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Comment on “Inelastic Collapse of a Randomly Forced Particle” by Cornell et al.

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Comment on "Inelastic Collapse of a Randomly Forced Particle"

In their Letter [1] Cornell *et al.* investigated the dynamics of a particle in a one-dimensional box of length l subject to a random force $\eta(t)$. The particle motion is governed by the equation

$$\frac{d^2x}{dt^2} = \eta(t), \tag{1}$$

where $\eta(t)$ is a Gaussian noise with zero mean and a delta-correlated time dependence $\langle\eta(t)\eta(t')\rangle = 2\delta(t - t')$. The collisions with the boundaries are inelastic, with coefficient of restitution r; that is, whenever the particle collides with a wall it rebounds with a speed which is r times smaller than the speed before collision. The authors report a remarkable transition in which below a critical value, $r_c = \exp(-\pi/\sqrt{3})$, the particle adheres to one of the walls after colliding with it an infinite number of times in a finite time. For $r > r_c$ the motion is ergodic. They also find that the value of r_c is independent of the box size and of the magnitude of a viscous damping term added to the equation of motion, Eq. (1). Through a series of rescaling of the position and time variables and followed by a transformation onto a stationary Gaussian process, the equation of motion is mapped onto that of a Brownian particle subject to a harmonic force, Eq. (6) of [1]. The critical value r_c was obtained from a mean-field analysis. They also performed a numerical integration of Eq. (6) and, upon transforming back onto the particle's original coordinates, they found evidence for the inelastic collapse for the particle trajectories, as predicted by their mean-field calculation. Numerical results for the collapse transition were displayed in Fig. 2 of their Letter.

We decided to numerically integrate Eq. (1) directly, in the hope to reproduce the results of Cornell *et al.* To our surprise we did not find *any instance of a collapse transition* for any values of r. In order to assert the robustness of the solutions, we used both second- and third-order stochastic Runge-Kutta methods [2] to integrate Eq. (1) for a variety of initial conditions on the positions and velocities. We used integration time steps in the interval 10^{-2} to 10^{-4}. We tested our numerical integration method against the exact result for both the mean square displacement and the mean square velocity of a Brownian particle. The agreement was within 2% or less of the exact values for 10^4 samples and times up to 150. In Fig. 1 we show our results for the trajectories of a particle with position $0 \leq x \leq 1$ and times up to $t = 100$ (same range of parameters used in Fig. 2 of [1]), for two values of the coefficient of restitution below and above the expected critical value $r_c = 0.163$. Clearly, no collapse transition is observed. We also find that the presence of a viscous damping term in Eq. (1) does not induce a collapse transition for *any* values of r and coefficient of friction.

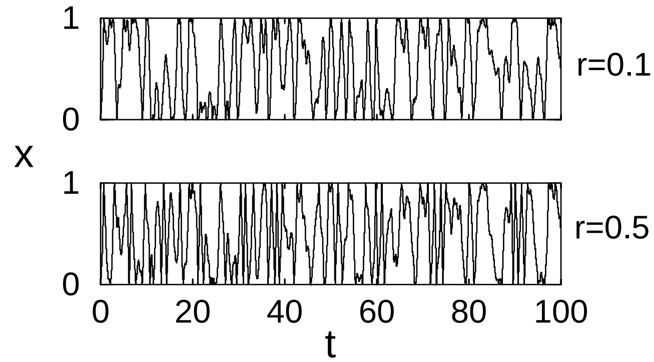

FIG. 1. Trajectories of a particle confined in a one-dimensional box subject to a random force. The trajectories correspond to values of the coefficient of restitution above and below $r_c = 0.163$. No evidence of the collapsed phase is observed. Numerical integrations were performed using a third-order stochastic Runge-Kutta method with integration time step 10^{-2}, and initial conditions $(x, v) = (0, 1)$.

We further analyze the model by calculating the power spectrum of $x(t)$. Our analysis shows that the averaged power spectrum is independent of the coefficient of restitution r. In addition, we find that the system exhibits two distinct $1/f^\alpha$ behaviors. For high frequencies we obtain a $1/f^2$ behavior for the power spectrum, as one would expect, which reflects the Brownian character of the particle motion at short times. On the other hand, at low frequencies we find a $1/f^4$ behavior for the power spectrum which can be attributed to the multiple collisions of the particle with the walls. We also find that in the low frequency regime the power α decreases as the damping coefficient increases. Details of these results will be presented elsewhere.

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