be re-expressed as (21).

2.1 Information for Managerial Decision-Makers and Policy-Makers

Before we present the algorithm to compute the equilibrium product flow, frequency, and Lagrange multiplier pattern, followed by numerical examples, it is worthwhile to identify the value of the model in terms of information provided to both managerial decision-makers as well as to policy-makers.

Through the equilibrium link flows (see also Figure 1), managers of the firms' respective supply chains have, at their disposal, the amounts of the product that they should produce using each available technology at each of their manufacturing plants, the amounts that should be shipped by each available mode to each of their distribution centers and/or directly from the manufacturing plants to the demand markets, and the volumes of the shipments via different modes to the demand markets so that their individual utilities, which consist of their profits and weighted emissions, are maximized. A given firm can also assess the potential impacts of changes in its data and various cost and emission functional forms, as well as those of the demand price functions, on its utility, and evaluate the impacts of the addition or deletion of demand markets, different manufacturing and storage technologies, modes of transport, etc. They can also assess the impacts of competitors leaving the markets as well as the addition of competitors. In addition, a firm can evaluate the impact on profits and of its emissions by varying its ω_i factor. This may provide venues for marketing its concerns about the environment and sustainability.

Policy-makers, in turn, may have, at their disposal, the ability to tax firms' environmental emissions and since the ω_is can also correspond to a tax, they can evaluate the impacts on emission reduction through the assessment of levied ω_is on firms under their jurisdiction. Firms, in turn, can determine the emissions throughout their supply chains and can see the redistribution of flows across manufacturing plants, distribution centers, modes of transport, etc., under different values of the ω_is.

3. The Algorithm

In this Section, we recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of the sustainable supply chain network oligopoly model with frequencies governed by the variational inequality (15) yields subproblems that can be solved explicitly and in closed form.

Specifically, recall that, at iteration $\tau+1$ of the Euler method (see also Nagurney and Zhang (1996)), one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{24}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{25}$$

where recall that $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein. Applications to the solution of network oligopolies can be found in Nagurney, Dupuis and Zhang (1994), Nagurney (2010), Nagurney and Yu (2012), and Nagurney and Li (2013).

Explicit Formulae for the Euler Method Applied to the Sustainable Supply Chain Network Variational Inequality (15)

The elegance of this procedure for the computation of solutions to the sustainable supply chain network problem modeled in Section 2 can be seen in the following explicit formulae. In particular, (24) for the sustainable supply chain network model governed by variational inequality problem (15) yields the following closed form expression, at iteration $\tau+1$, for all the product path flows x_p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$x_p^{\tau+1} = \max\Big\{0, x_p^\tau + a_\tau\big(\hat{\rho}_{ik}(x^\tau) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^\tau)}{\partial x_p} \sum_{q \in P_l^i} x_q^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \omega_i \frac{\partial \hat{E}_p(x^\tau, \gamma^\tau)}{\partial x_p} - \sum_{a \in L^i} \lambda_a^\tau \delta_{ap}\big)\Big\}, \tag{26a}$$

and the following closed form expression for all the activity frequencies γ_a; $a \in L^i$; $i = 1, \ldots, I$:

$$\gamma_a^{\tau+1} = \max\Big\{0, \gamma_a^\tau + a_\tau\big(\bar{u}_a \lambda_a^\tau - \frac{\partial \hat{g}_a(\gamma_a^\tau)}{\partial \gamma_a} - \omega_i \frac{\partial \hat{E}_p(x^\tau, \gamma^\tau)}{\partial \gamma_a}\big)\Big\}, \tag{26b}$$

with the Lagrange multipliers being computed for $a \in L^i$; $i = 1, \ldots, I$ according to:

$$\lambda_a^{\tau+1} = \max\Big\{0, \lambda_a^\tau + a_\tau\big(\sum_{q \in P} x_q^\tau \delta_{aq} - \bar{u}_a \gamma_a^\tau\big)\Big\}. \tag{26c}$$

As mentioned in the Introduction, this computational procedure can be interpreted as a discrete-time adjustment process where the iteration corresponds to a time period. According to (26a), if the marginal utility of a firm with respect to its product path flow minus the Lagrange multipliers on the path is positive, then it should increase its path flow at a given iteration; if it is sufficiently negative, so that according to (26a) the subsequent path flow would be negative, the max operator guarantees that the next iteration's path flow is zero, so as not to violate the nonnegativity constraint on the path flows. Note that, according to (26a), a firm needs to have information on the preceding iteration's path flows and frequencies.

A similar interpretation holds for the updates on the frequencies according to (26b) with (26c) guaranteeing, in turn, that the Lagrange multipliers are always nonnegative and decrease if the capacity times the frequency exceeds the link flow, at a given iteration.

In the next Section, we solve sustainable supply chain network problems using the above algorithmic scheme.

4. Numerical Examples

In this Section, we consider two firms, Firm 1 and Firm 2, each of which is involved in the production, storage, and distribution of a single product, which is differentiated by its brand. Each firm has, at its disposal, two manufacturing plants, two distribution centers, and serves a single demand market. Hence, the topology is as depicted in Figure 2. M_1^1 and M_1^2 are domestic manufacturing plants located in the United States, whereas M_2^1 and M_2^2 are off-shore manufacturing plants with lower operational costs. The distribution centers and the demand market are in the United States.

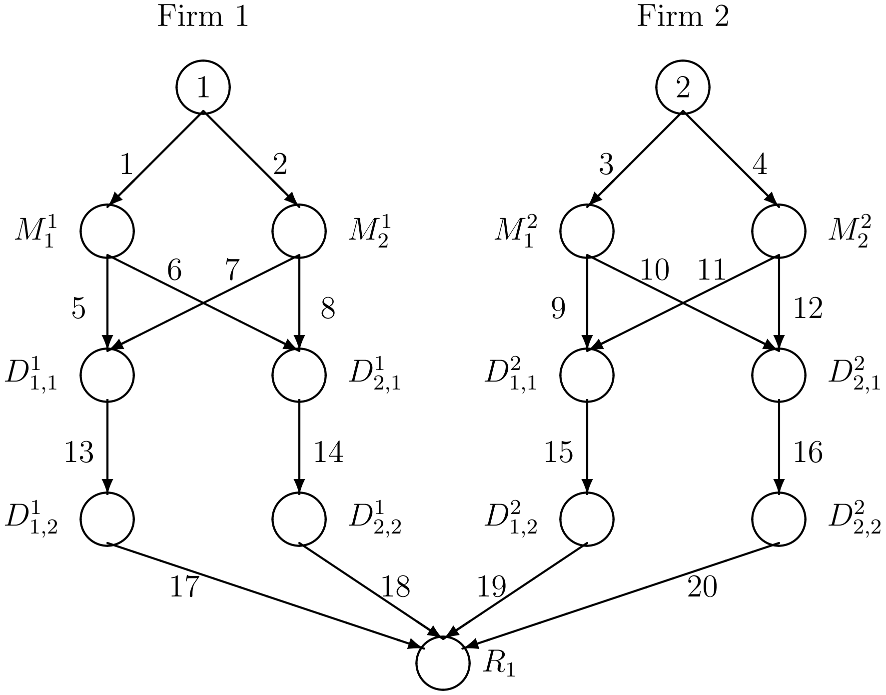

Figure 2: The Supply Chain Network Topology for Example 1

For the computation of solutions to the numerical examples, we implemented the Euler method, as discussed in Section 3, using Matlab. The convergence tolerance was $\epsilon = 10^{-6}$, and the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We considered the algorithm to have converged (cf. (26a)-(26c)) when the absolute value of the difference between successive path flows, link frequencies, and Lagrange multipliers differed by no more than the above ϵ. We initialized the algorithm by setting each path flow at 10.00, each activity frequency at 1.00, and each Lagrange multiplier at 0.00.

Example 1

In Example 1, Firm 1 cares about the emissions that it generates much more than Firm 2

Table 1: Link Capacities, Total Cost and Total Emission Functions for Example 1

Link a	$\bar{u}_a$	$\hat{c}_a(f)$	$\hat{g}_a(\gamma_a)$	$\hat{e}_a(f_a, \gamma_a)$
1	100	$5f_1^2 + 5f_1$	$\gamma_1^2 + 2\gamma_1$	$.05f_1^2 + .5f_1 + .5\gamma_1^2 + \gamma_1$
2	100	$.5f_2^2 + 4f_2$	$.5\gamma_2^2 + \gamma_2$	$.08f_2^2 + .8f_2 + .8\gamma_2^2 + 1.5\gamma_2$
3	100	$5f_3^2 + 4f_3$	$\gamma_3^2 + 1.5\gamma_3$	$.1f_3^2 + .5f_3 + \gamma_3^2 + 1.5\gamma_3$
4	100	$.5f_4^2 + 2f_4$	$.5\gamma_4^2 + .8\gamma_4$	$.15f_4^2 + .8f_4 + 2\gamma_4^2 + 2\gamma_4$
5	20	$.5f_5^2 + 2f_5$	$\gamma_5^2 + \gamma_5$	$.08f_5^2 + .5f_5 + \gamma_5^2 + \gamma_5$
6	20	$.5f_6^2 + 3f_6$	$\gamma_6^2 + \gamma_6$	$.08f_6^2 + .8f_6 + \gamma_6^2 + \gamma_6$
7	50	$f_7^2 + 10f_7$	$1.5\gamma_7^2 + .5\gamma_7$	$.05f_7^2 + .8f_7 + 1.5\gamma_7^2 + \gamma_7$
8	50	$f_8^2 + 8f_8$	$1.5\gamma_8^2 + .5\gamma_8$	$.05f_8^2 + .5f_8 + 1.5\gamma_8^2 + \gamma_8$
9	20	$.5f_9^2 + 1.5f_9$	$\gamma_9^2 + .8\gamma_9$	$.1f_9^2 + .5f_9 + \gamma_9^2 + 1.5\gamma_9$
10	20	$.5f_{10}^2 + 2f_{10}$	$\gamma_{10}^2 + .8\gamma_{10}$	$.1f_{10}^2 + .8f_{10} + \gamma_{10}^2 + 1.5\gamma_{10}$
11	50	$.8f_{11}^2 + 10f_{11}$	$1.5\gamma_{11}^2 + .3\gamma_{11}$	$.08f_{11}^2 + .8f_{11} + 1.75\gamma_{11}^2 + \gamma_{11}$
12	50	$.8f_{12}^2 + 8f_{12}$	$1.5\gamma_{12}^2 + .3\gamma_{12}$	$.08f_{12}^2 + .5f_{12} + 1.75\gamma_{12}^2 + \gamma_{12}$
13	100	$.5f_{13}^2 + 1.5f_{13}$	$\gamma_{13}^2 + .5\gamma_{13}$	$.01f_{13}^2 + .1f_{13} + .1\gamma_{13}^2 + .1\gamma_{13}$
14	100	$.5f_{14}^2 + 1.5f_{14}$	$\gamma_{14}^2 + .5\gamma_{14}$	$.01f_{14}^2 + .1f_{14} + .1\gamma_{14}^2 + .1\gamma_{14}$
15	100	$.5f_{15}^2 + f_{15}$	$.8\gamma_{15}^2 + \gamma_{15}$	$.05f_{15}^2 + .1f_{15} + .1\gamma_{15}^2 + .2\gamma_{15}$
16	100	$.5f_{16}^2 + f_{16}$	$.8\gamma_{16}^2 + \gamma_{16}$	$.05f_{16}^2 + .1f_{16} + .1\gamma_{16}^2 + .2\gamma_{16}$
17	20	$f_{17}^2 + f_{17}$	$\gamma_{17}^2 + \gamma_{17}$	$.1f_{17}^2 + f_{17} + 2\gamma_{17}^2 + 1.5\gamma_{17}$
18	20	$f_{18}^2 + 1.5f_{18}$	$\gamma_{18}^2 + \gamma_{18}$	$.1f_{18}^2 + 1.5f_{18} + 2\gamma_{18}^2 + 1.5\gamma_{18}$
19	20	$.8f_{19}^2 + f_{19}$	$\gamma_{19}^2 + .8\gamma_{19}$	$.2f_{19}^2 + f_{19} + 3\gamma_{19}^2 + 2\gamma_{19}$
20	20	$.8f_{20}^2 + 1.5f_{20}$	$\gamma_{20}^2 + .8\gamma_{20}$	$.2f_{20}^2 + 1.5f_{20} + 3\gamma_{20}^2 + 2\gamma_{20}$

does, which is indicated by the respective values of ω_1 and ω_2, where $\omega_1 = 5$ and $\omega_2 = 1$. In addition, Firm 1 utilizes more advanced technologies in its supply chain activities in order to lower the emissions that it generates, but at relatively higher costs.

Links 5, 6, 9, 10, and 17-20 correspond to the domestic shipment by small trucks, each with a capacity of 20, while links 7, 8, 11, and 12 represent international shipment by sea, followed by domestic rail, with a capacity of 50. Hence, the latter links correspond to intermodal transport. The link capacities, the total cost and the total emission functions for all the links are given in Table 1.

The demand price functions for the two products at demand market R_1 are:

$$\rho_{11}(d) = -d_{11} - .2d_{21} + 400, \quad \rho_{21}(d) = -2d_{21} - .5d_{11} + 400.$$

The computed equilibrium link flows, activity frequencies, and Lagrange multipliers are reported in Table 2. For completeness, below, we also provide the computed equilibrium path flows. There are four paths for each firm labeled as follows (please refer to Figure 2):

for Firm 1:

$$p_1 = (1,5,13,17), \quad p_2 = (1,6,14,18), \quad p_3 = (2,7,13,17), \quad p_4 = (2,8,14,18);$$

and for Firm 2:

$$p_5 = (3,9,15,19), \quad p_6 = (3,10,16,20), \quad p_7 = (4,11,15,19), \quad p_8 = (4,12,16,20).$$

The computed equilibrium path flow pattern is:
for Firm 1:

$$x^*_{p_1} = 6.97, \quad x^*_{p_2} = 5.26, \quad x^*_{p_3} = 21.17, \quad x^*_{p_4} = 22.31;$$

for Firm 2:

$$x^*_{p_5} = 4.84, \quad x^*_{p_6} = 3.71, \quad x^*_{p_7} = 19.42, \quad x^*_{p_8} = 20.41.$$

Table 2: Computed Equilibrium Link Flows, Activity Frequencies, and Lagrange Multipliers for Example 1

Link a	f^*_a	γ^*_a	λ^*_a
1	12.23	.1223	.0786
2	43.48	.4348	.1241
3	8.55	.0855	.0334
4	39.83	.3983	.0479
5	6.97	.3486	.5091
6	5.26	.2630	.4578
7	21.17	.4233	.2624
8	22.31	.4462	.2706
9	4.84	.2418	.1634
10	3.71	.1855	.1521
11	19.42	.3884	.0765
12	20.41	.4082	.0791
13	28.14	.2814	.0184
14	27.57	.2757	.0183
15	24.26	.2427	.0165
16	24.12	.2413	.0164
17	28.14	1.4069	1.9726
18	27.57	1.3784	1.9413
19	24.26	1.2130	.6252
20	24.12	1.2060	.6224

The computed demand for Firm 1's product is 55.71 and the price is 334.62, while the demand for Firm 2's product is 48.38 and the price is 275.39. Given Firm 1's effort to reduce its generated emissions, the consumers reveal their preferences for the product of Firm 1. Therefore, consumers are willing to pay more for Firm 1's product. Consequently, the profit of Firm 1 is $12,818.14$ with its total emissions being 549.68, while the profit of Firm 2 is $9,387.54$ with its total emissions being 754.66. The utilities (cf. (11)) for Firm 1 and for Firm 2 are: $10,069.74$ and $8,632.88$, respectively. Hence, Firm 1 emits less pollution and has both a higher profit and a higher utility than Firm 2. The total emissions generated by both firms in their supply chains is: $1,304.34$.

The equilibrium link flow, frequency, and Lagrange multiplier information reported in Table 2 provides valuable information for the managerial decision-makers responsible for the supply chain of Firm 1 and Firm 2. For example, Firm 1 now knows that its off-shore manufacturing plant should produce at a level $f_2^* = 43.48$ and at a level of $f_1^* = 12.23$ at its domestic plant. Firm 2, on the other hand, knows that it should produce at a level of $f_4^* = 39.83$ at its off-shore plant and at a level of $f_3^* = 8.55$ at its domestic plant. The values of the other equilibrium link flows let the respective firm identify how much to ship from each of its manufacturing plants to each of its distribution centers, and, finally, to the demand market. In addition, we note that the frequencies of all the distribution links (links 17-20) are greater than 1. In other words, due to the high volume of products to be distributed, the number of shipments from each distribution center to the demand market is greater than 1. For example, on link 17 (cf. Table 2), Firm 1, according to γ_{17}^*, would ship one full truck of its product to demand market R_1 and another one that would be just over 40% filled.

We investigate Firm 1 exploring other distribution options, so as to further reduce the emissions of its distribution activities, in Examples 2 and 3 below.

We also conducted sensitivity analysis by setting ω_1 and ω_2 equal to zero. In other words, Firm 1 and Firm 2 decide their product flows and activity frequencies without the consideration of their generated emissions. Equivalently, since, as mentioned earlier the ω_is can also play the role of environmental taxes imposed by the governmental regulatory body or policy-maker, having the flexibility to vary the ω_is is also useful from a policy perspective.

The computed demand for Firm 1's product is 72.31 and the price is 317.42, while the demand for Firm 2's product is 51.36 and the price is 261.12. The profit of Firm 1 is $13,551.23$ with its total emissions being 903.90, while the profit of Firm 2 is $9,023.13$ with its total emissions being 857.36. Due to consumers' preference, the profit of Firm 1 is still significantly higher than that of Firm 2. It is interesting to note that the profit of Firm 2 is lower without the consideration of the emissions! This analysis further supports that sacrificing of profit may not be necessary for accomplishment in sustainability.

However, we also note that with $\omega_1 = \omega_2 = 0$, the total emissions are now: $1,761.26$, a substantial increase from $1,304.34$ in which the weights (or taxes) were positive.

Example 2

In Example 2, Firm 1 is considering the utilization of large trucks for the distribution from its distribution center $D^1_{2,2}$ to the demand market. As shown in Figure 3, there is a new link 21 joining node $D^1_{2,2}$ with node R_1. The capacity of link 21 is 30, which is significantly larger than that of the other distribution links. The total cost and the total emission functions of link 21 are:

$$\hat{c}_{21}(f) = f_{21}^2 + 1.5f_{21},$$
$$\hat{g}_{21}(\gamma_{21}) = \gamma_{21}^2 + 1.5\gamma_{21},$$
$$\hat{e}_{21}(f_{21}, \gamma_{21}) = .1f_{21}^2 + 1.5f_{21} + 2\gamma_{21}^2 + 2\gamma_{21}.$$

The remaining data are identical to those in Example 1 with weights $\omega_1 = 5$ and $\omega_2 = 1$.

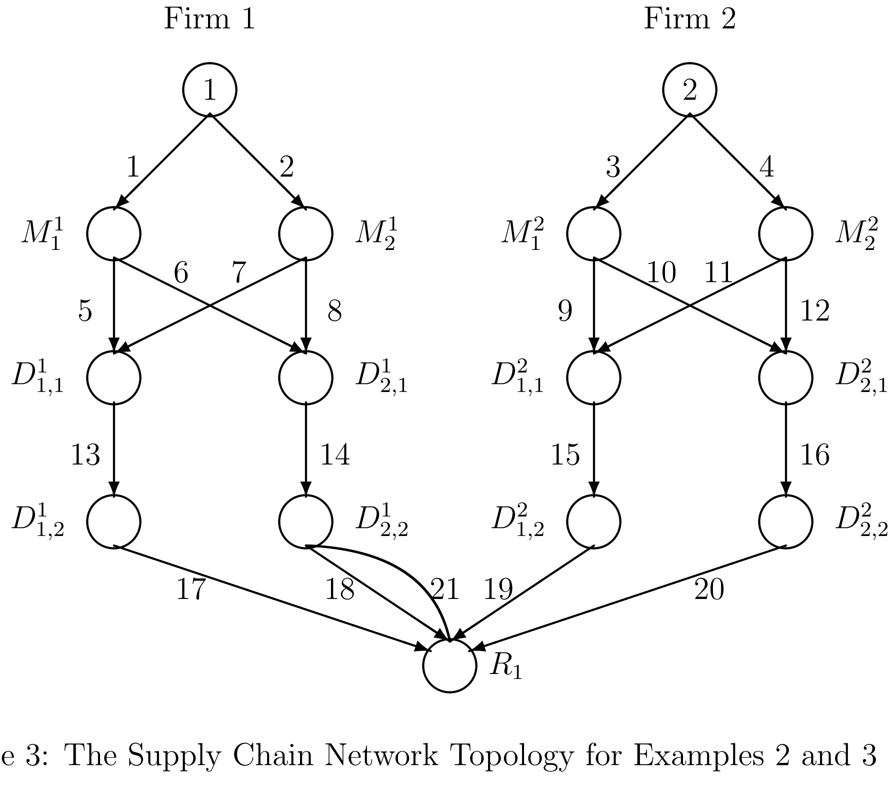

Figure 3: The Supply Chain Network Topology for Examples 2 and 3

Due to the added link 21, there are two new paths for Firm 1 labeled as follows:

$$p_9 = (1, 6, 14, 21), \quad p_{10} = (2, 8, 14, 21).$$

The computed equilibrium path flow pattern is now:

for Firm 1:

$$x^*_{p_1}=4.45,\quad x^*_{p_2}=4.28,\quad x^*_{p_3}=20.63,\quad x^*_{p_4}=13.00,\quad x^*_{p_9}=4.37,\quad x^*_{p_{10}}=13.09;$$

for Firm 2:

$$x^*_{p_5}=4.81,\quad x^*_{p_6}=3.69,\quad x^*_{p_7}=19.31,\quad x^*_{p_8}=20.29.$$

The computed equilibrium link flows, activity frequencies, and Lagrange multipliers are reported in Table 3.

The computed demand for Firm 1's product is 59.83 and the price is 330.55, while the demand for Firm 2's product is 48.10 and the price is 273.89. The profit of Firm 1 is $13,643.14$ with its total emissions being 566.85, while the profit of Firm 2 is $9,280.21$ with its total emissions being 746.74. The utilities for Firm 1 and for Firm 2 are: $10,808.91$ and $8,533.48$, respectively.

The total emissions for both supply chains are: $1,313.59$.

Example 3

Example 3 has the same data as Example 2 except that now link 21 represents the option of rail-truck intermodal transport with an even larger capacity of 50. The total cost and total emission functions are now:

$$\begin{aligned}
&\hat{c}_{21}(f)=f_{21}^2+f_{21},\\
&\hat{g}_{21}(\gamma_{21})=1.5\gamma_{21}^2+1.5\gamma_{21},\\
&\hat{e}_{21}(f_{21},\gamma_{21})=.01f_{21}^2+.5f_{21}+.5\gamma_{21}^2+.5\gamma_{21}.
\end{aligned}$$

The computed equilibrium link flows, activity frequencies, and Lagrange multipliers are also reported in Table 3. The computed equilibrium path flow pattern is:
for Firm 1:

$$x^*_{p_1}=3.64,\quad x^*_{p_2}=3.60,\quad x^*_{p_3}=20.46,\quad x^*_{p_4}=10.38,\quad x^*_{p_9}=6.15,\quad x^*_{p_{10}}=16.93;$$

for Firm 2:

$$x^*_{p_5}=4.81,\quad x^*_{p_6}=3.68,\quad x^*_{p_7}=19.27,\quad x^*_{p_8}=20.25.$$

The computed demand for Firm 1's product is 61.15 and the price is 329.25, while the demand for Firm 2's product is 48.01 and the price is 273.41. The profit of Firm 1 is $13,707.86$ with its total emissions being 518.91, while the profit of Firm 2 is $9,245.87$ with its total

Table 3: Computed Equilibrium Link Flows, Activity Frequencies, and Lagrange Multipliers for Examples 2 and 3

	Example 2			Example 3		
Link a	f_a^*	γ_a^*	λ_a^*	f_a^*	γ_a^*	λ_a^*
1	13.10	.1310	.0792	13.38	.1338	.0794
2	46.73	.4673	.1271	47.77	.4777	.1280
3	8.50	.0850	.0334	8.49	.0849	.0334
4	39.60	.3960	.0478	39.52	.3952	.0478
5	4.45	.2224	.4334	3.64	.1819	.4091
6	8.65	.4326	.5596	9.74	.4871	.5923
7	20.63	.4126	.2586	20.46	.4092	.2573
8	26.09	.5219	.2979	27.31	.5462	.3066
9	4.81	.2407	.1631	4.81	.2403	.1631
10	3.69	.1844	.1519	3.68	.1840	.1518
11	19.31	.3861	.0762	19.27	.3854	.0761
12	20.29	.4058	.0788	20.25	.4051	.0787
13	25.08	.2508	.0175	24.10	.2410	.0172
14	34.75	.3475	.0204	37.05	.3705	.0211
15	24.12	.2411	.0163	24.07	.2408	.0164
16	23.98	.2397	.0162	23.93	.2394	.0164
17	25.08	1.2540	1.8044	24.10	1.2049	1.7504
18	17.28	.8640	1.3754	13.97	.6987	1.1936
19	24.12	1.2060	.6224	24.07	1.2037	.6215
20	23.98	1.1990	.6196	23.93	1.1967	.6187
21	17.47	.5823	.8103	23.08	.4616	.1539

emissions being 744.20. The total emissions for both firms' supply chains are: $1,263.11$. The utilities for Firm 1 and for Firm 2 are: $11,113.33$ and $8,501.67$, respectively.

Comparing the results for Examples 1, 2, and 3, we observe that Firm 1 is able to provide more products at even lower prices with the multiple modes for distribution. Consequently, the profit of Firm 1 increases in both Examples 2 and 3, while the demand and the profit of Firm 2 decline slightly in those two examples. Due to the lower emission nature of intermodal transport, the rail-truck intermodal option (as discussed in Example 3) is more appealing than the utilization of large trucks (as discussed in Example 2) for distribution. In Example 2, the large truck transportation (link 21) accounts for about 50% of the distribution from the distribution center $D^1_{2,2}$ to the demand market, while in Example 3, the intermodal transport accounts for more than 60% of the distribution from the same distribution center to the demand market. Furthermore, the emissions generated by Firm 1 in Example 3 are lower than in Example 2.

In Example 3, we then asked the following question: At which value of ω_1, which represents Firm 1's environmental concern, would the equilibrium solution be such that the link flow $f^*_{18} = 0.00$? Hence, the distribution from the distribution center $D^1_{2,2}$ to the demand market R_1 would solely rely on the rail-truck intermodal transport. We varied the value of ω_1 , which was originally equal to 5, until we observed, computationally, that the equilibrium solution was such that the link flow $f^*_{18} = 0.00$, which means that there is no product flow on link 18. We found that when ω_1 is equal to 43 (or greater) then $f^*_{18} = 0.00$, and also then γ^*_{18} is equal to 0.00, which is reasonable, since there is no product flow on link 18, and, hence, the activity frequency of that link, γ^*_{18}, is also zero.

Also, for completeness, we also report the demands and the incurred prices, profits, emissions, and utilities for the two firms with $\omega_1 = 43$ and $\omega_2 = 1$. For Firm 1, the equilibrium demand is 22.38 and the price at the demand market of its product is 367.49. The profit of Firm 1 is $6,855.37$, the number of emissions that it generates is 85.02, and its utility is $3,199.65$. The equilibrium demand for Firm 2's product at the demand market is 50.64 at the incurred price of 287.53. The profit of Firm 2 is 10,278.00. The number of emissions that it generates in its supply chain is 820.22 and its utility is $9,457.78$. The total total emissions generated by both firms' supply chains is, hence, 905.24.

Note that $\omega_1 = 43$ could also be an environmental tax, under the imposition of which, the emissions, relative to those in the preceding example, have gone from $1,263.11$ to 905.24. This example demonstrates how a policy-maker can effect positive environmental change through such a policy instrument.

5. Summary and Conclusions

In this paper, we developed a new sustainable supply chain network model which captures competition among firms involved in the production, transport, storage, and distribution of products that are differentiated by brand. Examples of such products range from fast fashion to high technology products. Each firm weights the emissions generated in its supply chain network activities in an individual way and seeks to maximize its utility with the utility function of each firm consisting of its profits and its weighted emissions. We allow for multiple options for production, transport, storage, and distribution, so that the impact on the environment can be made and different options appropriately evaluated. In addition, we associate with each supply chain network link a capacity and each firm determines both its optimal product path flows and the frequency of operation of the supply chain activities. The emission functions associated with a link depend both on the flow on the link as well as on the frequency of the link. This provides flexibility in modeling the various supply chain activities in terms of the environmental impact. We emphasize that, although the focus here is on carbon emissions, the framework is sufficiently general to handle other types of emissions, including particulate matter, which have a big negative impact on air quality and human health globally.

The governing concept is that of Nash equilibrium. We derive alternative variational inequality formulations, in path flows and in link flows, of the equilibrium conditions and propose a computational procedure, which tracks the evolution of the path flows, frequencies, as well as the Lagrange multipliers associated with the capacity constraints. In our numerical examples we investigate the impact on profits, emissions, and utilities of the addition of different transport modes for distribution. We find that a firm can win in terms of profits as well as lower emissions. Also, we demonstrate the impact on emissions and profits if firms weight their environmental emissions more or not at all.

Possible extensions of our model can include the incorporation of risk associated with supply chain network activities, and the sharing of distribution and storage facilities, as well as transport modes. In addition, one could incorporate the full network topology associated with the transport and distribution links to include also route choice behavior. The exploration of asymmetric and/or imperfect information would also be interesting.

We leave such issues, along with empirical analyses, for future research.

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